Assignment 3
Statistical Methods in Applied Computer Science
DD2447

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In all cases you are supposed to provide as sound algorithms of the types described in the course. Summing over exponentially many terms and other solutions that always gives exponential time, will not be considered sufficient.

Exercise 2 MAP switch setting and position on model railway
Imagine a model railway with a single train. You know the map of the tracks including the position of all the switches, but you don't know current states of the switches, or where the train is currently located. Each switch has three connections: $\{0, L, R\}$. If the train comes from the direction of L or R, it always leaves in the direction 0. If the train comes from the direction 0, it will leave in either direction L or R, depending on the state of the switch. The switch has prior probability $1/2$ for each direction, but will remain the same throughout the train run. You are receiving a stream of signals from the train, each signal specifying the direction in which train has passed a switch: $\{0L, 0R, L0, R0\}$; you do not know, however, which switch the train has passed. Also, the sensors are noisy, and with a certain probability p, the train reports a random signal instead of a real direction in which it passed the switch.

Given the map of the railroad and a sequence of switch signals $\{y_i\}_{i=1}^{T}$. Your task is to compute MAP switch and current position.

Solution.

Exercise 3 Gibbs sampler for posterior of magic word generative model
The following generative model generates K sequences of length N: $\{s_i\}_{i=1}^{K}$ where $s_i = \{s_{i,j}\}_{j=1}^{N}$. All sequences are over the alphabet $[M]$. Each of these sequences has a "magic" word of length w hidden in it and the rest of the sequence is called background.

First $\forall i$, a start position r_i for the magic word is sampled uniformly from $[N - w + 1]$. Then the j:th positions in the words are sampled from $q_j(x)$, which is $Cat\,(x|\theta_j)$ where θ_j has a $Dir\,(\theta_j|\alpha)$ prior. All other positions in the sequences are sampled from the background distribution $q(x)$, which is $Cat\,(x|\theta)$ where θ has a $Dir\,(\theta|\alpha')$ prior.

Solution.