Exercise 1: 1.12.3.10 Intersection of sigma algebras
$\mathcal{F}_1$ and $\mathcal{F}_2$ are two sigma algebras of subsets of Ω. Show that

$$\mathcal{F}_1 \cap \mathcal{F}_2$$

is a sigma algebra of subsets of Ω.

Solution.

Exercise 2: 2.6.5.6 Use Chen's Lemma
$X \in \mathrm{Po}(\lambda)$. Show that

$$\mathrm{E}\left[X^n\right] = \lambda \sum_{i=0}^{n-1} \binom{n-1}{k} \mathrm{E}\left[X^k\right]. \tag{1}$$

Aid: Use Chen's Lemma with suitable $H(x)$.

Solution.
Lemma 0.1. **Chen's Lemma** $X \in \mathrm{Po}(\lambda)$ and $H(x)$ is a bounded Borel function, then

$$\mathrm{E}\left[XH(X)\right] = \lambda \mathrm{E}\left[H(X+1)\right].$$

Exercise 3: 3.8.3.1 Title of the problem

Solution.

Exercise 4: 3.8.3.14 Title of the problem

Solution.

Exercise 5: 4.7.2.4 Title of the problem

Solution.

Exercise 6: 5.8.3.11 Title of the problem

Solution.

Exercise 7: 7.6.1.1 Title of the problem

Solution.