**Exercise 1: 1.12.3.10 Intersection of sigma algebras**
$\mathcal{F}_1$ and $\mathcal{F}_2$ are two sigma algebras of subsets of $\Omega$. Show that

$$\mathcal{F}_1 \cap \mathcal{F}_2$$

is a sigma algebra of subsets of $\Omega$.

*Solution.*

**Exercise 2: 2.6.5.6 Use Chen's Lemma**
$X \in \mathrm{Po}(\lambda)$. Show that

$$\mathrm{E}\left[X^n\right] = \lambda \sum_{i=0}^{n-1} \binom{n-1}{k} \mathrm{E}\left[X^k\right]. \tag{1}$$

Aid: Use Chen's Lemma with suitable $H(x)$.

*Solution.*

*Lemma* 0.1. **Chen's Lemma** $X \in \mathrm{Po}(\lambda)$ and $H(x)$ is a bounded Borel function, then

$$\mathrm{E}\left[XH(X)\right] = \lambda \mathrm{E}\left[H(X+1)\right].$$

**Exercise 3: 3.8.3.1 Joint Distributions & Conditional Expectations**
Let $(X, Y)$ is a bivariate random variable, where $X$ is discrete and $Y$ is continuous. $(X, Y)$ has a joint probability mass - and density function given by

$$f_{X,Y}(k, y) = \begin{cases} \frac{\partial P(X=k, Y \leq y)}{\partial y} = \lambda \frac{(\lambda y)^k}{k!} e^{-2\lambda y} & , k \in \mathbb{Z}_{\geq 0},\ y \in [0, \infty) \\ 0 & . \end{cases}$$

(a) Check that

$$test$$

*Solution.*

**Exercise 4: 3.8.3.14** Title of the problem

*Solution.*

**Exercise 5: 4.7.2.4** Title of the problem

*Solution.*

**Exercise 6: 5.8.3.11** Title of the problem

*Solution.*

**Exercise 7: 7.6.1.1** Title of the problem

*Solution.*