HOMEWORK 2
MATH 115A-1

The following problems are due on **Thursday, July 12 at the beginning of the lecture**: Section 1.6: 11, 12, 20, 22, 23, 26, 28. Section 2.1: 4, 5. Also Question 1.

Question 1: Consider the vector space $\mathbb{R}^\infty$ of real sequences. The element of $\mathbb{R}^\infty$ look like $(a_n) = (a_1, a_2, a_3, \ldots)$. Consider the subset $S \subseteq \mathbb{R}^\infty$ defined as

$$S = \{(a_n) \in \mathbb{R}^\infty : a_1 = 0 \text{ or } 1\}.$$

(1) Prove that S generates V, i.e., $\text{Span}(S) = V$.
(2) Prove that S is linearly dependent. In particular, S is not a basis of $\mathbb{R}^\infty$ over $\mathbb{R}$.

(3) Let $\{e_1, e_2, e_3, \ldots, \}$ be the standard vectors in $\mathbb{R}^\infty$ defined in the previous homework. Prove that this set is linearly independent in $\mathbb{R}^\infty$. Furthermore, conclude from the work of previous homework that $\{e_1, e_2, e_3, \ldots, \}$ **not a basis** of $\mathbb{R}^\infty$.

Additional practice problems: Section 1.6: 1, 2, 3, 4, 6, 11, 12, 13, 15, 20, 22, 23, 26, 28. Section 2.1: 2, 4, 5, 6.