HOMEWORK 3
MATH 115A-1

The following problems are due on **Thursday, July 19 at the beginning of the lecture**: Section 2.1: 14, 17, 20, 18, 19. Section 2.2: 11, 16. Section 2.3: 11, 12. Also **Question 1** below.

Question 1: Consider the vector space $\mathbb{R}^\infty$ of real sequences. The element of $\mathbb{R}^\infty$ look like $(a_n) = (a_1, a_2, a_3, \ldots)$. Let $T : \mathbb{R}^\infty \to \mathbb{R}^\infty$ and $T' : \mathbb{R}^\infty \to \mathbb{R}^\infty$ be two linear transformation defined by

$$T(a_1, a_2, a_3, \ldots) = (0, a_1, a_2, a_3, \ldots) \quad \text{and} \quad T'(a_1, a_2, a_3, \ldots) = (a_2, a_3, a_4, \ldots).$$

For example, $T(1, 2, 3, \ldots) = (0, 1, 2, 3, \ldots)$ and $T'(1, 2, 3, \ldots) = (2, 3, 4, \ldots)$.

Then

(1) Prove that T is one–to–one but not onto.
(2) Prove that T' is onto but not one–to–one.

Additional practice problems: Section 2.1: 8-14, 16-20, 37. Section 2.2: 1, 4, 5, 8, 10, 11, 12, 13, 16. Section 2.3: 1, 3, 9, 11, 12.