HW 2. DUE THURSDAY JULY 5 IN CLASS

MATH 170A SUMMER 2018

No late HW is accepted. I must have your HW before I leave the classroom on Thursday.

The last two problems (especially the last problem) use material from Section 1.5 in the book. This material will be introduced in Monday's lecture.

(1) Out of all students enrolled in UCLA summer courses, 70% like peanut butter, 80% like jam, and 60% like both. What is the chance that a UCLA summer course student picked at random likes neither peanut butter nor jam?

(2) I am holding a raffle. There are 1000 tickets numbered 1-1000 and I pick one at random. Write down the sample space associated with this experiment. Then, determine the probability that the number that the ticket shows is divisible by 2 or 3.

(Hint: Let n be a positive integer less than 1000. Performing long division by n, we find that there are two positive integers, q ("the quotient") and r ("the remainder") such that,

$$1000 = qn + r,$$

where $0 \leq r < n$.

For example, if n is 7, then q is 142 and r is 6, because $1000 = 142 \cdot 7 + 6$.

The number of natural numbers that are less than 1000 and divisible by the natural number n is exactly q. You may use this fact without justification in your answer, but anyway try to convince yourself that it is true!)

(3) Supplementary problems, Chapter 1 number 18.

(4) You are on a game show. The host, Monty Hall, presents three closed doors. He says that behind one door is a car, and behind the other two are goats. Monty asks you to pick a door. You point to door 1. Monty opens door 3 and reveals a goat. He says, "now that you see a goat behind door 3, would you like me to give you the item behind door 1, or would you like to switch and receive the item behind door 2?"

Let C_i denote the event that the car is behind door i, for $i = 1, 2, 3$. You may assume $P(C_i) = 1/3$ for each i. Let G denote the event that Monty has revealed a goat behind door 3.

You would like to get the car, and have to decide whether or not to switch. So, we must compare $P(C_2|G)$ and $P(C_1|G)$.

(a) Use Bayes' Rule to express $P(C_2|G)$ in terms of $P(G|C_1)$, $P(G|C_2)$ and $P(G|C_3)$. Notice that, without further information, we do not know the values of $P(G|C_i)$!

(b) Before the show, the producer told you that

- Monty will always open a different door from the one you point to,

- no matter what door you chose initially, Monty will show you a goat,
- if Monty has a choice of two goats, he will pick one at random.

Compute $P(C_2|G)$ and $P(C_1|G)$. If you believe the producer, should you switch?

(c) You decide not to believe the producer. Your friend tells you,
- if a goat is behind door 1, then Monty will open door 1,
- if a car is behind door 1, then Monty will chose a different door.

If you believe your friend, should you switch?

(This question is a version of the so-called Monty Hall problem. See `https://en.wikipedia.org/wiki/Monty_Hall_problem` for a further discussion of this problem, along with its history. Example 1.12 on page 27 of the textbook is yet another version of the Monty Hall problem.)

(These resources are interesting and may familiarize you with the setting, but for the purposes of this HW you must answer the question as it is written on this sheet.)

(5) Supplementary problems, Chapter 1 number 21.

(6) In a bag there are three biased coins, c_1, c_2 and c_3. The probabilities that they show heads when tossed are, respectively, $\frac{1}{3}$, $\frac{2}{3}$, and 1.
(a) A coin is picked at random and tossed. It comes up heads. Let A_i be the event that it is coin i, for $i = 1, 2, 3$. Find the probability of A_i for each i.
(b) Given that a coin has been tossed and showed heads, what is the probability that a second toss of the same coin will yield heads?
(c) The coins are put back into the bag and one is picked at random (a new experiment is starting). It is tossed twice and comes up heads both times. Let B_i be the event that it is coin i, for $i = 1, 2, 3$. Find the probability of B_i for $i = 1, 2, 3$.
(d) Given that a coin has been tossed twice and has come up heads both times, what is the probability that a third toss of the same coin will yield heads?

Supplementary problems: http://www.athenasc.com/CH1-prob-supp.pdf