HW 5. DUE WEDNESDAY JULY 25 IN CLASS

MATH 170A SUMMER 2018

No late HW is accepted. I must have your HW before I leave the classroom on Wednesday.

The material for the last problem will be introduced in Monday's lecture. This material is covered in section 3.4 of the book.

(1) Chapter 2 Supplementary problems #19, (a), (b), (c), (e).

(2) A professor has a total of 400 students in her calculus classes. On a given day, each student has a $1/20$ chance of coming to her office hours. Let X be the number of students who come to office hours on a given day.
  - (a) Find the pmf of X.
  - (b) Repeat part (a) by approximating the pmf of X with a Poisson pmf.
  - (c) The professor's office accommodates only 10 students. What is the probability that 10 or less students come to office hour? Provide the exact formula using (a) and an approximation using (b).
  - (d) What is the probability that the number of students who come to office hour is exactly equal to the expected value of X? Use a calculator, if necessary, to write down your answer as one number.
  - (e) Find the probability in the previous part using the Poisson approximation. Use a calculator, if necessary, to write down your answer as one number.

  (This question is very similar to numbers 4 and 41 in the end of chapter 2 problems.)

(3) A function f is said to be "symmetric around μ" (for a real number μ) if $f(\mu + x) = f(\mu - x)$ for all x. Suppose f is a function such that:
  - $\int_{-\infty}^{\infty} x f(x)\, dx$ is well-defined, and
  - $\int_{-\infty}^{\infty} f(x)\, dx = 1$.
  - (a) Suppose f satisfies the two bullet-points above and that f is symmetric around 0. Show that $\int_{-\infty}^{\infty} x f(x)\, dx = 0$.
  - (b) Let μ be any real number. Suppose f satisfies the two bullet-points above and f is symmetric around μ. Show that $\int_{-\infty}^{\infty} x f(x)\, dx = \mu$. (Hint: apply part (a) to $g(x) = f(x + \mu)$.)
  - (c) Use part (b) to show that, if X is a normal random variable with parameters μ, σ, then $E[X] = \mu$.

(4) Chapter 3 Supplementary problems #2

(5) Chapter 3 Supplementary problems #3

(6) Let (X, Y) be the coordinates of a point picked uniformly at random from the shaded region R indicated in Figure 1. (If two subsets of R have the same area, then the point has equal probability of laying in either of them.)
  - (a) Find $f_{X,Y}$, the probability density function of X and Y.

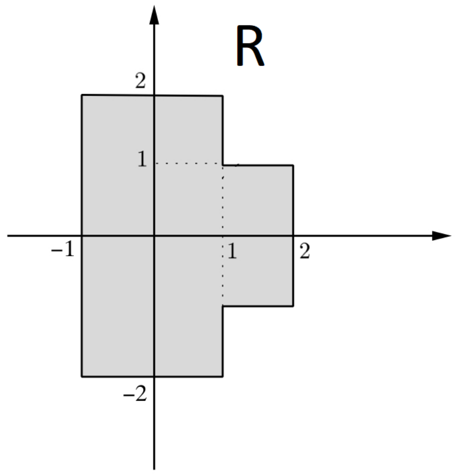

FIGURE 1. The shape for the last problem.

(b) Find f_X, the probability density function of X.
(c) Find f_Y, the probability density function of Y.
(d) Find $E[X]$, the expected value of X.
(e) Find $E[Y]$, the expected value of Y.

(Supplementary problems chapter 3: `http://www.athenasc.com/CH3-prob-supp.pdf`)