Resource Allocation for IRS Assisted mmWave Integrated Sensing and Communication Systems

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***Abstract*—This paper proposes an intelligent reflecting surface (IRS) assisted integrated sensing and communication (ISAC) system operating at the millimeter-wave (mmWave) band. Specifically, the ISAC system combines communication and radar operations and performs on the same hardware platform, detecting and communicating simultaneously with multiple targets and users. The IRS dynamically controls the amplitude or phase of the radio signal via the reflecting elements to reconfigure the radio propagation environment and enhance the transmission rate of the ISAC system in the mmWave band. By jointly designing the radar signal covariance (RSC) matrix, the beamforming vector of the communication system, and the IRS phase shift, the ISAC system transmission rate can be improved while matching the desired waveform for radar. The problem is non-convex due to multivariate coupling, and thus we decompose it into two separate subproblems. First, a closed-form solution of the RSC matrix is derived from the radar desired waveform. Next, the quadratic transformation (QT) technique is applied to the subproblem, and then alternating optimization (AO) is applied to determine the communication beamforming vector and the IRS phase shift. Also, we derive a closed-form solution for the formulated problem, effectively decreasing computational complexity. Finally, the simulations verify the effectiveness of the algorithm and demonstrate that the IRS can improve the performance of the ISAC system.**

***Index Terms*—Integrated sensing and communications, intelligent reflecting surface, waveform design.**

I. Introduction

Recently several new applications have emerged for the new generation of wireless communication systems, i.e. beyond 5G (B5G) and 6G, such as vehicle-to-everything (V2X), ultra high-definition (uHD) video and virtual reality (VR) [1]. The B5G/6G network can provide powerful support for sensing people, things, environment, and the connection of virtual spaces. Meanwhile, the increased demand for various sensing services has led to the problem of scarce spectrum resources. The communication and radar spectrum sharing (CRSS) techniques alleviate such spectrum crunch challenges [2]. Stepping towards B5G/6G, achieving a high transmission rate is an imperative requirement for the ISAC system. In recent years, the millimeter wave (mmWave) communication has been considered as one of essential techniques for 5G cellular system deployment due to its ultra-wide spectrum resources [3]. However, owing to high attenuation and weak penetration in the mmWave band, a large number of antennas are normally needed to implement high gain directional beamforming, which will incur hardware costs [4]. Thus, a low-cost scheme to develope the ISAC system in the mmWave band is urgently needed. Intelligent reflecting surfaces (IRS) have attracted extensive research interests from both academia and industry in the last few years, as an efficient and cost-effective way to enhance the performance of wireless network systems [5]. The IRS enables reconfiguration of the wireless propagation environment to support wireless communications and radar sensing [6]–[8]. The IRS consists of many low-cost passive reflecting elements and the desired signal phase can be adjusted without specialized RF processing [9]. Also, the IRS can provide an additional reflected link to the ISAC system from a different angle, potentially resulting in better sensing performance.

Two forms of radar and communication antenna deployment were proposed in [10], namely separated deployment and shared deployment. The former approach designs the radar signal in the downlink channel null space, while the communication beamforming is optimized so as to meet both the radar's and communication's performance requirements. In [11], the authors considered a joint communication and radar system incorporating base station (BS) and a multiple-input multiple-output (MIMO) radar that can provide communication services and simultaneously detect several targets. The ISAC system in the mmWave band for high-resolution imaging radar was investigated in [12] where the IRS-assisted wireless powered sensor network intelligently adjusts the phase shift of each reflecting element.

In this paper, we investigate an IRS-aided ISAC system for the multi-user downlink scenario in the mmWave band. We evaluate the effectiveness of the IRS by maximizing the sum rate of the downlink users subject to the constraints of the desired waveform for radar, the communication beamforming vector, and the IRS phase shift. As the desired beam pattern for radar is controlled by the radar signal covariance (RSC) matrix, we first derive a closed-form solution to the RSC

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matrix by relaxing the first sub-problem. The second sub-problem is then solved using the quadratic transformation (QT) technique, and its closed-form solutions are obtained iteratively by adopting the alternating iteration (AO) algorithm for the beamforming vector and the IRS phase shift. Finally, we present extensive simulation results to verify that the advantages of the IRS in improving the transmission rate of the ISAC system in the mmWave band.

Notations: Lowercase boldface and uppercase boldface indicate a vector and a matrix, respectively. $\mathbf{A}\succeq 0$ means that $\mathbf{A}$ is a semi-positive definite matrix. $\mathbf{A}^H$ and $\mathrm{vec}(\mathbf{A})$ represent conjugate transpose and the vectorization of $\mathbf{A}$, respectively. $\mathbf{A}^{-1}$ denotes matrix inverse. diag $(\mathbf{a})$ equals a diagnal matrix whose a diagnal elements are a vector $\mathbf{a}$. conj(a) and arg $(\mathbf{a})$ stand for the conjugate and the phase of a complex number $\mathbf{a}$, respectively. $\odot$ indicates the Hadamard product. $|\cdot|$ and $||\cdot||$ denote the absolute value and Euclidean norm, respectively. $\Re(\cdot)$ and $\mathrm{Im}(\cdot)$ define the real and imaginary parts, respectively.

II. System Model

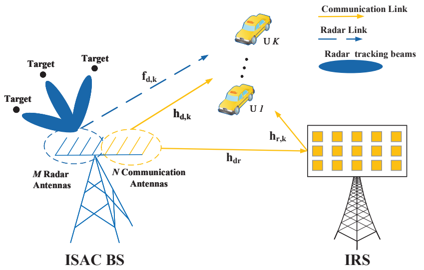

Fig. 1. System Model

We consider an IRS-assisted ISAC system with a BS with N communication antennas and M radar antennas, an IRS with L reflecting elements, and K single-antenna users as depicted in Fig. 1. The antennas are arranged in the BS as a unified linear array (ULA) pattern capable of transmitting statistically independent radar and communication signals to targets and K users. The IRS assists the BS in improving the signal strength in the communication coverage area.

A. MIMO Radar Signal

For the sensing purpose, we utilize a MIMO radar and assume that the target's prior position information has been obtained during the detection phase of the radar, and then is used to synthesize the desired beam pattern. The MIMO radar beamforming design aims to optimize the transmit power in a given direction or to match the desired beam pattern. The RSC matrix $\mathbf{R}$ gives a spatial beam pattern of a radar [13], which can be expressed as

$$\mathbf{R}=\frac{1}{T}\sum_{t=1}^{T}\mathbf{r}_t\mathbf{r}_t^H. \tag{1}$$

where T is the number of the total snapshots and $\mathbf{r}_t$ represents the the radar signal's t-th snapshot vector. The array steering vector is expressed as

$$\mathbf{a}(\theta_c)=\left[1,\exp(\frac{j2\pi d}{\lambda}\sin\theta_c),...,\exp(\frac{j2\pi d(M-1)}{\lambda}\sin\theta_c)\right]^T, \tag{2}$$

where d is the spacing between adjacent antenna elements, λ is the wavelength, and θ_c is the fine angle grid that covers the angular range of the interest target from $-\pi/2$ to $\pi/2$. Then according to [10], the transmit beam pattern is written as

$$P_t(\theta_c)=\mathbf{a}^H(\theta_c)\mathbf{R}\mathbf{a}(\theta_c). \tag{3}$$

B. MIMO Communication Signal

The signal received at the k-th user can be expressed as

$$x_k=\left(\mathbf{h}_{r,k}^H\mathbf{\Theta}_k\mathbf{h}_{dr}+\mathbf{h}_{d,k}\right)\sum_{k=1}^{K}\mathbf{w}_k d_k+\mathbf{f}_{d,k}\mathbf{r}_t+n_k, \tag{4}$$

where $\mathbf{h}_{dr}$, $\mathbf{h}_{d,k}$, $\mathbf{h}_{r,k}$ and $\mathbf{f}_{d,k}$ indicate the mmWave channels between the communication antennas and the IRS, between the communication antennas and the k-th user, and between the IRS and the k-th user, between the radar antennas and the k-th user, respectively. The $\mathbf{\Theta}_k=\mathrm{diag}(\exp(j\alpha_{k,1}),...,\exp(j\alpha_{k,L}))$ for $k=1,...,K$ denotes the diagonal IRS phase shift matrix and the $\alpha_{k,l}\in[0,2\pi], l=1,...,L$ indicates the phase shift of the l-th corresponding reflecting element. The d_k and $\mathbf{w}_k$ are communication symbol and the beamforming vector of the k-th user. All channels status information (CSI) are assumed to be perfectly known by transmitting pilot symbols. Based on the channel model [14], the channel coefficient $\mathbf{h}_{dr}$ can be expressed as

$$\mathbf{h}_{dr}=\sum_{\ell=0}^{N_p}v^{\ell}\mathbf{a}_B(\theta_B^{\ell})\mathbf{a}_I(\theta_I^{\ell}), \tag{5}$$

where v^{ℓ} represents the ℓ-th complex path gain, N_p is the non-line of sight paths, $\mathbf{a}_B(\theta)=\frac{1}{\sqrt{N}}[\exp(-j\frac{2\pi d}{\lambda}\theta i)]_{i\in I(N)}$ with $I(N)=\{n-(N-1)/2,\ \text{for}\ n=0,1,...,N-1\}$, and the $\mathbf{a}_I(\theta)$ is defined in the same way as $\mathbf{a}_B(\theta)$, the θ_B^{ℓ} and θ_I^{ℓ} indicate the ℓ^{th} azimuth angle for the BS and the IRS, respectively. $\ell=0$ denotes the line of sight (LoS) path and the IRS is generally deployed in a hotspot space resulting in a higher probability of LoS paths. Therefore, the channel between the IRS and the k-th user can be obtained as

$$\mathbf{h}_{r,k}=\sqrt{L}v^{\ell}\rho_r\rho_{t1}\mathbf{a}_{r,k}(\theta_k), \tag{6}$$

where ρ_r and ρ_{t1} denote the gains, the transmit elements and receive elements respectively, and $\mathbf{a}_{r,k}$ is the IRS steering vector. The channel coefficient $\mathbf{h}_{d,k}$ is expressed by

$$\mathbf{h}_{d,k}=\sqrt{N}v^{\ell}\rho_r\rho_{t2}\mathbf{a}_{d,k}(\theta_k), \tag{7}$$

where ρ_{t2}, and $\mathbf{a}_{d,k}$ represent the transmit gains and the steering vector of the communication antennas, respectively. Similarly, the radar channel coefficient with the k-th user can be written as

$$\mathbf{f}_{d,k}=\sqrt{M}v^{\ell}\rho_r\rho_{t3}\mathbf{a}_{s,k}(\theta_k), \tag{8}$$

where ρ_{t3} and $\mathbf{a}_{s,k}(\theta_k)$ indicate the transmit gains and the steering vector of the radar antennas, respectively. Thus, the signal to interference and noise ratio (SINR) of the k-th user can be derived as

$$\gamma_k = \frac{\left|\tilde{\mathbf{h}}_k \mathbf{w}_k\right|^2}{\sum\limits_{i=1, i\neq k}^{K} \left|\tilde{\mathbf{h}}_i \mathbf{w}_i\right|^2 + \mathbf{f}_{d,k}\mathbf{R}\mathbf{f}_{d,k}^H + \sigma^2}. \tag{9}$$

where $\tilde{\mathbf{h}}_k = \mathbf{h}_{r,k}^H \mathbf{\Theta}_k \mathbf{h}_{dr} + \mathbf{h}_{d,k}$.

C. Problem Formulation

In this paper, our objective is to maximize the sum transmission rate of the ISAC system, and evaluate the effectiveness of the IRS in improving the performance of communication system. This problem can be formulated as

$$\max_{\mathbf{\Omega}} \sum_{k=1}^{K} \log\left(1+\gamma_k\right) \tag{10a}$$

$$\text{s.t. } \left|\exp(j\alpha_{k,l})\right| = 1, \forall k \in [1,K], \forall l \in [1,L], \tag{10b}$$

$$\sum_{k=1}^{K} \mathbf{w}_k^H \mathbf{w}_k \leqslant P_c, \tag{10c}$$

$$\text{diag}(\mathbf{R}) = \frac{P_r}{M}\mathbf{I}, \tag{10d}$$

$$\mathbf{f}_{d,k}\mathbf{R}\mathbf{f}_{d,k}^H = 0, \tag{10e}$$

$$\mathbf{R} \succeq 0, \mathbf{R} = \mathbf{R}^H, \tag{10f}$$

$$\mathbf{\Omega} = \left\{\mathbf{\Theta}_1, ..., \mathbf{\Theta}_K, \mathbf{w}_1, ..., \mathbf{w}_k, \mathbf{R}\right\}. \tag{10g}$$

where P_c and P_r denote the transmit power of communication antennas and radar, respectively. (10b) denotes the each reflecting element phase shift constraint. (10c) is the maximum transmitting power limit of the communication system. (10d) indicates that all radar antennas keep the same transmitting power level. (10e) aims to eliminate interference from the radar to the downlink user via a zero-forcing operation.

Proposition 1: Problem (10a) is equivalent to

$$\max_{\mathbf{w_k}, \mathbf{\Theta}_k, \mathbf{R}, \mathbf{p}} f_1 = \frac{1}{\ln 2}\sum_{k=1}^{K} \ln(1+p_k) - p_k + \frac{(1+p_k)\gamma_k}{1+\gamma_k}, \tag{11}$$

where $\mathbf{p} = [p_1, p_2, ..., p_K]$ represents an introduced auxiliary vector by the Lagrangian dual transformation in [15].

Proof: Refer to Appendix A.

We can find that the k-th optimal auxiliary variable $\hat{p}_k = \gamma_k$ when (11) takes a partial derivative with respect to p_k and set to zero. For given $\mathbf{w}_k$, $\mathbf{\Theta}_k$ and $\mathbf{R}$, the auxiliary vector $\mathbf{p}$ can be updated by solving (9) at each iteration. Given $\mathbf{p}$, the optimization of $\mathbf{w}_k$, $\mathbf{\Theta}_k$ and $\mathbf{R}$ in (11) can be formulated as

$$\begin{aligned} \max_{\mathbf{w_k}, \mathbf{\Theta}_k, \mathbf{R}} f_2 &= \sum_{k=1}^{K} \frac{(1+p_k)\gamma_k}{1+\gamma_k} \\ &= \sum_{k=1}^{K} \frac{\bar{p}_k \left|\tilde{\mathbf{h}}_k^H \mathbf{w}_k\right|^2}{\sum\limits_{i=1}^{K} \left|\tilde{\mathbf{h}}_i^H \mathbf{w}_i\right|^2 + \mathbf{f}_{d,k}^H \mathbf{R}\mathbf{f}_{d,k} + \sigma^2} \end{aligned} \tag{12}$$

$$s.t. \text{ (10b), (10c), (10d), (10e), (10f),}$$

where $\bar{p}_k = \omega_k(1+p_k)$. By converting to (12), the logarithm in (10a) can be handled in a more straightforward manner.

III. Achievable Sum Rate Maximization

Problem (12) is non-convex due to multivariate coupling. To solve this problem, we split the problem into two sub-problems. First, we derive a closed-form expression for the RSC matrix. We then apply the QT variation and utilize the AO algorithm to derive closed form expressions for the beamforming vector and the IRS phase shift matrix.

A. Design Radar Signal Covariance Matrix

The radar beam pattern plays a pivotal role in radar detection and tracking, which can be obtained by the RSC matrix $\mathbf{R}$ and formulated as a constrained least-squares problem [10]

$$\begin{aligned} &\min_{\beta, \mathbf{R}} \sum_{c=1}^{C} \left|\beta P(\theta_c) - \mathbf{a}^H(\theta_c)\mathbf{R}\mathbf{a}(\theta_c)\right|^2 \\ &s.t.\ \beta > 0,\ \text{(10d), (10e), (10f),} \end{aligned} \tag{13}$$

where β is a scale factor, $\{\theta_c\}_{c=1}^{C}$ indicates the fine angular grid covers the detection angle range of $[-\pi/2, \pi/2]$, and $P(\theta_c)$ represents thedesired beam pattern of MIMO radar. Motivated by the pseudo-covariance matrix synthesis algorithm in [16], we adopt a scheme that can significantly reduce the RSC matrix's complexity. First, neglecting the constraint (10e), the objective function can be vectorized as

$$f(\mathbf{R}, \beta) = \sum_{c=1}^{C} \left|\beta P(\theta_c) - \text{vec}(\mathbf{V}(\theta_c))^H \mathbf{r}_v\right|^2, \tag{14}$$

where $\mathbf{V}(\theta_c) = \mathbf{a}(\theta_c)\mathbf{a}(\theta_c)^H$ and $\mathbf{r}_v = \text{vec}(\mathbf{R})$. With respect to the symmetry of $\mathbf{R}$ and constraint (10d), we define the $\mathbf{V}_{i,j}$ and $\mathbf{R}_{i,j}$ as the elements of the i-th row and j-th column of the $\mathbf{V}$ and $\mathbf{R}$ matrixes.

Denoting $\mathbf{A}_{ij}$ as the $(i,j)^{\text{th}}$ element of a matrix $\mathbf{A}$, let us define $\mathbf{v}_1 = [\text{Re}(\mathbf{v}_d^T)\ \text{Im}(\mathbf{v}_d^T)]^T$ and $\mathbf{r}_1 = [\text{Re}(\mathbf{r}_d^T)\ \text{Im}(\mathbf{r}_d^T)]^T$, where $\mathbf{v}_d = [\mathbf{V}_{12}, ..., \mathbf{V}_{1M}, \mathbf{V}_{23}, ..., \mathbf{V}_{(M-1)M}]^T$, and $\mathbf{r}_d = [\mathbf{R}_{12}, ..., \mathbf{R}_{1M}, \mathbf{R}_{23}, ..., \mathbf{R}_{(M-1)M}]^T$. Also we define $\mathbf{v}_2 = [\mathbf{V}_{11}, ..., \mathbf{V}_{MM}]^T = \mathbf{1}_M, \mathbf{r}_2 = [\mathbf{R}_{11}, ..., \mathbf{R}_{MM}]^T = \frac{P_r}{M}\mathbf{1}_M$, where $\mathbf{1}_M$ represents an all one vector of length M.

Thus, we can transform (14) to

$$\begin{aligned} f(\mathbf{r}_x) &= \sum_{c=1}^{C} \left(\left[2\mathbf{v}_1^T\ \mathbf{v}_2^T\right]\begin{bmatrix}\mathbf{r}_1 \\ \mathbf{r}_2\end{bmatrix} - \beta P(\theta_c)\right)^2 \\ &= 4\sum_{c=1}^{C} \left(\left[\mathbf{v}_1^T\ -\tfrac{1}{2}P(\theta_c)\right]\begin{bmatrix}\mathbf{r}_1 \\ \beta\end{bmatrix} + \tfrac{1}{2}P_r\right)^2 \\ &= 4\sum_{c=1}^{C} \left(\mathbf{v}_x^T(\theta_c)\mathbf{r}_x + \tfrac{1}{2}P_r\right)^2, \end{aligned} \tag{15}$$

where $\mathbf{v}_x^T = \left[\mathbf{v}_1^T - \frac{1}{2}P(\theta_c)\right]$ and $\mathbf{r}_x = \begin{bmatrix}\mathbf{r}_1 \\ \beta\end{bmatrix}$. The optimal closed-form solution can be derived by obtaining the derivative for $\mathbf{r}_x$ in (15) and setting it to zero as

$$\mathbf{r}_x = -\frac{P_r}{2}\mathbf{V}_x^{-1}\mathbf{v}_u, \tag{16}$$

where $\mathbf{V}_x = \sum\limits_{c=1}^{C} \mathbf{v}_x(\theta_c)\mathbf{v}_x^H(\theta_c)$ and $\mathbf{v}_u = \sum\limits_{c=1}^{C} \mathbf{v}_x(\theta_c)$. Then, we can rearrange the $\mathbf{r}_x$ to construct the RSC matrix $\mathbf{R}_{rad}$. However, $\mathbf{R}_{rad}$ is not guaranteed to be a positive definite matrix here, and thus we apply eigenvalue decomposition (ED) as

$$\mathbf{R}_{rad} = \mathbf{U}\mathrm{diag}(\tilde{\sigma})\mathbf{U}^H, \tag{17}$$

where $\tilde{\sigma}$ is a vector replaces negative eigenvalues of the origianl matrix with their absolute values or zeros, and $\mathbf{U} = [\mathbf{u}_1, ..., \mathbf{u}_{Nr}]$ represents the corresponding eigenvectors [16]. Then, the null space of the interfering channel by considering the constraint (10e) is formed as

$$\mathbf{Z}_{proj} = \mathbf{I} - \mathbf{F}(\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H, \tag{18}$$

where $\mathbf{F} = [\mathbf{f}_{d,1}, \mathbf{f}_{d,2}, ..., \mathbf{f}_{d,K}]$. With the classical null-space projection (NSP) approach, the RSC matrix for eliminating radar interference to downlink users is given by

$$\mathbf{R}_n = \mathbf{Z}_{proj}\mathbf{R}_{rad}\mathbf{Z}_{proj}^H, \tag{19}$$

where $\mathbf{R}_n$ is a semi-definite positive Hermitian matrix, but it is not guaranteed that all its diagonal elements are $\frac{\mathrm{P}_r}{N_r}$. Therefore, we apply the diagonal normalization (DN) method to normalize its diagonal elements as

$$\mathbf{R} = \mathbf{T}\mathbf{R}_n\mathbf{T}^H, \tag{20}$$

where $\mathbf{T}$ is a diagonal matrix with diagonal elements of $\sqrt{\frac{P_r}{N_r}\mathrm{diag}(\mathbf{R}_{com})}$.

B. Design for Communication Beamforming

For a given RSC matrix, the problem (12) can be reformulated as

$$\max_{\mathbf{w_k},\boldsymbol{\Theta}_k} \ f_3 = \sum_{k=1}^{K} \frac{\bar{p}_k\left|\tilde{\mathbf{h}}_k^H\mathbf{w}_k\right|^2}{\sum\limits_{i=1}^{K}\left|\tilde{\mathbf{h}}_i^H\mathbf{w}_i\right|^2 + \mathbf{f}_{d,k}^H\mathbf{R}\mathbf{f}_{d,k} + \sigma^2}, \tag{21a}$$

$$s.t. \ (10\text{b}), (10\text{c}), \tag{21b}$$

where the optimization variables $\boldsymbol{\Theta}_k$ and $\mathbf{w}_k$ are coupled with each other in (21a), which is a multi-ratio fractional programming problem. Thus, given the IRS phase shift first, we apply the AO algorithm and the quadratic transformation (QT) technique in [15] to rewrite (21) in the form of a biconvex problem as

$$\begin{aligned} f_4(\mathbf{w_k}, \mathbf{s}) = &\sum_{k=1}^{K} 2\sqrt{\bar{p}_k}\Re\{s_k^*\tilde{\mathbf{h}}_k^H\mathbf{w}_k\} \\ &-|s_k|^2\left(\sum_{i=1}^{K}\left|\tilde{\mathbf{h}}_i^H\mathbf{w}_i\right|^2 + \mathbf{f}_{d,k}^H\mathbf{R}_J\mathbf{f}_{d,k} + \sigma^2\right), \end{aligned} \tag{22}$$

where $\mathbf{s} = [s_1, ..., s_K]^T$ is as auxiliary vector for the QT technique. For a given $\mathbf{w_k}$, we can obtain the optimal s_k by setting $\partial f_4/\partial s_k = 0$ as

$$s_k = \frac{\sqrt{\bar{p}_k}\tilde{\mathbf{h}}_k^H\mathbf{w}_k}{\sum\limits_{i=1}^{K}\left|\tilde{\mathbf{h}}_i^H\mathbf{w}_i\right|^2 + \mathbf{f}_{d,k}^H\mathbf{R}_J\mathbf{f}_{d,k} + \sigma^2}. \tag{23}$$

Since the problem (22) is a convex problem with respect to $\mathbf{w_k}$, given the auxiliary vector $\mathbf{s}$, the optimal $\mathbf{w_k}$ can be derived from the Lagrangian multiplier method as

$$\mathbf{w}_k = \sqrt{\bar{p}_k}s_k\left(\mu\mathbf{I} + \sum_{i=1}^{K}|s_k|^2\tilde{\mathbf{h}}_i\tilde{\mathbf{h}}_i^H\right)^{-1}\tilde{\mathbf{h}}_k, \tag{24}$$

where μ is the Lagrange multiplier of the power constraint in (10c), which can be computed by the following lemma via the bisection search method.

Lemma 1: The optimal μ in (24) is determined as

$$\mu = \left\{\mu \geqslant 0 \mid \sum_{k=1}^{K}\mathbf{w}_k^H\mathbf{w}_k = P_{\max}\right\}. \tag{25}$$

Proof: Refer to Appendix B

C. Design for phase shift of the IRS

In this subsection, we proceed to optimize the IRS phase shift of the k-th user after deriving the optimal $\mathbf{w_k}$. We express $\tilde{\mathbf{h}}_k$ as

$$\begin{aligned} \tilde{\mathbf{h}}_k &= \mathbf{h}_{r,k}^H\boldsymbol{\Theta}_k\mathbf{h}_{dr} + \mathbf{h}_{d,k} = \mathbf{e}_k\mathrm{diag}\left(\mathbf{h}_{r,k}^H\right)\mathbf{h}_{dr} + \mathbf{h}_{d,k} \\ &= \begin{bmatrix} \mathbf{e}_k & 1 \end{bmatrix}\begin{bmatrix} \mathrm{diag}\left(\mathbf{h}_{r,k}^H\right)\mathbf{h}_{br} \\ \mathbf{h}_{d,k} \end{bmatrix} = \tilde{\mathbf{e}}_k\mathbf{A}_k, \end{aligned} \tag{26}$$

where $\tilde{\mathbf{e}}_k = [\tilde{e}_{k,1}, \ldots, \tilde{e}_{k,L}] = \begin{bmatrix} \mathbf{e}_k & 1 \end{bmatrix}$ with $\mathbf{e}_k = [\exp(j\alpha_{k,1}), \ldots, \exp(j\alpha_{k,L})]$. Then, the problem (21) can be reformulated as

$$\max_{\tilde{\mathbf{e}}_k} f_5(\tilde{\mathbf{e}}_k) = \sum_{k=1}^{K}\frac{\bar{p}_k|\tilde{\mathbf{e}}_k\mathbf{A}_k\mathbf{w}_k|^2}{\sum\limits_{i=1}^{K}|\tilde{\mathbf{e}}_k\mathbf{A}_i\mathbf{w}_i|^2 + \mathbf{f}_{d,k}^H\mathbf{R}_J\mathbf{f}_{d,k} + \sigma^2}. \tag{27a}$$

$$s.t. \ |\tilde{\mathbf{e}}_k(l)| = 1, \tag{27b}$$

Similarly, we apply the QT technique to (27a) as

$$\begin{aligned} \max_{\tilde{\mathbf{e}}_k} \mathrm{f}_6(\tilde{\mathbf{e}}_k, \mathbf{b}) = &\sum_{k=1}^{K} 2\sqrt{\bar{p}_k}\Re\left\{b_k{}^*\tilde{\mathbf{e}}_k\mathbf{C}_k\right\} \\ &-|b_k|^2\left\{\sum_{i=1}^{K}|\tilde{\mathbf{e}}_k\mathbf{C}_i|^2 + \mathbf{f}_{d,k}^H\mathbf{R}_J\mathbf{f}_{d,k} + \sigma^2\right\}, \end{aligned} \tag{28a}$$

$$s.t. \ (27\text{b}), \tag{28b}$$

where $\mathbf{b} = [b_1, ..., b_k]$ is an auxiliary vector for the QT technique, and $\mathbf{C}_k = \mathbf{A}_k\mathbf{w}_k$. We can derive the optimal $\hat{b}_k$ by the Lagrange multiplier method as

$$\hat{b}_k = \frac{\sqrt{\bar{p}_k}\tilde{\mathbf{e}}_k\mathbf{C}_k}{\sum_{i=1}^{K}|\tilde{\mathbf{e}}_k\mathbf{C}_i|^2 + \mathbf{f}_{d,k}^H\mathbf{R}_J\mathbf{f}_{d,k} + \sigma^2}, \tag{29}$$

Thus, for a given b_k, we further manipulate the mathematical derivation of (28) as

$$\max_{\tilde{\mathbf{e}}_k} f_7(\tilde{\mathbf{e}}_k) = -\tilde{\mathbf{e}}_k^H\bar{\mathbf{E}}\tilde{\mathbf{e}}_k + 2\Re\left\{\tilde{\mathbf{e}}_k\mathbf{F}\right\} - d_1, \tag{30a}$$

$$s.t. (27\text{b}), \tag{30b}$$

where $\bar{\mathbf{E}} = \sum\limits_{k=1}^{K}|b_k|^2\sum\limits_{i=1}^{K}\mathbf{C}_i\mathbf{C}_i^H$, $\mathbf{F} = \sum\limits_{k=1}^{K}\sqrt{\bar{p}_k}b_k^*\mathbf{C}_k$ and $d_1 = \sum\limits_{k=1}^{K}|b_k|^2(\sigma^2 + \mathbf{f}_{d,k}^H\mathbf{R}_J\mathbf{f}_{d,k})$. The problem (30) is a quadratical constraint quadratic programming (QCQP) problem and its objective becomes

$$\min_{\tilde{\mathbf{e}}_k} f_7(\tilde{\mathbf{e}}_k) = \tilde{\mathbf{e}}_k^H\mathbf{E}\tilde{\mathbf{e}}_k - 2\Re\left\{\tilde{\mathbf{e}}_k\mathbf{F}\right\}, \tag{31}$$

This problem remains intractable due to the constant modulus constraint. Next, we employ the MM algorithm and consider a series of solvable subproblems to address the problem (31) iteratively through approximating its objective function and constraint set [17].

Proposition 2: The objective function (31) at the m-th iteration for any given $\tilde{\mathbf{e}}_k^{(m)}$ is approximated as

$$\begin{aligned}&f_7(\tilde{\mathbf{e}}_k) \leqslant \tilde{\mathbf{e}}_k \mathbf{P} \tilde{\mathbf{e}}_k^H - 2\Re\left\{\tilde{\mathbf{e}}_k\left[(\mathbf{P}-\mathbf{E})\bar{\mathbf{e}}_k^H + \mathbf{F}\right]\right\} + \bar{\mathbf{e}}_k(\mathbf{P}-\mathbf{E})\bar{\mathbf{e}}_k^H \\ &= \lambda_{\max}(\mathbf{E})||\tilde{\mathbf{e}}_k||^2 - 2\Re\left\{\tilde{\mathbf{e}}_k\left[(\lambda_{\max}(\mathbf{E})\mathbf{I}_{L+1\times L+1} - \mathbf{E})\bar{\mathbf{e}}_k^H + \mathbf{F}\right]\right\} \\ &+ \tilde{d} = g(\tilde{\mathbf{e}}_k|\tilde{\mathbf{e}}_k^{(m)}),\end{aligned} \tag{32}$$

where $\bar{\mathbf{e}}_k$ is an approximate solution to $\tilde{\mathbf{e}}_k$ derived in the previous iteration, the $\lambda_{\max}(\mathbf{E})$ represents the maximum eigenvalue of $\mathbf{E}$. The $\tilde{d}$ and $\mathbf{P}$ are defined as $\bar{\mathbf{e}}_k[\lambda_{\max}(\mathbf{E})\mathbf{I}_{L+1\times L+1} - \mathbf{E}]\bar{\mathbf{e}}_k^H$ and $\lambda_{\max}(\mathbf{E})\mathbf{I}_{L+1\times L+1}$, respectively. Adopting a surrogate function of (31), the problem (31) becomes

$$\begin{aligned}&\min_{\tilde{\mathbf{e}}_k} \lambda_{\max}(\mathbf{E})||\tilde{\mathbf{e}}_k||^2 - 2R\{\tilde{\mathbf{e}}_k\bar{\mathbf{F}}\}, \\ &s.t. \text{ (27b)},\end{aligned} \tag{33}$$

where $\bar{\mathbf{F}} = (\lambda_{\max}(\mathbf{E})\mathbf{I}_{L+1\times L+1} - \mathbf{E})\bar{\mathbf{e}}_k^H + \mathbf{F}$. We can find that $||\tilde{\mathbf{e}}_k||^2 = L+1$ since the phase shift of the l-th element $|\tilde{\mathbf{e}}_k(l)| = 1$ in the IRS. The term $R\{\tilde{\mathbf{e}}_k\bar{\mathbf{F}}\}$ can be maximized when $\tilde{\mathbf{e}}_k(l)$ and the $\bar{\mathbf{F}}(l)$ are identical. Thus, defining $\mathbf{c} = [c_1, \ldots, c_N] = (\lambda_{\max}(\mathbf{E})I_{L+1\times L+1} - \mathbf{E})\,\bar{\mathbf{e}}_k^H + \mathbf{F}$ as the optimal solution to (33) is derived as

$$\tilde{\mathbf{e}}_k = [\exp(j\arg(c_1)), ..., \exp(j\arg(c_{L+1}))], \tag{34}$$

We summarize the MM algorithm as *Algorithm 2* below.

Algorithm 1 MM Algorithm

Initialization: $m = 0$.
repeat
  Obtain $\bar{\mathbf{F}}^{(m)} = (\lambda_{\max}(\mathbf{E})\mathbf{I}_{L+1\times L+1} - \mathbf{E})(\bar{\mathbf{e}}_k^{(m)})^H + \mathbf{F}$.
  Obtain the optimal phase shift $\tilde{\mathbf{e}}_k^{(m)*}$.
  Update $\tilde{\mathbf{e}}_k^{(m+1)} = \tilde{\mathbf{e}}_k^*$ and calculate $f_7(\tilde{\mathbf{e}}_k^{(m+1)})$.
  Set $m \leftarrow m+1$.
until convergence

IV. Performance Evaluation

In this section, we carry out numerical simulations to verify the performance of the IRS assisted ISAC system. According to [18], the channel gain is given by $r_k \sim \mathcal{CN}(0, 10^{-0.1PL(\eta)})$, where $PL(\eta) = \sigma_a + 10\sigma_b \log(\eta) + \upsilon$ with $\upsilon \sim \mathcal{N}(0, \sigma_\kappa^2)$. In the channel realizations, we set the noise power $\sigma^2 = -50$ dBm, $\sigma_a = 61.4$, and $\sigma_b = 2$, $\sigma_\kappa = 5.8$ dB. The BS is equipped with $N = 8$ communication antennas and $M = 8$ radar antennas, which are deployed at (-30 m, 0 m), the users $K = 5$ are randomly deployed in a circle of radius 5 m centered at the origin, and is located deployed at (30 m, 0 m) with $L = 10$ reflecting elements. The transmit power of the communication and the radar are $Pc = 30$ dBm, and $Pr = 30$ dBm, respectively. We assume that there are five targets located at the angle $-50°, -25°, -0°, 25°$, and $50°$ with respect to the BS in the mesh grid with $1°$ gap from $-90°$ to $90°$.

The following benchmark schemes and considered for comparison with the proposed algorithm.

1) *Discrete phase shifts:* We consider the scheme of a discrete phase shift for IRS, which is given as $\mathcal{F}_d = \left\{\mathbf{e}_d(l) = \exp(j\alpha_{d,l}),\ \text{for } \alpha_{d,l} \in \left\{\frac{2\pi i}{2^B}\right\}_{i=0}^{2^B-1}\right\}$, where $\mathcal{F}_d$ is the finite set of phase shifts and B represents the phase resolution in bits.
2) *Random phase shifts:* The phase shift is randomly generated, while the precoding vector and the radar RSC matrix are optimally derived.
3) *Without IRS:* the system considers the transmission between the ISAC BS and the user with no IRS, the RSC matrix and the communication precoding vector are derived accordingly.

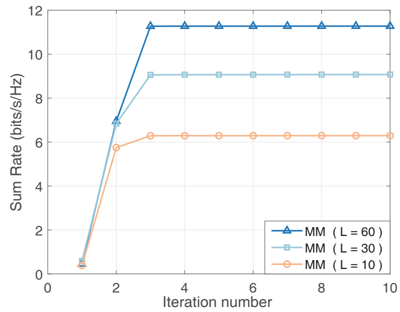

Fig. 2. Convergence of the proposed algorithm.

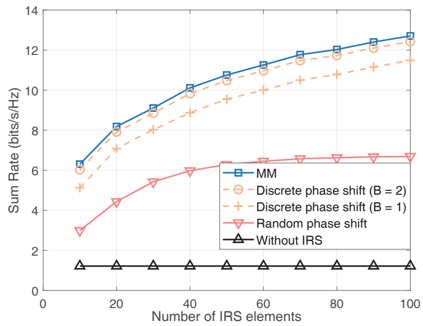

Fig. 3. Sum Rate with respect L.

First, we demonstrate the convergence property in Fig. 2. From the figure, one can see that the sum rate converges within three iterations, confirming the effectiveness of our proposed AO algorithm.

In Fig. 3, we unveil the effect of the IRS reflecting elements L in improving the sum rate. It can be observed that the sum rate increases with the IRS reflecting elements in all schemes. By jointly designing the BS beamforming and the IRS phase shift matrix, the BS-user channels and the beamforming can be better matched to each other. When the number of the IRS reflecting elements grows, the sum rate with the IRS improves considerably compared to the scheme without IRS. This follows from the fact that a large L will introduce a stronger reflected signal to enhance the signal reception.

Finally, we present the radar beam patterns in Fig. 4, where the BP denotes our desired beam pattern. The peak sidelobe level (PSL) with IRS is better than that without IRS. This is because the IRS assists the BS to optimize its beamforming. The IRS is a more effective way to increase the system rate. Also, according to [19], the IRS can be utilized to reduce the

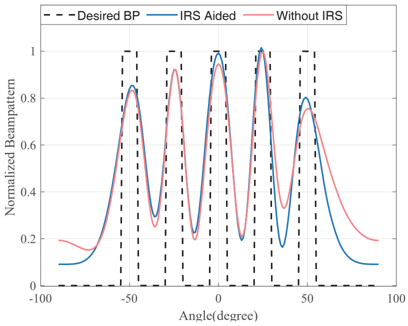

Fig. 4. Radar beam patterns.

multi-user interference and minimize the difference between the obtained waveform and the desired waveform.

V. CONCLUSION

This paper has focused on the IRS-assisted ISAC system in the mmWave scenario and formulated a sum-rate maximization problem by jointly optimizing the radar RSC matrix, the communication beamforming vector, and the IRS phase shift. We have derived closed-form expressions for the radar RSC matrix, the communication beamforming vector of the communication system, and the IRS phase shift. The MM algorithm have been deployed to solve the IRS phase shift, which can effectively reduce the computational complexity. Finally, the numerical simulations have verified the effectiveness of our proposed scheme, and demonstrated proved that the IRS can be eployed as an effective way to improve the ISAC system transmission rate.

APPENDIX

A. Proof to Proposition 1

When λ_k is given, (11) is a concave differentiable function with respect to p_k. Taking the derivative of (11) with respect to p_k and setting it to zero, we have

$$\frac{\partial f_1}{\partial p_k} = \frac{1}{\ln 2}\sum_{k=1}^{K}\frac{1}{1+p_k} - 1 + \frac{\gamma_k}{1+\gamma_k} = 0. \tag{35}$$

Then, we obtain $\hat{p}_k = \gamma_k$. Taking p_k back to (11) yields (10a). As such, the two problems have equal optimal objective values, ensuring their equivalence.

B. Proof to Lemma 1

For brevity, let $\mathbf{W} = [\mathbf{w}_1, ..., \mathbf{w}_k]$ and (24) can be rearranged as

$$\hat{\mathbf{W}} = (\mu\mathbf{I} + \mathbf{K})^{-1}\mathbf{H}\mathbf{S} \tag{36}$$

where $\mathbf{K} = \sum_{i=1}^{K} |s_k|^2 \tilde{\mathbf{h}}_i \tilde{\mathbf{h}}_i^H$, $\mathbf{H} = [\tilde{\mathbf{h}}_1, ..., \tilde{\mathbf{h}}_k]$ and $\mathbf{S} = \mathrm{diag}([\sqrt{\bar{p}_1}s_1, ..., \sqrt{\bar{p}_K}s_K])$. We have $\mathrm{tr}\left(\mathbf{W}\mathbf{W}^H\right) = \sum_1^K \mathbf{w}_k^H \mathbf{w}_k$ and then take the derivation of the transit power with respect to μ as

$$\begin{aligned}\frac{\partial \mathrm{tr}(\mathbf{W}\mathbf{W}^H)}{\partial \mu} &= tr\left(\left(\frac{\partial \mathrm{tr}(\mathbf{W}\mathbf{W}^H)}{\partial \mathbf{W}}\right)^H \frac{\partial \mathbf{W}}{\partial \mu}\right) \\ &= -2 \cdot \mathrm{tr}\left(\mathbf{P}^H (\mu\mathbf{I} + \mathrm{K})^{-1}\mathbf{P}\right),\end{aligned} \tag{37}$$

which shows that (36) is monotonically decreasing with μ and that the optimal solution $\hat{\mu}$ in the critical case of the power constraint (10c) is precisely its minimum value.

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