Max-Min Fairness Precoder Design for Rate-Splitting Multiple Access: Impact of Imperfect Channel Knowledge

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***Abstract*—Rate-splitting multiple access (RSMA) is a key enabling technology for beyond 5G networks due to its robustness against imperfect channel knowledge. In this paper, we consider both the imperfect channel state information (CSI) at the receiver (CSIR) and imperfect CSI at the transmitter (CSIT) and propose a max-min fairness optimization framework relying on RSMA to guarantee each user's minimum level of quality of service (QoS). First, we formulate the max-min fairness optimization problem that has non-convexity. To transform the non-convex problem into a convex problem, we apply the semi-definite relaxation (SDR) and convex-concave procedure (CCP) to the proposed algorithm. From simulation results, we confirm that RSMA is still a robust scheme over conventional multiple access schemes for both imperfect CSIT and CSIR.**

***Index Terms*—Rate-splitting multiple access (RSMA), imperfect channel knowledge, max-min fairness, precoder design.**

I. Introduction

In practical cellular systems, orthogonal multiple access (OMA) techniques are used. However, in order to support the massive connectivity of various form-factor devices in the future, it is needed to employ more effective multiple access techniques such as non-orthogonal multiple access (NOMA) and rate-splitting multiple access (RSMA) [1], [2]. Over the past few years, RSMA has emerged as a robust multiple access framework for beyond 5 G networks due to its robustness to imperfect channel knowledge [2]. A primary operation of RSMA at the transmitter side is to divide each user's data stream into two parts: common message and private message. For a 1-layer RSMA case, all the common messages are combined into one super-common message and encoded into a single common data stream. The private messages are encoded into a corresponding private data stream. Thus, when considering a K-user downlink cellular system, $K + 1$ streams are simultaneously transmitted via multiple antennas. At each receiver side, the common data stream is decoded first by treating the private data streams as noise. The common data stream can be removed through successive interference cancellation (SIC) operation and then the intended private data stream can be decoded by the corresponding user by also treating the other private data streams as noise. The original message can be reconstructed at each user by combining the decoded common data stream and private data stream.

The flexibility of RSMA allows easy control of the power portion of the common data stream and private data streams [2], resulting in a better performance compared to the conventional multiple access techniques in a variety of multi-antenna wireless networks. It has recently been shown in the literature that RSMA has superiority in the cloud radio access network (C-RAN) [3], and non-orthogonal unicast and multicast (NOUM) transmission [4]. The robustness under imperfect channel state information (CSI) case is one of the main advantages of RSMA systems. By providing a higher robustness against imperfect channel state information, RSMA can be an attractive solution in many different application scenarios, such as satellite communications, integrated radar/sensing and communication, and intelligent reconfigurable surface-aided communications [2]. One can apply the RSMA technique in non-terrestrial networks in which perfect CSI is more difficult to achieve with rapid channel variations over time due to the relative movement of non-terrestrial flying/moving objects with respect to the ground stations [5].

Most of previous works have revealed that RSMA provides better performance on imperfect CSI at the transmitter (CSIT) [6]. A max-min fairness optimization problem is carried out in the worse-case scenario under bounded error assumption [6]. However, only channel feedback errors at the transmitter were studied in [2], and the imperfect CSI at the receiver (CSIR) case is not considered. In practice, channel estimation errors occur at the receiver side. Under the imperfect CSIR assumption, the common message cannot be entirely removed by SIC since the receiver cannot obtain the true channel. In this paper, we consider RSMA for both imperfect CSIT and CSIR. First, we propose a max-min fairness optimization problem based on the rate derived from the imperfect SIC operation. We apply an alternating optimization algorithm relying on semi-definite relaxation (SDR) and convex-concave procedure (CCP) since the original max-min fairness problem is non-convex. Then, by turning this problem into a convex optimization problem, we can obtain a near optimal solution. It is worth noting that our objective function is different from the sum-rate maximization problem [7], [8]. In the sum-rate maximization problem, it is not needed to determine a specific value of each common rate portion which does not affect the sum rate. However, in this paper, the main objective is to maximize the minimum rate among all users with regard to the precoding vectors as well as the common rate portions, which brings quite challenging research issues. Through this, the fairness can be achieved since the optimal solution of the max-min fairness is achieved when every user has the same rate. From simulation results, we show that RSMA is a more robust multiple access technique than any other conventional techniques for both imperfect CSIT and CSIR.

The rest of this paper is organized as follows. Section II briefly describes the system model, channel knowledge assumption, and the rate derived from imperfect CSIR. Section III presents the max-min fairness problem and alternating optimization algorithm to convert the non-convex problem into the convex problem. Section IV provides the simulation results and lastly Section V concludes this paper.

We briefly outline the notations used in this paper. $\mathcal{CN}(m, \sigma^2)$ indicates a complex Gaussian random variable with mean m and variance σ^2 and $\mathrm{tr}(\cdot)$ is the trace operation. A matrix $\mathbf{I}_n$ indicates a n by n identity matrix. blkdiag$(\cdot)$ indicates a block-diagonal operation that generates a

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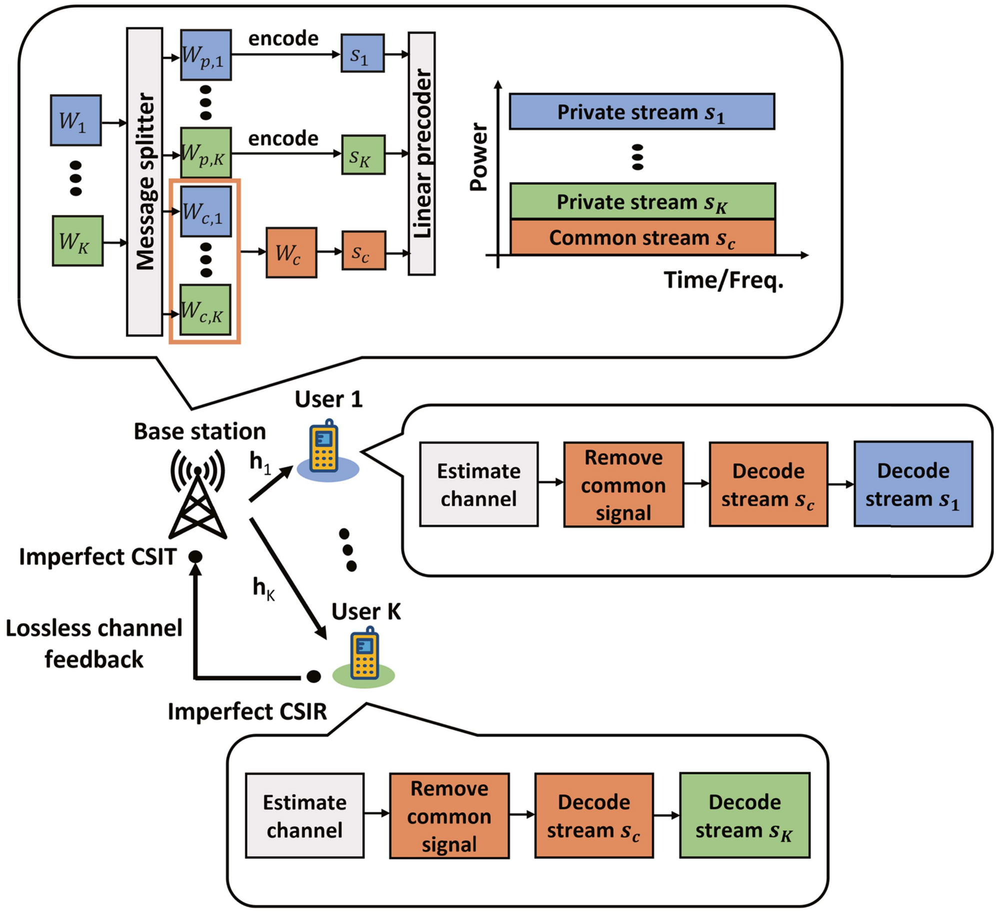

Fig. 1. System structure of RSMA under imperfect channel knowledge.

block-diagonal matrix $\mathbf{X} = \text{blkdiag}\,(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N) \in \mathbb{C}^{NK \times NK}$ by concatenating N matrices $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N \in \mathbb{C}^{K \times K}$.

II. System Model

We consider a multi-user multiple-input single-output (MISO) system where a base station (BS) has N_t antennas and serves K users with a single antenna each. The received signal at k-th user, $k \in \mathcal{K} = \{1, 2, \ldots, K\}$, is written as

$$y_k = \mathbf{h}_k^H \mathbf{x} + n_k, \tag{1}$$

where $\mathbf{x}$ is the transmitted signal, $\mathbf{h}_k \in \mathbb{C}^{N_t \times 1}$ is the channel vector from the BS to k-th user, and $n_k \sim \mathcal{CN}(0, \sigma_{n,k}^2)$ is additive white Gaussian noise (AWGN). We assume that each user's noise variance is equal, $\sigma_{n,k}^2 = \sigma_n^2$, and $\mathbf{h}_k$ has an independent and identically distribution (i.i.d), i.e., $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_{h,k}^2 \mathbf{I}_{N_t})$. The transmitted power constraint is $\mathbb{E}\{\mathbf{x}\mathbf{x}^H\} \leq P_t$.

As shown in Fig. 1, in (1-layer) RSMA, the message for the k-th user, W_k, is divided into the common message $W_{\mathrm{c},k}$ and the private message $W_{\mathrm{p},k}$, i.e., $W_k = \{W_{\mathrm{p},k}, W_{\mathrm{c},k}\}$. All the common messages split from $W_k, \forall k \in \mathcal{K}$ are combined into one super-common message W_{c}. W_{c} is then encoded into one common stream which is designed to be decodable by all users. On the other hand, each private message is encoded into one private stream that is only decodable by its corresponding user. It is worth noting that the common data stream can be eliminated by the SIC operation. Therefore, the common data stream is firstly decoded then after the SIC operation private data stream is decoded. We assume that the data symbol in each stream has a Gaussian distribution with zero mean and unit variance, i.e., $s_{\mathrm{c}} \sim \mathcal{CN}(0, 1)$ and $s_k \sim \mathcal{CN}(0, 1)$, $\forall k \in \mathcal{K}$. The transmitted signal is given by

$$\mathbf{x} = \mathbf{p}_{\mathrm{c}} s_{\mathrm{c}} + \sum_{k=1}^{K} \mathbf{p}_k s_k, \tag{2}$$

where $\mathbf{p}_{\mathrm{c}} \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{p}_k \in \mathbb{C}^{N_t \times 1}$ are the precoding vectors for the data symbols of the common stream and private streams, respectively. The transmitted signal power constraint with the total power P_t is expressed as $\|\mathbf{p}_{\mathrm{c}}\|^2 + \sum_{k \in \mathcal{K}} \|\mathbf{p}_i\|^2 \leq P_t$.

A. Channel Knowledge Assumption

We assume a channel model for both imperfect CSIT and CSIR. In this scenario, each user obtains the channel information with the presence of channel estimation error. We assume that the BS has the same channel information owing to the lossless channel feedback. Thus, actual channel of the k-th user is expressed as

$$\mathbf{h}_k = \hat{\mathbf{h}}_k + \mathbf{e}_k \tag{3}$$

where $\hat{\mathbf{h}}_k$ is the estimated channel $\mathbf{e}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Phi}_k)$ is the channel estimation error. Note that both the base station and all users know the expectation of the channel as $\mathbb{E}[\mathbf{h}_k] = \hat{\mathbf{h}}_k$. The covariance of the channel becomes

$$\mathbb{E}[(\mathbf{h}_k - \mathbb{E}[\mathbf{h}_k])(\mathbf{h}_k - \mathbb{E}[\mathbf{h}_k])^H] = \mathbf{\Phi}_k. \tag{4}$$

We assume that the covariance matrix of channel error $\mathbf{e}_k(= \mathbf{h}_k - \hat{\mathbf{h}}_k = \mathbf{h}_k - \mathbb{E}[\mathbf{h}_k])$ is $\mathbf{\Phi}_k = \sigma_{e,k}^2 \mathbf{I}_{N_t}$. The received signal in (1) can be reformulated as

$$\begin{aligned} y_k &= \hat{\mathbf{h}}_k^H \mathbf{x} + \mathbf{e}_k^H \mathbf{x} + n_k \\ &= \hat{\mathbf{h}}_k^H \mathbf{p}_c s_c + \mathbf{e}_k^H \mathbf{p}_c s_c + \sum_{j=1}^{K} \left(\hat{\mathbf{h}}_k^H \mathbf{p}_j s_j + \mathbf{e}_k^H \mathbf{p}_j s_j \right) + n_k. \end{aligned} \quad (5)$$

B. The Rate Derived From Imperfect CSIR

It is challenging to indicate the achievable rate at the receiver side under imperfect CSIR since each user cannot have exact information about the genie channel. To tackle this problem, one can apply the generalized mutual information (GMI) [9]. First, we start with the point-to-point case to represent a general GMI form. When the input signal s follows a Gaussian distribution, i.e., $s \sim \mathcal{CN}(0, S)$, the output signal is expressed as

$$y = hs + z \quad (6)$$

where h is the fading coefficient and z is the additional noise ($z \sim \mathcal{CN}(0, Z)$). Having knowledge of the expectation of channel $\mathbb{E}[h]$ and the variance of channel σ_h^2, h can be represented as $h = \hat{h} + e$ where $\mathbb{E}[h] = \hat{h}$ and $\mathbb{E}[e] = 0$. By considering $\hat{h}$ as channel estimate and e as channel error ($e \sim \mathcal{CN}(0, \sigma_h^2)$), GMI under imperfect channel knowledge is defined as

$$I_{\text{GMI}} = \log_2 \left(1 + \frac{|\hat{h}|^2 S}{\mathbb{E}[|e|^2] S + Z} \right), \quad (7)$$

where $\mathbb{E}[|e|^2] = \sigma_h^2$. Note that there is a self-interference generated by imperfect CSIR as shown in (7). In the case of perfect CSIR, $\sigma_h^2 = 0$, $h = \hat{h}$, thus GMI equals to Shannon capacity. Whereas, in the case of imperfect CSIR, GMI matches the achievable rate if the input follows a Gaussian distribution and the user employs the nearest neighbor decoder [10].

We then apply GMI to RSMA under imperfect channel knowledge scenario. The above received signal equation can be transformed into a similar form of (6), which can be expressed as

$$\begin{aligned} y_k &= h_{k,c} s_c + z_c \\ &= \hat{h}_{k,c} s_c + e_{k,c} s_c + z_c \end{aligned} \quad (8)$$

where $h_{k,c} = \hat{h}_{k,c} + e_{k,c}$, $\hat{h}_{k,c} = \hat{\mathbf{h}}_k^H \mathbf{p}_c$, $e_{k,c} = \mathbf{e}_k^H \mathbf{p}_c$, and $z_c = \sum_{j=1}^K (\hat{\mathbf{h}}_k^H \mathbf{p}_j s_j + \mathbf{e}_k^H \mathbf{p}_j s_j) + n_k$. The rate for the common stream can be expressed as

$$\begin{aligned} &R_{c,k} \\ &= \log_2 \left(1 + \frac{|\hat{\mathbf{h}}_k^H \mathbf{p}_c|^2}{\mathbb{E}[|\mathbf{e}_k^H \mathbf{p}_c|^2] + \sum_{j=1}^K (|\hat{\mathbf{h}}_k^H \mathbf{p}_j|^2 + \mathbb{E}[|\mathbf{e}_k^H \mathbf{p}_j|^2]) + \sigma_n^2} \right) \end{aligned} \quad (9)$$

where $\mathbb{E}[|\mathbf{e}_k^H \mathbf{p}_j|^2] = \mathbf{p}_j^H \mathbf{\Phi}_k \mathbf{p}_j$ and $\mathbb{E}[|\mathbf{e}_k^H \mathbf{p}_c|^2] = \mathbf{p}_c^H \mathbf{\Phi}_k \mathbf{p}_c$. Note that $|\mathbf{e}_k^H \mathbf{p}_c|^2$ is related to the desired stream as well as the channel error term, which is considered to be the interference when decoding the desired stream. The common rate is derived by

$$R_c \triangleq \min_k R_{c,k} \quad (10)$$

since all the users need to decode the common stream. However, since all the users obtain knowledge of the estimated channel not the actual channel, the common stream cannot be eliminated completely. Therefore, the received signal after applying SIC is derived as

$$\begin{aligned} y_k^{\text{SIC}} &= y_k - \hat{\mathbf{h}}_k^H \mathbf{p}_c s_c \\ &= \mathbf{e}_k^H \mathbf{p}_c s_c + \sum_{j=1}^{K} \left(\hat{\mathbf{h}}_k^H \mathbf{p}_j s_j + \mathbf{e}_k^H \mathbf{p}_j s_j \right) + n_k, \end{aligned} \quad (11)$$

where the part of the common stream $\mathbf{e}_k^H \mathbf{p}_c s_c$ remains after the SIC operation, referred to as residual interference. Similar to the common stream, the rate of the private stream can be obtained. The received signal after SIC is rewritten as

$$\begin{aligned} y_{k,\text{SIC}} &= h_k s_k + z_k \\ &= \hat{h}_k s_k + e_k s_k + z_k \end{aligned} \quad (12)$$

where $h_k = \hat{h}_k + e_k$, $\hat{h}_k = \hat{\mathbf{h}}_k^H \mathbf{p}_k$, $e_k = \mathbf{e}_k^H \mathbf{p}_k$, and $z_k = \mathbf{e}_k^H \mathbf{p}_c s_c + \sum_{j=1, j\neq k}^K (\hat{\mathbf{h}}_k^H \mathbf{p}_j s_j + \mathbf{e}_k^H \mathbf{p}_j s_j) + n_k$. Thus, the rate for the k-th user's private stream is given by

$$R_k = \log_2 \left(1 + \frac{|\hat{\mathbf{h}}_k^H \mathbf{p}_k|^2}{\mathbb{E}[|\mathbf{e}_k^H \mathbf{p}_c|^2] + \mathbb{E}[|\mathbf{e}_k^H \mathbf{p}_k|^2] + I_k + \sigma_n^2} \right), \quad (13)$$

where $I_k = \sum_{j=1, j\neq k}^K (|\hat{\mathbf{h}}_k^H \mathbf{p}_j|^2 + \mathbb{E}[|\mathbf{e}_k^H \mathbf{p}_j|^2])$. There are additional interference terms (i.e., $\mathbb{E}[|\mathbf{e}_k^H \mathbf{p}_c|^2]$ and $\mathbb{E}[|\mathbf{e}_k^H \mathbf{p}_k|^2]$) compared to the achievable rate of the perfect channel knowledge case since all the users treat the signal associated with channel estimation error as interference,.

III. The Proposed User Rate Fairness Optimization Techniques

Our objective in this section is to optimize a max-min fairness problem to guarantee each user's quality of service (QoS) when using the RSMA framework. However, it is difficult to solve since the max-min fairness problem has non-convexity. To this end, we present a relaxed optimization problem based on SDR and CCP that modifies a non-convex problem to a convex problem.

Note that the common stream is a combination of each user's common message. As such, the common rate R_c can be split into K parts, $C_1, C_2, \ldots, C_K$ in which C_k is the portion of the common rate for the k-th user's message and $\sum_{k=1}^K C_k = R_c$. Lastly, the total achievable rate of the k-th user is the sum of the private rate and the portion of the common rate, i.e., $R_k + C_k$. To handle the min-max operation in an efficient manner, we introduce an auxiliary variable R_t that acts as a lower bound for the minimum achievable rate among all users. Then, the max-min fairness optimization problem becomes

$$\begin{aligned} (\mathscr{P}_1) \quad & \max_{\mathbf{p}, \mathbf{c}, R_t} R_t \\ & \text{s.t. } R_k + C_k \geq R_t, \ \forall k \in \mathcal{K}, \\ & R_{c,k} \geq \sum_{k=1}^K C_k, \ \forall k \in \mathcal{K}, \\ & C_k \geq 0, \forall k \in \mathcal{K}, \ \|\mathbf{p}\|^2 \leq P_t, \end{aligned}$$

where $\mathbf{p} = [\mathbf{p}_1^H, \ldots, \mathbf{p}_K^H, \mathbf{p}_c^H]^H \in \mathbb{C}^{N_t(K+1)\times 1}$ and $\mathbf{c} = [C_1, C_2, \ldots, C_K] \succcurlyeq 0$ is a vector for the portion of common rate. Note that $(\mathscr{P}_1)$ is a non-convex problem, which is difficult to solve. Therefore, we need to transform the original non-convex problem into the modified convex problem. Firstly, by employing a stacking method [11], we convert the rates for the common stream and private stream into a matrix form as

$$R_{c,k} = \log_2\left(\frac{\mathbf{p}^H\mathbf{A}_k\mathbf{p}}{\mathbf{p}^H\mathbf{B}_k\mathbf{p}}\right), R_k = \log_2\left(\frac{\mathbf{p}^H\mathbf{B}_k\mathbf{p}}{\mathbf{p}^H\mathbf{D}_k\mathbf{p}}\right), \tag{14}$$

where the matrices $\mathbf{A}_k$, $\mathbf{B}_k$, and $\mathbf{D}_k \in \mathbb{C}^{N_t(K+1)\times N_t(K+1)}$ are block-diagonal and positive definite, which can be defined as

$$\mathbf{A}_k = \text{blkdiag}(\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H, \hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H, \ldots, \hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H) + \left(\frac{\sigma_n^2}{P_t} + \sigma_{e,k}^2\right)\mathbf{I}_{N_t(K+1)} \tag{15}$$

$$\mathbf{B}_k = \mathbf{A}_k - \text{blkdiag}\left(\mathbf{0}, \mathbf{0}, \ldots, \hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H\right) \tag{16}$$

$$\mathbf{D}_k = \mathbf{B}_k - \text{blkdiag}\left(\mathbf{0}, \ldots, \underbrace{\hat{\mathbf{h}}_k\hat{\mathbf{h}}_k^H}_{k\text{-th block}}, \ldots, \mathbf{0}\right). \tag{17}$$

Note that the power constraint $\|\mathbf{p}\|^2 \leq P_t$ can be removed due to the fact that the rate is not affected by the magnitude of $\mathbf{p}$. Also note that each rate $R_k = \log_2(\mathbf{p}^H\mathbf{B}_k\mathbf{p}) - \log_2(\mathbf{p}^H\mathbf{D}_k\mathbf{p})$ is the difference of concave functions, which is a non-convex function. From this, we then propose an alternating optimization algorithm to convert the problem that has non-convexity. To this end, we derive the upper and lower bounds of (14) using exponential terms with auxiliary variables $a_k, b_{1,k}, b_{2,k}, d_k$:

$$\mathbf{p}^H\mathbf{A}_k\mathbf{p} \geq e^{a_k}, \quad \mathbf{p}^H\mathbf{B}_k\mathbf{p} \geq e^{b_{2,k}}, \tag{18}$$

$$\mathbf{p}^H\mathbf{B}_k\mathbf{p} \leq e^{b_{1,k}}, \quad \mathbf{p}^H\mathbf{D}_k\mathbf{p} \leq e^{d_k}. \tag{19}$$

Using these four bounds, we can derive the lower bound of $R_{c,k}$ and R_k as

$$R_{c,k} \geq \frac{1}{\ln 2}\left(a_k - b_{1,k}\right), R_k \geq \frac{1}{\ln 2}\left(b_{2,k} - d_k\right). \tag{20}$$

Using these lower bounds, we can transform the problem $(\mathscr{P}_1)$ into a convex problem by modifying the non-convex constraints (18)–(19). To solve the non-convexity by quadratic constraints, $\mathbf{p}^H\mathbf{A}_k\mathbf{p} \geq e^{a_k}$ and $\mathbf{p}^H\mathbf{B}_k\mathbf{p} \geq e^{b_{2,k}}$, we apply the SDR technique [12]. The quadratic term of the constraint (18) becomes

$$\mathbf{p}^H\mathbf{A}_k\mathbf{p} = \mathsf{tr}\left(\mathbf{p}^H\mathbf{A}_k\mathbf{p}\right) = \mathsf{tr}\left(\mathbf{A}_k\mathbf{p}\mathbf{p}^H\right) = \mathsf{tr}\left(\mathbf{A}_k\mathbf{X}\right) \tag{21}$$

by substituting $\mathbf{X} = \mathbf{p}\mathbf{p}^H$ with the constraints $\mathbf{X} \succeq 0$ and $\mathsf{rank}(\mathbf{X}) = 1$. The constraint (19) becomes convex when the rank constraint is removed.

Also, we apply CCP to the non-convex exponential terms, $\mathbf{p}^H\mathbf{B}_k\mathbf{p} \leq e^{b_{1,k}}$ and $\mathbf{p}^H\mathbf{D}_k\mathbf{p} \leq e^{d_k}$ [13]. These constraints are the difference of convex functions, which are in general non-convex. By using the first-order Talyor series approximation, we can transform the exponential terms in (19), which is concave. Thus, the relaxed constraints of (18)–(19) are given by

$$\mathsf{tr}\left(\mathbf{A}_k\mathbf{X}\right) \geq e^{a_k}, \;\; \mathsf{tr}\left(\mathbf{B}_k\mathbf{X}\right) \geq e^{b_{2,k}}, \tag{22}$$

$$\mathsf{tr}\left(\mathbf{B}_k\mathbf{X}\right) \leq e^{b_{1,k}^{(m-1)}}\left(b_{1,k} - b_{1,k}^{(m-1)} + 1\right), \tag{23}$$

$$\mathsf{tr}\left(\mathbf{D}_k\mathbf{X}\right) \leq e^{d_k^{(m-1)}}\left(d_k - d_k^{(m-1)} + 1\right). \tag{24}$$

where m is the iteration number. The following optimization problem $(\mathscr{P}_2)$ at the m-th iteration is a convex problem and is expressed as

$$\begin{aligned}(\mathscr{P}_2): \max_{\mathbf{X}, \bar{\mathbf{c}}, \bar{R}_t, a, b_1, b_2, d} \;& \bar{R}_t \\ \text{s.t.} \;\; & b_{2,k} - d_k + \bar{C}_k \geq \bar{R}_t, \;\; \forall k \in \mathcal{K}, \\ & \bar{C}_k \geq 0, \;\; \forall k \in \mathcal{K}, \\ & a_k - b_{1,k} \geq \sum_{k=1}^{K} \bar{C}_k, \;\; \forall k \in \mathcal{K}, \\ & \mathbf{X} \succeq 0, (22), (23), (24)\end{aligned}$$

where $\bar{R}_t = R_t \ln 2$ and $\bar{C}_k = C_k \ln 2$. The values of $b_{1,k}^{(m-1)}$ and $d_k^{(m-1)}$ are updated at every optimization step by solving the problem $(\mathscr{P}_2)$. The convergence to local optimum solution is guaranteed through a few iterations. Note that in order to satisfy the rank constraint, one can use a sufficiently large number of random vectors [12]. Overall step of the proposed algorithm is described in Algorithm 1. It is worth noting that the convex problem can be solved through CVX toolbox [14].[1]

Algorithm 1: RSMA-MMF.

initialize: $b_{1,k}^{(0)}, d_k^{(0)}$, set the iteration count $m = 0$.
repeat
  $m \leftarrow m + 1$
  Given $b_{1,k}^{(m-1)}, d_k^{(m-1)}$, solve problem $(\mathscr{P}_2)$
until convergence of $b_{1,k}^{(m)}, d_k^{(m)}$
Generate enough random vectors $\mathbf{f} \sim \mathcal{CN}(0, \mathbf{I}_{N_t(K+1)})$
Decomposition of the optimal $\mathbf{X}^\star = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$ from $(\mathscr{P}_2)$
Choose the best $\mathbf{p}^\star = \mathbf{U}\mathbf{\Sigma}^{1/2}\mathbf{f}$ as for maximizing $\bar{R}_t$
output: $\mathbf{p}^\star, \bar{\mathbf{c}}^\star$

TABLE I
POWER ASSIGNMENT FOR OTHER ACCESS SCHEMES

Multiple Access Scheme	s_1	s_2	s_c
RSMA	P_1	P_2	P_c
SDMA	P_1	P_2	0
NOMA	P_1	0	P_c
OMA	P_1	0	0
Multicasting	0	0	P_c

IV. Simulation Results and Discussions

In this section, we measure the numerical results to validate the proposed algorithm. We deploy a multi-user MISO system consisting of $N_t = 2$ transmit antennas and $K = 2$ with a single receive antenna. We assume that the actual channel has i.i.d complex Gaussian distribution, $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_{h,k}^2\mathbf{I}_{N_t})$. The channel distribution for the estimated channel follows $\hat{\mathbf{h}}_k \sim \mathcal{CN}(\mathbf{0}, (\sigma_{h,k}^2 - \sigma_{e,k}^2)\mathbf{I}_{N_t})$. The variance of AWGN is set to $\sigma_n^2 = 1$.

In the simulations, we compare the proposed RSMA technique with existing multiple access strategies: spatial division multiple access (SDMA), NOMA, OMA, and multicasting [15]. Table I shows that RSMA can work as existing strategies by adjusting the power allocated to the stream. It is assumed that the first user (strong user) has a larger channel gain than the second user (weak user). Therefore, the power of the private stream of the weak user should be set to zero in the NOMA case, i.e., $P_2 = 0$. RSMA is shown to reduce to SDMA when the common stream is turned off, i.e., $P_c = 0$. RSMA also becomes OMA by assigning the total transmit power to one of the private streams within the designated time slot.

[1]In CVX, SDR problems are typically solved by a computational efficient interior point algorithm, which runs in probably polynomial time [12].

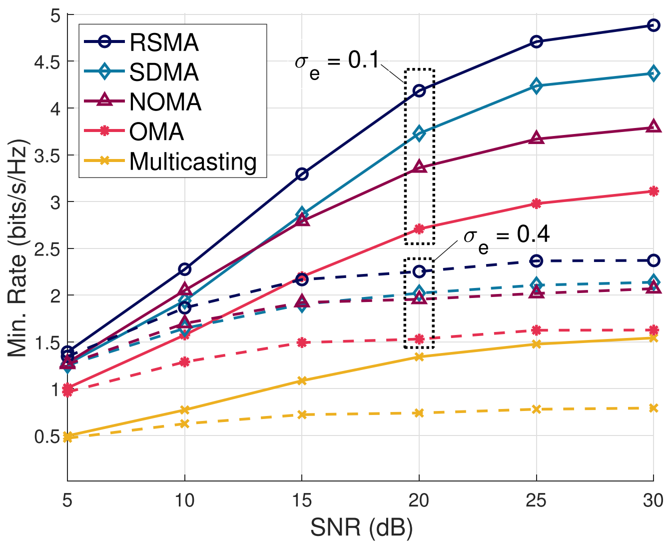

Fig. 2. Max-min rate performance comparison of the proposed RSMA and existing strategies versus SNR when $\sigma_e = 0.1$ and $\sigma_e = 0.4$.

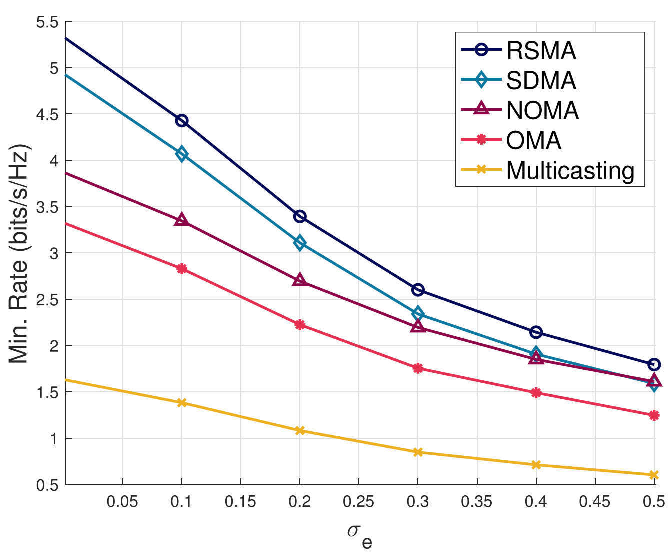

Fig. 3. Max-min rate performance of the proposed RSMA and existing strategies with respect to σ_e.

In Fig. 2, we compare the max-min rate of the proposed RSMA method with existing multiple access schemes with respect to signal-to-noise ratio (SNR) when $\sigma_e = 0.1$ and $\sigma_e = 0.4$. The variance of the user channel is set to where $\sigma^2_{h,k} = 1, \forall k \in \mathcal{K}$. We observe that RSMA shows the best performance at all SNR regimes among various existing multiple access techniques. Secondly, we access the max-min rate performance of the RSMA and conventional multiple access schemes when σ_e varies and the SNR is 20 dB. It is observed from Fig. 3 that the robustness of the RSMA over SDMA against the channel estimation error increases. Specifically, the max-min rate gap is around 8% when $\sigma_e = 0$ while the max-min rate gap is around 13% when $\sigma_e = 0.5$. Also, RSMA achieves a better max-min rate performance compared to NOMA, OMA, and multicasting, regardless of σ_e.

V. Conclusion

In this paper, we investigated the max-min fairness optimization with RSMA technique which is quite robust to imperfect channel knowledge than existing multiple access schemes. To solve the max-min fairness problem that has non-convexity, we proposed an optimization algorithm based on CCP and SDR methods. Through simulation results, we confirmed that the proposed RSMA shows robustness for both imperfect CSIT and CSIR. An important future direction is to apply the proposed RSMA method in non-terrestrial networks in which it is hard to perfectly track 3D channel structure owing to high mobility of satellites and unmanned aerial vehicles (UAVs).

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