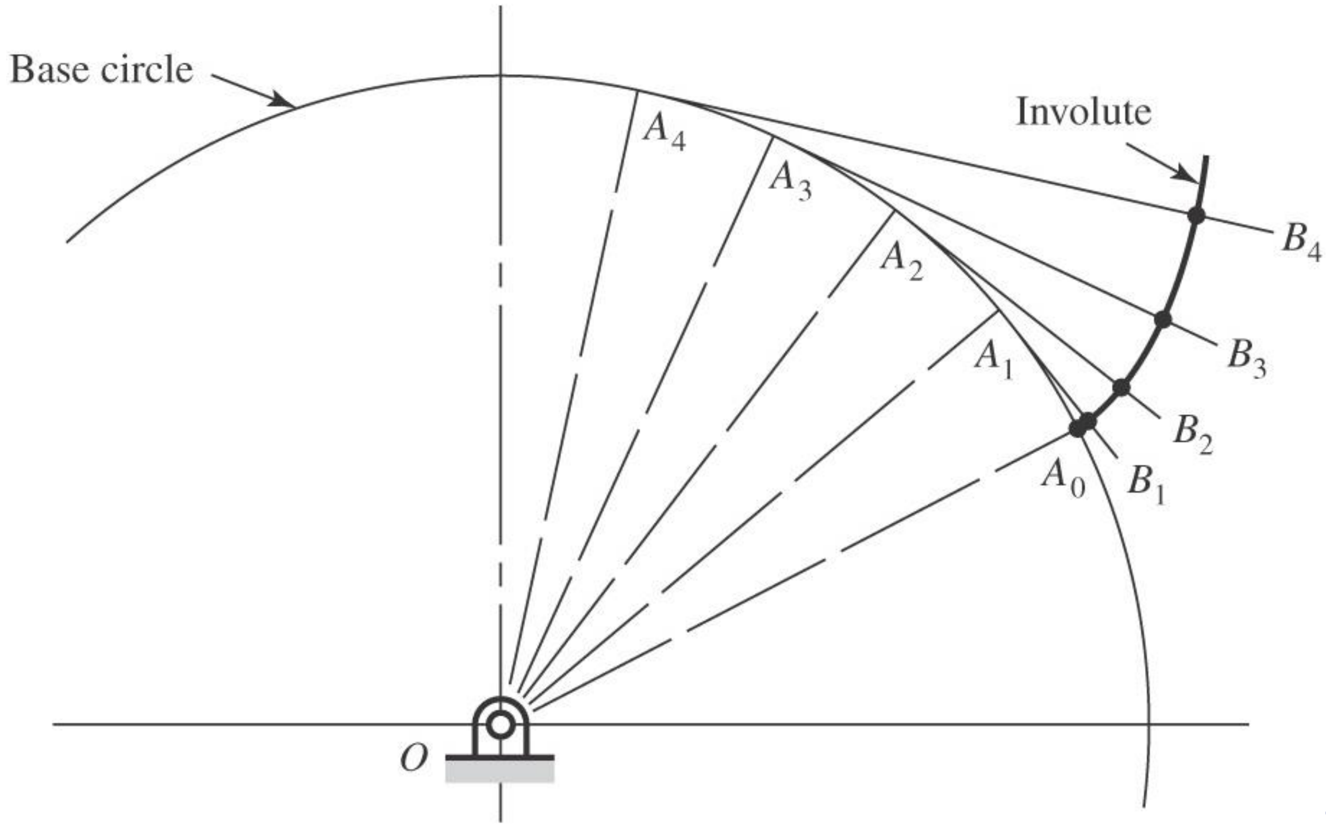
Gears and Gearing
Part 2

Involute (Tooth) Profile

- The most common conjugate action tooth profile is the ***involute*** profile.
- Can be generated by unwrapping a string from a cylinder, keeping the string taut and tangent to the cylinder.
- Circle is called *base circle*.



Why an involute tooth surface?

- Constant speed ratio between gears
  - Smoother speeds
- Constant line of action of forces between gears
  - Which means constant torque transmitted
- Theoretical contact at tangents of pitch circles
- Allow multiple teeth in contact
  - Higher loads
- Tolerant of center distance errors

Common Terminology

where P = diametral pitch, teeth per inch

N = number of teeth

d = pitch diameter, in or mm

p = circular pitch, in or mm

$$P = \frac{N}{d} \qquad (13\text{–}1)$$

$$p = \frac{\pi d}{N} = \pi m \qquad (13\text{–}3)$$

$$pP = \pi \qquad (13\text{–}4)$$

Nomenclature of Spur-Gear Teeth

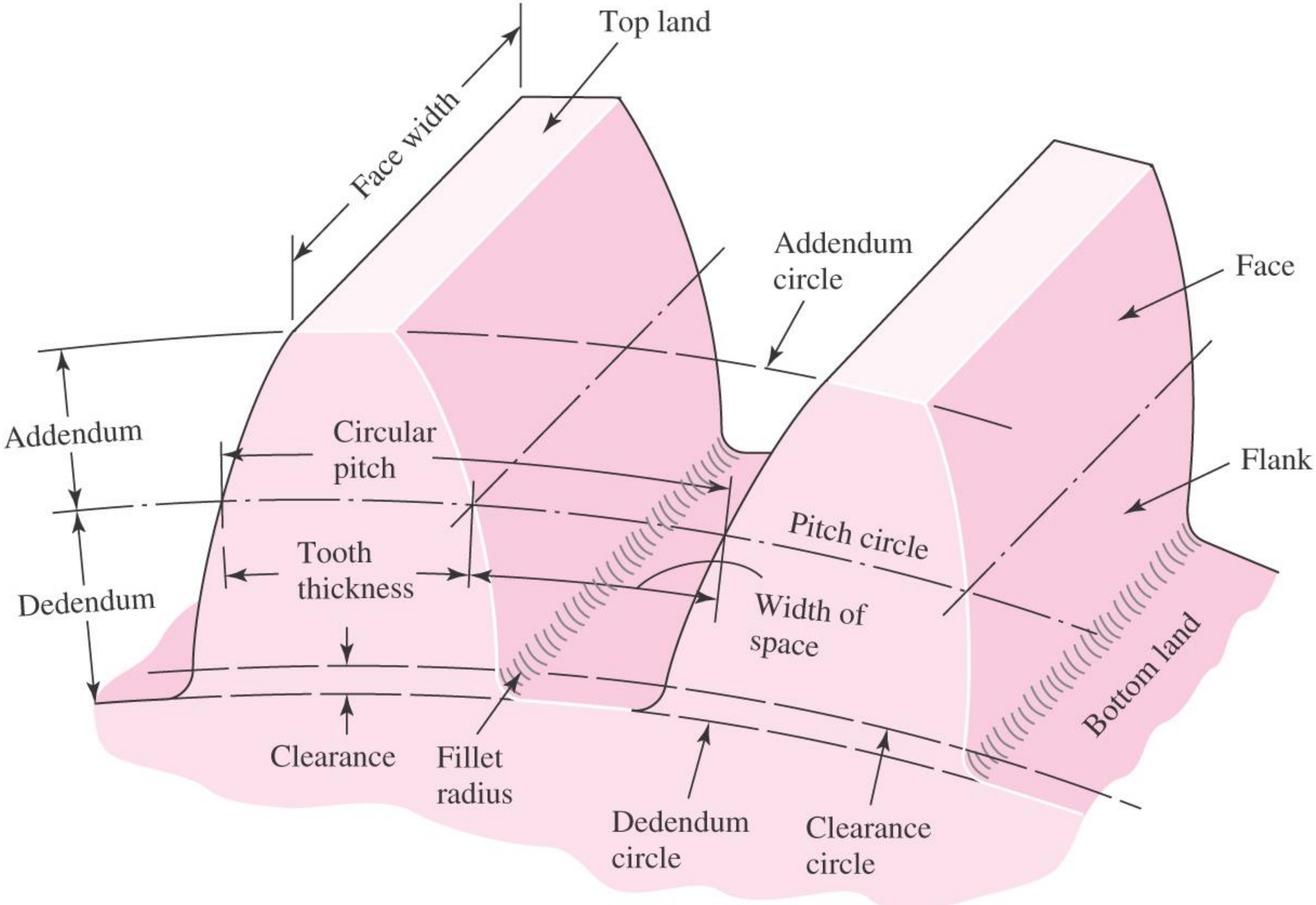


Fig. 13–5

Involute Profile Producing Conjugate Action

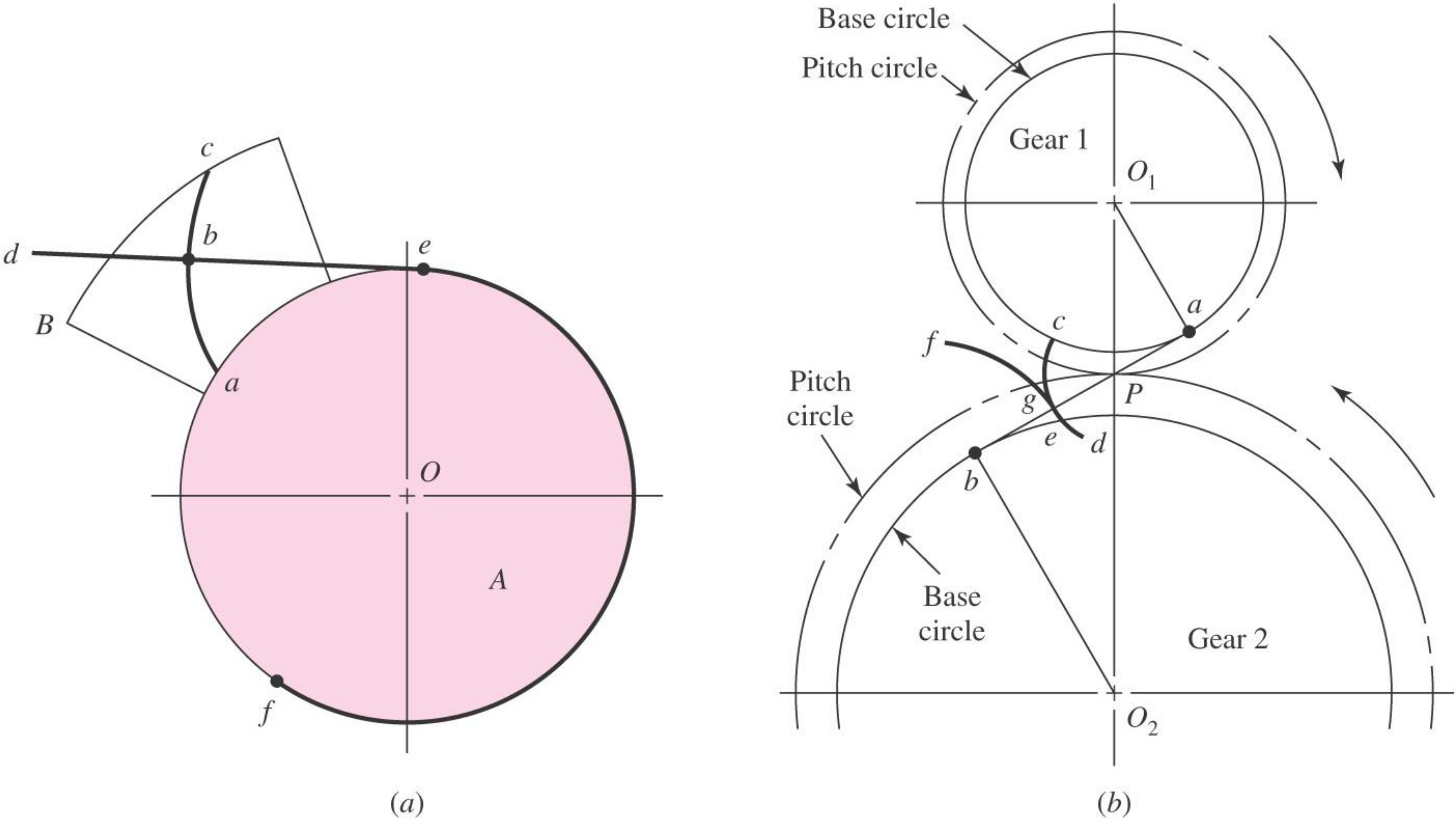


Fig. 13–7

Sequence of Gear Layout

- Pitch circles in contact
- Pressure line at desired pressure angle
- Base circles tangent to pressure line
- Involute profile from base circle
- Cap teeth at addendum circle at 1/P from pitch circle
- Root of teeth at dedendum
circle at 1.25/P from pitch circle (clearance)
- Tooth spacing from circular pitch, $p = \pi / P$

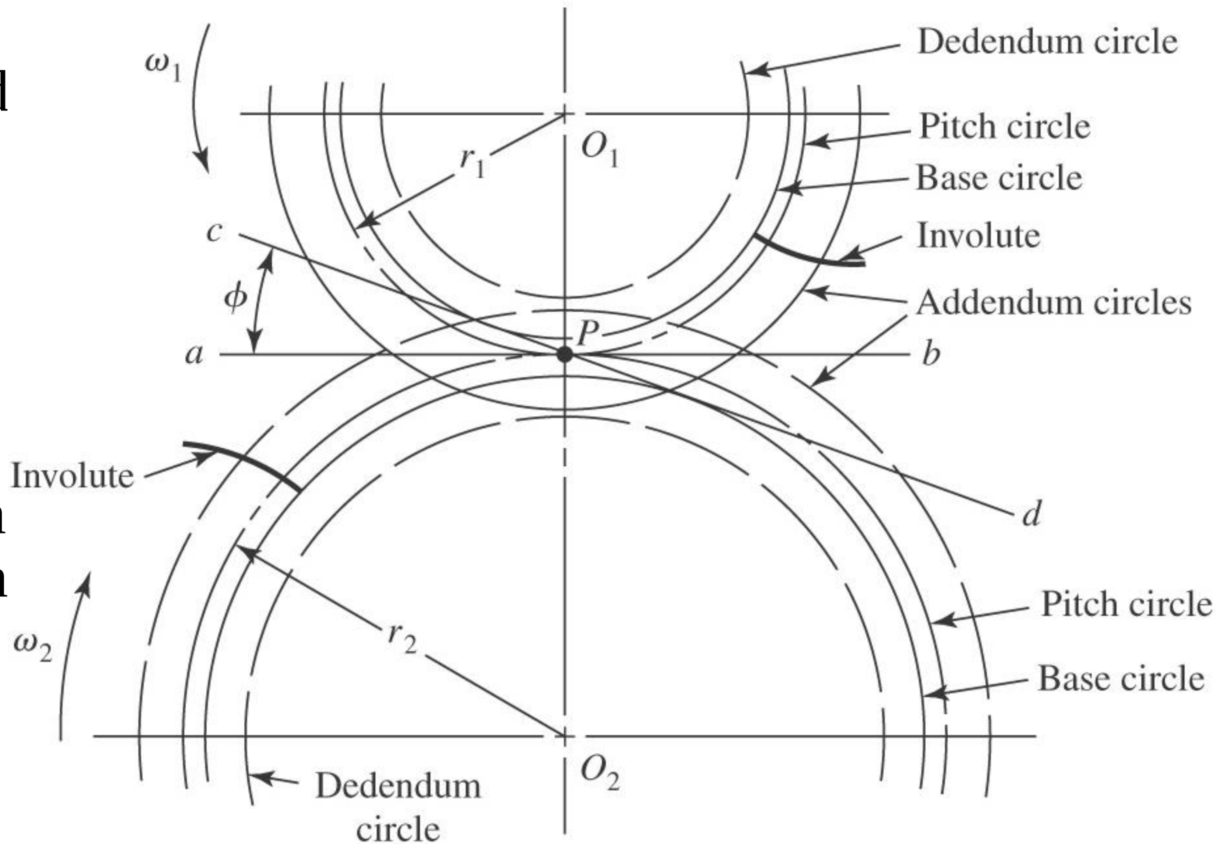


Fig. 13–9

Relation of Base Circle to Pressure Angle

$$r_b = r\cos\phi \qquad (13\text{–}6)$$

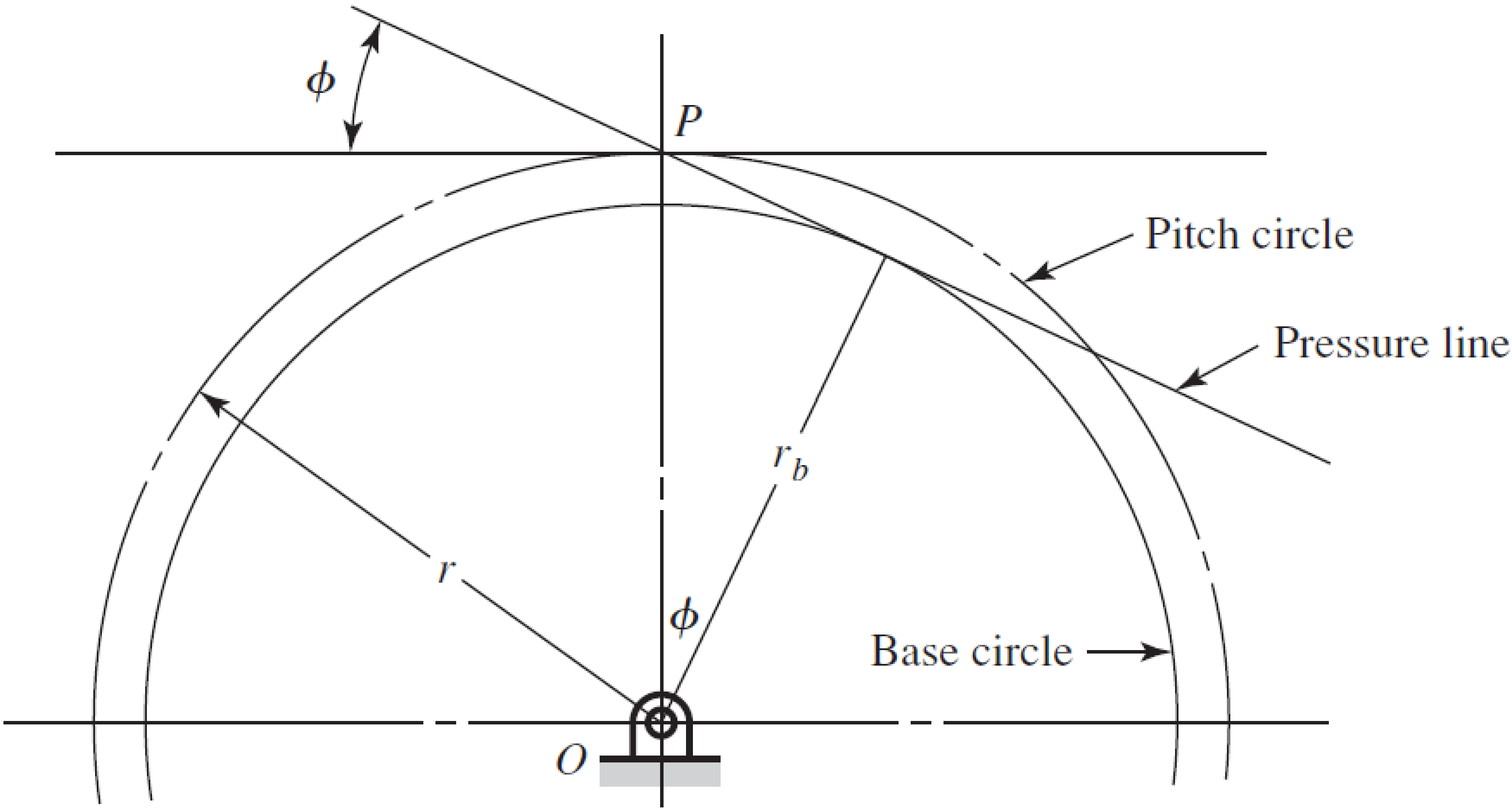


Fig. 13–10

Tooth Action

- First point of contact at *a* where flank of pinion touches tip of gear
- Last point of contact at *b* where tip of pinion touches flank of gear
- Line *ab* is *line of action*

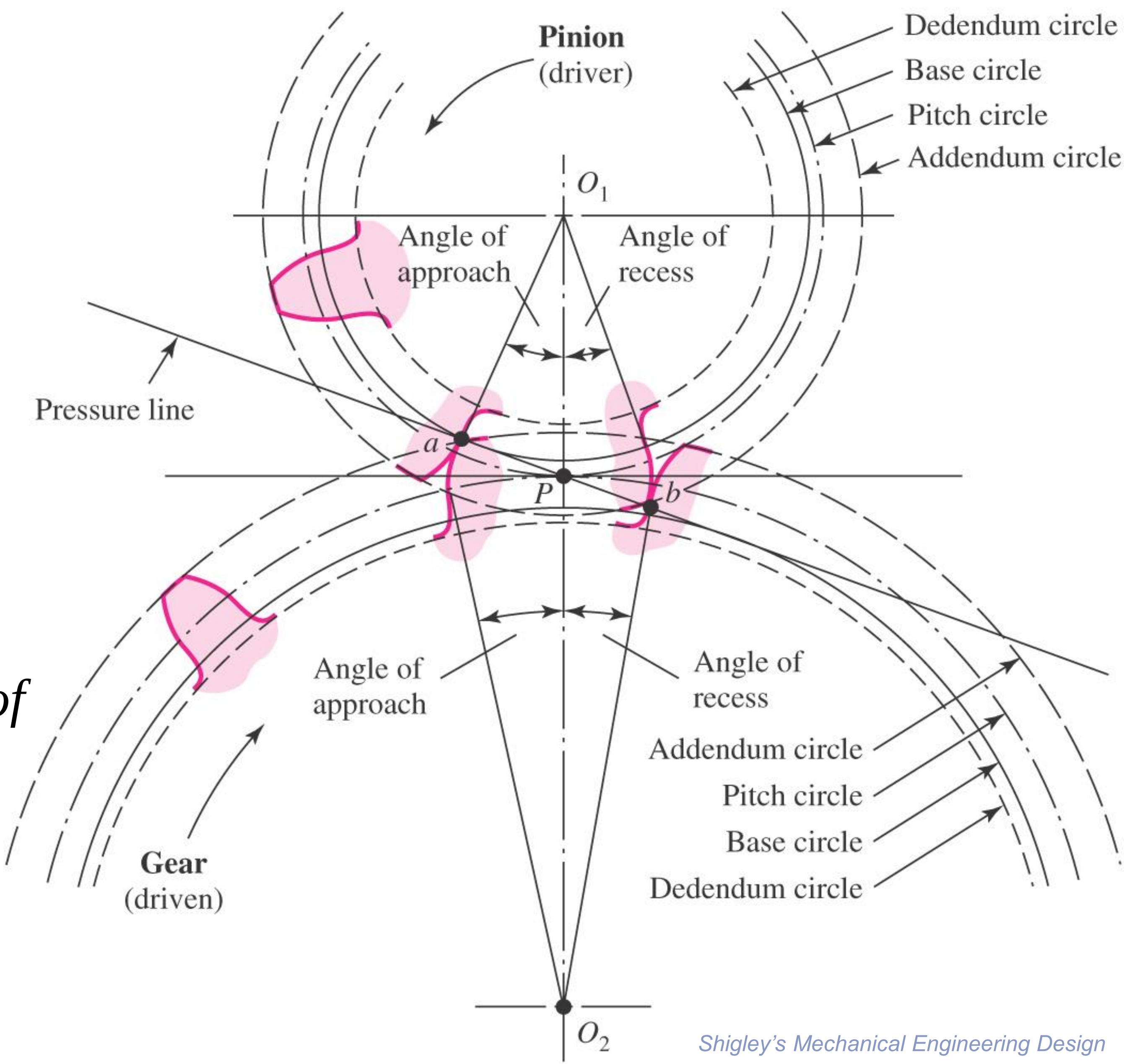


Fig. 13–12

Rack

- A *rack* is a spur gear with an pitch diameter of infinity.
- The sides of the teeth are straight lines making an angle to the line of centers equal to the pressure angle.
- The *base pitch* and *circular pitch*, shown in Fig. 13–13, are related by

$$p_b = p_c \cos \phi \qquad (13\text{–}7)$$

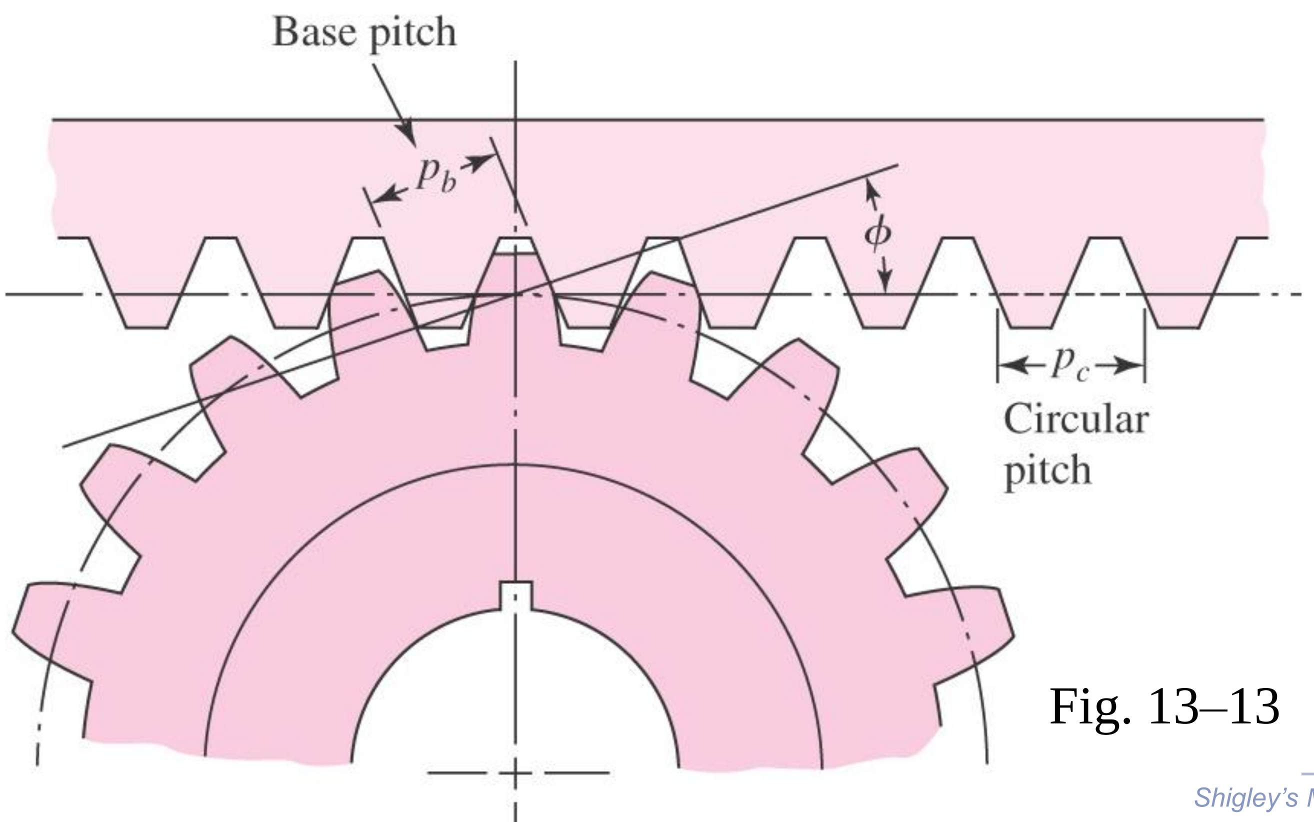


Fig. 13–13

Internal Gear

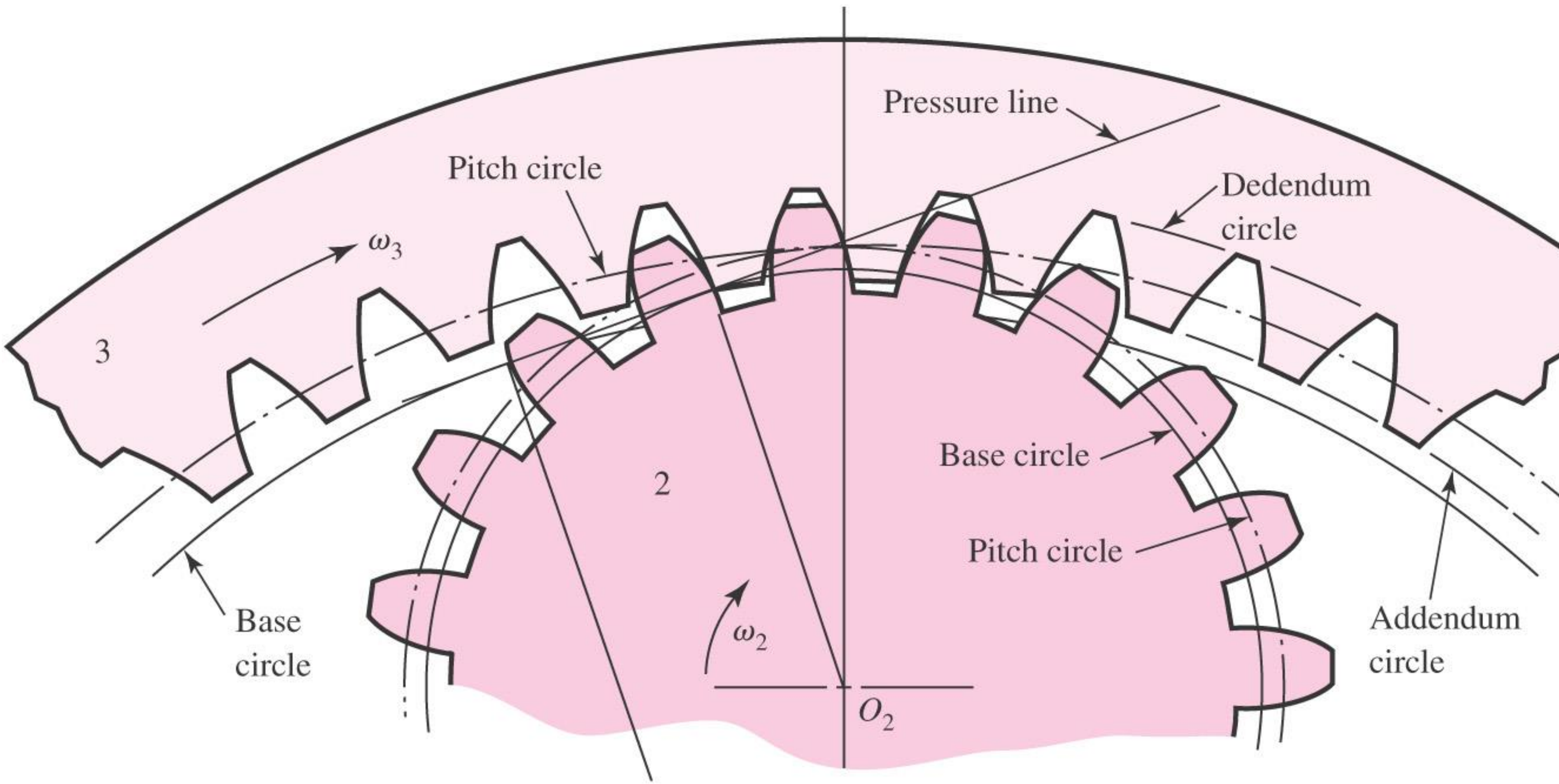


Fig. 13–14

Example 13–1

A gearset consists of a 16-tooth pinion driving a 40-tooth gear. The diametral pitch is 2, and the addendum and dedendum are $1/P$ and $1.25/P$, respectively. The gears are cut using a pressure angle of 20°.
(*a*) Compute the circular pitch, the center distance, and the radii of the base circles.
(*b*) In mounting these gears, the center distance was incorrectly made $\frac{1}{4}$ in larger. Compute the new values of the pressure angle and the pitch-circle diameters.

Solution

(*a*)

$$p = \frac{\pi}{P} = \frac{\pi}{2} = 1.571 \text{ in}$$ **Answer**

The pitch diameters of the pinion and gear are, respectively,

$$d_P = \frac{N_P}{P} = \frac{16}{2} = 8 \text{ in} \qquad d_G = \frac{N_G}{P} = \frac{40}{2} = 20 \text{ in}$$

Therefore the center distance is

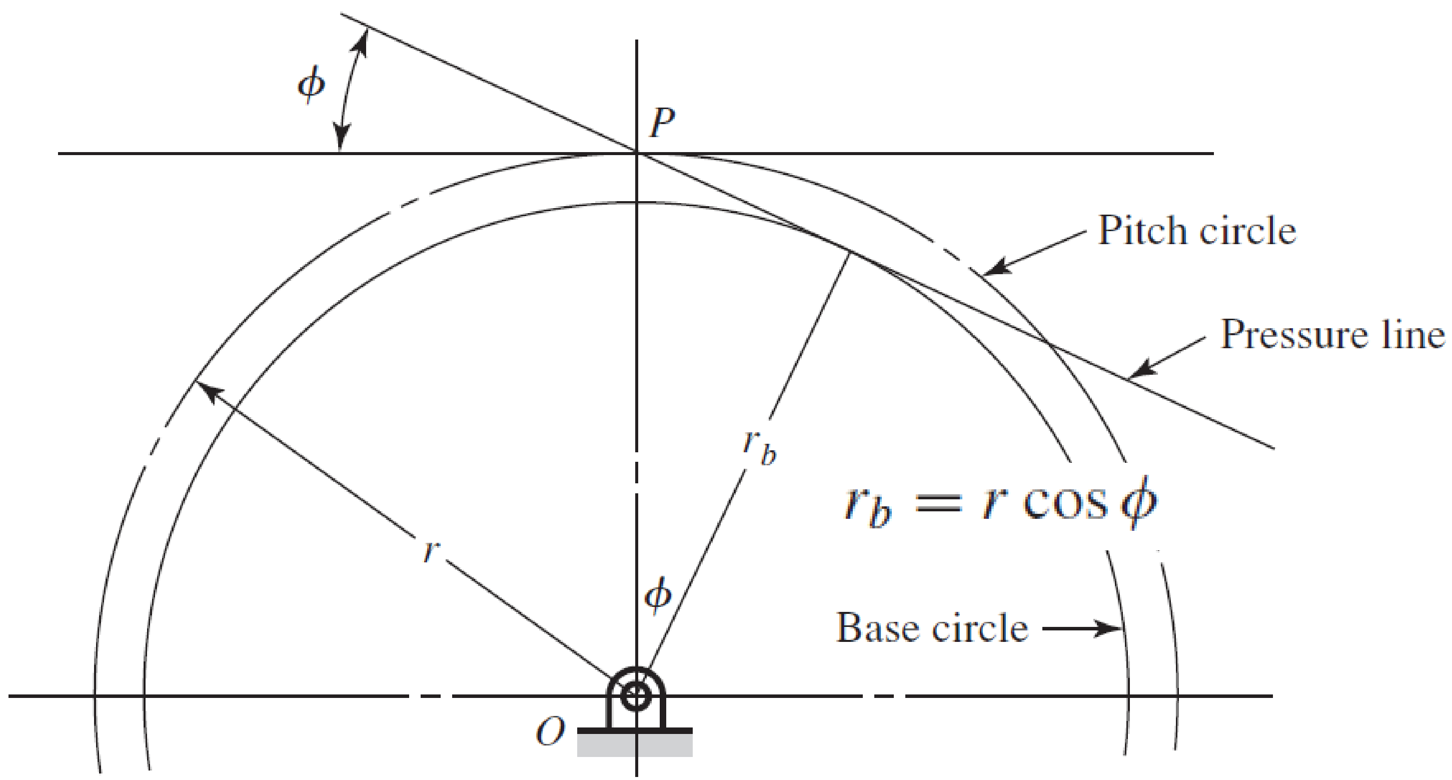
$$\frac{d_P + d_G}{2} = \frac{8 + 20}{2} = 14 \text{ in}$$ **Answer**

Example 13–1

Since the teeth were cut on the 20° pressure angle, the base-circle radii are found to be, using $r_b = r\cos\phi$,

$$r_b(\text{pinion}) = \frac{8}{2}\cos 20^\circ = 3.759\ \text{in} \qquad \text{Answer}$$

$$r_b(\text{gear}) = \frac{20}{2}\cos 20^\circ = 9.397\ \text{in} \qquad \text{Answer}$$



Example 13–1

(*b*) Designating d'_P and d'_G as the new pitch-circle diameters, the $\frac{1}{4}$-in increase in the center distance requires that

$$\frac{d'_P + d'_G}{2} = 14.250 \tag{1}$$

Also, the velocity ratio does not change, and hence

$$\frac{d'_P}{d'_G} = \frac{16}{40} \tag{2}$$

Solving Eqs. (1) and (2) simultaneously yields

$$d'_P = 8.143 \text{ in} \qquad d'_G = 20.357 \text{ in}$$ Answer

Since $r_b = r \cos \phi$, using either the pinion or gear, the new pressure angle is

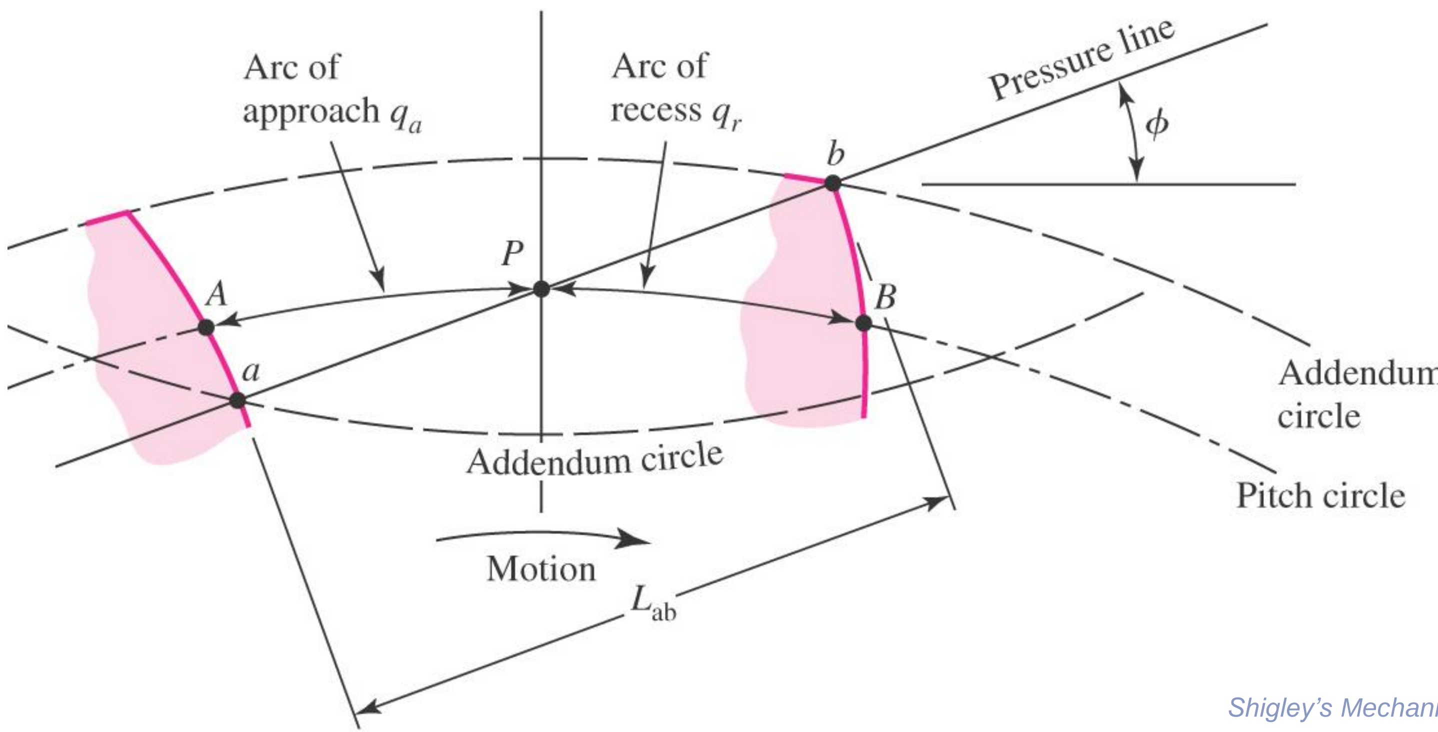
$$\phi' = \cos^{-1} \frac{r_b(\text{pinion})}{d'_P/2} = \cos^{-1} \frac{3.759}{8.143/2} = 22.59°$$ Answer

Contact Ratio

- Arc (distance) of action q_t is the sum of the arc of approach q_a and the arc of recess q_r, that is $q_t = q_a + q_r$
- The *contact ratio* m_c is the ratio of the arc of action and the circular pitch.

$$m_c = \frac{q_t}{p} \tag{13–8}$$

- <u>The contact ratio is the average number of pairs of teeth in contact.</u>



Contact Ratio

- Contact ratio can also be found from the length of the line of action

$$m_c = \frac{L_{ab}}{p \cos \phi} \qquad (13\text{–}9)$$

- The contact ratio should be at least 1.2

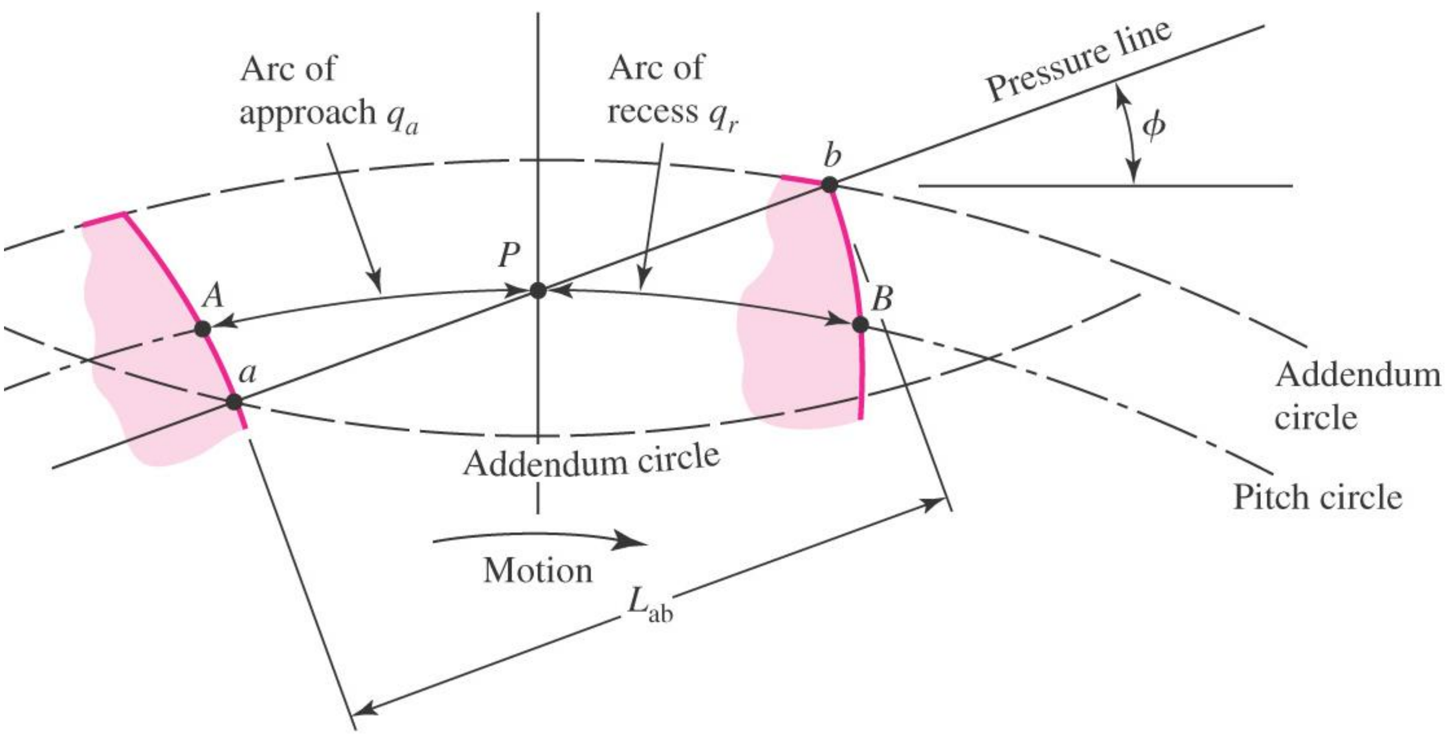


Fig. 13–15

Interference

- Contact of portions of tooth profiles that are not conjugate is called *interference*.
- Occurs when contact occurs below the base circle
- If teeth were produced by generating process then the generating process removes the interfering portion; known as *undercutting*.

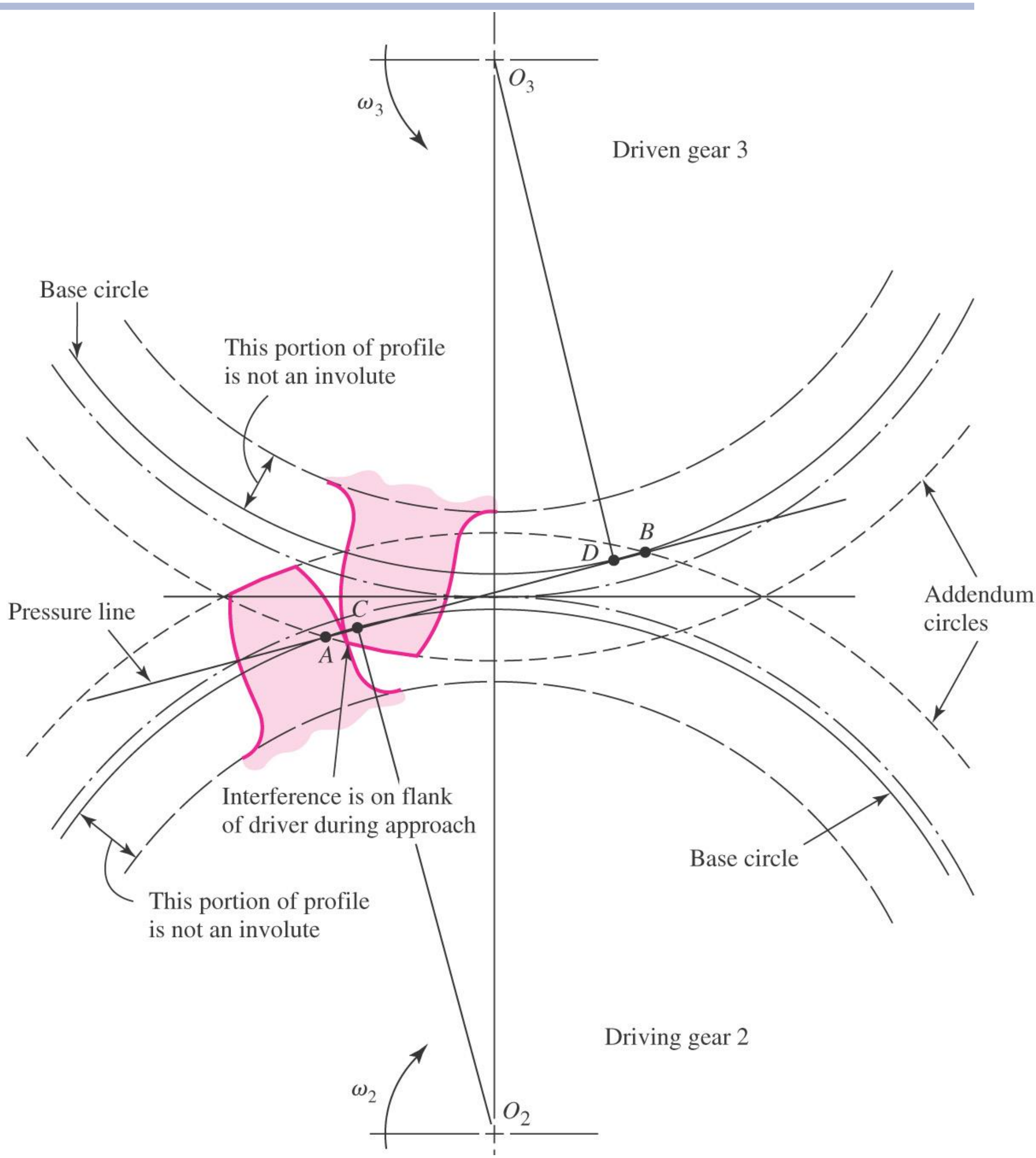


Fig. 13–16

Interference of Spur Gears

- On spur and gear with one-to-one gear ratio, smallest number of teeth which will not have interference is

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \qquad (13\text{–}10)$$

- k =1 for full depth teeth. k = 0.8 for stub teeth
- On spur meshed with larger gear with gear ratio $m_G = N_G/N_P = m$, the smallest number of teeth which will not have interference is

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right) \qquad (13\text{–}11)$$

Interference of Spur Gears

- Largest gear with a specified pinion that is interference-free is

$$N_G = \frac{N_P^2 \sin^2 \phi - 4k^2}{4k - 2N_P \sin^2 \phi} \qquad (13\text{–}12)$$

- Smallest spur pinion that is interference-free with a rack is

$$N_P = \frac{2(k)}{\sin^2 \phi} \qquad (13\text{–}13)$$

Interference

- For 20º pressure angle, the most useful values from Eqs. (13–11) and (13–12) are calculated and shown in the table below.

Minimum N_P	Max N_G	Integer Max N_G	Max Gear Ratio $m_G = N_G/N_P$
13	16.45	16	1.23
14	26.12	26	1.86
15	45.49	45	3
16	101.07	101	6.31
17	1309.86	1309	77

Interference

- Increasing the pressure angle to 25º allows smaller numbers of teeth

Minimum N_P	**Max N_G**	**Integer Max N_G**	**Max Gear Ratio $m_G = N_G/N_P$**
9	13.33	13	1.44
10	32.39	32	3.2
11	249.23	249	22.64

Interference

- Interference can be eliminated by using more teeth on the pinion.
- However, if tooth size (that is diametral pitch P) is to be maintained, then an increase in teeth means an increase in diameter, since $P = N/d$.
- Interference can also be eliminated by using a larger pressure angle. This results in a smaller base circle, so more of the tooth profile is involute.
- This is the primary reason for larger pressure angles.
- Note that the disadvantage of a larger pressure angle is an increase in radial force for the same amount of transmitted force.

Forming of Gear Teeth

- Common ways of forming gear teeth
  - Sand casting
  - Shell molding
  - Investment casting
  - Permanent-mold casting
  - Die casting
  - Centrifugal casting
  - Powder-metallurgy
  - Extrusion
  - Injection molding (for thermoplastics)
  - Cold forming

Cutting of Gear Teeth

- Common ways of cutting gear teeth
  - Milling
  - Shaping
  - Hobbing

Shaping with Pinion Cutter

Fig. 13–17

Shaping with a Rack

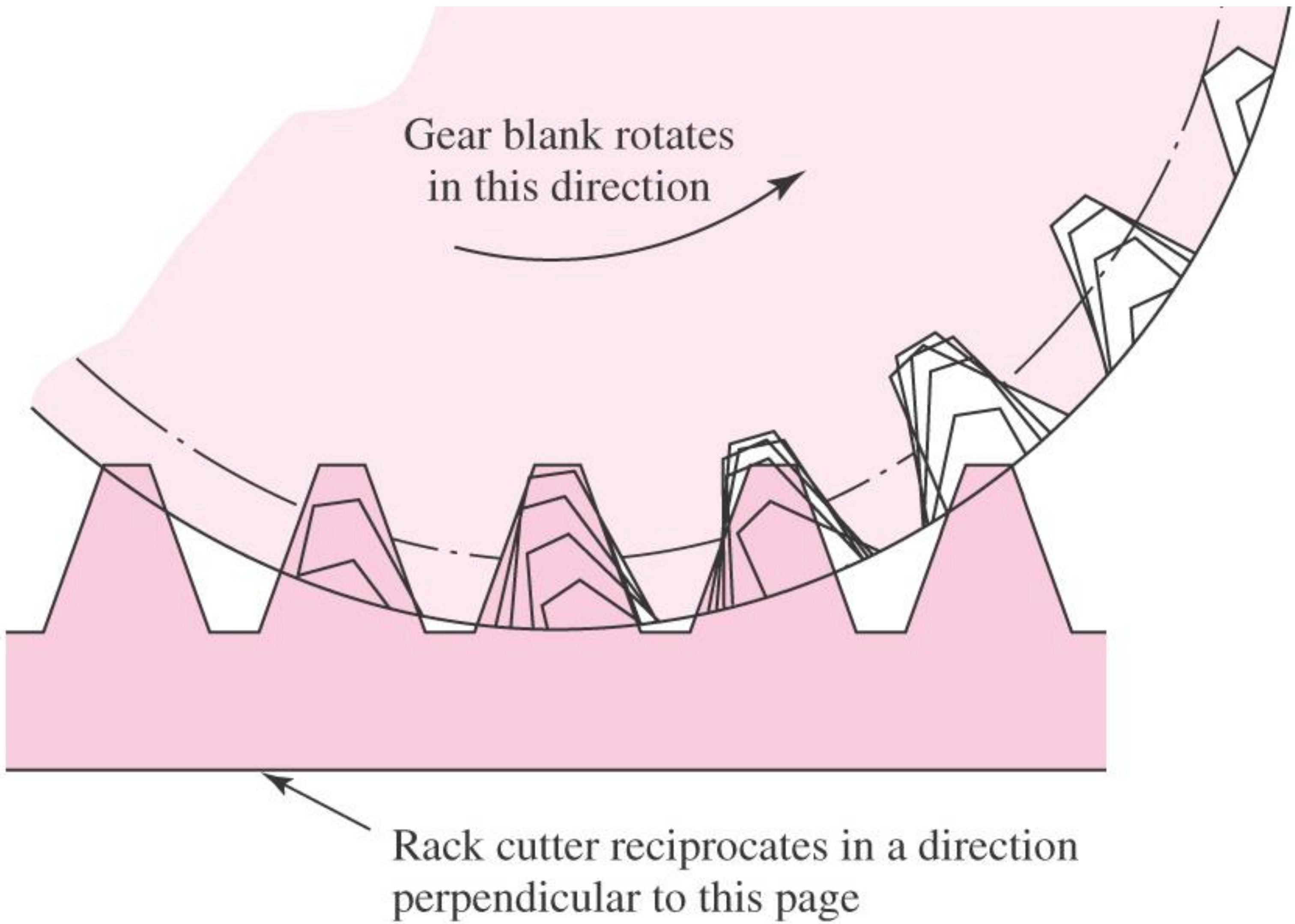


Fig. 13–18

Hobbing a Worm Gear

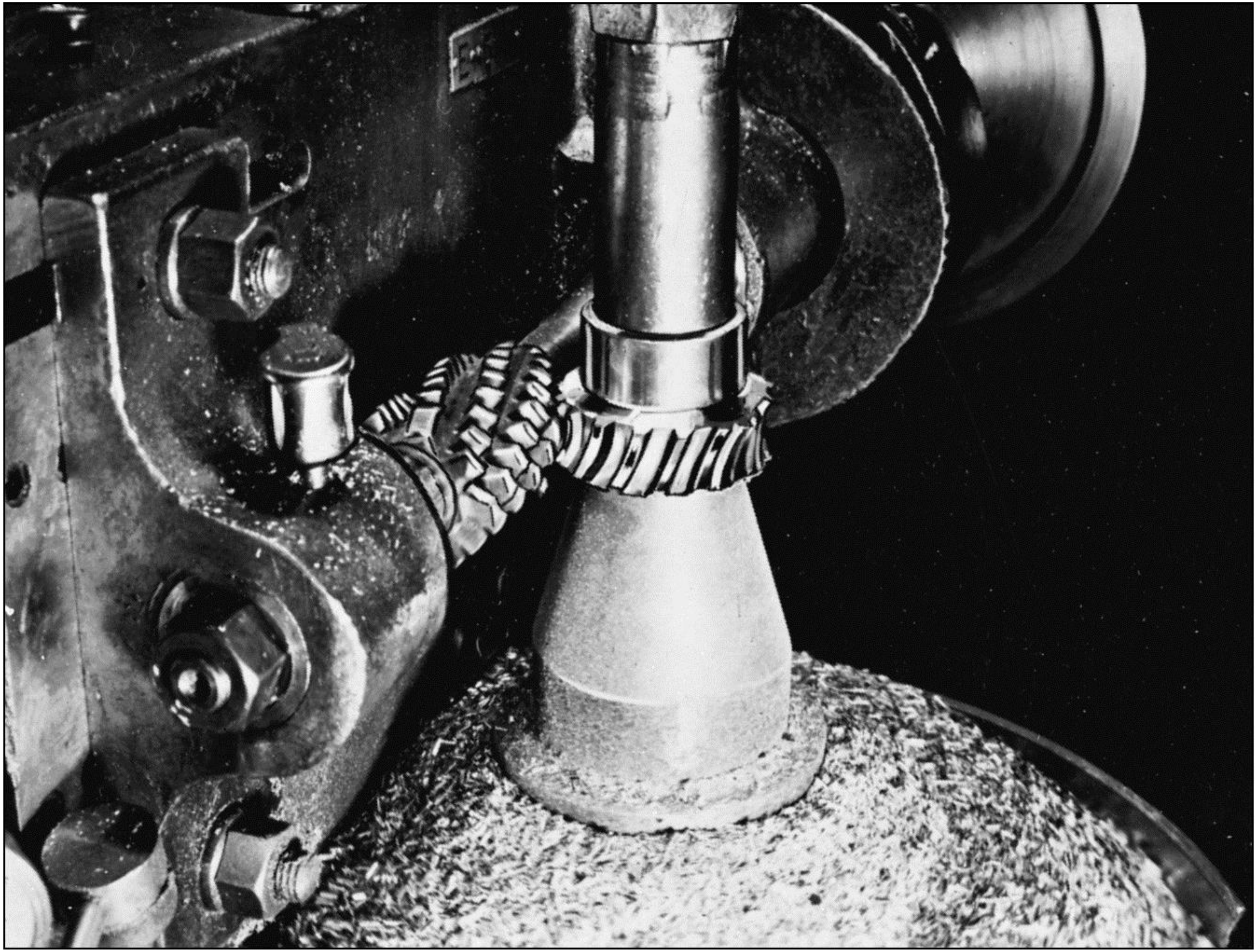


Fig. 13–19