Analytical Aspects of Small Format Surveys Using Oblique Aerial Photographs

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ABSTRACT: Appropriate analytical techniques to make use of small format oblique photographs for photogrammetry are outlined, including several transformation methods and associated problems relating to the inclusion of reseau measurements. Various options are compared by applying the techniques to a series of hand-held, small format oblique aerial photographs.

The accuracies of derived coordinates are examined and show that the small format camera can be considered to be of photogrammetric importance. Some of the transformation methods associated with the use of reseau photography to correct for image deformation are outlined and compared in terms of their ability to model specific problems associated with semi-metric photography.

1. Introduction

Conventional aerial photography for topographic mapping is taken using survey cameras mounted in aircraft. Flight planning results in a regular series of stereopairs, forming overlapping strips of photographs covering the terrain to be mapped (Kennie and Matthews, 1985).[1] The cameras are expensive and calibrated for geometrical departures from the ideal perspective projection of the terrain. The photographic images are used in plotting machines which make use of optical and mechanical analogue devices to simulate the geometrical relationships between the terrain, a stereopair and the photographic images (Burnside, 1985; Wolf, 1983).[2,3] Contoured maps are produced at analogue plotting machines by an operator who is skilled at manipulation, stereo-viewing and interpretation of the photographic images.

Digital photogrammetry on the other hand makes use of linear or matrix arrays of sensors which transform electromagnetic energy from an object into electrical signals. Image correlation algorithms are under development which, when implemented in high-speed computers will allow the object which has been sensed to be reconstructed in terms of its spatial and qualitative characteristics (Muller, 1989).[4]

Between these two extremes there is a growing use of analytical methods instead of analogue methods for producing a digital representation of an object which has been photographed (Ghosh, 1979; Granshaw, 1980).[5,6] This analytical approach frees the photography from restrictions placed upon it by the use of analogue plotters. These restrictions are mainly on the type of camera which can be used and on the geometrical relationships between the cameras and the object. Analytical plotters are now in widespread use, but their introduction has been justified largely on the improvement in speed and accuracy which they give in relation to analogue plotters and also on the fact that they produce digital data primarily and graphical data secondarily.

Opportunities to use unconventional cameras exist when analytical methods are to be used, but one of the main problems is lack of knowledge about the internal geometry of a camera which was not designed to be used for photogrammetry. Nevertheless the use of such a camera for photogrammetry can be much cheaper than conventional aerial photography. Some recent experimental work to devise methods for modelling some aspects of the internal geometry of these non-metric cameras (i.e. cameras not designed for photogrammetry), is described.

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2. Basis of the Analytical Method

It is generally assumed by photogrammetists that an ideal photograph is a perspective projection (Albertz and Kreiling, 1975; Slama, 1980).[7,8] The mathematical description of the geometrical relationships between the three-dimensional object, the perspective centre of the photograph and the two dimensional image is a pair of collinearity equations which express the concept that a point *'A'* on the object, the perspective centre *'O'* and the image *'a'* of point *'A'* are collinear (Fig. 1), (Ghosh, 1979; Slama, 1980).[5,8]

The definition of a plane rectangular photo-coordinate system can be made by one of the following methods (in increasing order of difficulty).

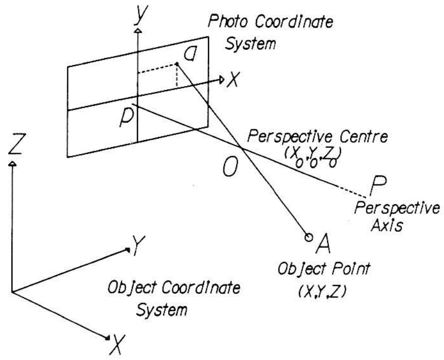

Fig. 1. A perspective projection.

(a) by calibrated fiducial marks, in the camera body, whose images are exposed on each photograph either by light entering the lens or by independent illumination of the marks; or
by a calibrated reseau plate fixed to the camera body and situated in front of and adjacent to the focal plane;

(b) by the corners of the format (if sharp); or

(c) by the format edges.

Other aspects of the internal geometry of the camera must be modelled mathematically if high accuracy is required of the photogrammetry. These other aspects are:

(a) the position of p of the intersection of the axis POp with the image plane, i.e. the photo-coordinates (x_p, y_p) of p, the **principal point**;

(b) the distance *'Op'* from the inner perspective centre of the lens to the focal plane, which is the **principal distance**, or loosely, the **calibrated focal length** of the lens;

(c) the radial and tangential geometrical lens distortion;

(d) the distortion of the photographic medium which takes place between the time of exposure and the time of measurement of the photo-coordinates of image points; and

(e) the lack of flatness of the photographic medium at the time of exposure and its lack of orthogonality to the perspective axis.

A **metric camera** is one which has been designed and calibrated (Faig, 1976)[9] so that the foregoing aspects of internal geometry are either known, and so can be used to correct measurements of photo-coordinates, or are insignificant and can be often ignored. The Zeiss (Jena) UMK is one example of a metric camera. It has independent illumination of calibrated fiducial marks, calibrated principal distances, calibrated principal point coordinates and known geometrical lens distortions, the latter amounting to about 10 to 20 μm in the corners of the format, depending on the principal distance setting. A film-flattening device or glass plates can be used to reduce the effects of aspects (d) and (e) to some extent.

A **semi-metric camera** is one which has not been designed for photogrammetry, but which has had some modifications and calibration to enable some of the aspects (a) to (e) above to be modelled mathematically (Wester-Ebbinghaus, 1986).[10] The Rollei-Metric 6006 is one example, it having been fitted with a reseau and click stops corresponding to calibrated focal lengths.

A **non-metric camera** is one which has been neither designed nor modified for photogrammetry and whose internal geometry is unknown and frequently unstable, (Slama, 1980).[8] Standard 35 mm cameras and an unmodified Rollei 6006 are examples of non-metric cameras.

In general, internal camera geometry is modelled mathematically by *inner orientation parameters*. These are:

(a) the calibrated principal distance, c;
(b) the photo-coordinates (x_p, y_p) of the principal point;
(c) coefficients K_1, K_2, and K_3 of an even-powered polynomial to represent radial geometrical lens distortion and/or coefficients P_1 and P_2 to represent tangential geometrical lens distortion;
(d) parameters which are deemed to be appropriate for modelling in-plane deformation of the photographic medium after exposure (derived from measurements of the images of the reseau crosses); and
(e) parameters which express unflatness of the registration plate behind the film and of local areas of the film arising from air trapped between the film, its backing sheet and/or the registration plate.

Such a mathematical model is often referred to as **inner orientation** to distinguish it from **exterior orientation** which is a mathematical description of the position of the perspective centre and the direction of the camera axis in the object-coordinate system.

The photographic images of fiducial marks (or of format corners or sides if the camera has no fiducial marks) of reseau cross images and of images of object points are measured in a mono or stereo-comparator to give **comparator coordinates** (x_c, y_c in Fig. 2). These coordinates are often measured with standard deviations of between 3 and 5 μm and are the basic measurements used for analytical photogrammetry.

If a fully calibrated metric camera with a reseau is used, then it is possible to compute refined photo-coordinates by explicit corrections and transformations of comparator coordinates using calibrated values for x_p, y_p, K_1, K_2, K_3, P_1, P_2 and reseau data. If a semi-metric camera is used, some explicit corrections might be possible (using x_p, y_p and reseau data for example). With a non-metric camera, no explicit corrections can generally be made.

The lack of knowledge of some or all of the *a priori* inner orientation parameters can be overcome by including these parameters in the analytical procedure as unknowns to be estimated.

Referring to section 1 and Fig. 1 the basic mathematical expressions are collinearity equations which express the condition that an object point A, with coordinates (X, Y, Z) in the object space, the perspective centre O, with coordinates (X_0, Y_0, Z_0) in the object space, and the image point a, with refined photo-coordinates (x, y), are collinear. For each object point such as A and its **homologous point** a, two such equations can be written, but each equation contains several unknowns. These unknowns are usually six elements of exterior orientation, namely (X_0, Y_0, Z_0) and the three direction cosines (or equivalent) of the perspective axis (POp).

There may also be unknown inner orientation parameters such as c, x_p, y_p, K_1, K_2, K_3, P_1, P_2 if a non-metric camera is used. In addition, the object coordinates (X, Y, Z) of A may be unknown, but there must be at least three non-collinear points in the object whose coordinates are known (from a survey for example) and which are called **control points**. If the object is photographed from at least two positions, and if the comparator coordinates of the images of several object points (including control points) are measured , then it is possible to carry out a least squares estimation of all unknowns (interior orientation parameters, exterior orientation parameters and object point coordinates) based on linearized collinearity equations. Such a least squares estimation is usually referred to as a **bundle adjustment** (Brown, 1976)[11] and when estimation of interior orientation parameters is part of the solution, instead of having been applied explicitly, the method is referred to as a **self calibrating bundle adjustment** (Kenefick *et al.*, 1972).[12]

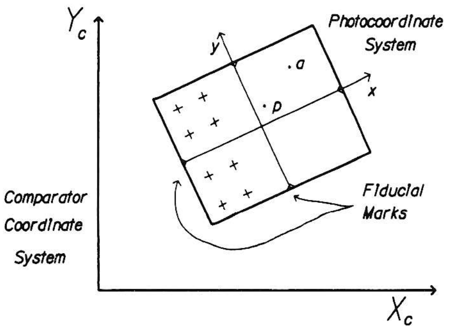

Fig. 2. Comparator and Photocoordinate Systems.

Typically, for small format photogrammetry, there might be four camera positions and 50 object points. Thus, if each object point is imaged on each photograph there will be 400 collinearity equations to be used to estimate 24 elements of exterior orientation, not more than 141 object coordinates and perhaps 8 interior orientation parameters giving at least 227 degrees of freedom.

There are two missing features in the foregoing approach, namely correction for in-plane deformation and lack of flatness of the photographic medium. The use of glass plates in a metric camera can reduce the effects of these aspects, but a reseau in a small format semi-metric camera can be used to correct for the former and measurement of the unflatness of a registration plate can give an indication of the effects of the latter. Some experimental work has been carried out to see to what extent a semi-metric camera can be used for photogrammetry. The results of the work follow.

3. Correction Methods for In-Plane Deformation

Three methods have been investigated and are summarized in Fig. 3. In each case comparator coordinates are transformed first to give **unrefined photo-coordinates** by a similarity transformation without scale change.

3.1 The first correction method is a mathematical treatment applied to all reseau image points at the same time. The x_r and y_r corrections to each measured coordinate can be computed, for example, from a second-order polynomial ($x_r = a_1 + a_2x + a_3y + a_4xy + a_5x^2 + a_6y^2$). The parameters of the polynomial can be estimated by minimizing the sum of squares of the residual deformations at all reseau points, for displacements in x and, subsequently, displacements in y.

The refined or corrected photo-coordinates of any measured image point can be

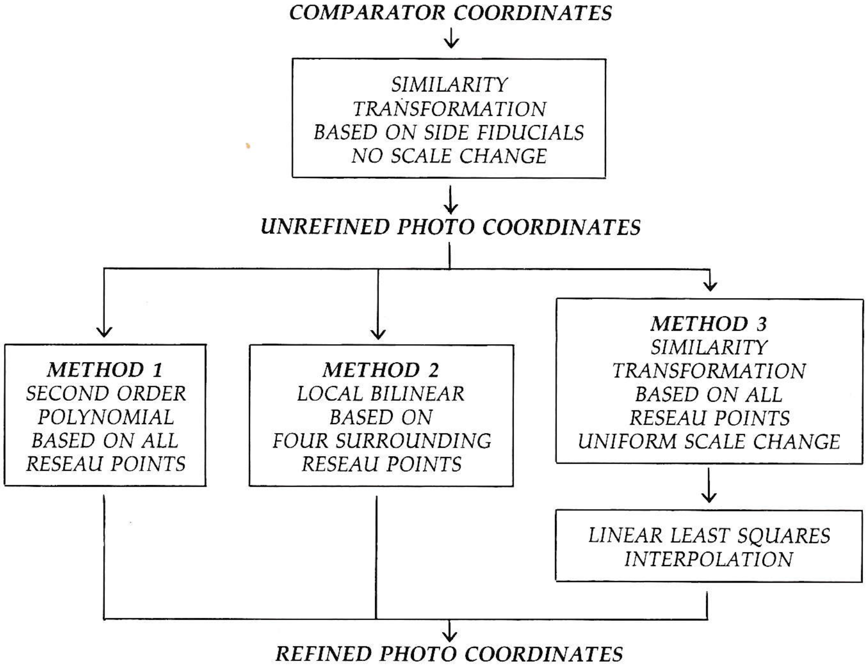

Fig. 3. Film Deformation Correction Methods.

computed by substituting its raw photo-coordinates into the polynomial.

3.2 The second correction method is to use the deformations at the four reseau crosses surrounding an image point to derive a local correction. Bilinear equations ($x_r = a_1 + a_2x + a_3y + a_4xy$) can be used to model the distortions at the four reseau crosses, again applied in both x and y, giving a unique fit, since for each imaged point there are four parameters and four observations. Again, refined photo-coordinates of the image point are computed by substitution into the bilinear equations.

3.3 The third correction method, linear least squares interpolation, involves the use of empirically derived covariance functions to model the relationship between the distortion components of pairs of points. This method of interpolation is based on the theory of stationary random functions, devised by Yaglom (1962)[13] in which the information at each reseau point is considered to be composed of two classes of random variables, namely, correlated stochastic variables and observational errors. Although the theory has been extended to multidimensional cases, an assumption that the deformations in x and y are mutually independent can be made to simplify computation (Kraus, 1971; Schut, 1974; Hardy, 1977).[14–16]

The method involves four main steps:

(a) initially, to meet theoretical considerations, any trends in the data must be removed, in this case by a similarity transformation;

(b) the covariance function is derived independently for both x and y displacements, from the known displacement error at each reseau point by computing the covariance between pairs of points at given separations;

(c) each covariance function is then modelled by a least squares fit of the data to a Gaussian curve;

(d) corrections to each image point can then be derived by linear least squares interpolation using the minimum variance estimate for the refined image point coordinates, i.e. that which minimizes the sum of the variances of the corrections. This estimation procedure utilizes the covariance between the image point to be corrected and each reseau point, as computed using the Gaussian curve since the separation between points is known.

These various approaches will produce three different sets of refined photo-coordinates, which can then be processed in a bundle adjustment program. The results are given in the next section.

4. Assessment of the Correction Methods for In-plane Deformation

4.1 Test Site

To assess the merits of the three methods of correction it is necessary to apply them to typical working conditions. A series of small format oblique aerial photographs were taken of the Black Ven landslide in Dorset in order to complete a sequence of historical photography (Chandler and Cooper, 1988).[17] The photography was obtained in June 1988 using a hand held Rollei 6006 semi-metric camera, with reseau, kindly loaned by Mr M. Kafetz of AV Distributors Ltd.

A light aircraft was hired and all flying, field work and return travel to London was carried out in one day. Nine targets were distributed around the object space and a control survey carried out using a Wild TC1 tacheometer. The coordinates of these targets were established on a local coordinate system using a three dimensional variation of coordinates estimation program. The standard deviations of these object control points were less than ± 0.01 metre. The photography and the control point field appeared to provide a useful case study for the examination of the methods discussed in section 3.

4.2 Measurement

Four photographs were selected for measurement, in which seven coordinated targets appeared on at least two photographs. The four photographs had been taken sequentially on a single pass of the site and, although slightly convergent, permitted stereoscopic measurement. Flying altitude was approximately 250 metres and the area of interest was at a range of between 350 and 750 metres from the camera positions.

T-MAX 400 ASA 120 roll film was used, with an exposure time of 1/500th of a second. This fine grained emulsion produced a high quality image even under magnification by a factor of 40. The four negatives were measured using a Zeiss (Jena) Stecometer stereo-comparator with a resolution of 1 μm. Points measured included all targeted points, some additional natural points and the 121 imaged reseau crosses. Each point was measured four times and this gave standard deviations of measured points of approximately ± 3 μm in x, and ± 5 μm in y. These values were compatible with the quoted precision of the Stecometer of ± 3 μm, and the larger value reflects the comparative difficulty of removing y parallax.

4.3 Data Processing

The three methods for correcting for in-plane deformation described in section 3 and illustrated in Fig. 3 were used to derive three sets of refined photo-coordinates (B, C and D in Table 1). These, together with unrefined photo-coordinates (A), gave four sets of data for bundle adjustments.

Four different bundle adjustments were carried out with each set of photo-coordinates. The first bundle adjustment (I) was made using the calibrated principal distance as the only non-zero interior orientation parameter. The second (II) was carried out with interior orientation parameters c, K_1, K_2 and K_3 estimated by the adjustment; it was called a **self calibrating bundle adjustment**. The third adjustment (III) was carried out by first applying explicit corrections to the photo-coordinates according to values from the independent camera calibration. The fourth (IV) used the same corrected photo-coordinates, but included interior orientation parameters c, K_1, K_2 and K_3 as unknowns to be estimated. In this case, these parameters were not included to represent radial lens distortion (which was corrected explicitly by the independent camera calibration) but to see whether or not they could be used to represent any image deformation uncorrected by the three methods used to refine the raw photo-coordinates.

The coordinates of one of the control points were disregarded in the adjustment so that the accuracy of the estimated coordinates of this check point could be assessed. To enable meaningful comparisons to be made, each bundle adjustment was carried out with identical sets of photo-coordinates. Stochastic properties of all measurements and of all control coordinates were the same for each procedure.

In order to summarize and examine results it was necessary to identify a single comparative measure. The variance factor is derived from the residuals of the photo-coordinates and control points and the standard deviations that were associated with them. Assuming the removal of all gross errors, if the estimated variance factor is significantly greater than unity then the stochastic model assigned *a priori* might be erroneous or certain systematic errors remain uncompensated, or both.

An identical stochastic model was used for all permutations so the estimated *a posteriori* variance factor crudely indicates the presence of uncompensated systematic errors. The estimated variance factor was used as a means of comparing the various permutations and the sixteen computed values are contained in Table 1.

It is clear from this table that some form of camera calibration is essential for use with a small format camera and that self-calibrating techniques appear suitable if there is no alternative. Interestingly, this type of correction appears generally more appropriate than applying corrections explicitly using the calibrated data. It is thought that the parameters used to model lens distortion also remove a significant proportion of the distortions associated with film deformation. This was shown to be so by the fact that the radial lens distortion deduced from II(A) did not agree particularly well with that from the independent camera calibration. The use of parameters K_1, K_2 and K_3 in II(A) modelled not only the radial lens distortion, but also a significant component of film deformation.

TABLE 1
A posteriori variance factors derived from all processing options

Photo-Coordinates		Explicit c (I)	Self Cal. c, K_1, K_2, K_3 (II)	Explicit c radial lens distortion (III)	Self Cal. K_1, K_2, K_3 Explicit c rad. lens dst. (IV)
Unrefined	(A)	18.04	1.72	1.78	1.74
Polynomial	(B)	17.88	1.89	2.03	1.90
Least Squares Interpolation	(C)	20.17	0.64	0.62	0.63
Bilinear	(D)	19.85	0.63	0.67	0.63

Of the three methods of compensating for film deformation, both the bilinear and the least squares interpolation methods appear to lower the *a posteriori* variance factor significantly, provided that camera inner orientation is modelled either explicitly or with self-calibration. The second order correction appears to be unfavourable but it is thought that this is mainly due to the problems associated with film unflatness, (section 5).

The coordinates of one of the targeted points were unused in each of the bundle adjustments. This was done so that the discrepancies between the known coordinates of this point and those derived by the various analytical methods could be compared. This check point was at the upper limit of the camera-object range, at a distance of approximately 670 metres. The maximum base:distance ratio was 1:3, and because the point was imaged on all photographs, its coordinates derived from photogrammetry posed a critical test.

Fig. 4 illustrates the standard deviations of the coordinates of the check point from each bundle adjustment. These values are broadly identical for each processing option as the values are dependent upon geometry and the stochastic model, which remain essentially unchanged. The precision of the determination of the point in the Y direction is ± 0.29 metre. This is greater than either the X or Z directions because the camera axes are orientated in the general direction of the Y axis of the ground coordinate system.

The geometry provided by the four photographs is only slightly convergent and so precision in the general direction of the camera axes is lower. In general, the use of convergent oblique photographs will lead to greater homogeneous precision (Granshaw, 1980).[6]

The discrepancies between the estimated coordinates of the check point and the coordinates of the point from the control survey are illustrated in Figs. 5–8 inclusive.

Smaller discrepancies result from processing options which use photo-coordinates defined by least squares interpolation (C) and bilinear interpolation (D) in conjunction with some form of calibration (bundle adjustments II, III and IV). These options correspond to those which result in the smaller estimated variance factor (Table 1). The particular processing option, which results in the smallest set of discrepancies, is for photo-coordinates refined by least squares interpolation (C) and used in a bundle adjustment with

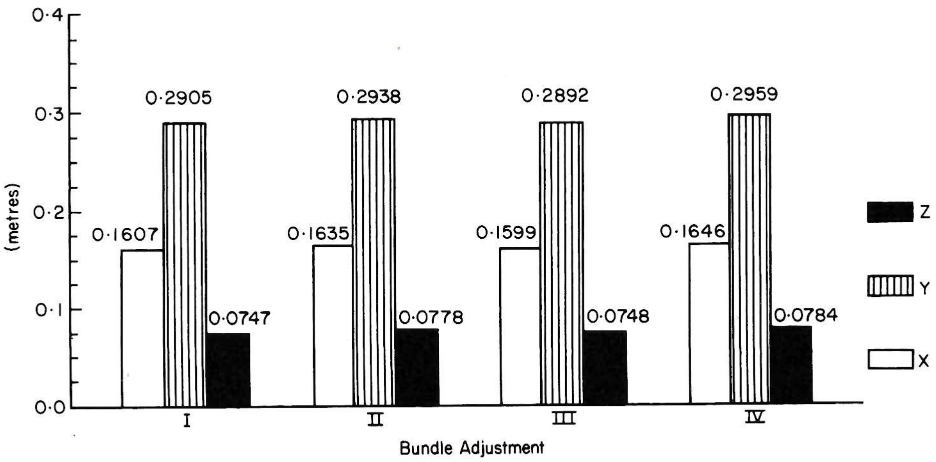

Fig. 4. Standard Deviations of Estimated Coordinates of Check Point.

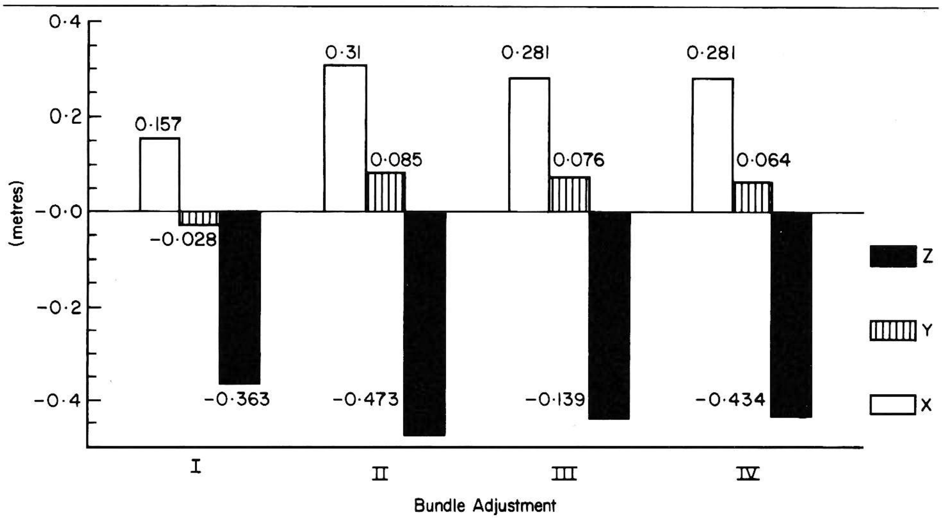

Fig. 5. Actual Coordinate—Computed Coordinate Raw Photo-coordinates A (unrefined).

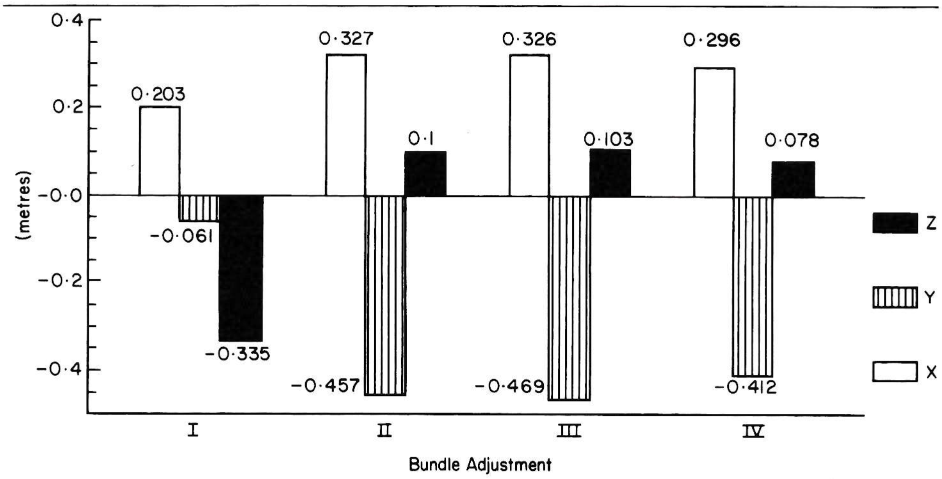

Fig. 6. Actual Coordinate—Computed Coordinate Photo-coordinates refined by Second Order Polynomial (B).

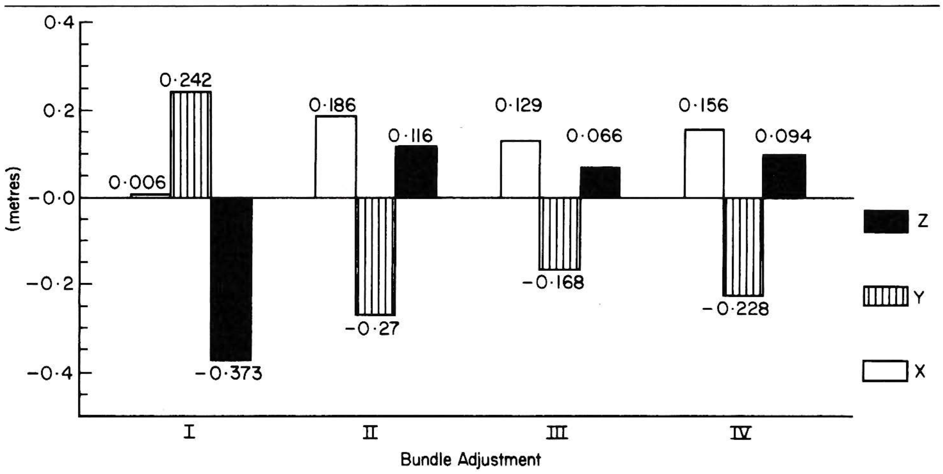

Fig. 7. Actual Coordinate—Computed Coordinate Photo-coordinates Refined by Least Squares Interpolation (C).

explicitly applied interior orientation parameters (III) in Fig. 7, although there is very little difference between these discrepancies and those from processing option III(D). In each of these cases, the largest discrepancy is in the Y coordinate which has the largest standard deviation (Fig. 4).

If an independent camera calibration has not been made, then self-calibration can give a reasonable solution if photo-coordinates are refined by either a bilinear or a least squares interpolation. This is indicated by a variance factor of about 0.64 for processing options II(C) and II(D) in Table 1 and and by the discrepancies shown in Figs. 7 and 8. The self-calibration however can be unsatisfactory if the geometrical relationships between the camera and the object point array are inappropriate. An extreme case of inappropriate geometry for self-calibration is when the object point array is planar and the camera axes are normal to that plane. For the configuration used in this test, the geometry was far from that limiting case.

5. Out of Plane Deformation

One aspect of the interior orientation that has not so far been considered is lack of flatness of the photographic material at the time of exposure. This can be due to lack of contact between the film and reseau plate, probably caused by trapped air and often identifiable in gross cases since not all reseau cross images will appear sharp. The degree of correction possible for such undulations is related to their period and the density of the reseau grid. Probable causes of lack of contact include air trapped between the film and the reseau plate and similarly between the film and pressure plate, the backing paper used in 120 roll film appears to compound the problem by providing an extra layer. Lack of flatness of the pressure plate, the pressure applied to the film and any non-uniformities in the thickness of the emulsion and film base will also contribute.

Another cause can be unflatness of the register glass. A series of interferometric measurements were taken of the reseau of a Rollei 6006, at the National Physical Laboratory. The contour plot (Fig. 9) was derived from these measurements and indicates the magnitude of unflatness of the rear surface of this particular reseau plate. However, unflatness of the rear surface is not the only consideration, since departures from parallelism between the surfaces of the plate were also found. These departures were of the order of 15 to 20 μm.

Results obtained by exposing a variety of film types in this camera suggest that out-of-plane displacements are significantly greater than in-plane displacements, probably causing the local radial distortions seen in Fig. 10, derived from one of the images used in this series of bundle adjustments. Magnitudes of these deformations are typical, maximum displacements from the calibrated positions being of the order of 40 to 50 μm.

6. Conclusions

A range of analytical procedures are available to enable small format oblique photography to be used for photogrammetric measurement.

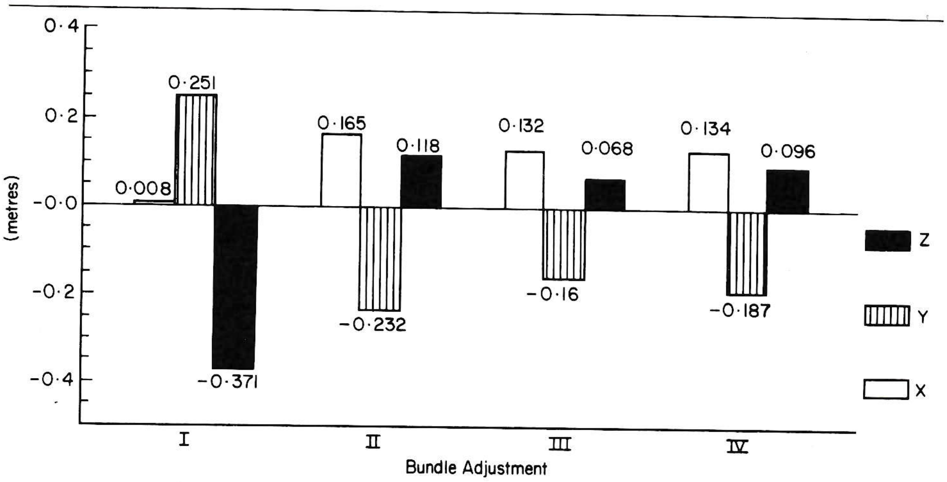

Fig. 8. Actual Coordinate—Computed Coordinate Photo-coordinates Refined by Bilinear Interpolation (D).

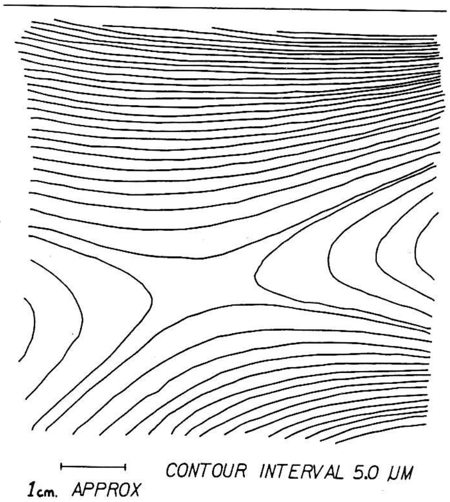

Fig. 9. Unflatness of Reseau Plate Rear Surface, Rolleimetric 6006 Camera No. 005298811.

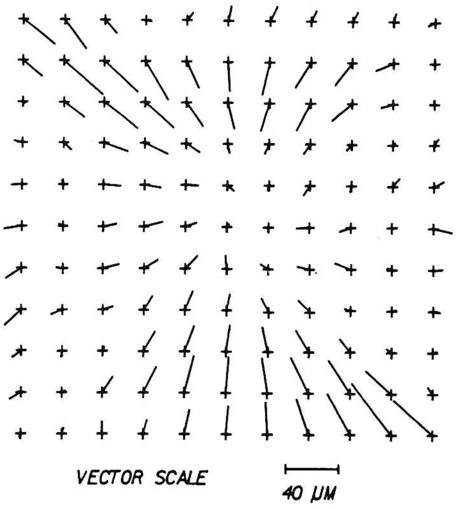

Fig. 10. Vector Plot of Deformations Negative 14 Black Ven June 1988 (Kodak TMAX 400 120 Roll Film, 130 μm thick cellulose triacetate film base, exposed in Rollei 6006 No. 005298811, processed using Ilford IDII 1+1 for 11 minutes at 21°C).

The selection of a suitable procedure should depend upon the type of camera that has been used. Small format cameras can generally be classified as non-metric or semi-metric. If the former has been used then a self-calibrating bundle adjustment provides one approach. This may be of slightly lower accuracy but may be adequate for many purposes, and also seems to model certain elements associated with film deformation.

If the camera is semi-metric, small improvements can possibly be made by refining the photo-coordinates explicitly for the interior orientation of the camera and using a standard bundle adjustment. More significant improvements can be made if the camera is provided with a reseau and suitable computational procedures used to incorporate these additional measurements. In this example the local bilinear correction is not only the most simple computationally, but also provides better results. However any errors in measurement of the reseau are directly transferred to the refined photo-coordinates.

Least squares interpolation involving a covariance function has only been given a basic treatment here, and there are many possibilities for improvement, both statistical in smoothing out measuring errors and deterministic in that a variety of functions may be used, based on the physical characteristics of the image deformations occurring.

The separation of the systematic effects caused by in-plane deformations and out-of-plane deformations will also be an important area for further work. Following the work discussed in section 5, it may be possible to calibrate the camera so that some of the out-of-plane deformation can be removed explicitly before processing the refined photo-coordinates in a bundle adjustment.

The results obtained from this particular analysis show that small format oblique aerial photography can be used and give acceptable results in many cases, but that a proper selection of procedures to meet precision and accuracy requirements economically is necessary. Further improvements in accuracy can be probably made by further research into the physical reasons for both in-plane and out-of-plane deformation of the photographic image.

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