Agricultural Water Management, 13 (1988) 25-32
Elsevier Science Publishers B.V., Amsterdam — Printed in The Netherlands

A Simple Dated Water-Production Function for Use in Irrigated Agriculture

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(Accepted 29 June 1987)

ABSTRACT

Rao, N.H., Sarma, P.B.S. and Chander, S., 1988. A simple dated water-production function for use in irrigated agriculture. *Agric. Water Manage.*, 13: 25-32.

Dated water-production functions are useful in evaluating alternative irrigation strategies. The dated water-production functions commonly used in irrigation-optimization models are briefly reviewed. Based on the heuristic assumption that the Boolean principle is applicable, a simple multiplicative dated water-production function is proposed. The function was compared for a number of crops, with the Stewart-Hagan-Pruitt and Jensen models. The proposed model is shown to be applicable over a wide range of stress conditions.

INTRODUCTION

When available water supplies are limited, increased irrigation efficiency can result from optimal sequencing of crop water-deficits (Stewart et al., 1975). This is possible if the effects of different feasible irrigation regimes on crop growth are quantitatively evaluated and compared. Water-production functions provide a convenient means to quantify crop responses to irrigation. This paper deals with the comparison of water-production functions for general applicability in irrigation management. It is assumed that all inputs other than water such as fertilizers, pesticides etc. are at levels which do not limit crop yields.

DATED WATER-PRODUCTION FUNCTIONS

The functional relationship between crop yield (Y) and water use (X) is called the water-production function. X may reasonably be represented (Stewart, 1980) by: the depth of irrigation water (IRR); the total field water supply (FWS = effective rainfall + IRR + soil water storage); or the consumptive use of

0378-3774/88/$03.50

water estimated as the actual evapotranspiration, AET (this includes the water loss by crop transpiration and soil evaporation). Water-production functions relating seasonal irrigation depth and crop yield have been derived for several crops by polynomial regression analyses (Hexem and Heady, 1978). The total field water supply (FWS) and yield (Y) can also be related in a similar fashion (Stewart, 1980). The relationship between seasonal actual evapotranspiration (AET) and crop growth, on the other hand, has been found to be linear (Stewart and Hagan, 1973; Hanks and Hill, 1980).

However, for sequencing crop water-deficits optimally, the timing of irrigations is important (Downey, 1972; Vaux and Pruitt, 1983). The crop water-use/yield relations which include the effects of both timings and quantities of irrigation are called dated water-production functions. These relationships are complex as they must include the effects of crop water-stress in different periods of the growing season. The length of these periods is arbitrary and may be chosen to coincide with some regular intervals (months, weeks, etc.) or with crop-related physiological growth stages. The magnitude of crop water-deficit is assessed in terms of the extent by which the actual evapotranspiration (AET) falls short of its potential value (PET) or the actual soil moisture content is short of a critical threshold value. The specific indices used to quantify stress are relative evapotranspiration (AET/PET), relative evapotranspiration deficit (1 − AET/PET) or soil moisture deficit (SMD).

The effects of stress, as defined by these indices, in different periods of the growing season interact in a complex manner. Simplifications are introduced by assuming that the stress effects in each period are independent. The combined effects of stress in several periods are evaluated by postulating that these effects are additive or multiplicative. Several production functions derived on this basis have been used in irrigation optimization models. Some of the well-known dated production functions are listed in Table 1. The crop yields predicted by different models in Table 1 were compared with the yields obtained in field experiments by several workers (Howell and Hiler, 1976; Cordova, 1979; Saxton and Bluhm, 1982). Based on these results, the following general observations may be made: both additive and multiplicative forms of dated water-production functions can predict crop yield within reasonable limits; the parameters of these functions (Table 1) need to be determined locally (Hill et al., 1983), that is, dated production functions are location-specific; and despite their approximate nature, they can be usefully employed in irrigation-optimization models (Jones, 1983; Rao, 1985).

GENERALIZED WATER-YIELD RELATIONSHIPS

The development of dated production functions of the kind listed in Table 1 requires extensive local experimental data. This is not always available in irrigation project areas in developing countries. Recognizing this, Doorenbos

TABLE 1

Some well known dated water production functions

Source	Dated water production functions (Y/Y_M)	Independent variable
Hall and Butcher (1968)	$\prod_{i=1}^{n} a_i(\text{SMD}_i)$	Soil moisture deficit index
Jensen (1968)	$\prod_{i=1}^{n} (\text{AET/PET})^{\lambda_i}$	Relative ET
Hiler and Clark (1971)	$1-\frac{A}{Y_M}\sum_{i=1}^{n} \text{CS}_i(1-\text{AET/PET})_i$	Relative ET deficit
Hanks (1974)	$\prod_{i=1}^{n} (T/T_p)^{\lambda_i}$	Relative transpiration
Minhas et al. (1974)	$\prod_{i=1}^{n} [1-(1-\text{AET/PET})_i^2]^{b_i}$	Relative ET deficit
Blank (1975)	$\sum_{i=1}^{n} A_i\ (\text{AET/PET})_i$	Relative ET
Stewart et al. (1976)	$1-\sum_{i=1}^{n} \text{YRR}_i\ (1-\text{AET/PET})_i$	Relative ET deficit
Sudar et al. (1981)	$Y_M \sum_{i=1}^{n} (1-\text{AET/PET})_i Y_{S_i}$	Relative ET deficit

T and T_p refer to actual and potential transpiration, respectively.

and Kassam (1979) analysed the available information on crop yield (Y) response to water and empirically derived yield-response factors, K_i. These factors are sensitivity indices for water stress in specified physiological growth stages (i) of crops, and are given by:

$$1-Y/Y_M=K_i\ (1-\text{AET/PET}) \qquad (1)$$

PET is governed by climatic conditions alone when soil moisture availability does not limit evapotranspiration. Y_M is the maximum crop yield that can be obtained when water is not limiting (i.e. when AET = PET). Equation (1) is valid for most crops for water deficits in the range ($1-\text{AET/PET}\leqslant 0.5$). Water deficits of this order are not usually exceeded for well-adapted varieties grown under irrigated conditions. About 80–85% of the observed yield variation at different locations was explained by this relationship. The response factors K_i were, therefore, recommended for planning and operation of irrigation systems. Since then (1) has been validated and used by several others to predict crop-yield responses at several locations in the U.S.A., China, Korea etc. (Hayes et al., 1982; Terjung et al., 1984a, b).

In spite of its wide applicability, equation (1) is not directly useful in irrigation scheduling with limited water supplies. This is because the yield-response factors K_i quantify the effect of water stress in specified growth stages.

For application in deriving optimal irrigation schedules, they need to be combined into a dated production function.

The crop-growing season is divided into N growth stages ($i=1, N$) which coincide with the vegetative, flowering, grain-formation and maturity stages, etc., of crop growth. Then, if a water deficit occurs in the first (vegetative) growth period alone, the resulting yield Y_1 will be given by:

$$f_1 = Y_1/Y_M = 1 - K_1 (1 - \mathrm{AET}/\mathrm{PET})_1 \tag{2}$$

in which the suffix 1 refers to the first growth stage. The response to water deficits occurring in any one of the other growth stages ($i=2, 3, 4$) will be given by relationships similar to (2).

For water deficits in more than one crop growth stage, the dated water-production function quantifying their combined effects is:

$$F = F(f_1, f_2, \ldots, f_N) \tag{3}$$

in which N is the number of growth stages for which the values of K_i are known. Three formulations of the function F may be considered:

(A) The additive model:

$$F = Y/Y_M = 1 - \sum_{i=1}^{N} K_i \ (1 - \mathrm{AET}/\mathrm{PET})_i \tag{4}$$

Equation (4) is identical to the dated production function of Stewart et al. (1976) in which K_i replaces the yield-response ratios YRR_i (Table 1).

(B) The general multiplicative power function form proposed by Jensen (1968):

TABLE 2

Values of λ_i, corresponding to different values of K_i

Yield response factor K_i	λ_i
0.20	0.15
0.25	0.19
0.30	0.24
0.40	0.32
0.45	0.37
0.50	0.42
0.55	0.47
0.60	0.52
0.75	0.68
0.80	0.74
1.00	1.00
1.50	1.95

$$F = Y/Y_{\mathrm{M}} = \prod_{i=1}^{N} (\mathrm{AET/PET})^{\lambda_i} \tag{5}$$

λ_i is a sensitivity factor which can be determined graphically using the equations (Tsakiris, 1982):

$$Y_i/Y_{\mathrm{M}} = (\mathrm{AET/PET})^{\lambda_i} \tag{6}$$

or

$$\ln(Y_i/Y_{\mathrm{M}}) = \lambda_i \ln(\mathrm{AET/PET}) \tag{7}$$

In equations (6) and (7) values of Y_i/Y_{M} are determined using (1) for each value of K_i. The values of λ_i corresponding to each value of K_i listed by Doorenbos and Kassam (1979) are obtained in this fashion (Table 2).

(C) Models (A) and (B), though they use yield-response factors K_i are based on well-known versions of the dated production functions listed in Table

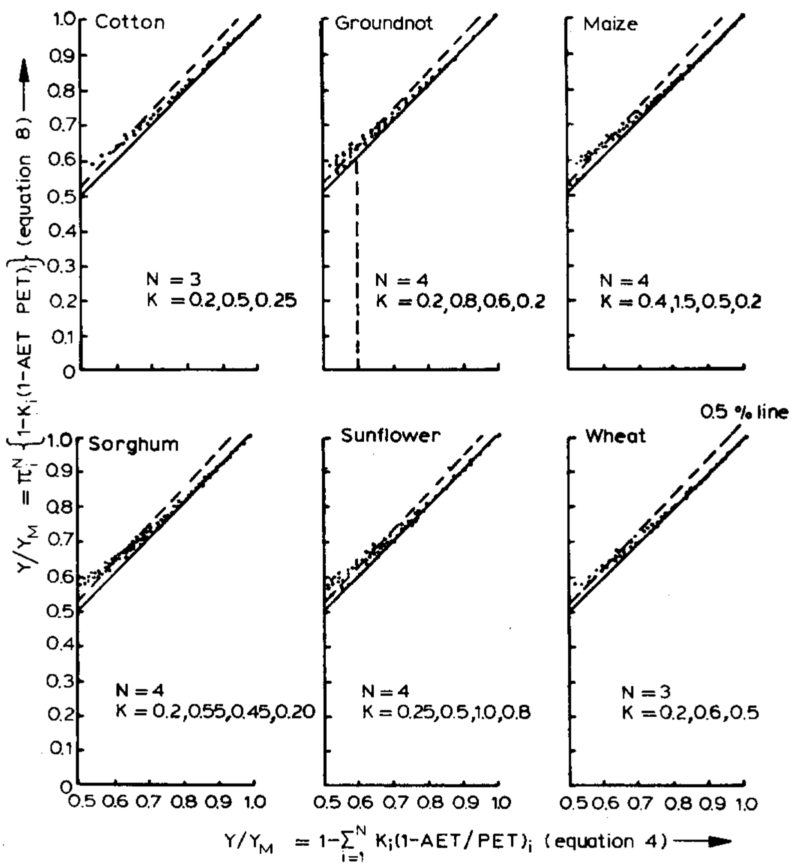

Fig. 1. Comparisons between relative yield values for different crops determined from equations (4) and (8).

1. A third formulation is proposed in this study as a simple multiplicative model:

$$F = Y/Y_M = \prod_{i=1}^{N} [1 - K_i\ (1 - \text{AET/PET})_i] \tag{8}$$

This is based on the heuristic assumption that the Boolean principle is applicable and the yield expected at the end of any growth stage is determined with respect to the maximum yield expected at the beginning of that stage.

The Hall and Butcher (1968) model (Table 1) has a similar heuristic basis (Hall and Dracup, 1969). The difference is in the stress index used in (8). In this equation relative evapotranspiration deficit quantifies the moisture stress, whereas Hall and Butcher used soil moisture deficit to quantify stress.

COMPARISONS OF THE PROPOSED DATED WATER-PRODUCTION FUNCTIONS

The three mathematical formulations of the dated production functions considered above, namely, equations (4), (5) and (8), are compared. Several

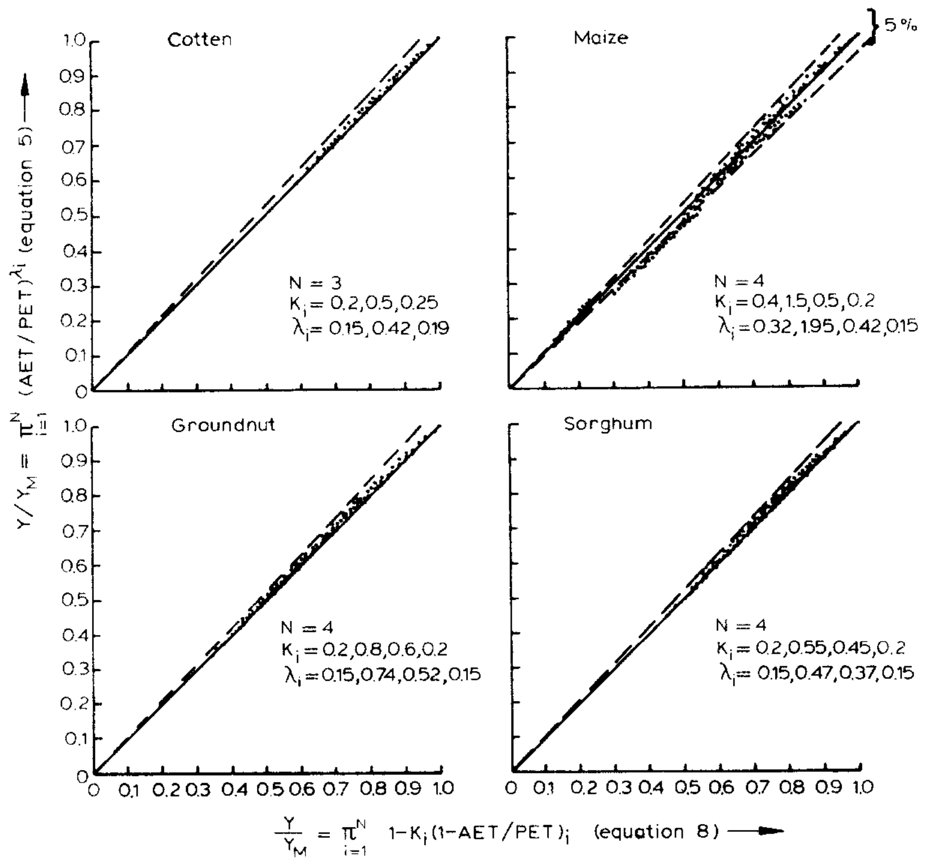

Fig. 2. Comparisons between relative yield values for different crops determined from equations (5) and (8).

crops for which the values of K_i are available (Doorenbos and Kassam, 1979, p.39) were considered. The feasible range of stress conditions studied is limited to ($1-\text{AET}/\text{PET} \leqslant 0.5$). This is also the range for which the K_i values were determined. A large number of feasible combinations of stress in different growth stages were considered. The results of the above comparisons for some crops are presented in Figs. 1 and 2.

Equations (4) and (8) do not differ appreciably in the relative yield range of 50–100% (Fig. 1). In the 60–100% range they differ by less than 5%. However, for crops with relatively large values of K_i (e.g. maize for which $K_2=1.5$) the additive model (A) leads to unrealistically low (sometimes negative) values of Y/Y_M, even when the relative evapotranspiration deficit in any stage does not exceed 50%. Equations (5) and (8) differ by less than 5% in the entire range of relative evapotranspiration deficit studied (Fig. 2).

Thus, while all three production functions tend to predict identical values of relative yield at lower levels of soil moisture stress, equation (4) gives difficulties when higher values of K_i are involved and relative ET is low. Equation (5) or (8) may be used in the entire feasible range of evapotranspiration deficit, but the former has the disadvantage that λ_i needs to be determined before it is used.

CONCLUSION

Three dated water-production function models derived from crop growth stage yield response factors given by Doorenbos and Kassam (1979) are compared. It is concluded that a simple heuristic multiplicative form is applicable over a wide range of stress conditions. This form can be used in locations where experimental data on crop water use and yield are sparse.

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