CT561: Systems Modelling & Simulation

Lecture 11: Policy Analysis with the SIR Model, and Course Summary

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https://github.com/JimDuggan/SDMR

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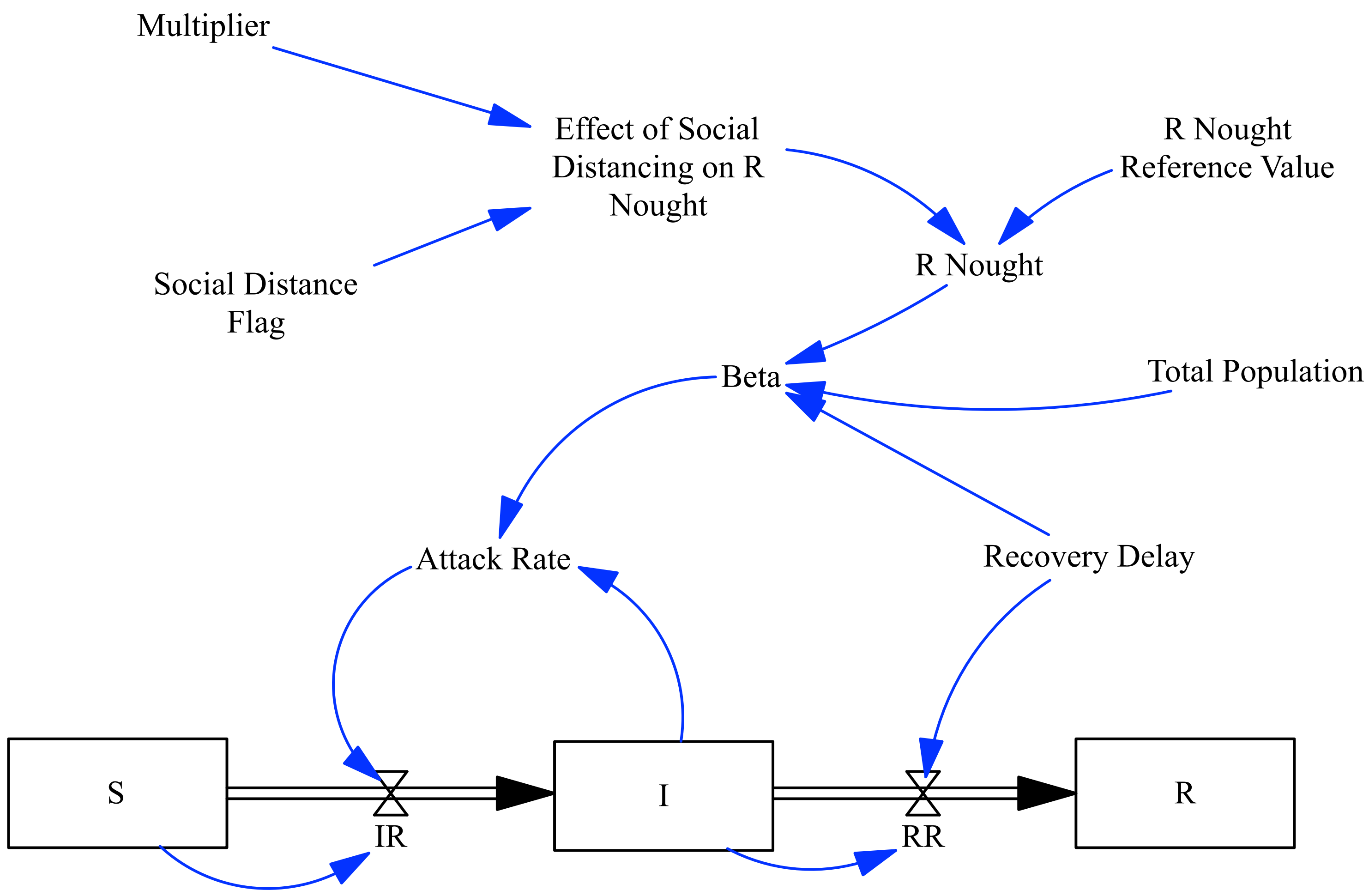
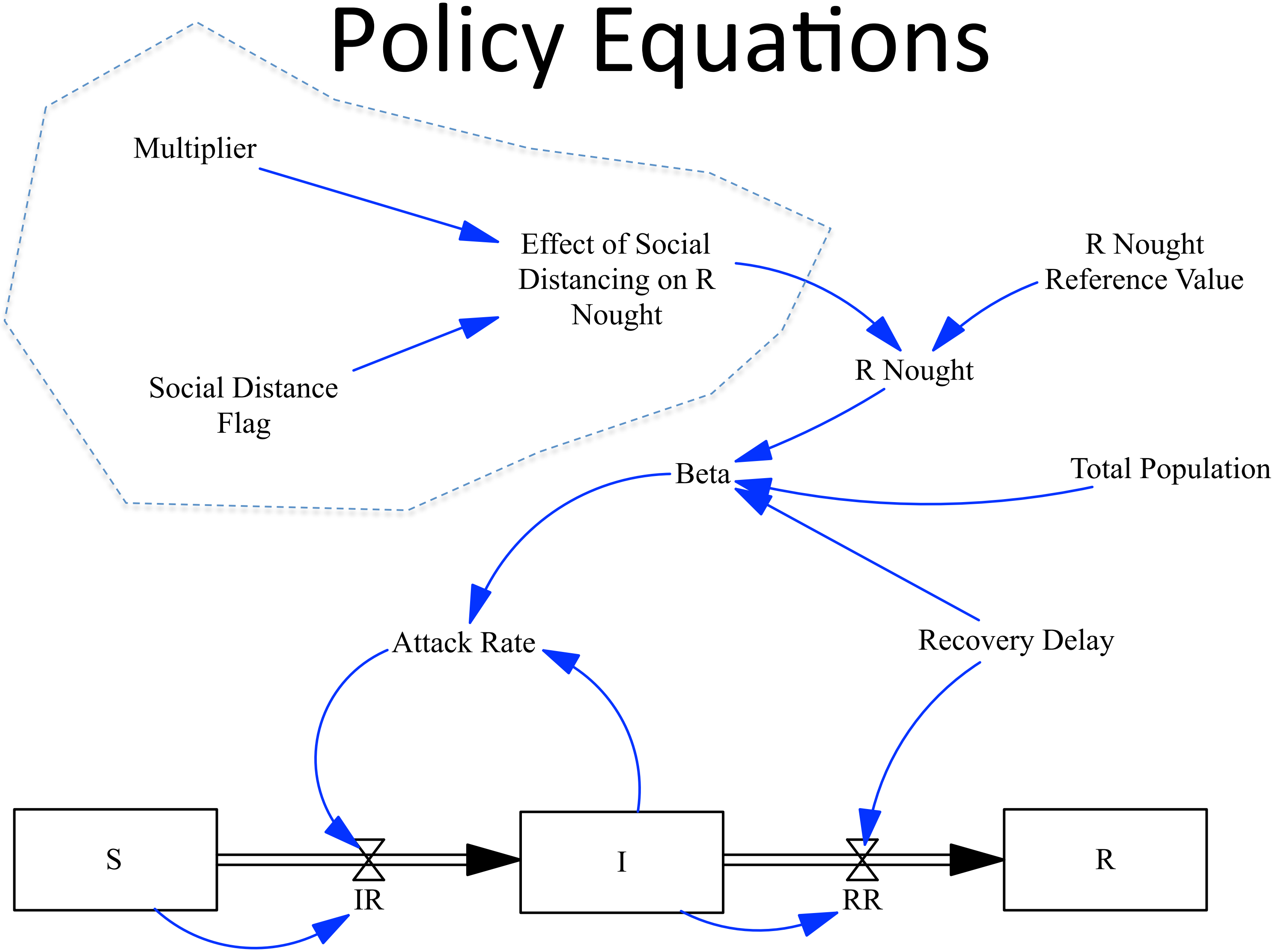
The SIR Model



Policy Equations



Model Equations (1)

S = INTEG(- IR , 9999)

I = INTEG(IR - RR , 1)

R = INTEG(RR , 0)

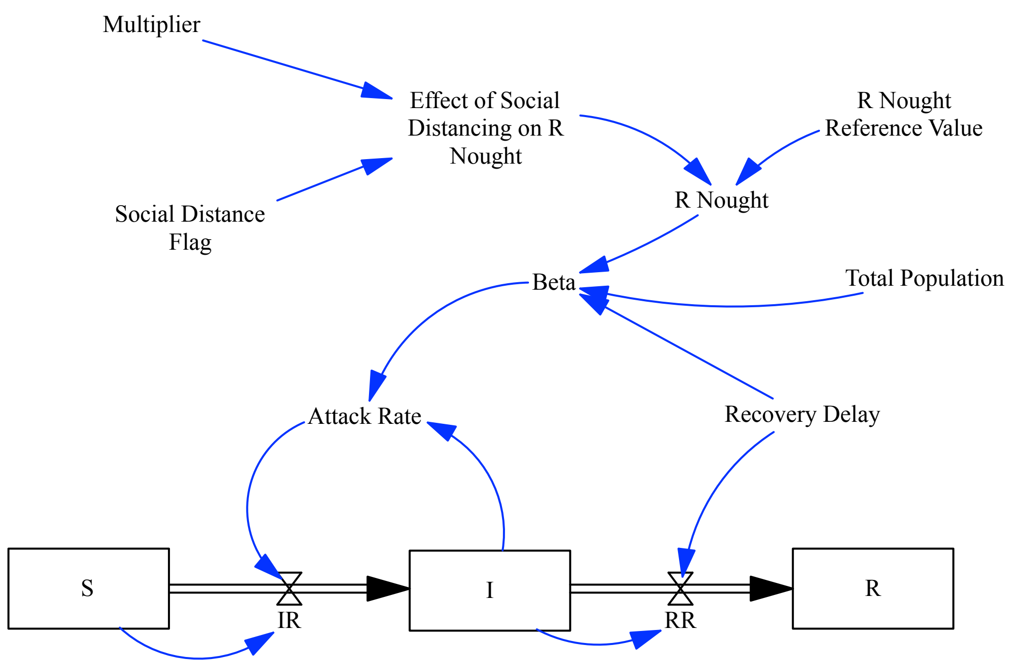
Attack Rate = Beta * I

IR = Attack Rate * S

Recovery Delay = 2

RR = I / Recovery Delay

Total Population = 10000



R Nought Reference Value = 1.6

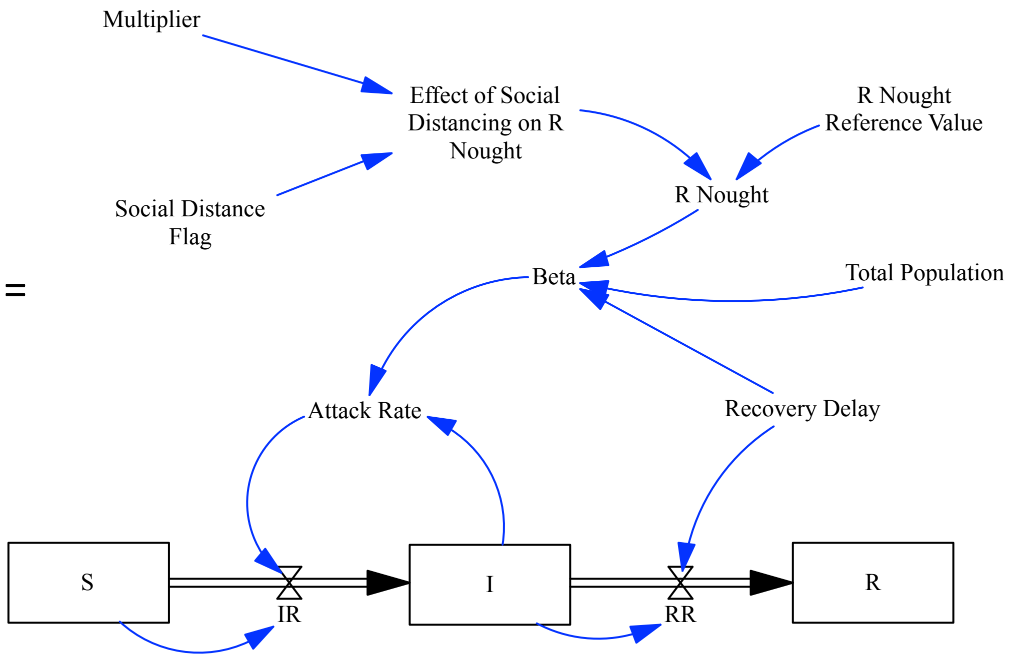
R Nought = R Nought Reference Value * Effect of Social Distancing on R Nought

Social Distance Flag = 1

Multiplier = 1 - step (0.05, 15)

Effect of Social Distancing on R Nought = if then else (Social Distance Flag = 1, Multiplier , 1)

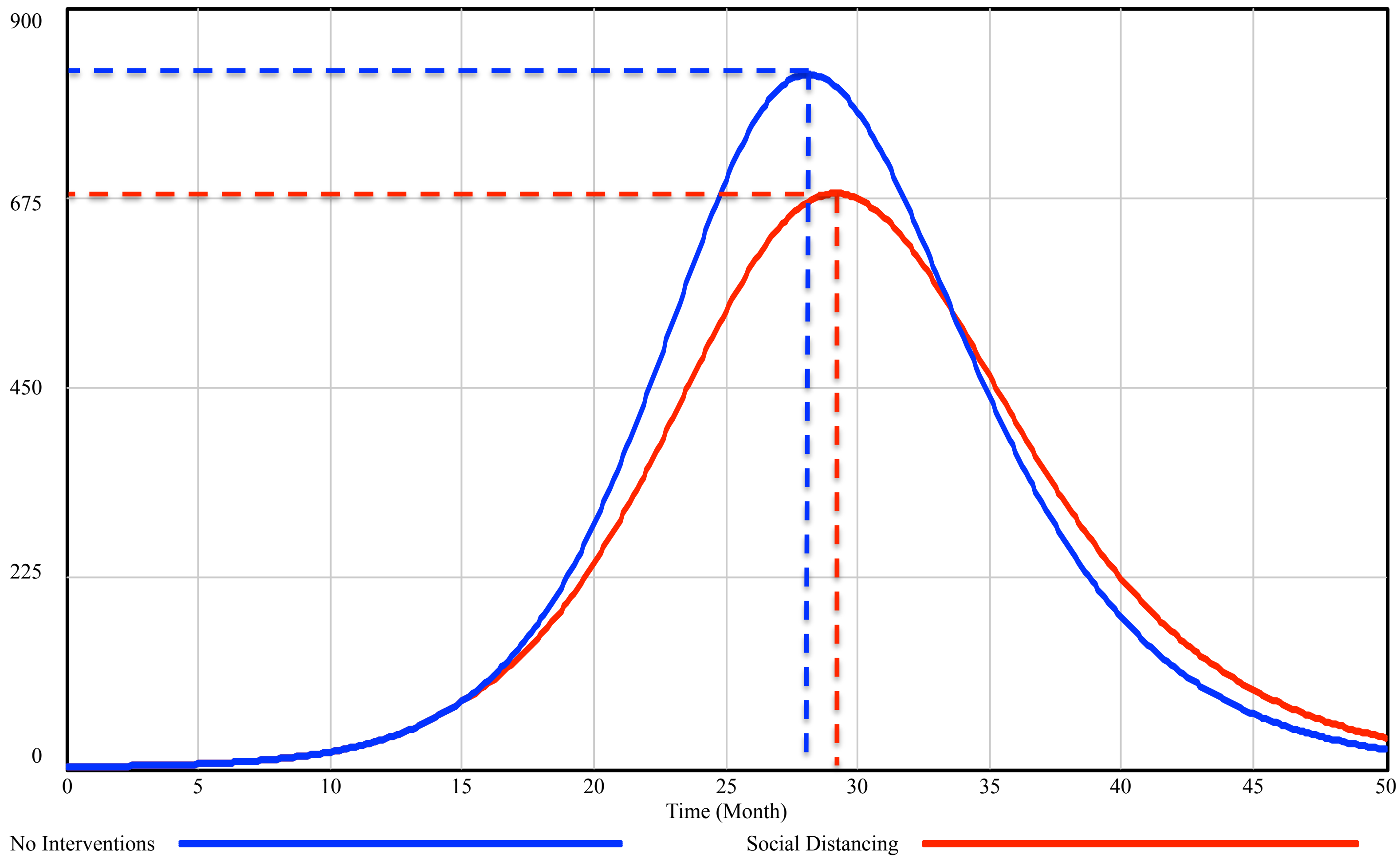
Beta = R Nought / (Total Population * Recovery Delay)



Scenarios

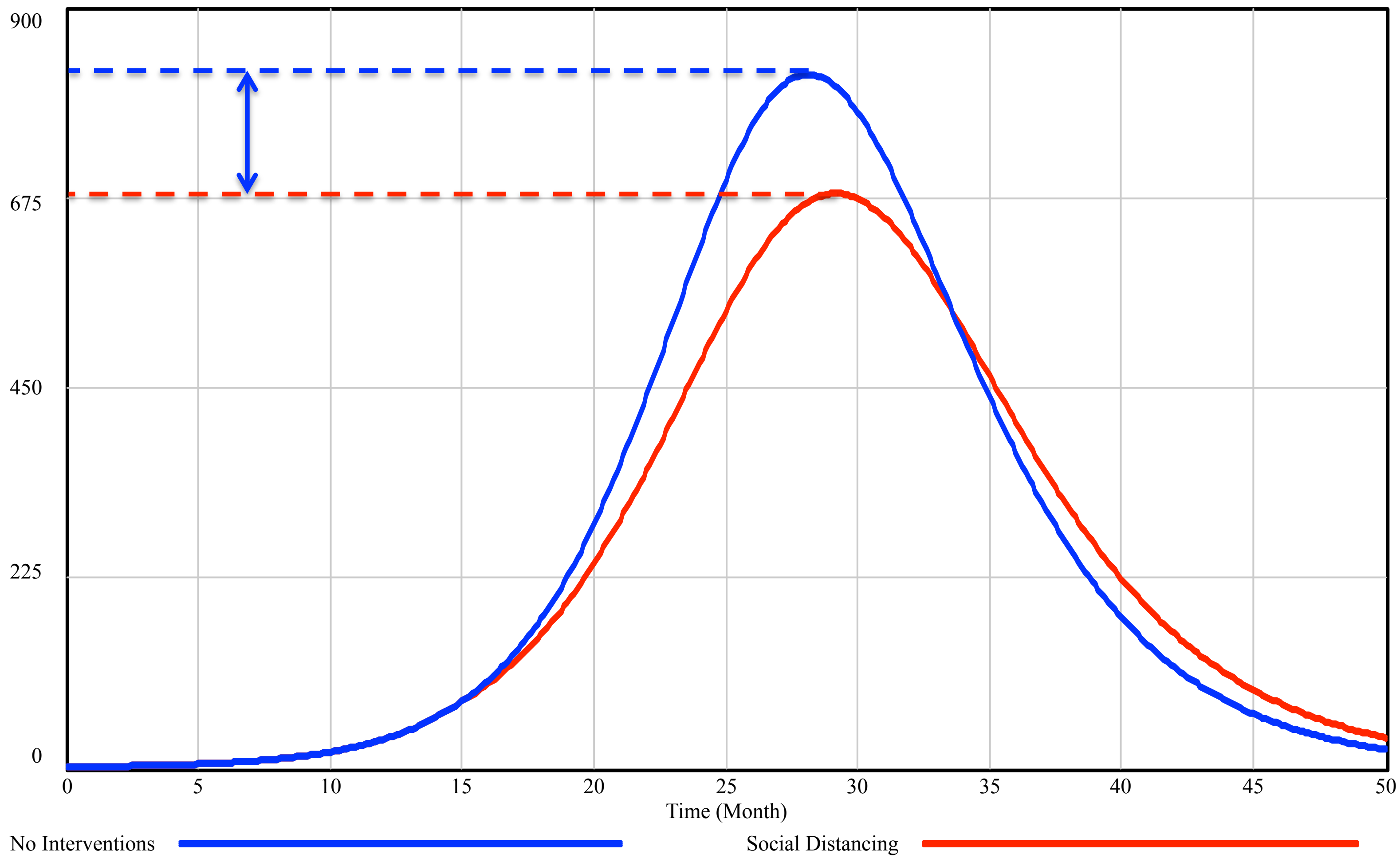
1. Run the model with no interventions
2. Run the model with social distancing added after time 15, where the measures reduce R_0 by 5%
   – School closures
   – Event cancellations
   – More people work from home

Results

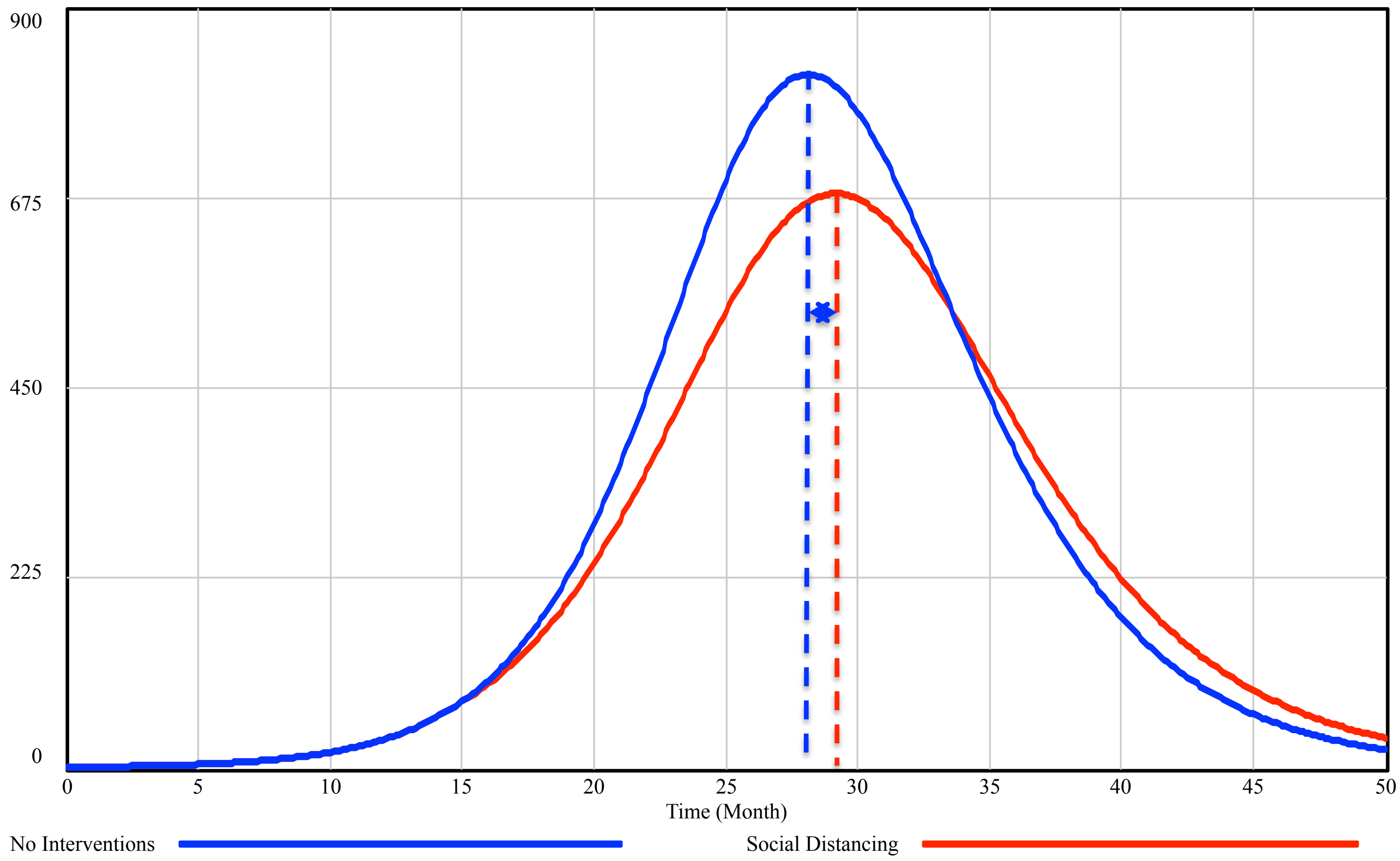


900
675
450
225
0
0
5
10
15
20
25
30
35
40
45
50
Time (Month)
No Interventions
Social Distancing

Max value reduced

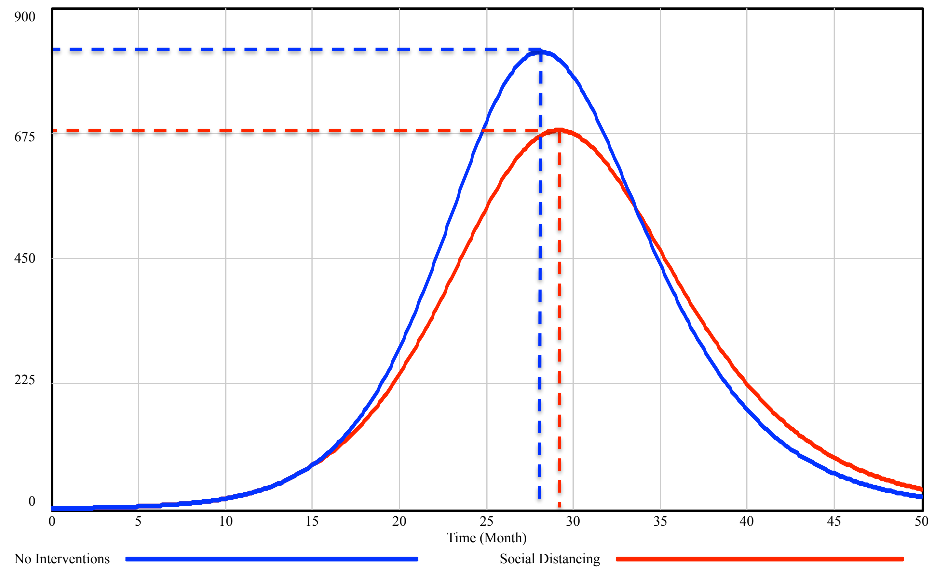


Time to peak extended



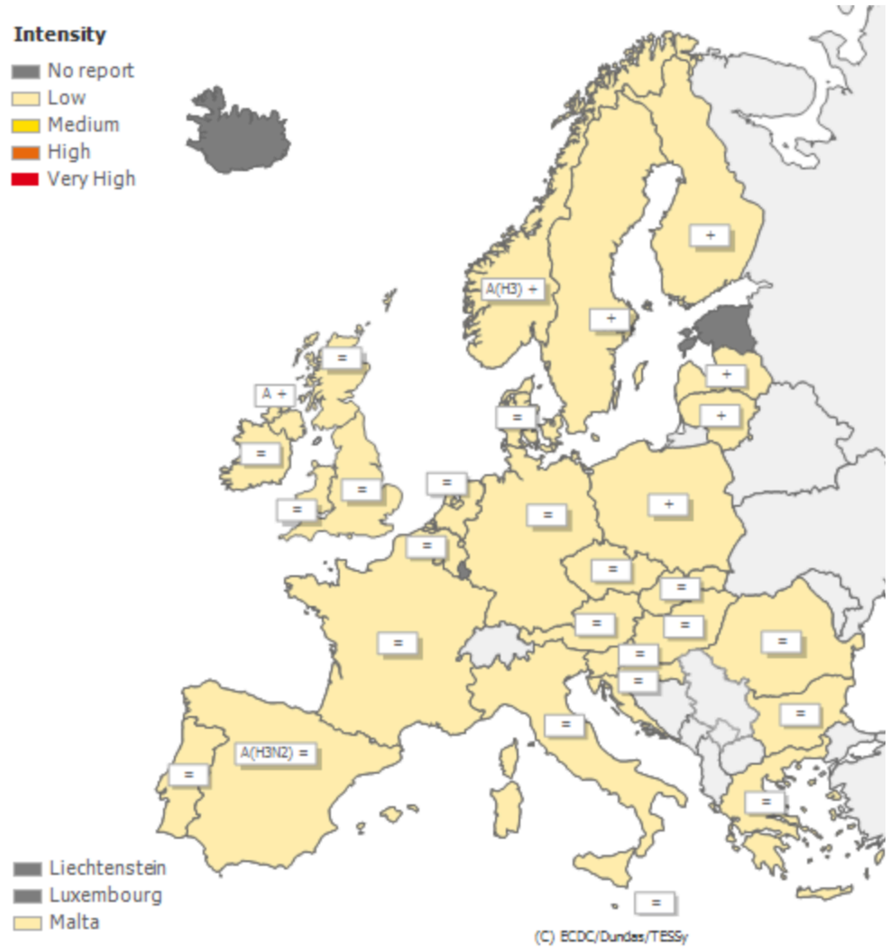
Intervention Benefits

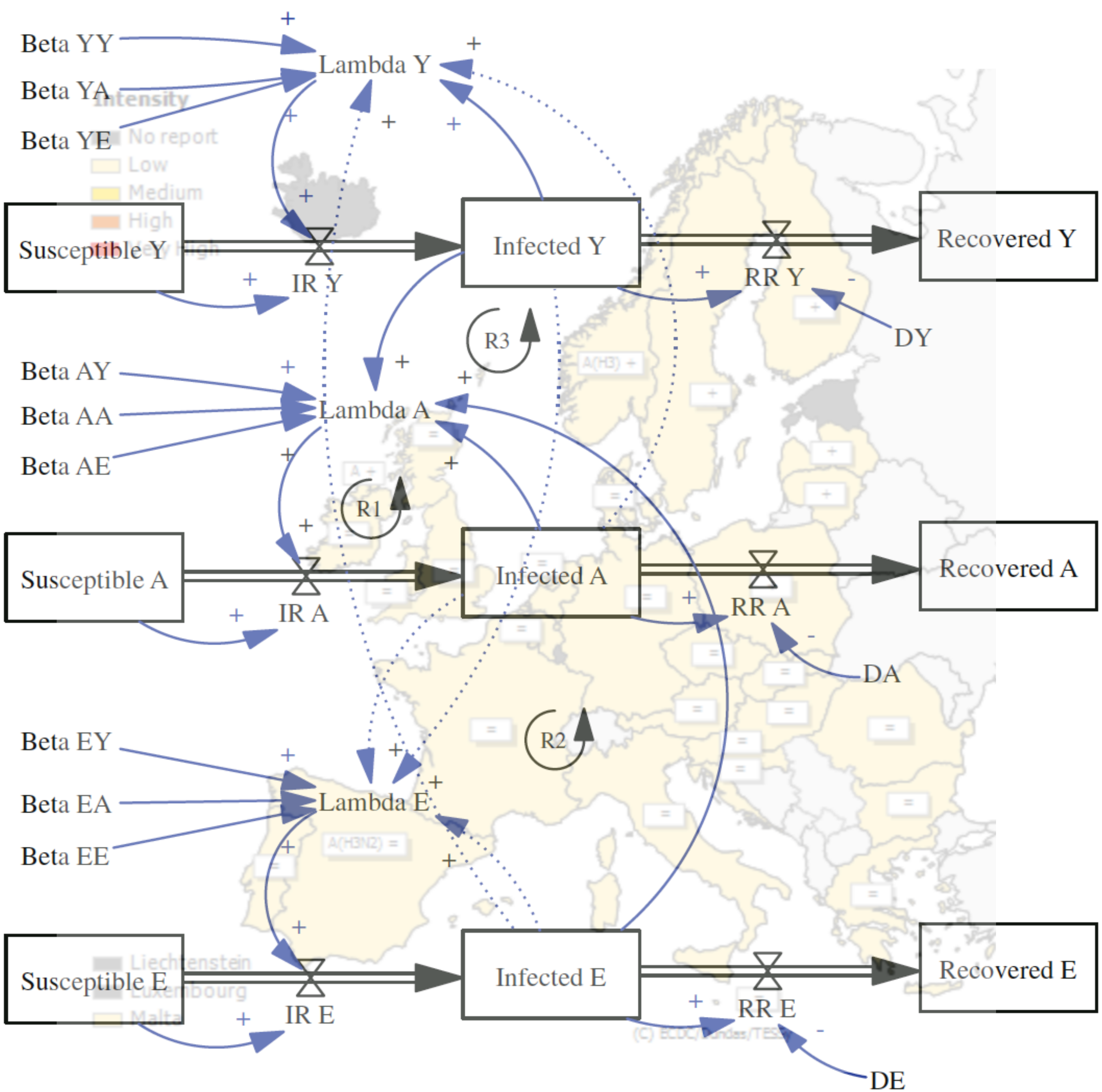
- Lowers the volume of peak demand on health services (height of curve reduced)
- Pushes out peak time to give more time for other countermeasures (Vaccine supply)



Further Possible Extensions to Model

- Disaggregate by region
- Disaggregate by cohort
- Add virulence of virus, with types of reactions
  - Mild
  - Moderate
  - Severe
- Add medical resources
  - Anti-virals
  - Hospital beds
  - ICU Resources




Beta YY
Beta YA
Beta YE
Lambda Y
Susceptible Y
IR Y
Infected Y
RR Y
Recovered Y
DY
R3
Beta AY
Beta AA
Beta AE
Lambda A
R1
Susceptible A
IR A
Infected A
RR A
Recovered A
DA
R2
Beta EY
Beta EA
Beta EE
Lambda E
Susceptible E
IR E
Infected E
RR E
Recovered E
DE
Intensity
No report
Low
Medium
High

Force of Infection…

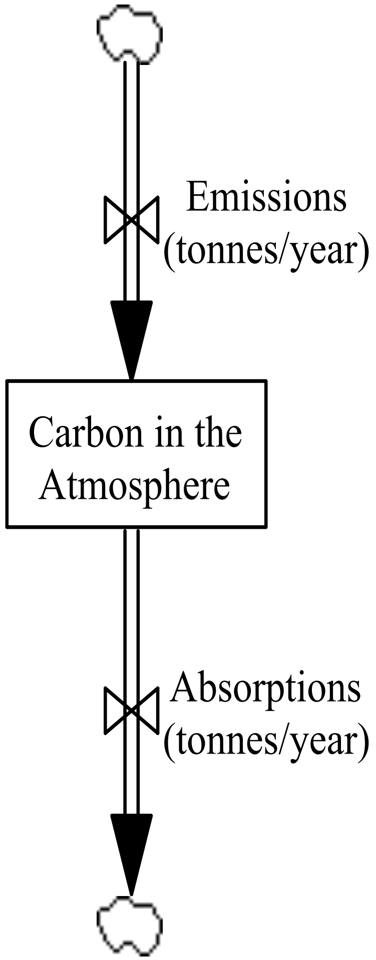
$$\begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_N \end{bmatrix} = \begin{bmatrix} ce_{11}/N_1 & \cdots & ce_{1n}/N_1 \\ \vdots & \ddots & \vdots \\ ce_{n1}/N_N & \cdots & ce_{nn}/N_N \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix}$$

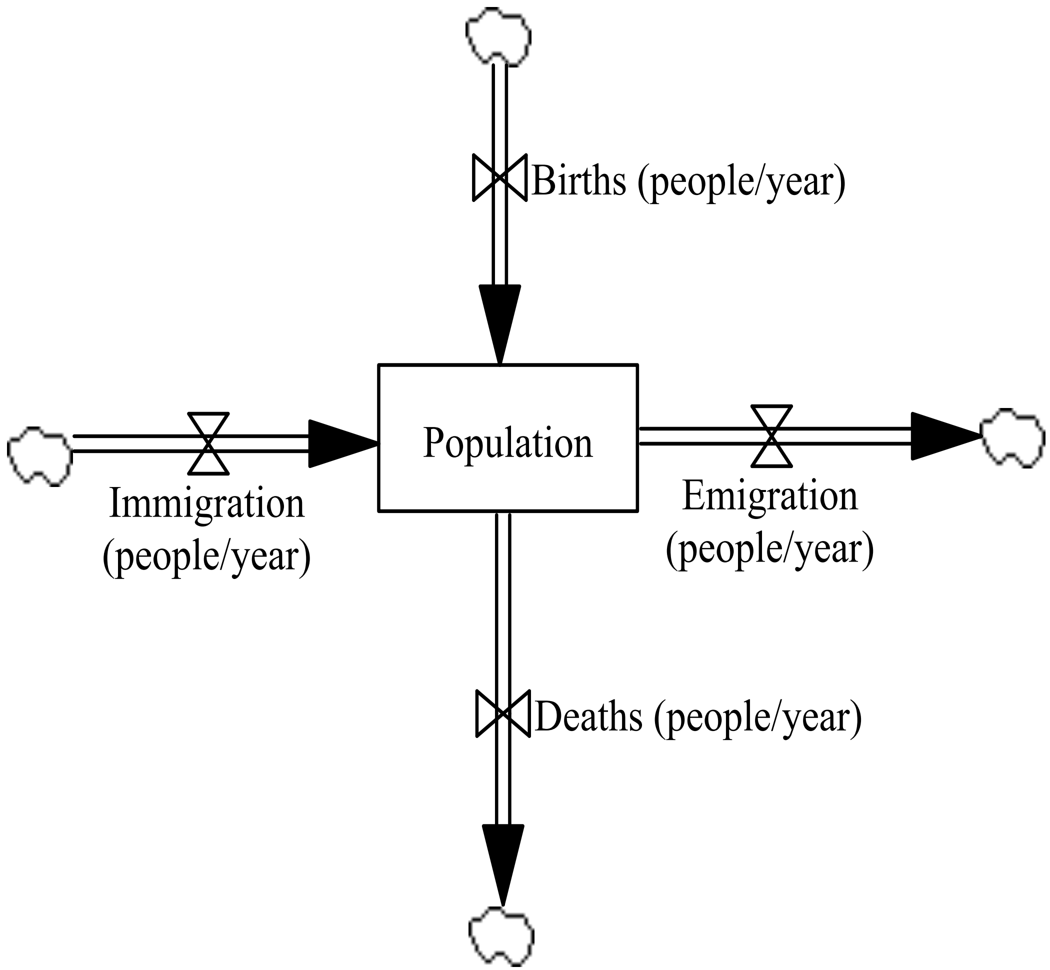
$$\begin{pmatrix} \lambda_Y \\ \lambda_A \\ \lambda_E \end{pmatrix} = \begin{pmatrix} CE_{YY}/N_Y & CE_{YA}/N_Y & CE_{YE}/N_Y \\ CE_{AY}/N_A & CE_{AA}/N_A & CE_{AE}/N_A \\ CE_{EY}/N_E & CE_{EA}/N_E & CE_{EE}/N_E \end{pmatrix} \begin{pmatrix} I_Y \\ I_A \\ I_E \end{pmatrix}$$

Course Summary
Exam: 4 Questions, Answer 3

- Stocks and flows
- Feedback
- Graphical & Numerical Integration
- Effect Variables
- Delays and Information Delays
- Stock Management Structure
- Infectious Disease Models (SIR)
- R Implementation of Models, deSolve, Sensitivity Analysis.

(1) Stocks and Flows





Stocks

- A **stock** is the foundation of any system.
- **Stocks** are the elements of the system that you can see, feel, count, or measure at any given time.
- A **system stock** is, an accumulation of material or information that has built up over time
- Dimensions are units (litres, people, lines of code)

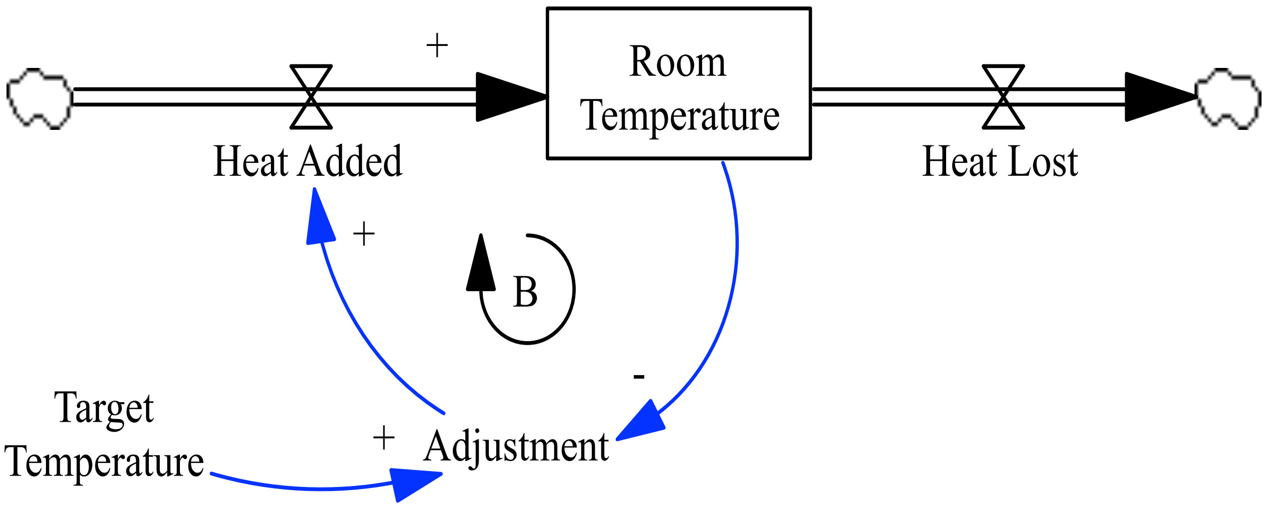
Meadows, Donella H. *Thinking in systems: A primer*. Chelsea Green Publishing, 2008.

Flows

- Stocks change over time through the actions of a **flow**.
- Flows are:
  - filling and draining,
  - births and deaths,
  - purchases and sales,
  - deposits and withdrawals
  - enrolments and graduations
- Dimensions are units/ time period (litres/day, people/year)

(2) Feedback

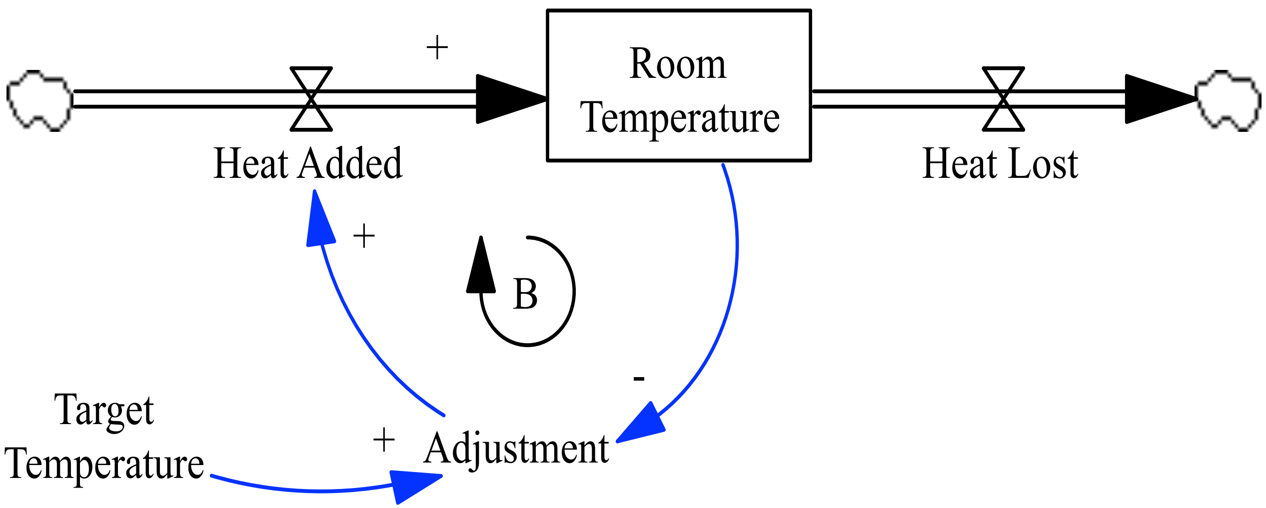
A closed chain of causal connections from a stock, through a set of decisions or rules or physical laws or actions that are dependent on the level of the stock, and back again through a flow to change the stock.



Balancing Loop

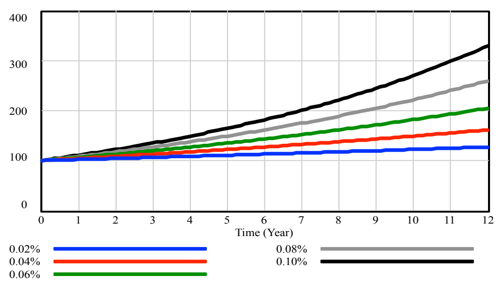
Balancing feedback loops are goal-seeking structures in systems and are:

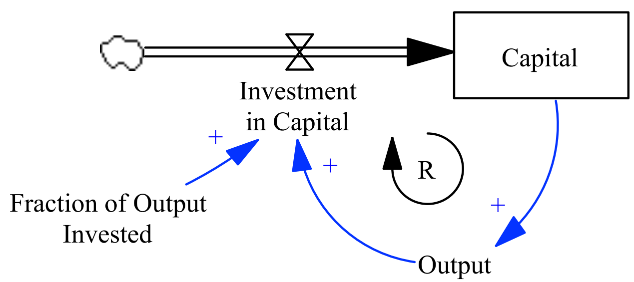
- sources of stability and
- sources of resistance to change.

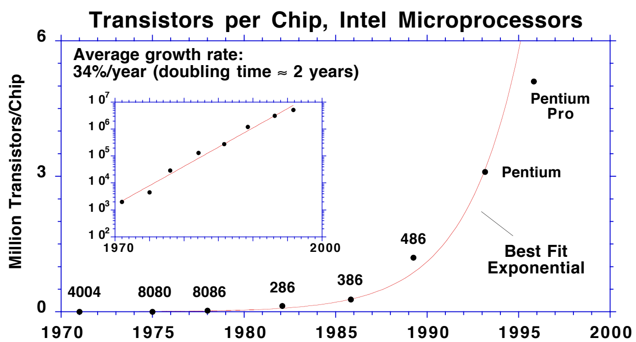


Reinforcing Loops

- **“Reinforcing feedback loops are self-enhancing, leading to exponential growth or to runaway collapses over time.**
- They are found whenever a stock has the capacity to reinforce or reproduce itself.”



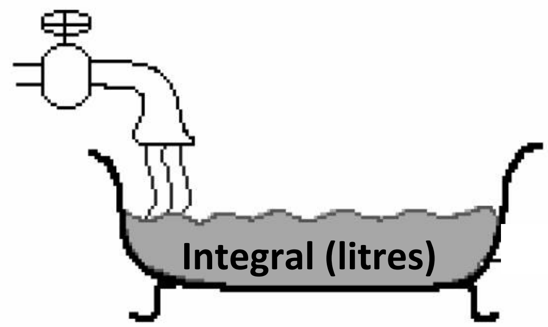


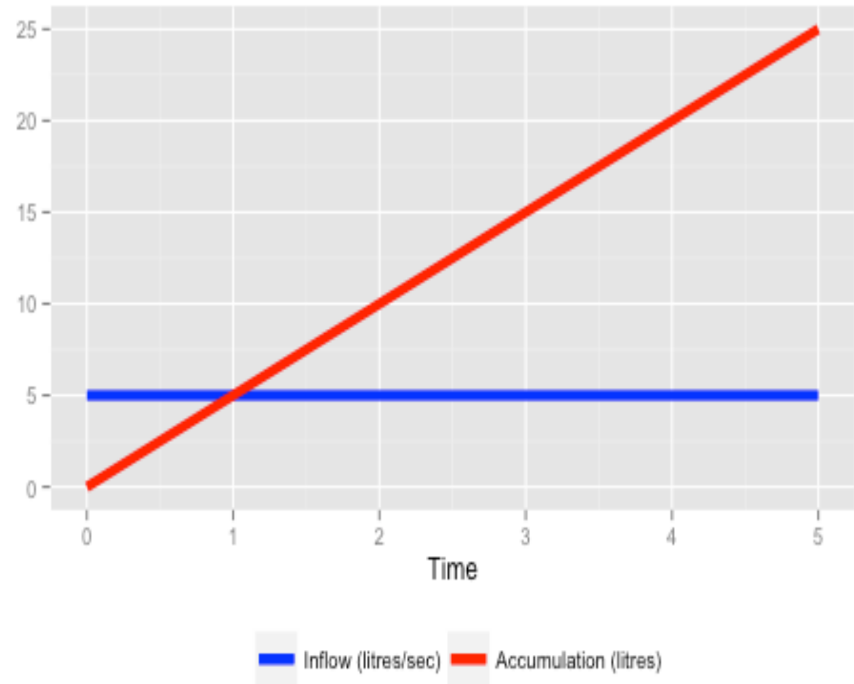


(3) Integration

- Calculus is the study of how things **change over time**, and is described by Strogratz (2009) as *"perhaps the greatest idea that humanity has ever had."*
- Integration is the mathematical process of calculating the area under the net flow curve, between initial and final times.

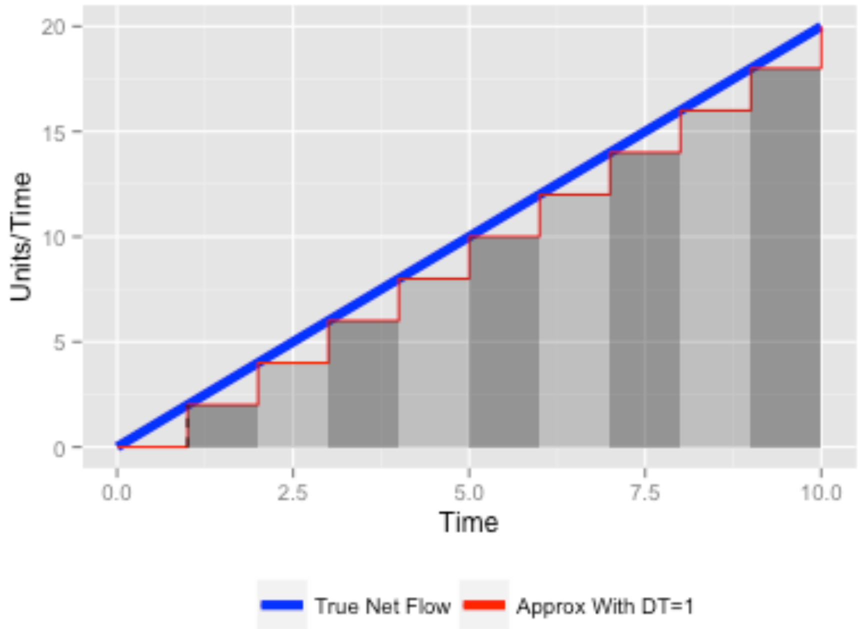
Derivative (litres/minute)





Numerical Integration

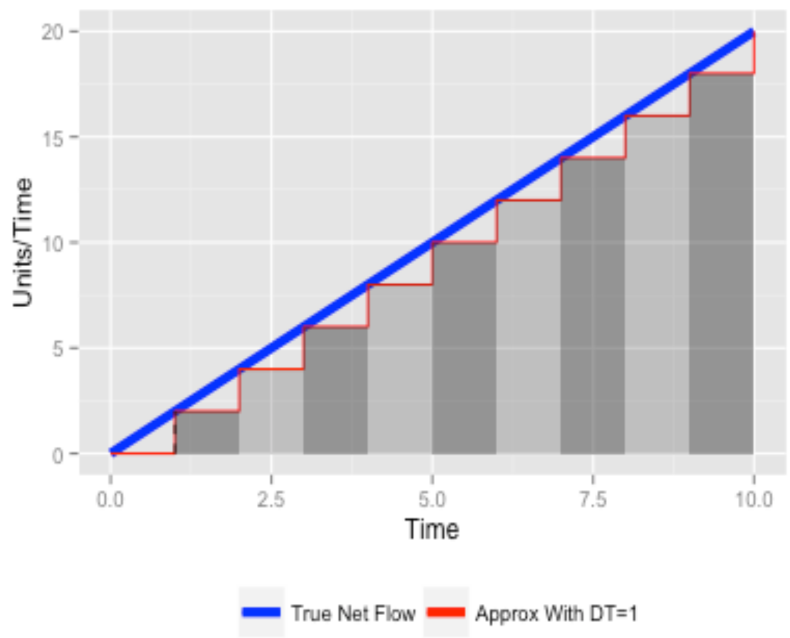
- Euler Method
- Approximate area under the net flow curve as a summation of rectangles, of width DT
- The smaller DT, the more accurate the result



$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$

Solution, DT=1

Time	$Stock_t$	Net Flow
0	0	0
1	0+0=0	2
2	0+2=2	4
3	2+4=6	6
4	6+6=12	8
5	12+8=20	10
6	20+10=30	12
7	30+12=42	14
8	42+14=56	16
9	56+16=72	18
10	72+18=90	20

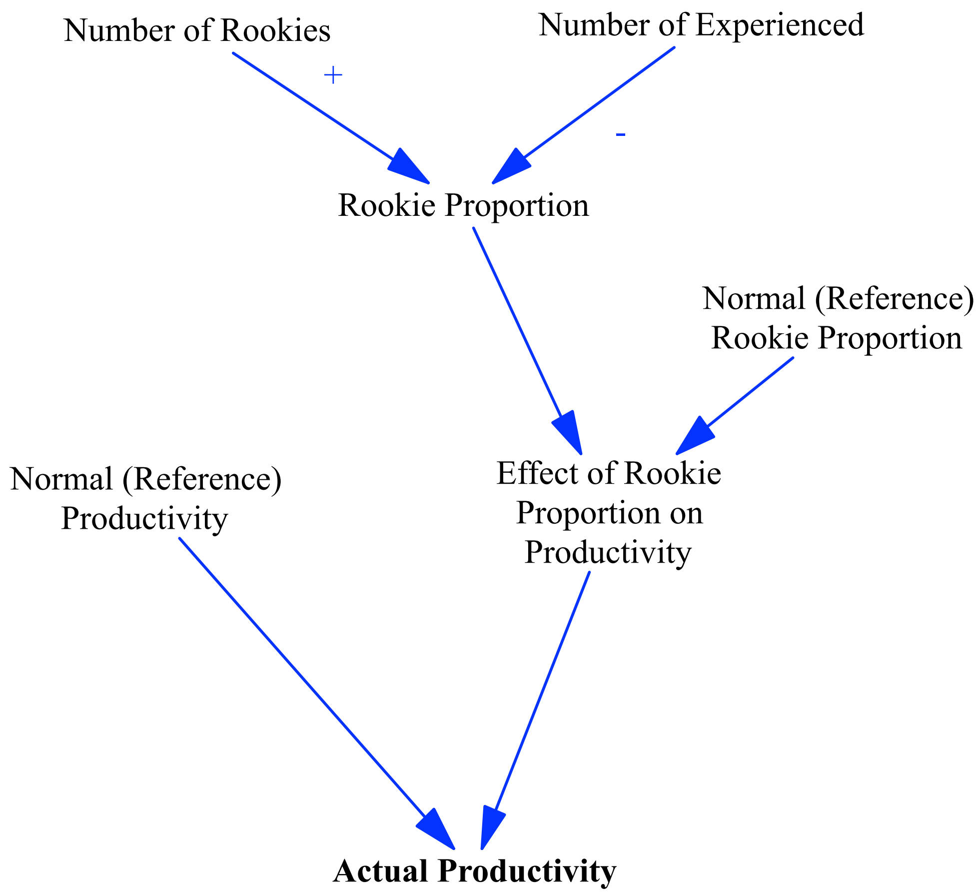


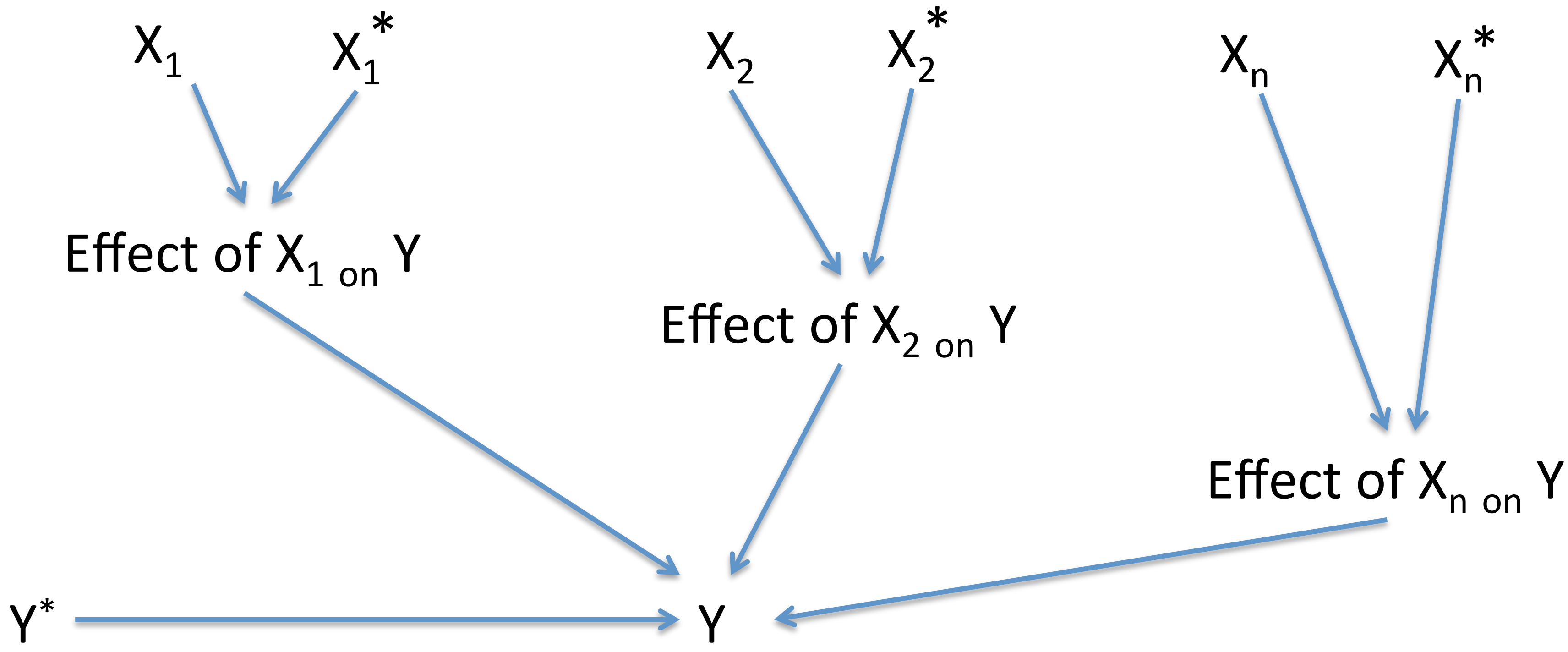
$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$

Note: Stock only depends on previous stock and net flows

(4) Formulating Effects

- An important building block for models is to capture how variables influence one another over time.
- System dynamics offers a convenient structure for modeling effect variables (Sterman 2000).





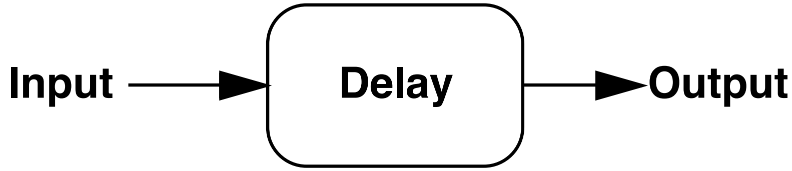
$$Y = Y^{*} \times Effect\,(X_1 on\, Y) \times \ldots \times Effect(X_n on\, Y)$$

$$Effect\,(\,X_i\; on\; Y) = f\left({}^{X_i}\!/_{X_i^{*}}\right)$$

(5) Delays

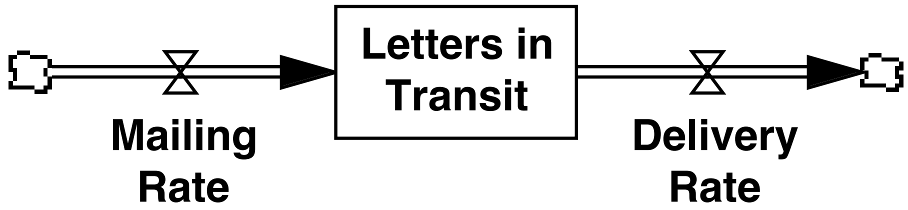
- "Delays are pervasive.
  - It takes time to measure and report information.
  - It takes time to make decisions.
  - It takes time for decisions to affect the state of the system" (Sterman 2000)
- We need to use delays in many of our models

The output of a delay lags behind the input:



General structure of a material delay:

The post office as a delay:



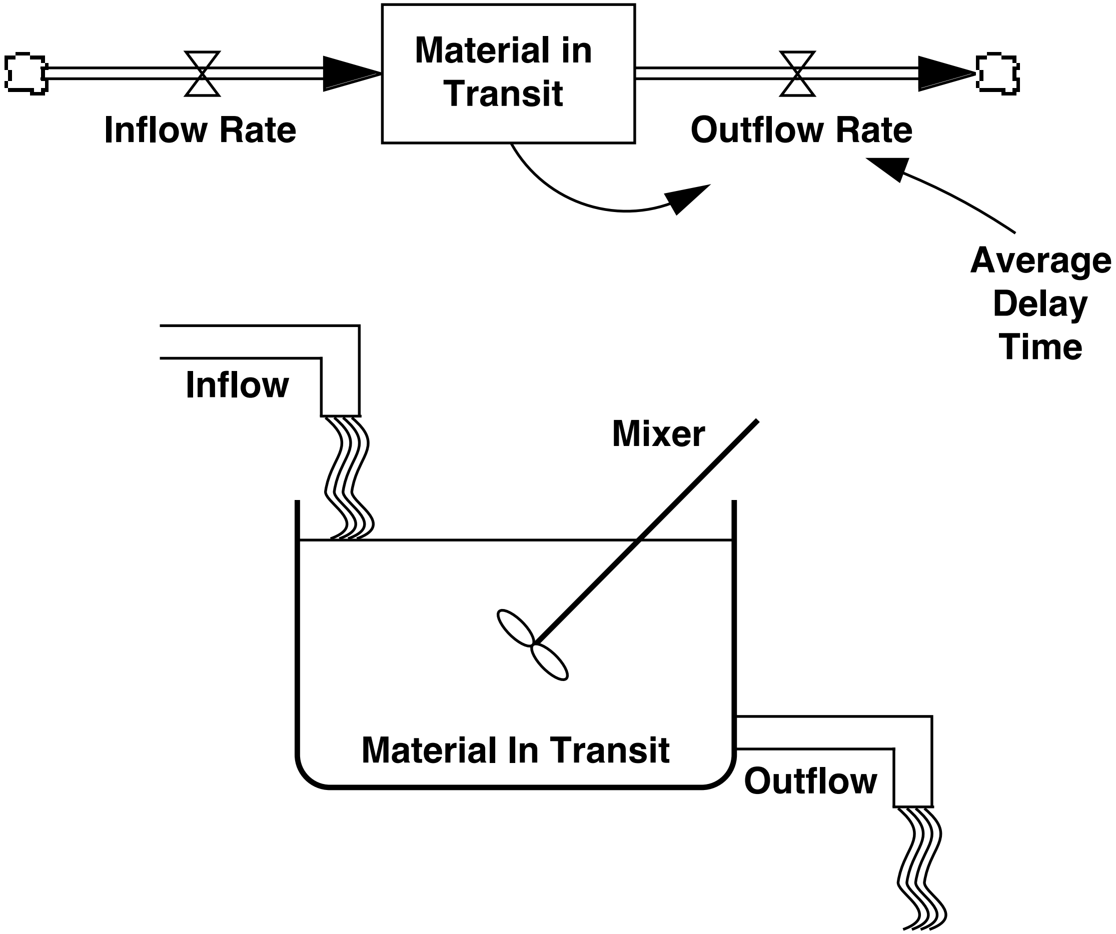
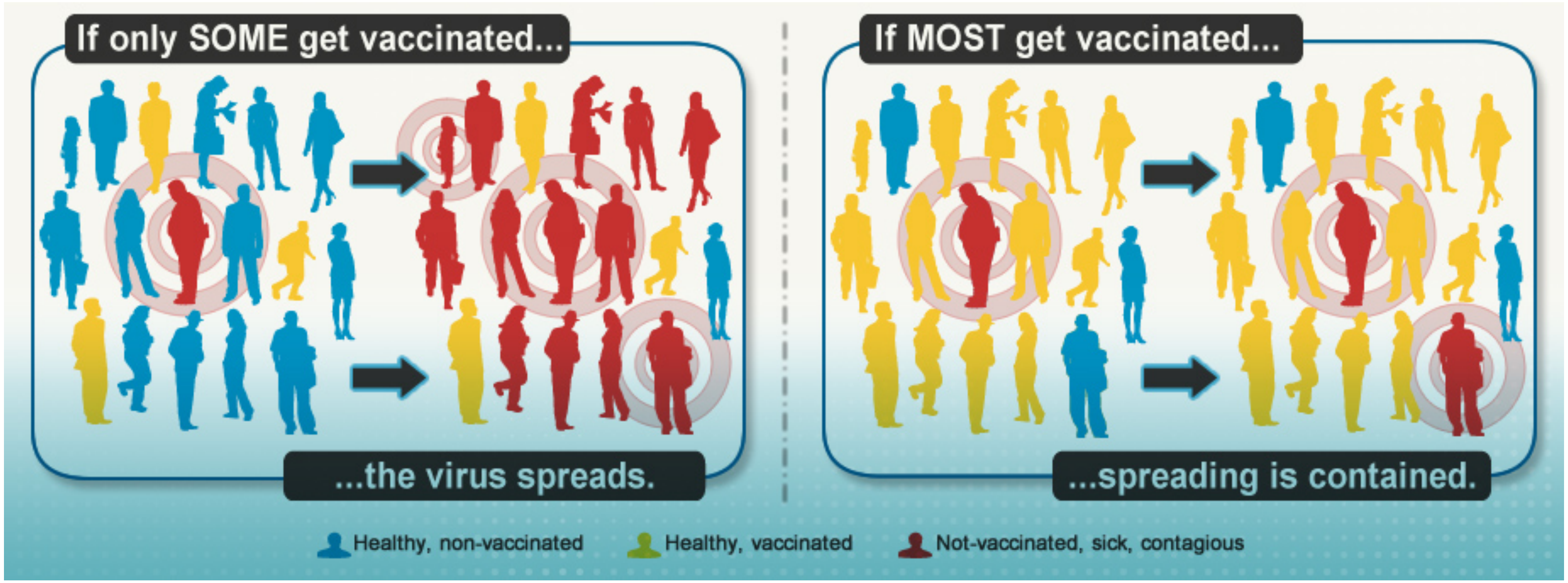


Figure 11-4 First-order material delay: structure

The outflow is proportional to the stock of material in transit. The contents of the stock are perfectly mixed at all times, so all items in the stock have the same probability of exit, independent of their arrival time.

Resource Constrained Flows

- Some flows are resource constrained
- No resources, no flow
- Examples: Coders write code, Health workers administer vaccines

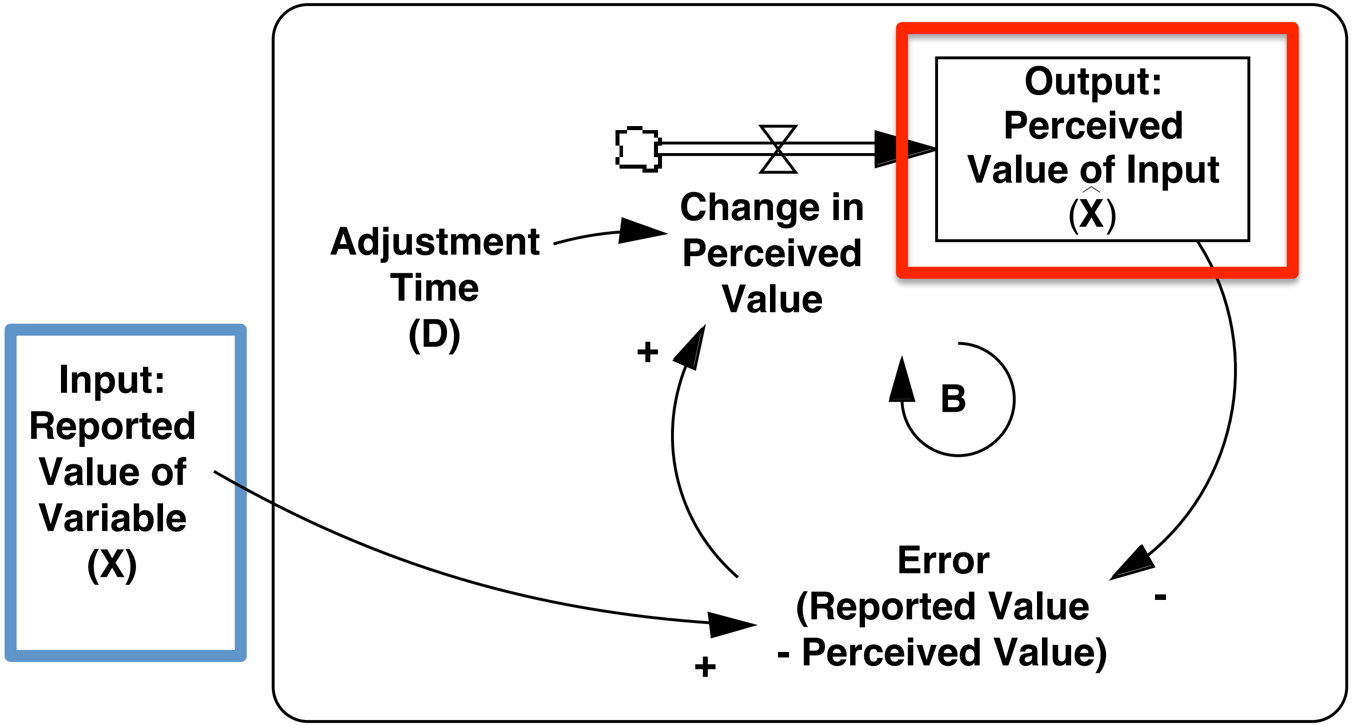


http://www.cdc.gov/vaccines/vac-gen/images/vaccines-protect.jpg

Structure of information delays

- Cannot be modeled with the same structure as material delays (where material is conserved)
- Simplest information delay and one of the most widely used models of belief adjustment and forecasting is *exponential smoothing* or ***adaptive expectations***
- Key idea: the belief gradually adjusts to the most recent value of the variable (negative feedback)

Information Delay Stock and Flow Model (D = 1 /α)



Sterman 2000
Business Dynamics

$\hat{X}$ = INTEGRAL(Change in Perceived Value, $\hat{X}$(0))

Change in Perceived Value = Error/D = (X – $\hat{X}$)/D

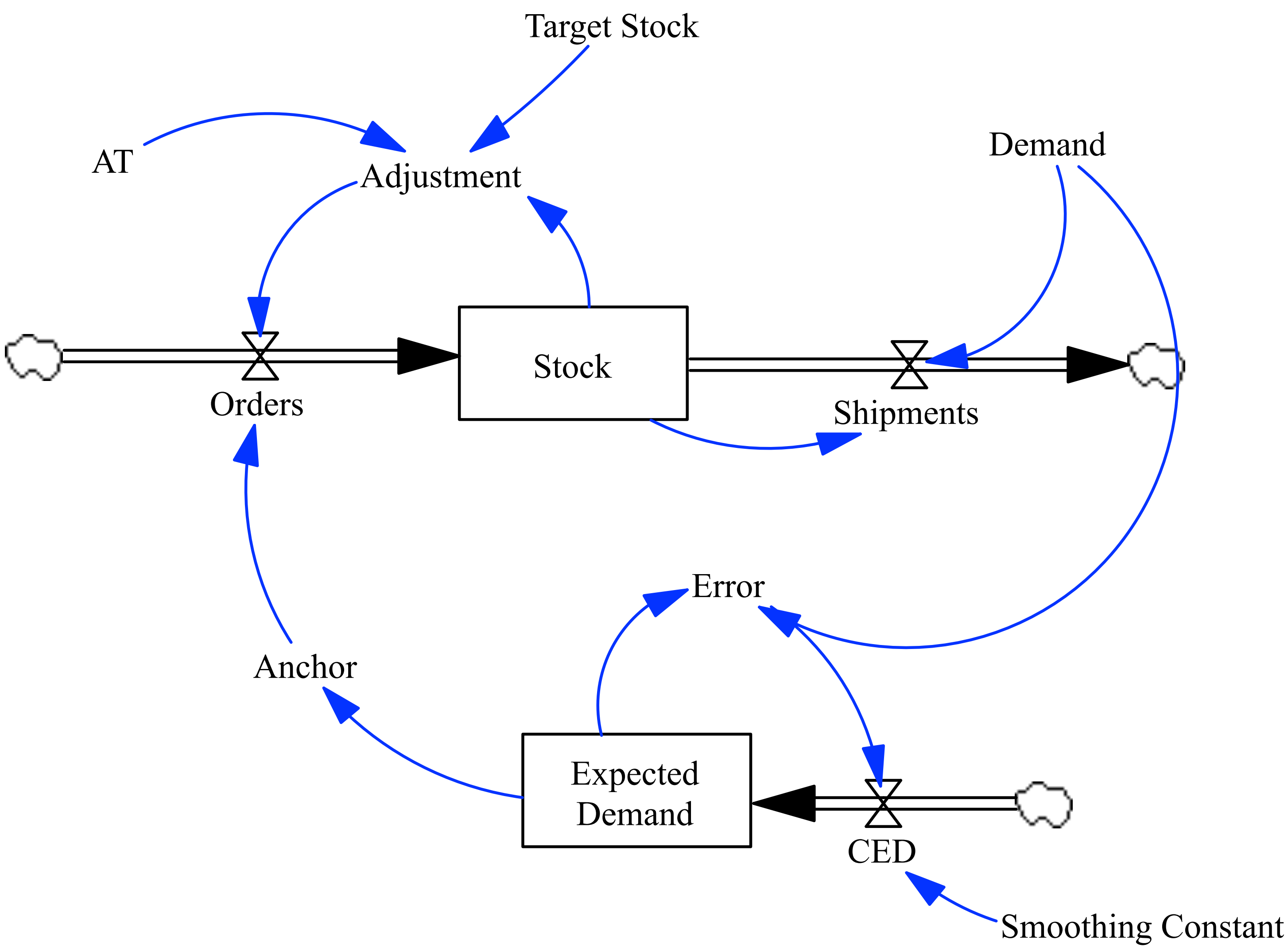
(6) Rules for managing a stock

(1) Managers should replace <u>expected losses</u> from the stock **(the anchor)**

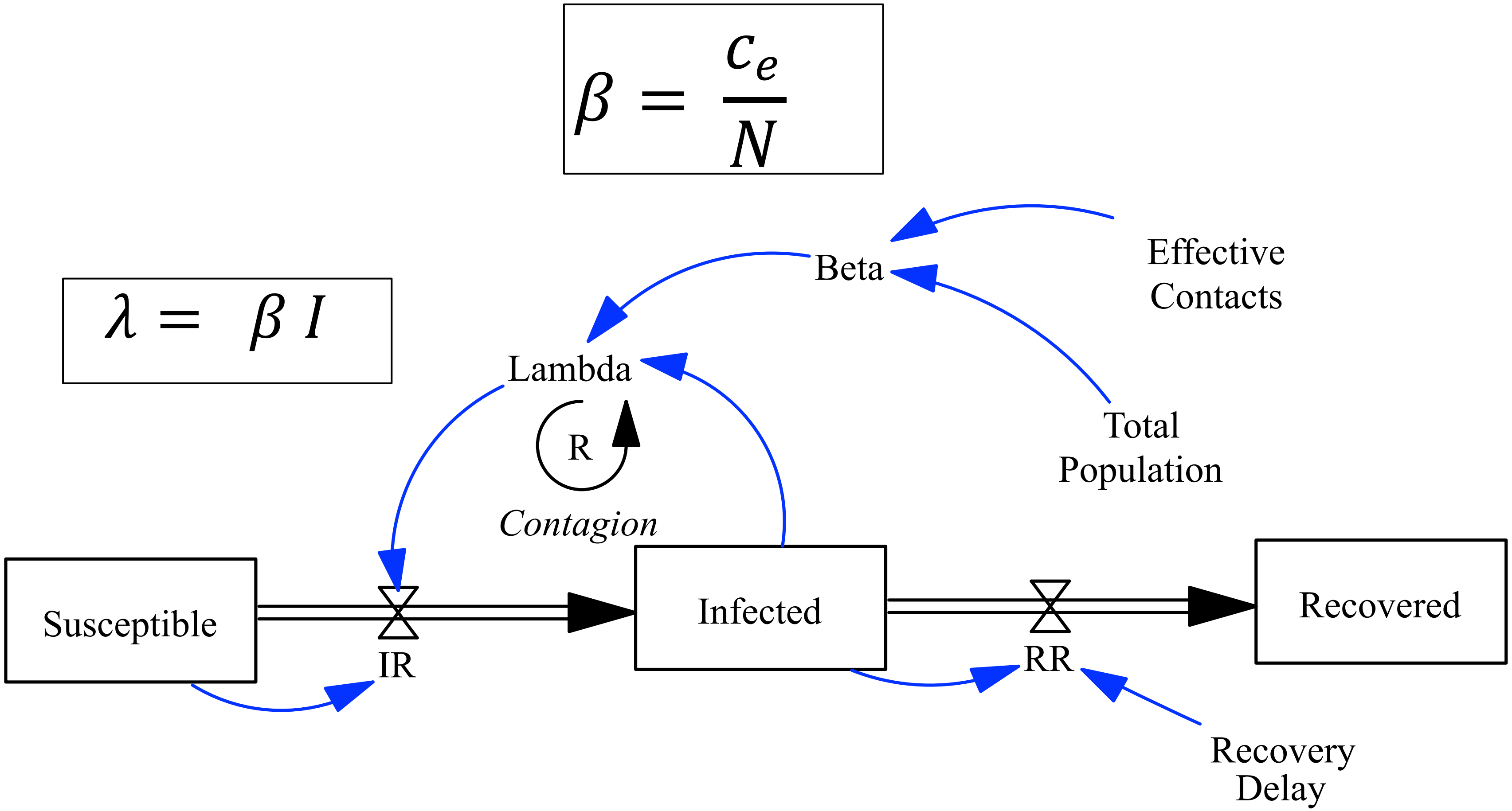
(2) Managers should <u>reduce the discrepancy</u> between the desired and actual stock **(the Adjustment).** Acquire:

- more than the expected losses when the stock is less that the desired,
- less than the expected losses when there is a surplus.

The Stock and Flow Model

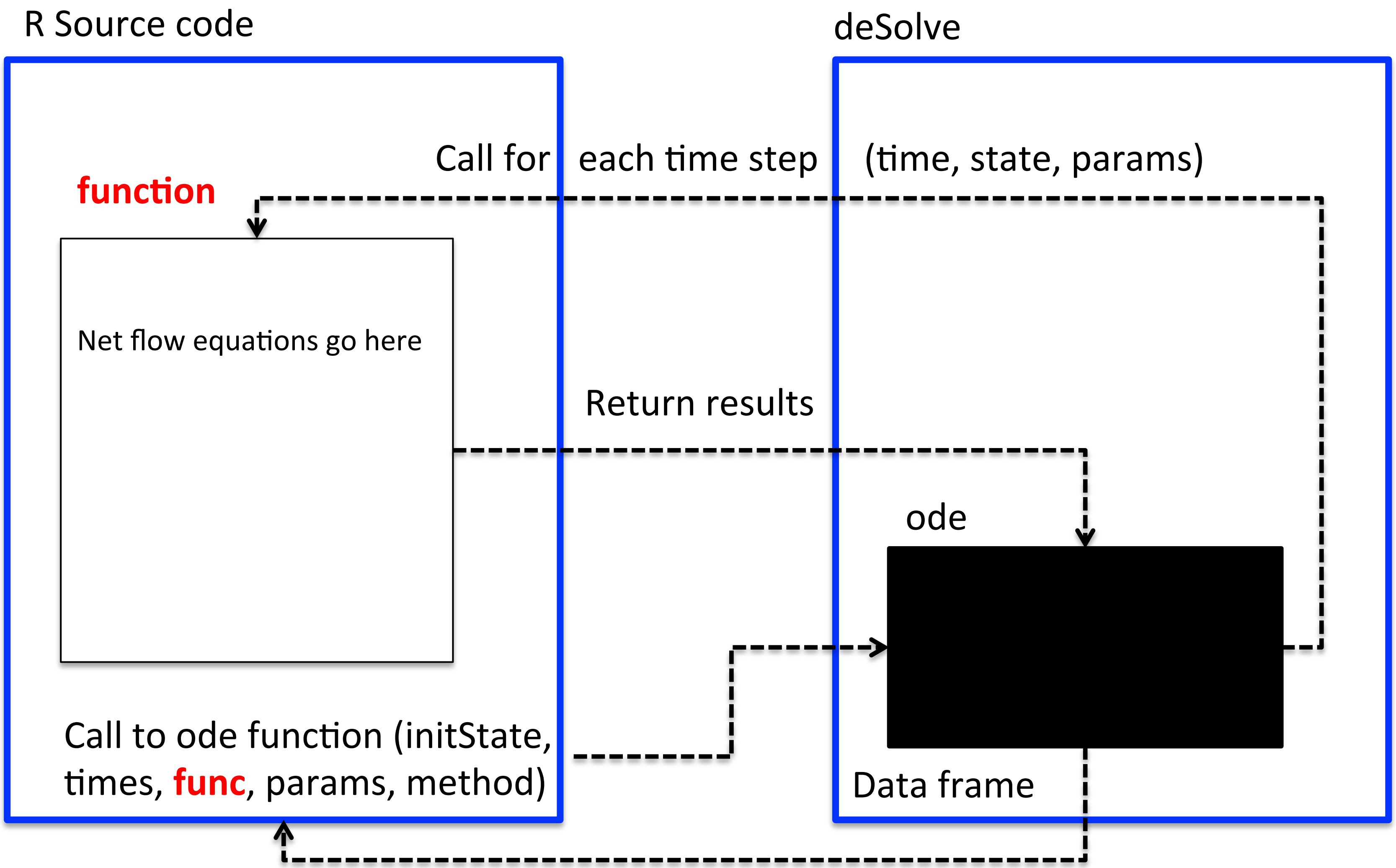


(7) SIR Model

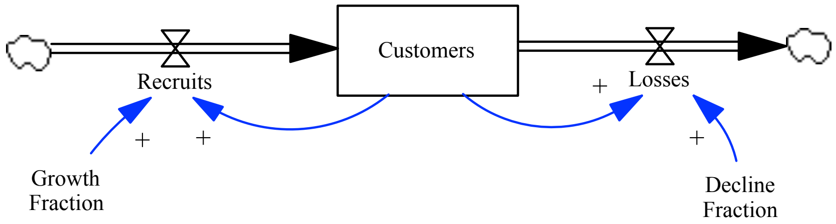


$$\beta = \frac{c_e}{N}$$
$$\lambda = \beta I$$
Beta
Effective Contacts
Lambda
R
Contagion
Total Population
Susceptible
IR
Infected
RR
Recovered
Recovery Delay

(8) deSolve & R


R Source code
deSolve
Call for each time step (time, state, params)
function
Net flow equations go here
Return results
ode
Call to ode function (initState, times, func, params, method)
Data frame

R Code (1/4)




```
library(deSolve)
library(ggplot2)

# Setup simulation times and time step
START<-2015; FINISH<-2030; STEP<-0.25

# Create time vector
simtime <- seq(START, FINISH, by=STEP)

# Create stock and auxs
stocks  <- c(sCustomers=10000)
auxs    <- c(aGrowthFraction=0.08, aDeclineFraction=0.03)
```

R Code (2/4)

```
# Model function
model <- function(time, stocks, auxs){
 with(as.list(c(stocks, auxs)),{

   fRecruits<-sCustomers*aGrowthFraction

   fLosses<-sCustomers*aDeclineFraction

   dC_dt <- fRecruits - fLosses

   return (list(c(dC_dt),
               Recruits=fRecruits, Losses=fLosses,
               GF=aGrowthFraction,DF=aDeclineFraction))
 })
}
```

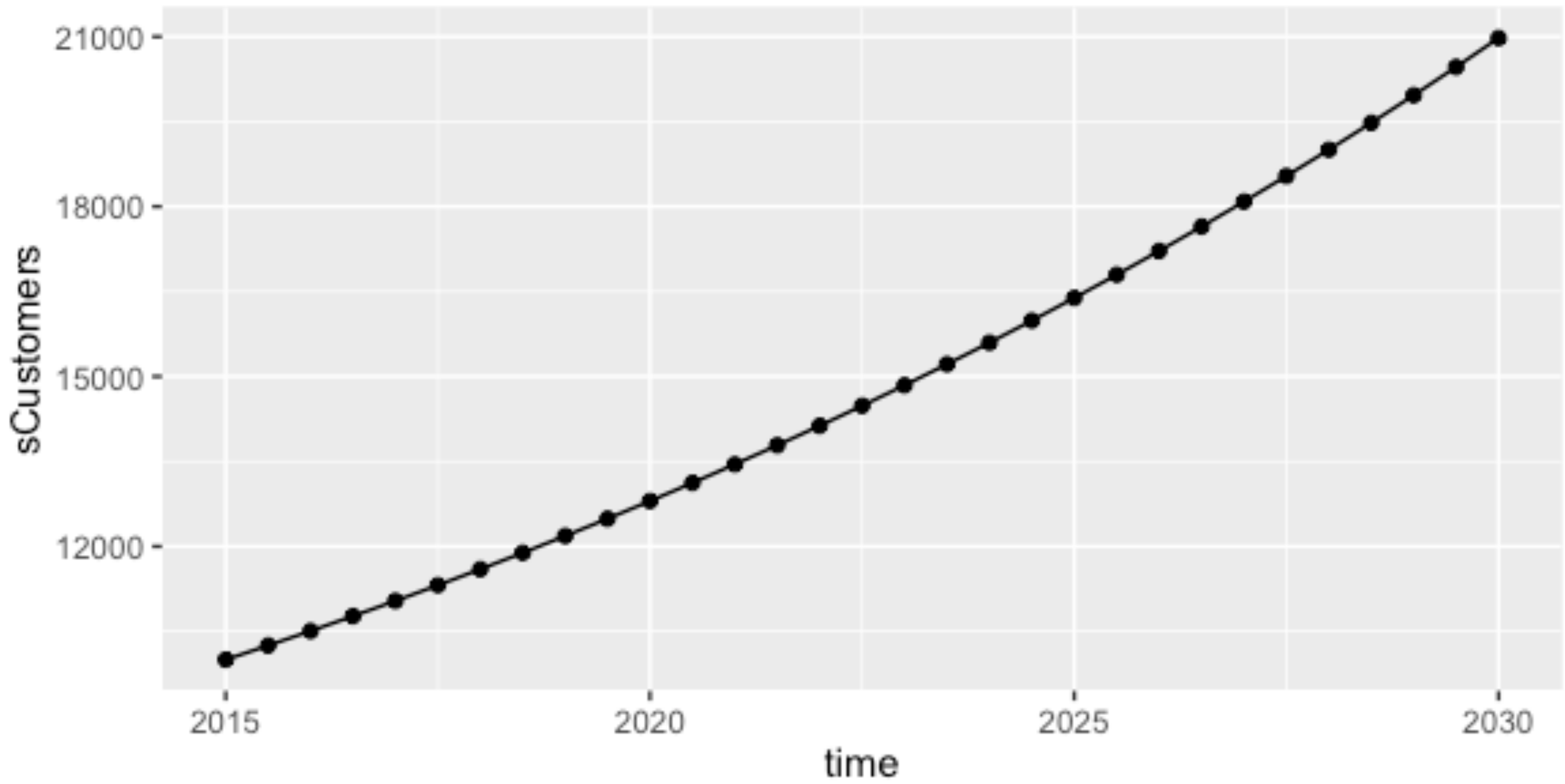
R Code (3/4)

```
# Run simulation
o<-data.frame(ode(y=stocks, times=simtime, func = model,
              parms=auxs, method="euler"))
```

```
> o[1:10,]
   time    sCustomers Recruits  Losses   NetFlow   GF   DF
1  2015.00 10000.00  800.0000 300.0000 500.0000 0.08 0.03
2  2015.25 10125.00  810.0000 303.7500 506.2500 0.08 0.03
3  2015.50 10251.56  820.1250 307.5469 512.5781 0.08 0.03
4  2015.75 10379.71  830.3766 311.3912 518.9854 0.08 0.03
5  2016.00 10509.45  840.7563 315.2836 525.4727 0.08 0.03
6  2016.25 10640.82  851.2657 319.2246 532.0411 0.08 0.03
7  2016.50 10773.83  861.9065 323.2150 538.6916 0.08 0.03
8  2016.75 10908.50  872.6804 327.2551 545.4252 0.08 0.03
9  2017.00 11044.86  883.5889 331.3458 552.2431 0.08 0.03
10 2017.25 11182.92  894.6337 335.4877 559.1461 0.08 0.03
```

Plot results (4/4)

```
qplot(x=time,y=sCustomers,data=o) + geom_line()
```



2015 Q1 (b)

(b) Consider the function $y = 2t + 10$

Show how this would be integrated over the interval [0,10] using Euler's method, with DT = 1. Also, assume the initial value of the integral is 0.

Would this solution underestimate or overestimate the true value of the integral?

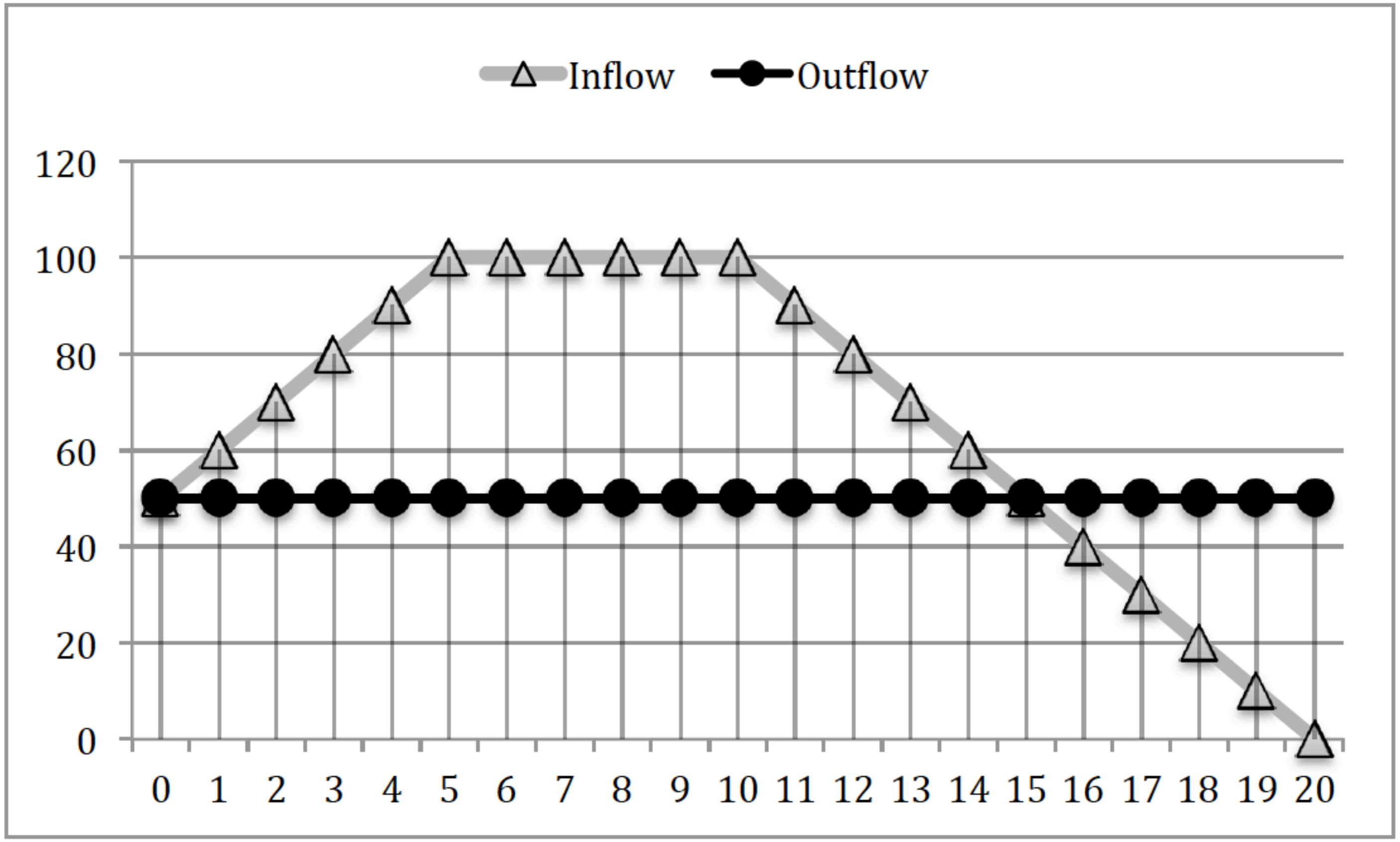
(6)

2015 Q1 (c)

(c) Based on the flow variables shown in the diagram (each flow starts at 50 units), and assuming that the initial value of the stock is 100, perform the following:

- Calculate, and show, the net flow
- Use graphical integration to plot the stock over time, clearly showing the behaviour mode for each segment (e.g. *increasing at an increasing rate, etc.)*

2015 Q1 (c)


Inflow
Outflow
120
100
80
60
40
20
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20