CT561: Systems Modelling and Simulation

Week 1: Introduction

https://github.com/JimDuggan/CT561

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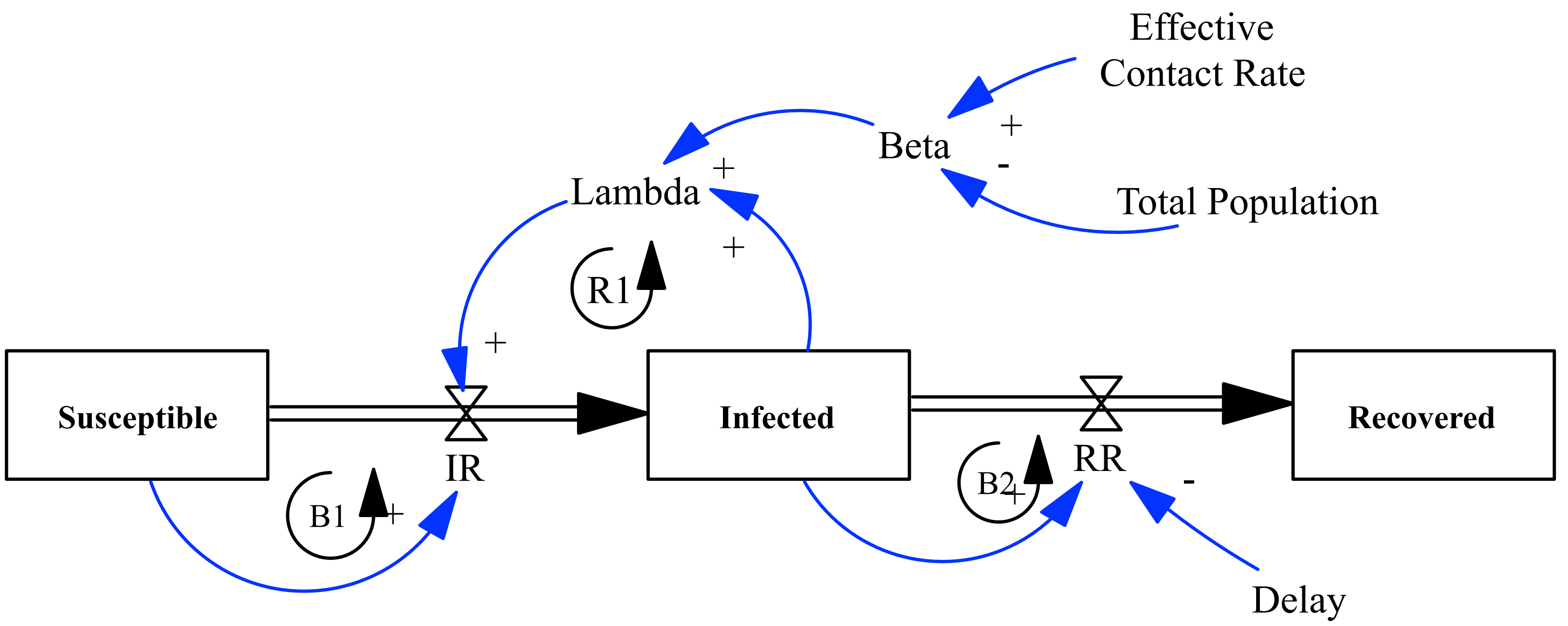
Motivation

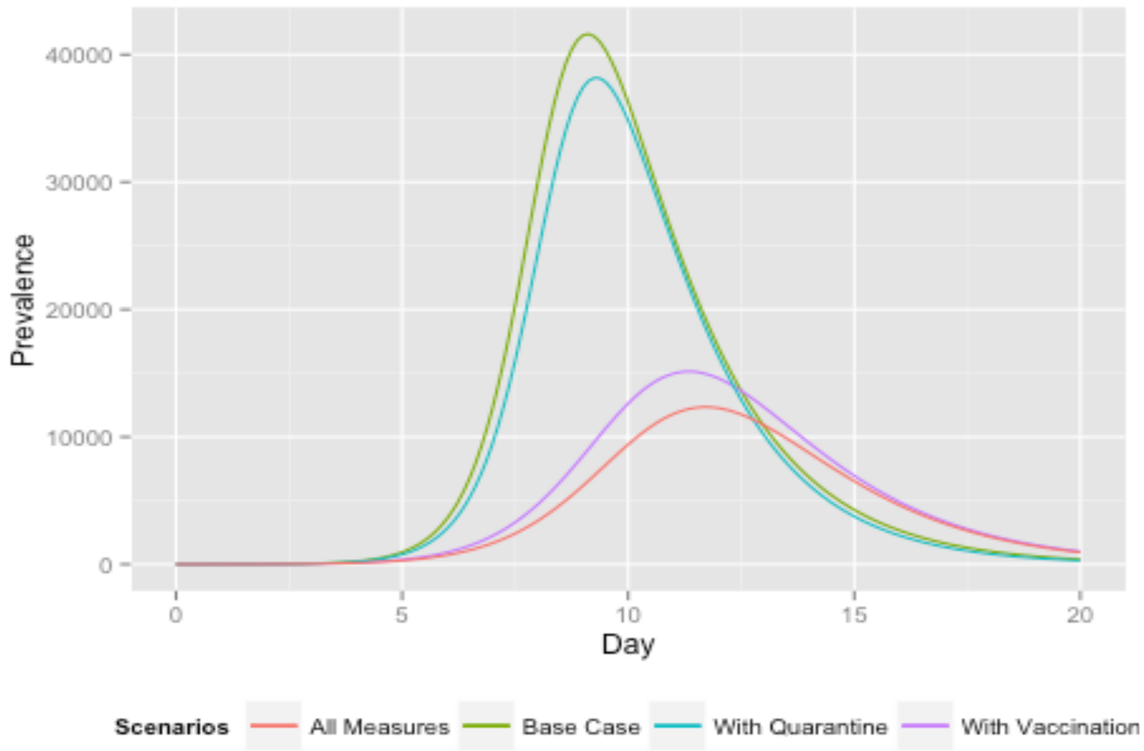
A model should always be created for a purpose.

Jay W. Forrester, Urban Dynamics (1969), p.113.



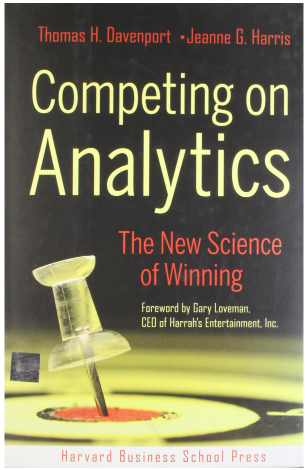
Useful Models

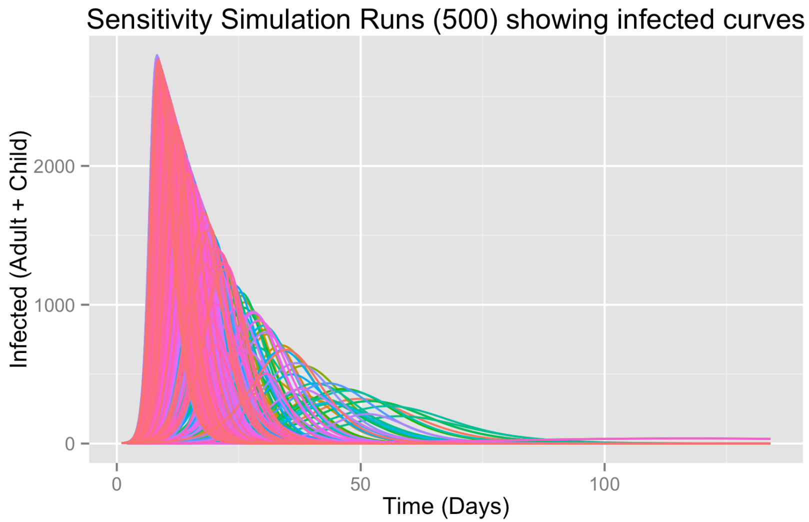
- In order for a model to be useful to decision makers, it must provide some view on future behavior (Meadows 1974)



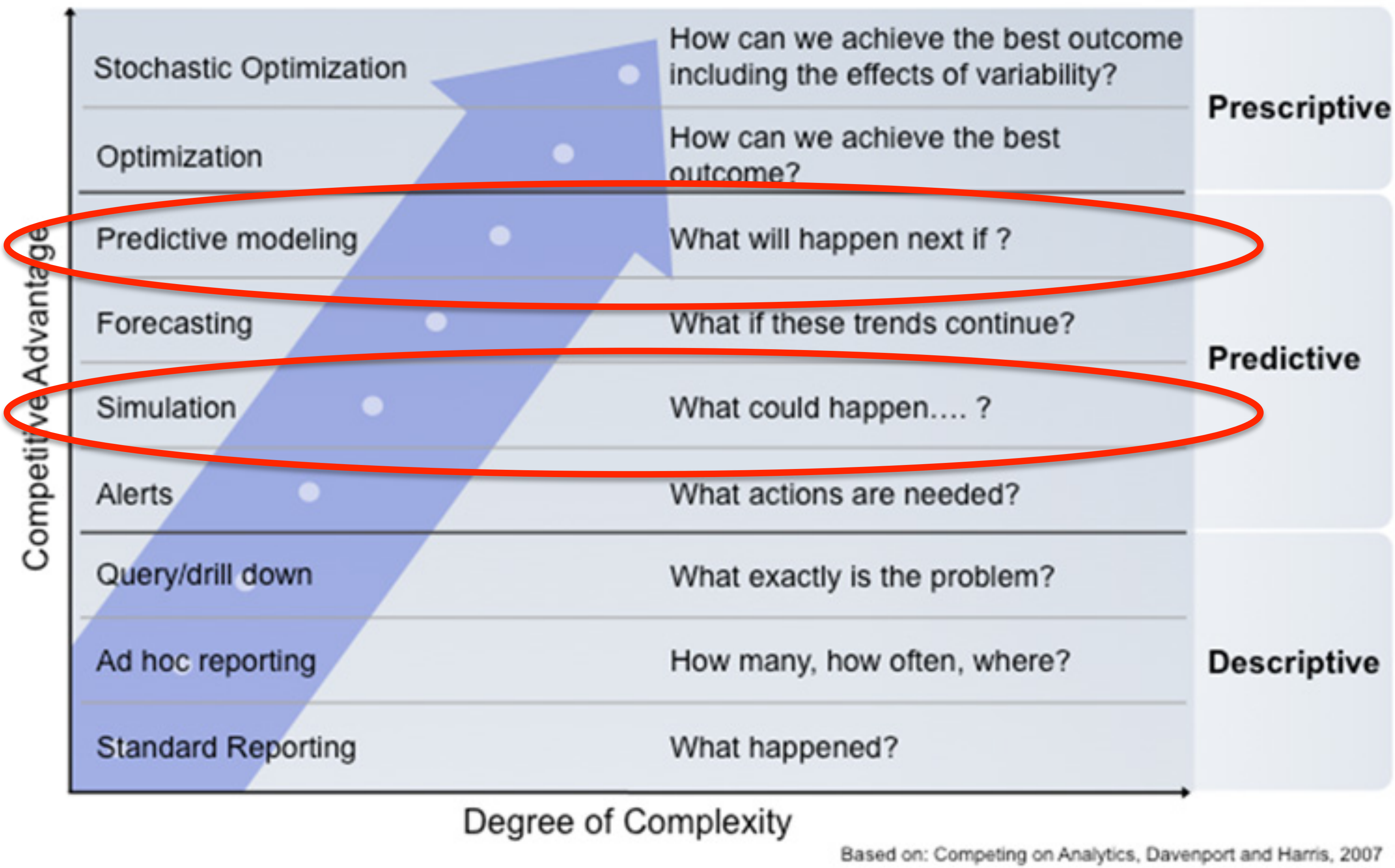
Context - Analytics

"The extensive use of ***data****, statistical and quantitative analysis, explanatory and* ***predictive models****, and* ***fact-based management*** *to* ***drive decisions and actions****."* Davenport and Harris (2007).





Our Focus: Systems Simulation



http://www.scdigest.com

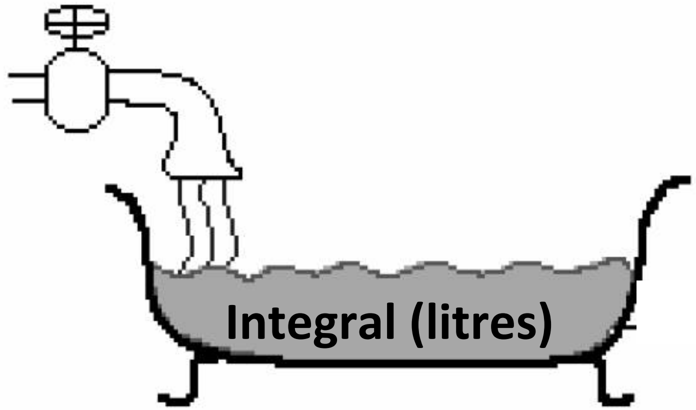
Model Types (Meadows 1974)

- *Absolute, precise predictions*, for example, when and where will the next solar eclipse be observable?
- *Conditional, precise predictions,* for example, if a cooling systems fails in a nuclear power plant, what will be the maximum pressure exerted on the reactor's containment vessel?
- ***Conditional, imprecise projections of dynamic behavior***, for example, if an infectious disease spreads through a population, what is the likely future burden of demand on intensive care facilities one month from the outbreak date?

Calculus

- Calculus is the study of how things **change over time**, and is described by Strogratz (2009) as “perhaps the greatest idea that humanity has ever had.”
- Integration is an intuitive concept that can be understood without reference to formal mathematics.

Derivative (litres/minute)



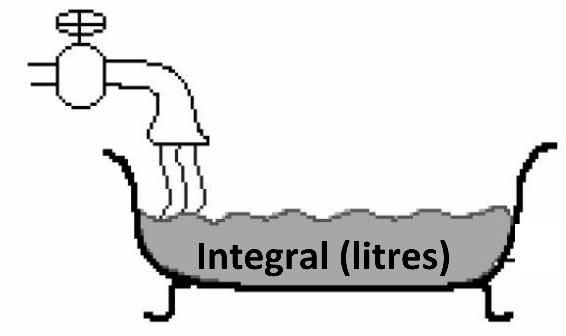
After five minutes of constant flow of five litres/minute, if the bathtub is initially empty, how much water will the bathtub contain?

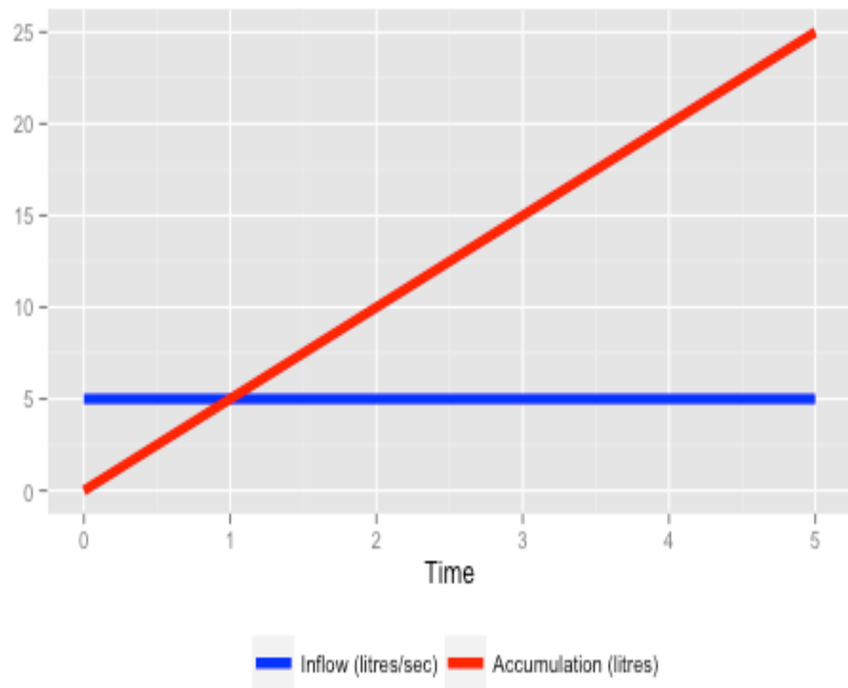
http://systems-sciences.uni-graz.at/etextbook/sw2/stocks_flows.html

Calculus in Action

- The tap represents an inflow (derivative) as 5 litres/minute
- The bathtub represents a stock (integral) that accumulates the net flow (inflows minus outflows)
- If we know the inflow and the initial stock, we can predict the behaviour over time

Derivative (litres/minute)





Method 1: Analytical Approach

$$\int x^n \; dx = \frac{1}{n+1} x^{n+1} + c$$

$$f(x) = \; 5\,x^0 \longrightarrow$$

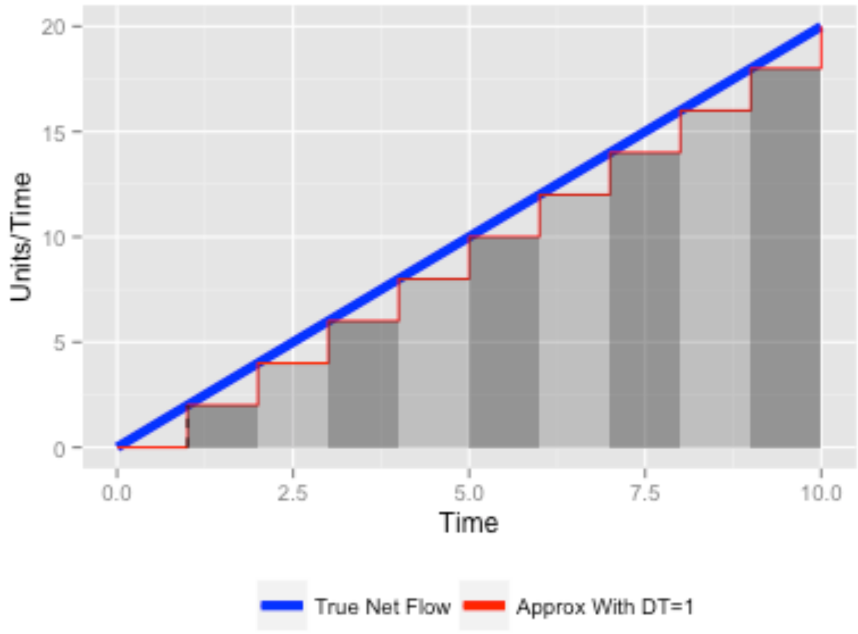
$$\int 5\,x^0\,dx = 5\int x^0\,dx = 5x^1 + c$$

$$\int_0^5 5x^0 dx = 5(5) - 5(0) = 25 \longrightarrow$$

$$\int_0^{1000} 5x^0 dx = 1000(5) - 5(0) = 5{,}000$$

Method 2: Numerical Integration

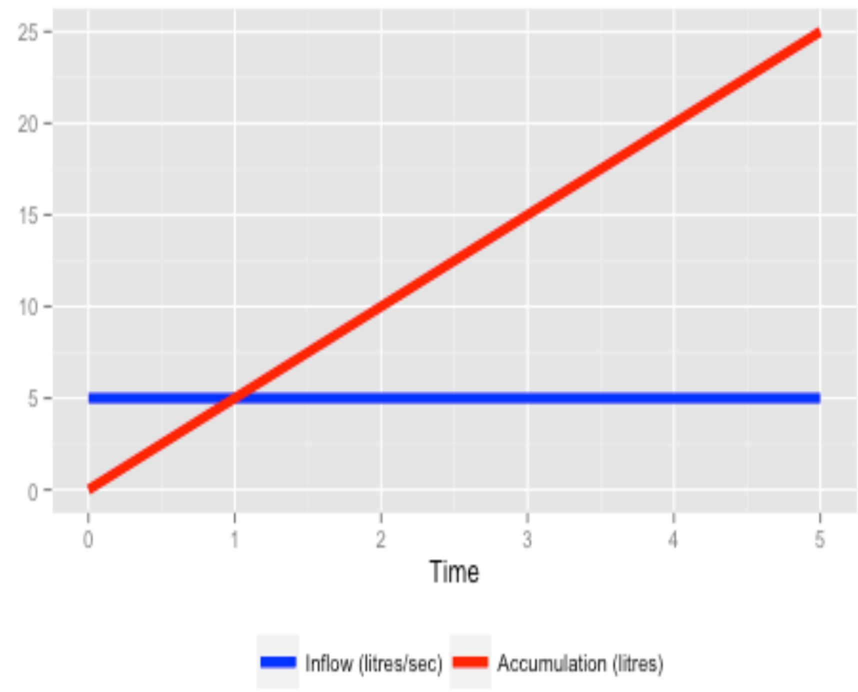
- Euler Method
- Approximate area under the net flow curve as a summation of rectangles, of width DT
- The smaller DT, the more accurate the result



$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$

The Algorithm

- Set Time = START
- Initialise Stocks
- Calculate Flows
- Time = Time + DT
- While (Time <= END)
  - Calculate Stocks
  - Calculate Flows
  - Time = Time + DT



Excel Solution, DT=1

Time	Bathtub (Litres)	Tap (Litres/Minute)
0	0	5
1	5	5
2	10	5
3	15	5
4	20	5
5	25	5
6	30	5
7	35	5
8	40	5
9	45	5
10	50	5

$$S_t = S_{t-dt} + NF_{t-dt} \times DT$$

Note: Stock only depends on previous stock and net flows

Challenge 1.1

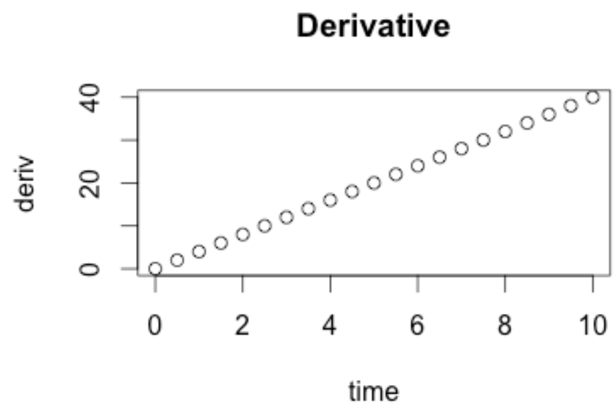
- Solve, analytically and numerically, the following derivative. Assume the integral has an initial value of 100, and the time interval is [0-10]. Set DT = 0.5 for the numerical solution.

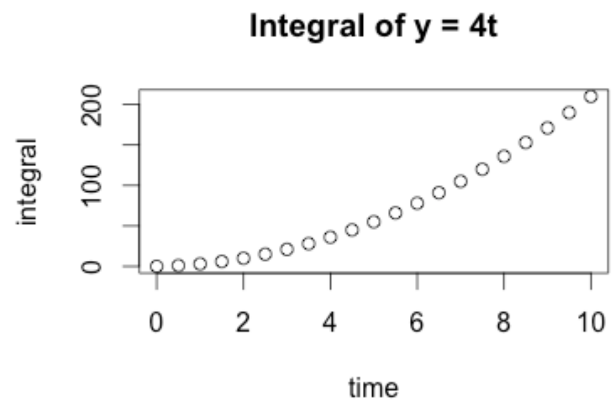
$$\frac{dy}{dt} = 4t$$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c$$

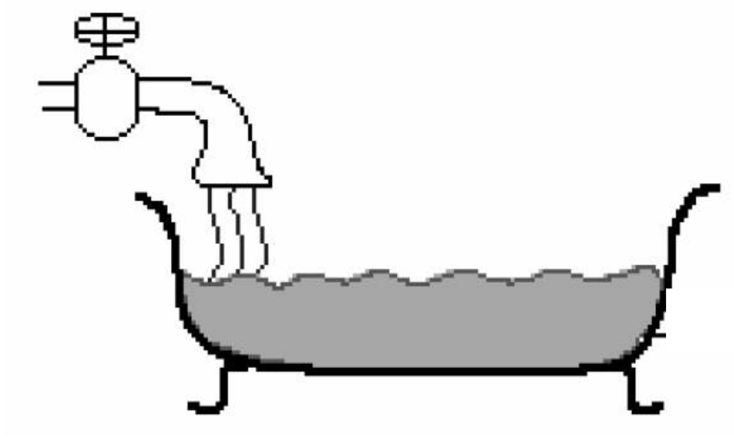
Euler’s Algorithm in R

```r
INIT<-100
DT<-0.5
time<-seq(0,10,by=DT)
deriv<-4*time # dy/dt = 4t
integral<-time*0 # just to setup the vector size

#initialise stock for the first time unit
integral[1]<-INIT

for(i in 2:length(time))
{
  integral[i]<-integral[i-1]+deriv[i-1]*DT
}

plot(time,deriv, main = "Derivative")
plot(time,integral, main = "Integral of y = 4t")
```

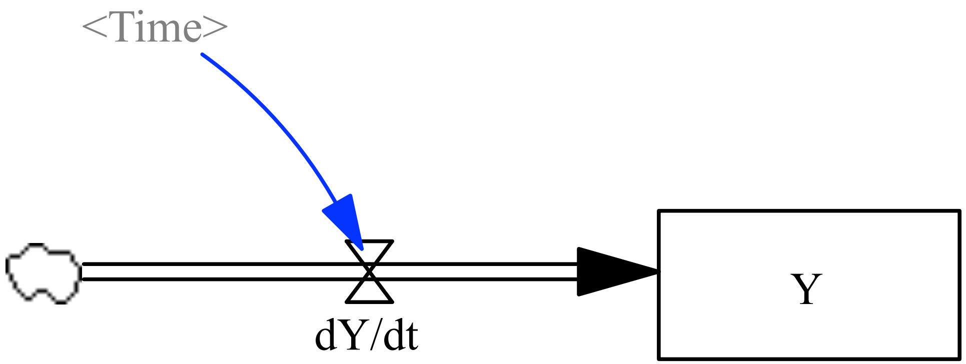




Integrals and Derivatives



$$\frac{dy}{dt} = 4t$$



Stocks and Flows

Stocks

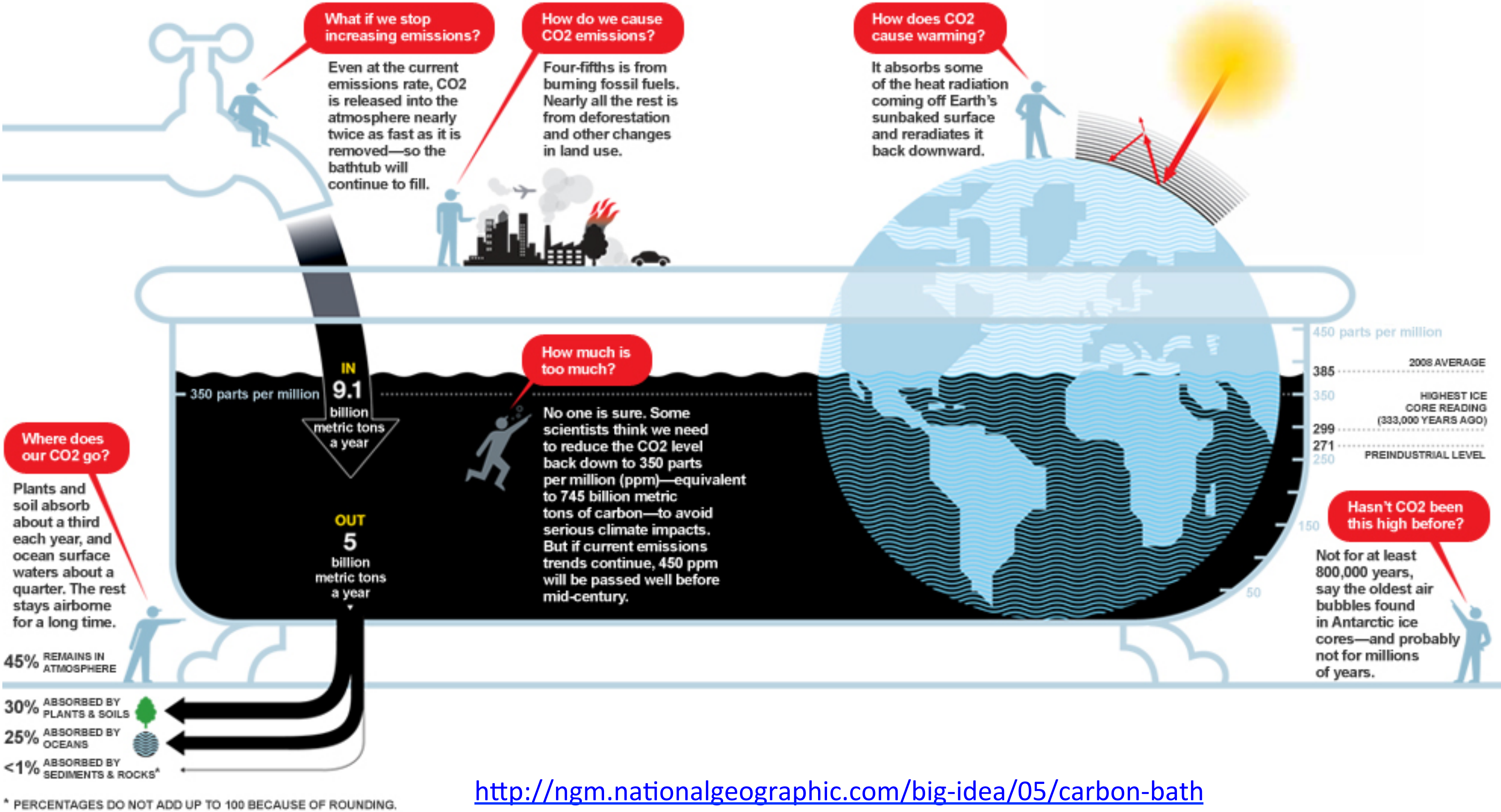
- A **stock** is the foundation of any system.
- **Stocks** are the elements of the system that you can see, feel, count, or measure at any given time.
- A **system stock** is, an accumulation of material or information that has built up over time
- Dimensions are units (litres, people, lines of code)

Meadows, Donella H. *Thinking in systems: A primer*. Chelsea Green Publishing, 2008.

Flows

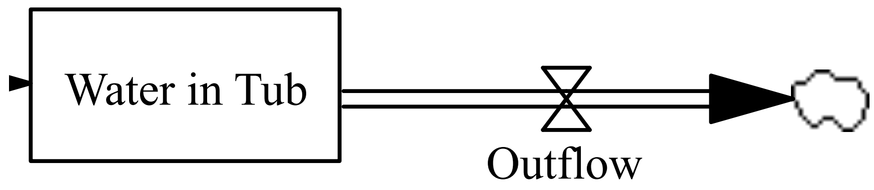
- Stocks change over time through the actions of a **flow**.
- Flows are:
  - filling and draining,
  - births and deaths,
  - purchases and sales,
  - deposits and withdrawals
  - enrolments and graduations
- Dimensions are units/ time period (litres/day, people/year)

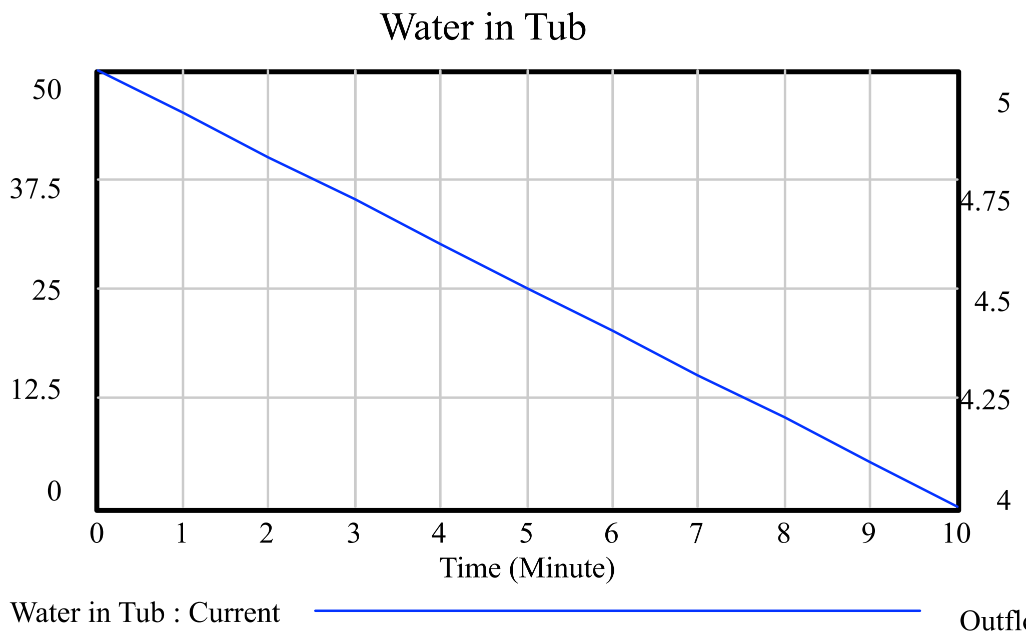
A Stock and Flow Model of Carbon in the Atmosphere

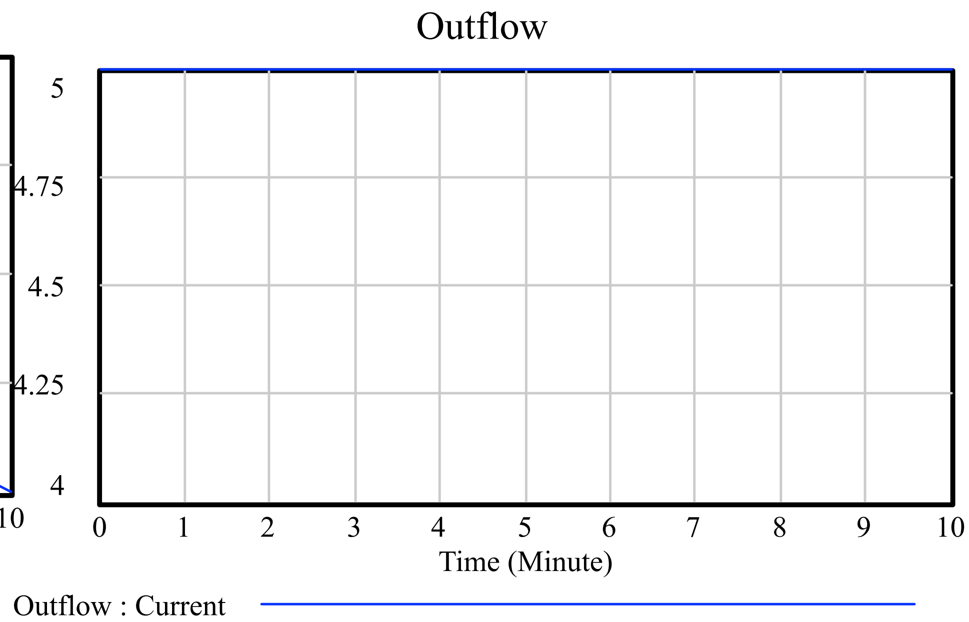


http://ngm.nationalgeographic.com/big-idea/05/carbon-bath

Stock and Flow Model

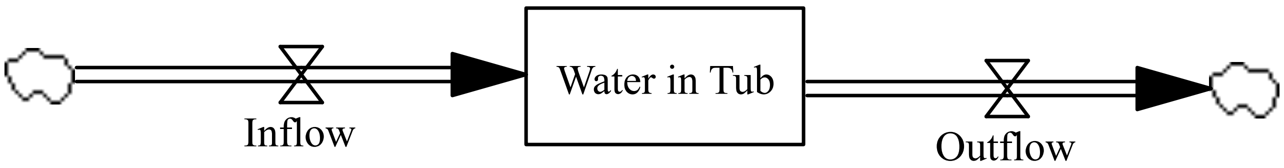






What happens next?

- Now imagine starting again with a full tub, and again open the drain, but this time, when the tub is about half empty, turn on the inflow faucet so the rate of water flowing in is <u>just equal</u> to that flowing out.



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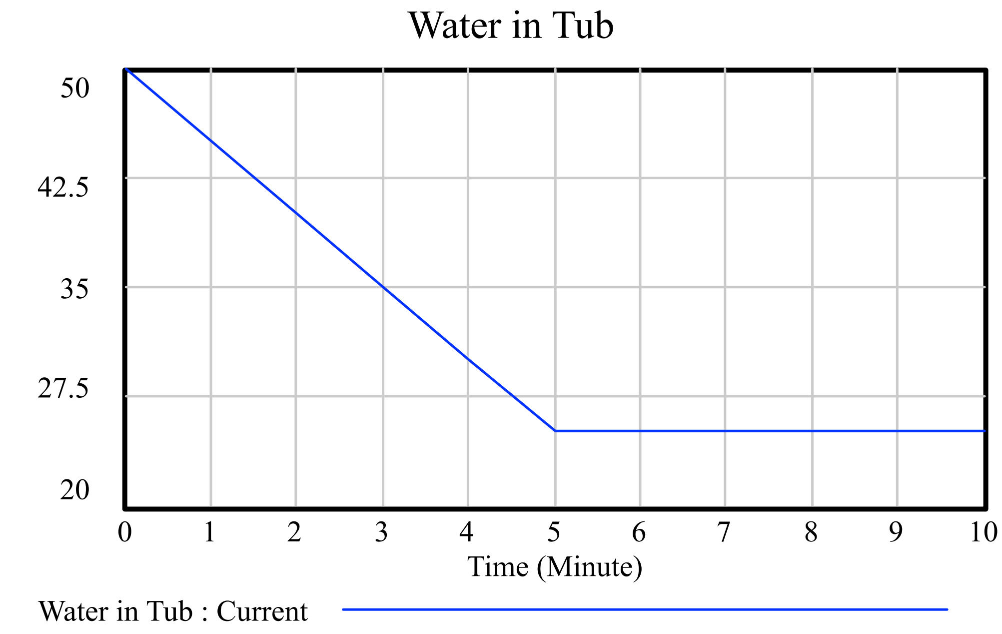

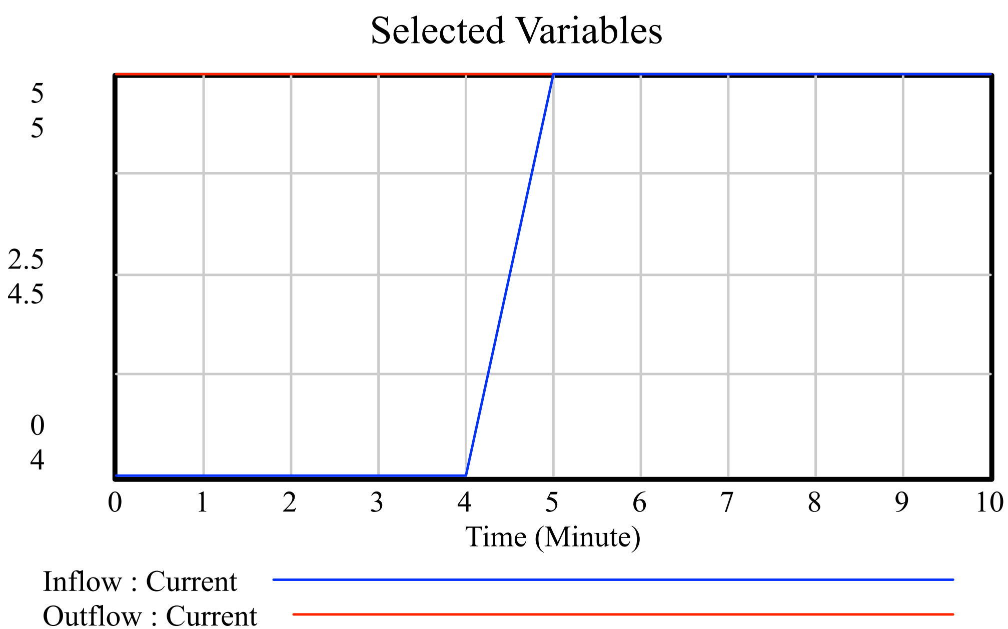

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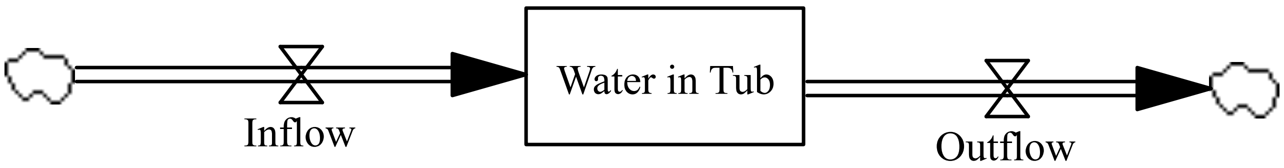
- The amount of water in the tub stays constant at whatever level it had reached when the inflow became equal to the outflow.
- It is in a state of **dynamic equilibrium**—its level does not change, although water is continuously flowing through it.

What happens next?

- Now imagine starting again with a full tub, and again open the drain, but this time, when the tub is about half empty, turn on the inflow faucet so the rate of water flowing in is <u>greater than</u> that flowing out.



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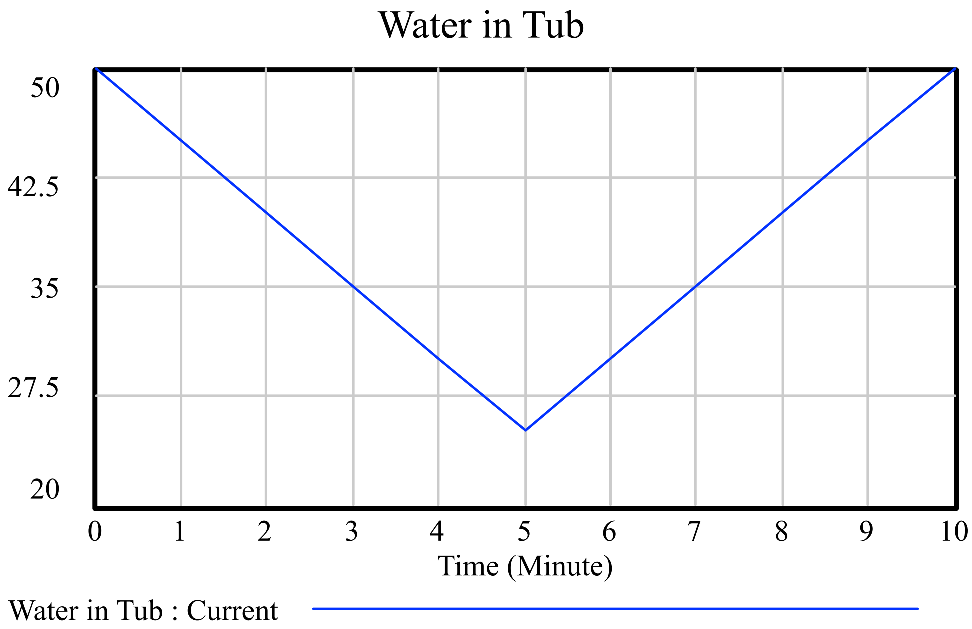

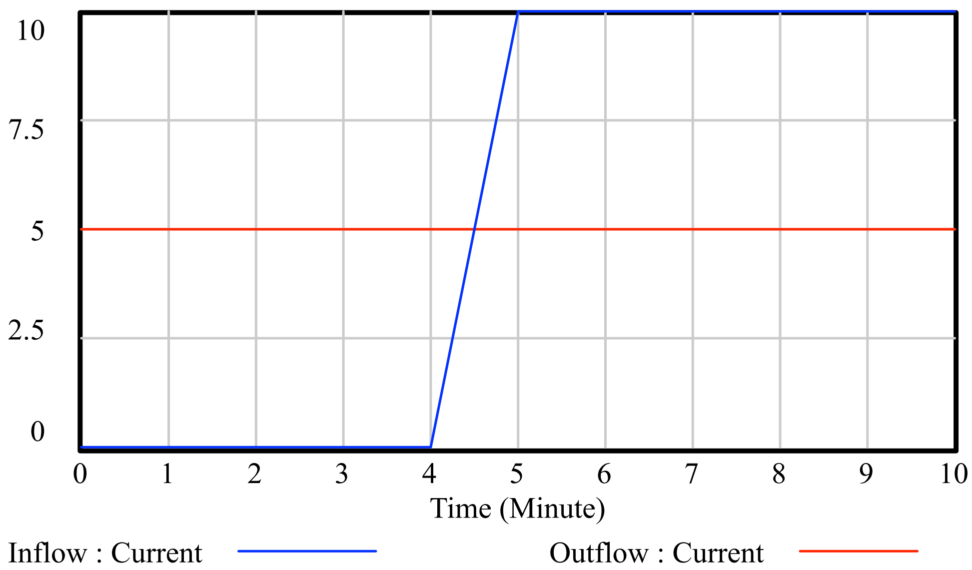

- The level of water in the tub slowly rises.
- This model of a bathtub is a very simple system with just one stock, one inflow, and one outflow.

Meadows, Donella H. *Thinking in systems: A primer*. Chelsea Green Publishing, 2008.

General Principle of Stock/Flow Systems

- From this simple bathtub model you can deduce **several important principles** that extend to more complicated systems:
  - As long as the sum of all inflows exceeds the sum of all outflows, the level of the stock will **rise**.
  - As long as the sum of all outflows exceeds the sum of all inflows, the level of the stock will **fall**.
  - If the sum of all outflows equals the sum of all inflows, the stock level **will not change**; it will be held in dynamic equilibrium at whatever level it happened to be when the two sets of flows became equal.

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Regulating Stocks

- Most individual and institutional decisions are designed to regulate the levels in stocks.
  - *If the stock of food in your kitchen gets low, you go to the store.*
- People monitor stocks constantly and make decisions and take actions designed to **raise** or **lower** stocks or to keep them within **acceptable ranges**.

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Summary

- **Systems thinkers see the world as a collection of stocks along with the mechanisms for regulating the levels in the stocks by manipulating flows.**

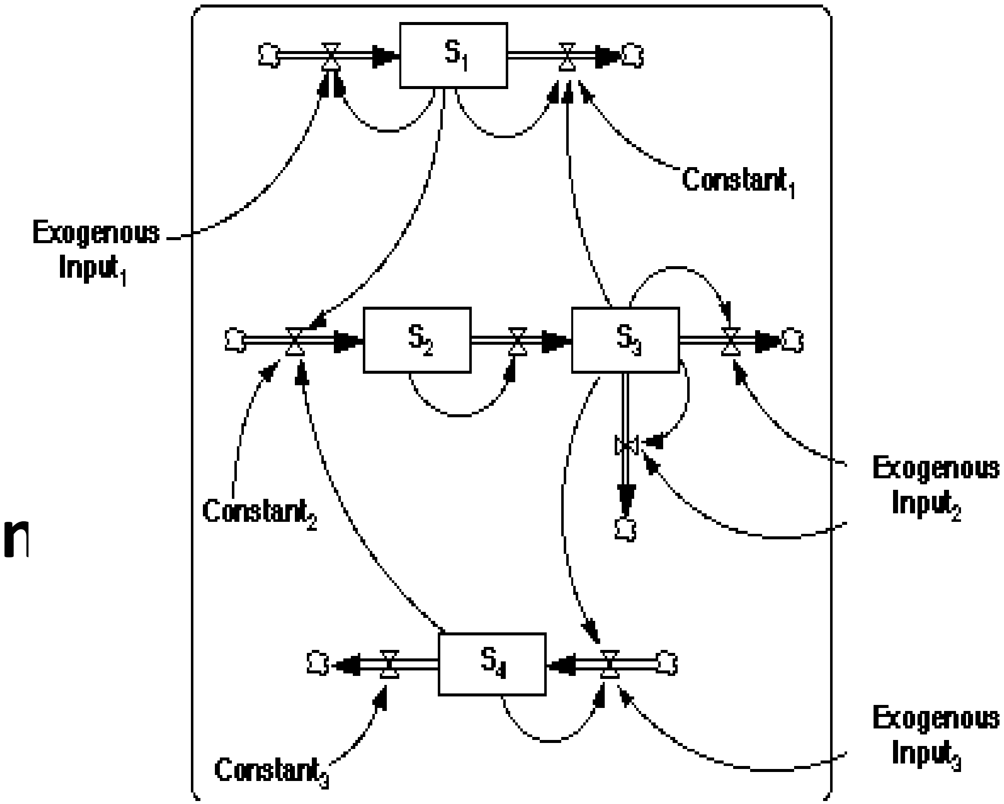

Diagram source: J.D. Sterman,Business Dynamics: Copyright © 2001 by the McGraw-Hill Companies

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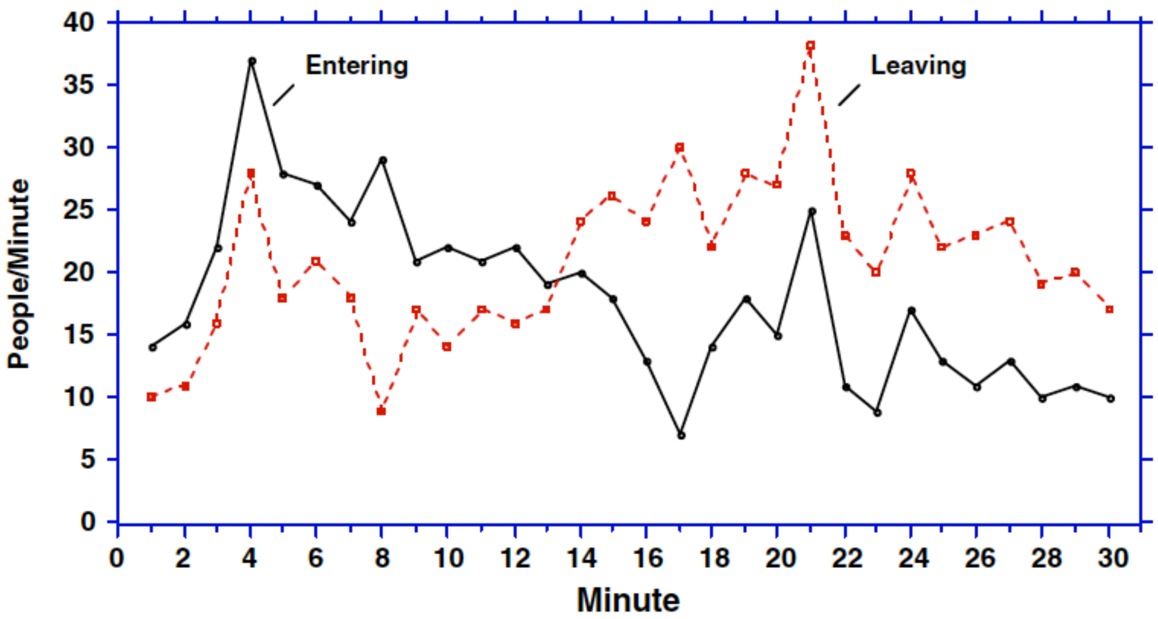
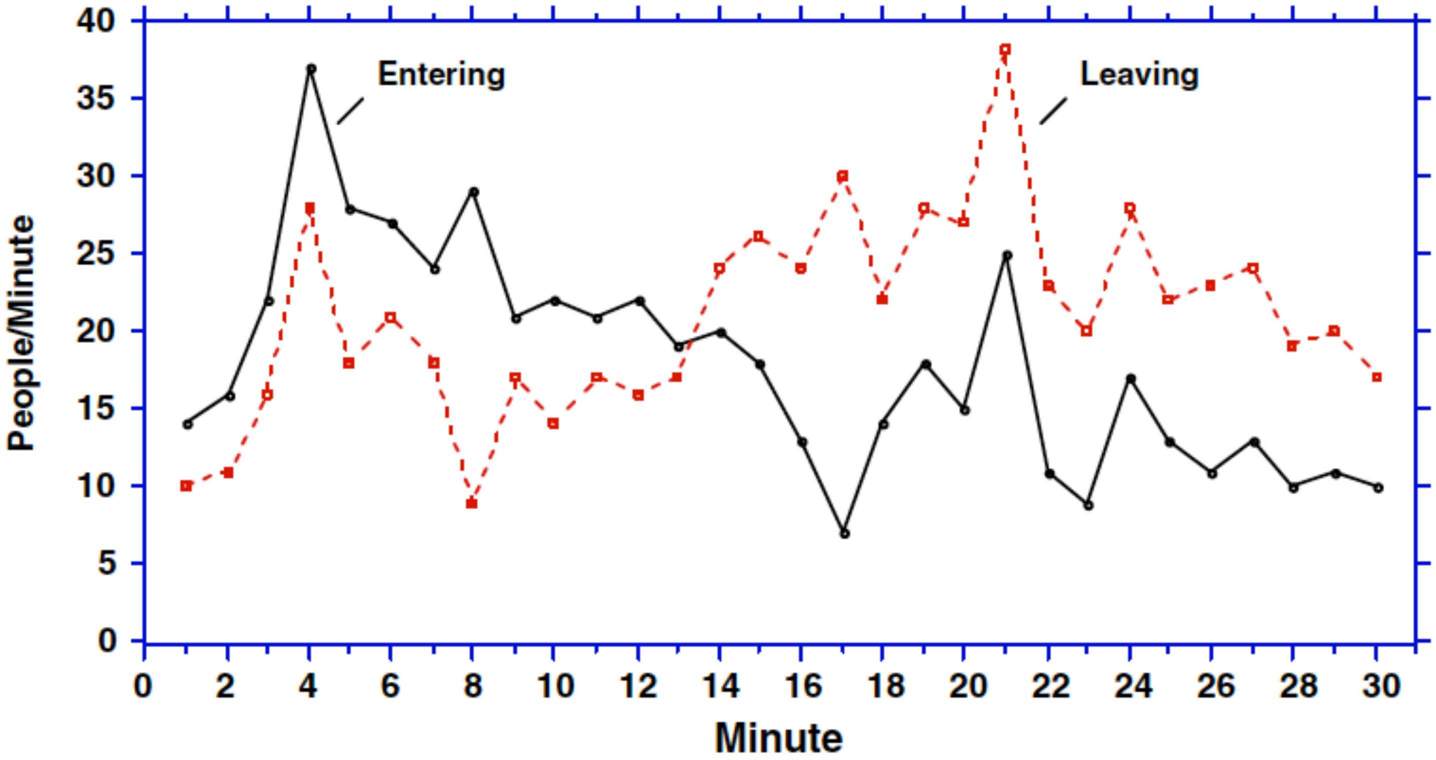
Challenge 1.2

The Department Store problem (Cronin et al. 2009)



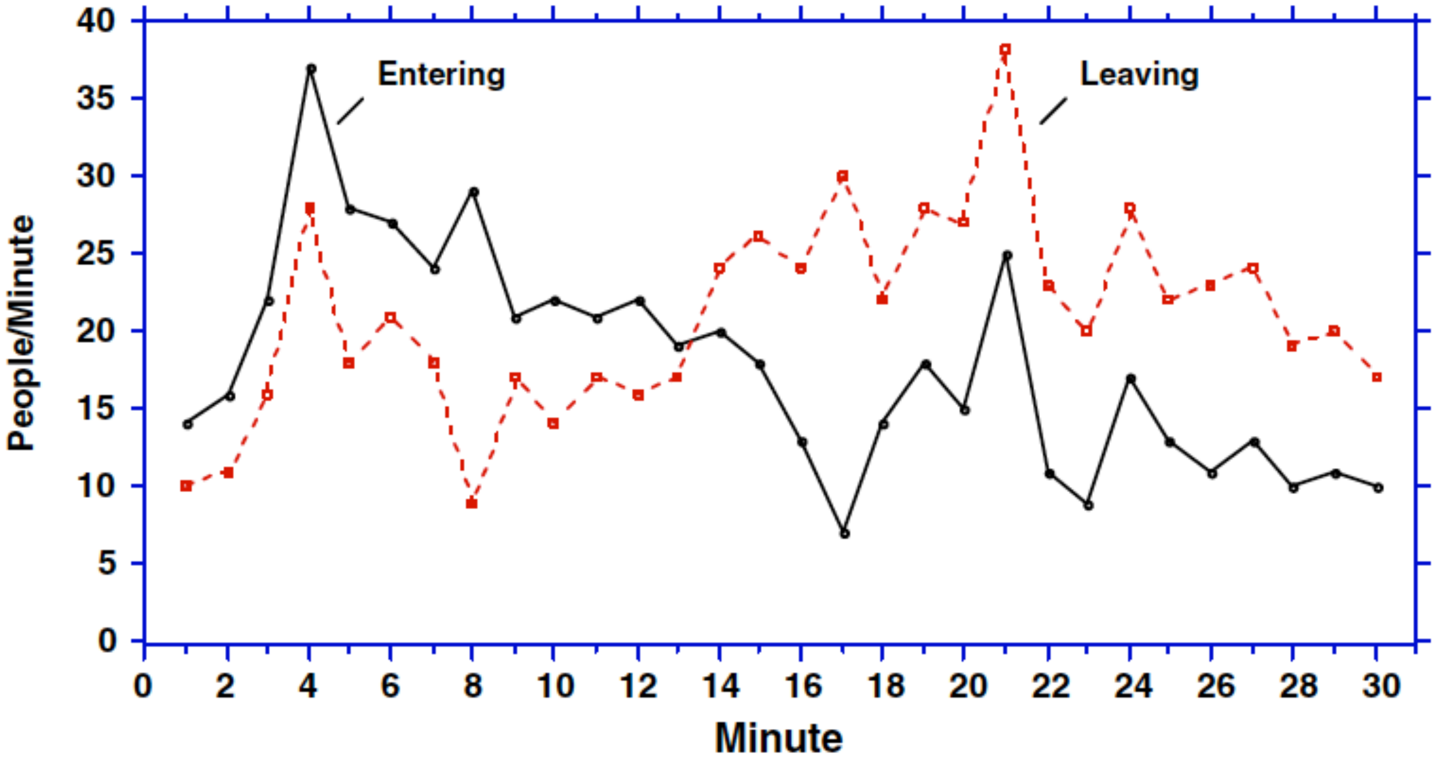
Question 1



1. During which minute did the most people enter the store?

Minute ________ ❑ Can't be determined

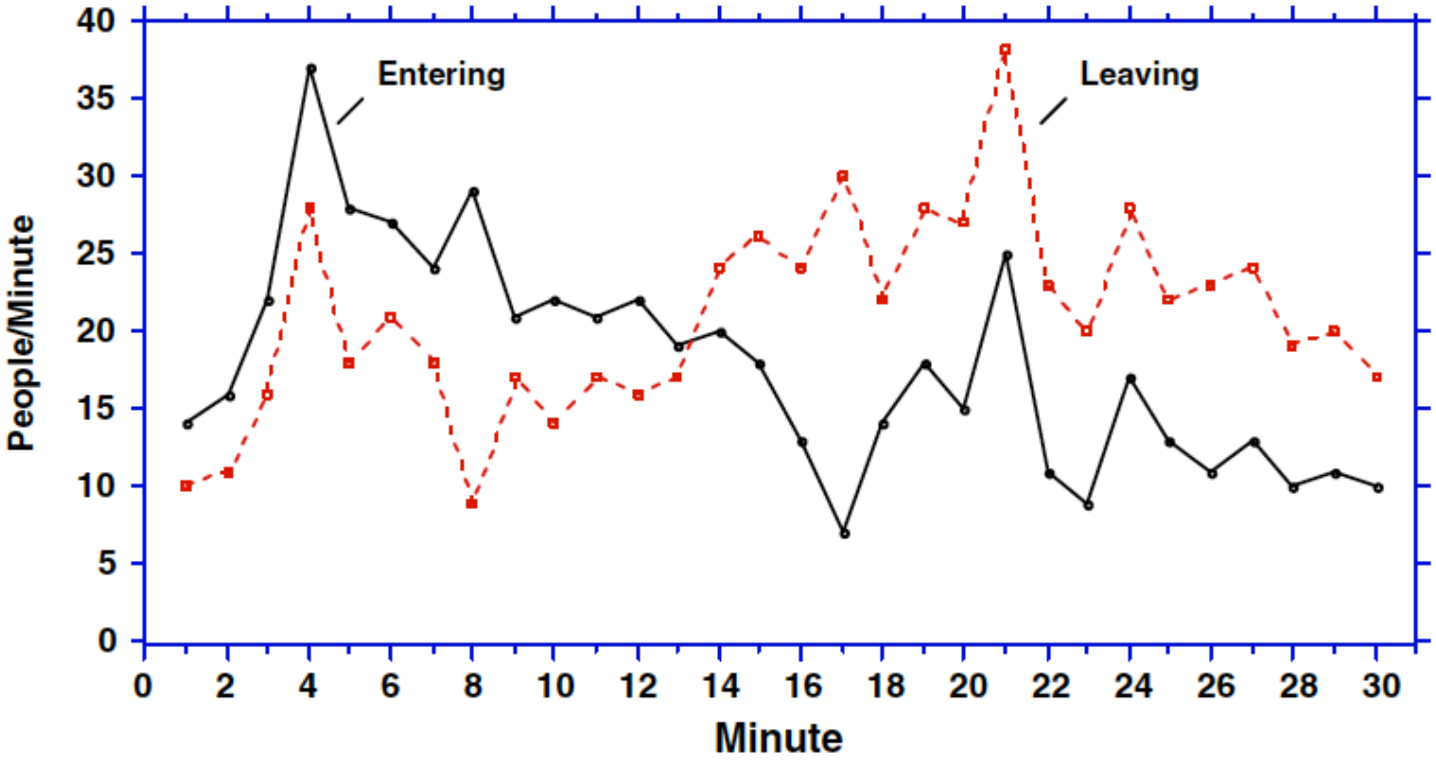
Question 2



2. During which minute did the most people leave the store?

Minute _________ ❑ Can’t be determined

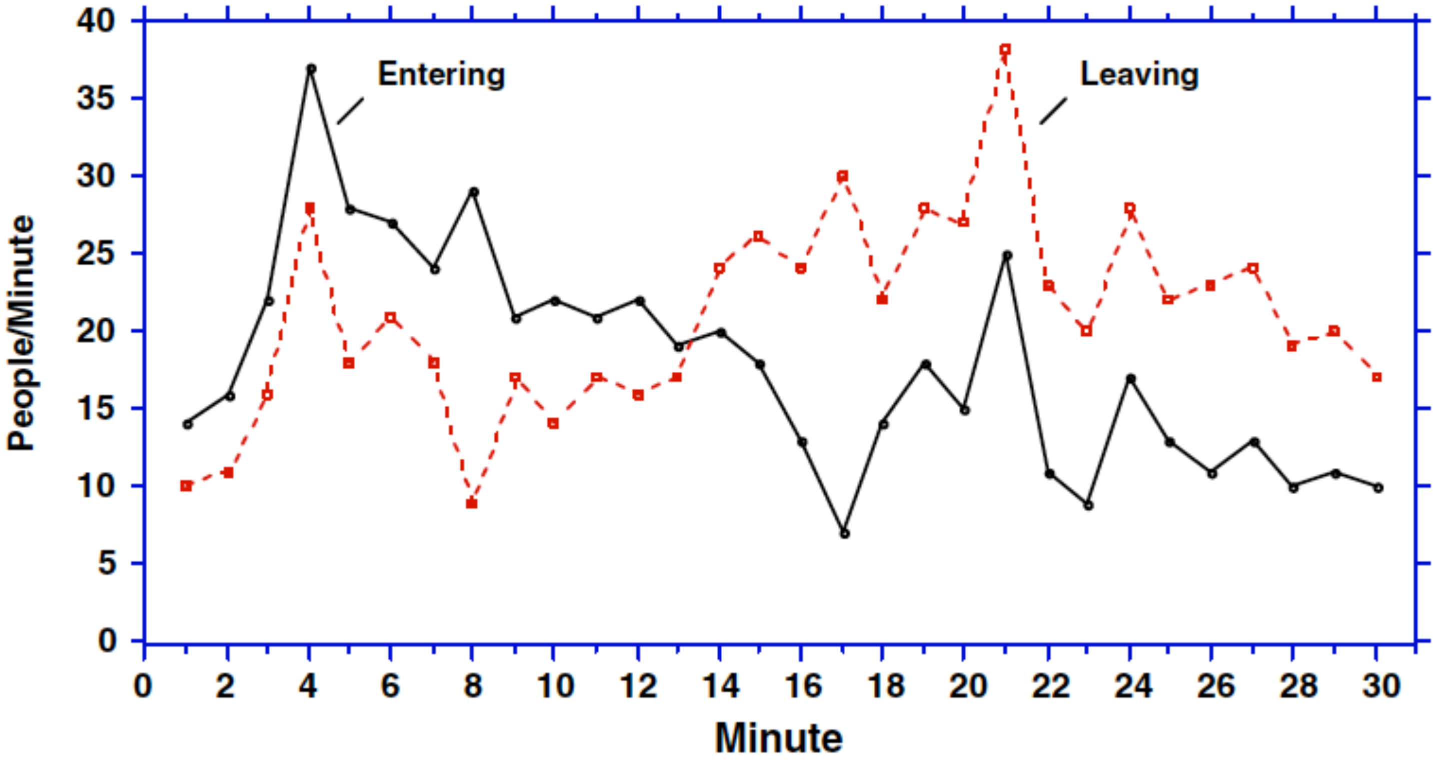
Question 3



3. During which minute were the most people in the store?

Minute __________ ❑ Can't be determined

Question 4

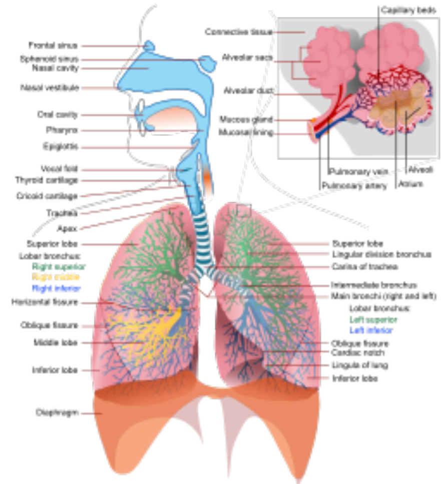


4. During which minute were the fewest people in the store?

Minute ________ ❑ Can't be determined

What is a system?

- A **system** is an interconnected set of elements that is coherently organized in a way that achieves something.
- It must consist of three kinds of things:
  - Elements
  - Interconnections
  - Function/Purpose



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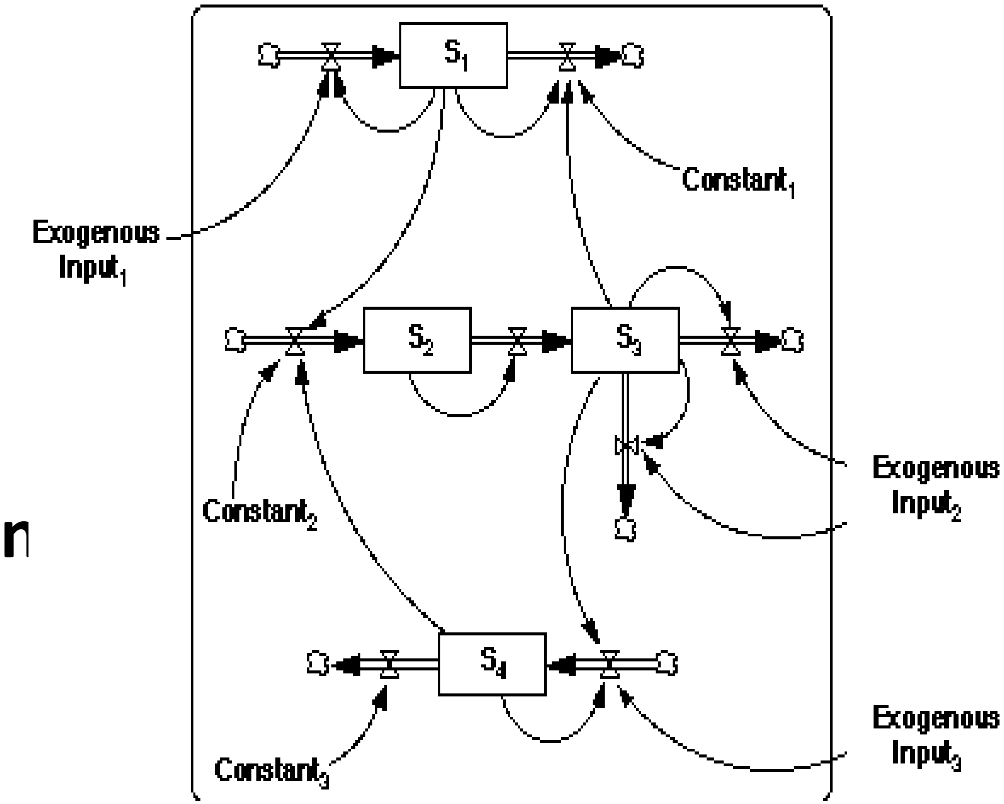

Diagram source: J.D. Sterman,Business Dynamics: Copyright © 2001 by the McGraw-Hill Companies

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Challenge 1.3

- Where are the stocks in a University System?
- For each stock
  - What is the inflow
  - What is the outflow

References

- Meadows, Donella H. *Thinking in systems: A primer*. Chelsea Green Publishing, 2008.
- Matthew A. Cronin, Cleotide Gonzalez and John D. Sterman. 2009. “Why don’t well-educated adults understand accumulation? A challenge to researchers, educators and citizens.” *Organization Behavior and Human Decision Processes.* 108. (2009). 116-130.
- Sterman, J. D. (2000). Business dynamics: systems thinking and modeling for a complex world. Boston: Irwin/McGraw-Hill.

SOLUTIONS

Solution to 1.1

$$\int t^n \; dt = \frac{1}{n+1}\, t^{n+1} + c$$

$$\frac{dy}{dt} = 4t$$

$$y = \int 4t \, dt = 4 \int t \, dt = 4 \left(\frac{1}{2}\right) t^2 + c$$

$$y_{10} = (100) + (2 \times 10^2) - (2 \times 0^2) = 300$$