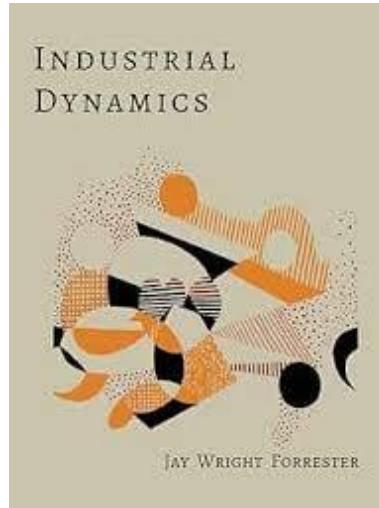


Data Science for Operational Researchers Using R Online

9. Operations Research Examples

Prof. Jim Duggan,
School of Computer Science
University of Galway.

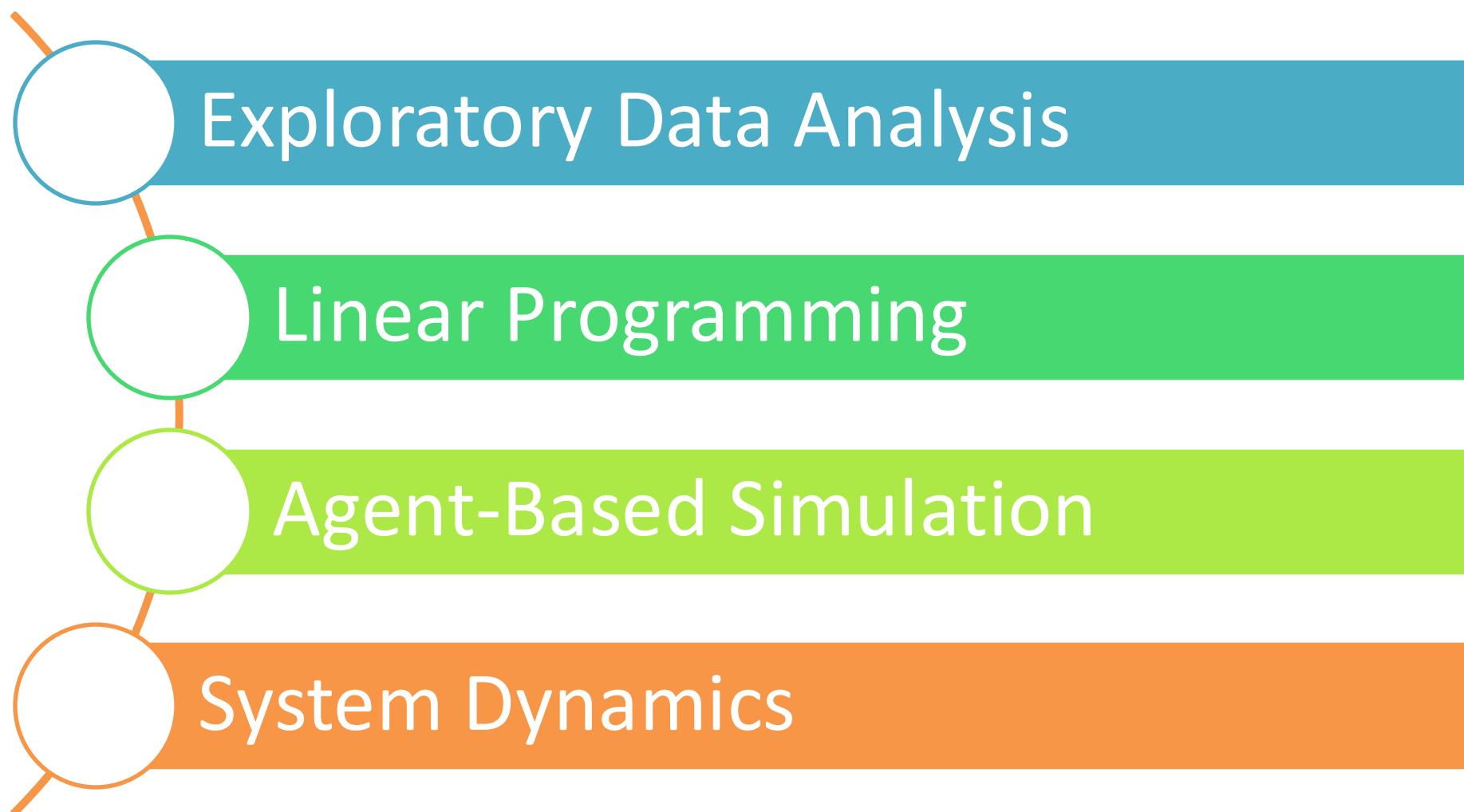
https://github.com/JimDuggan/explore_or



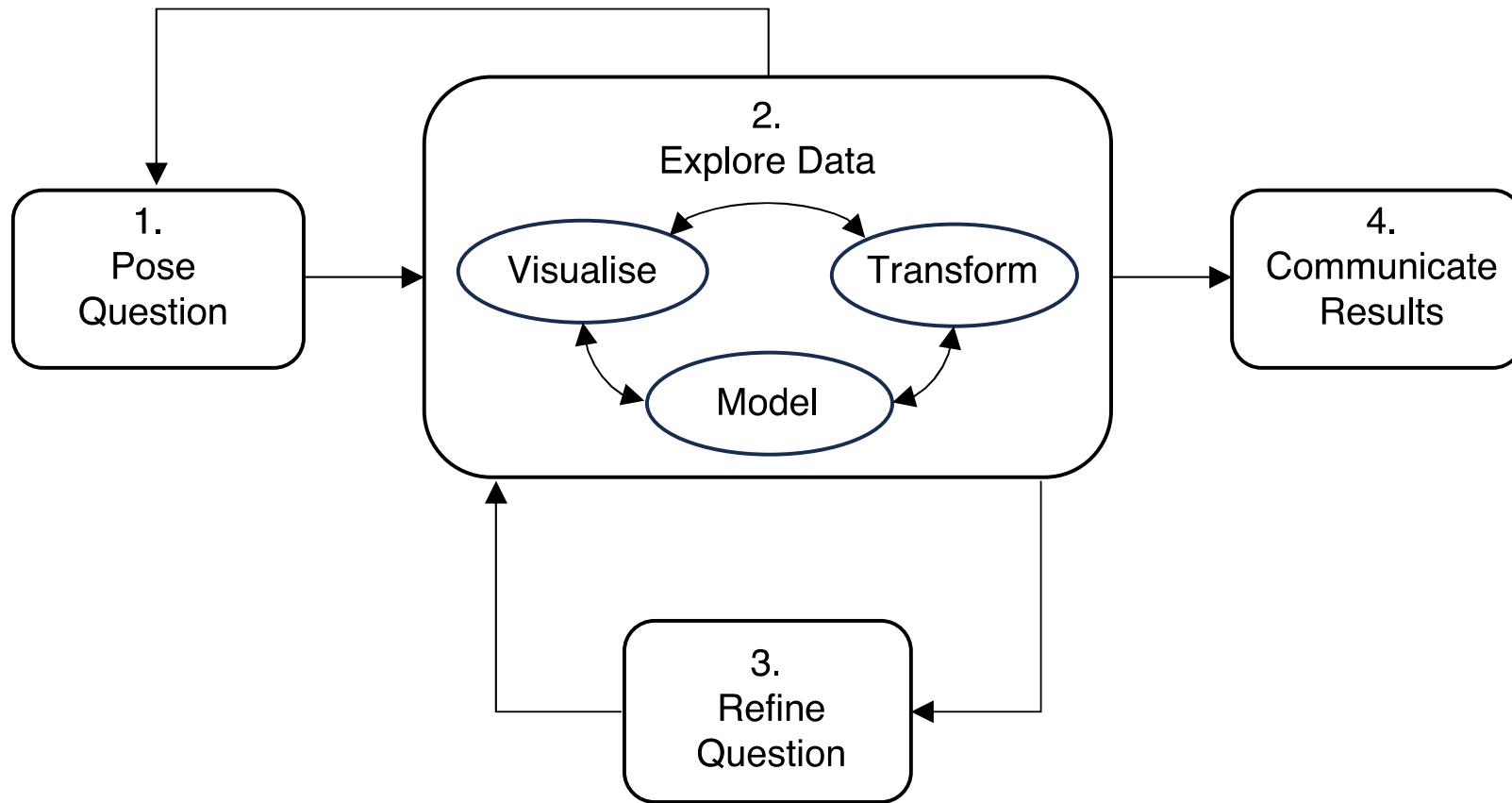
Management is the process of converting information into action.
The conversion process we call decision making.

— Jay W. Forrester (Forrester, 1961)

OR Examples



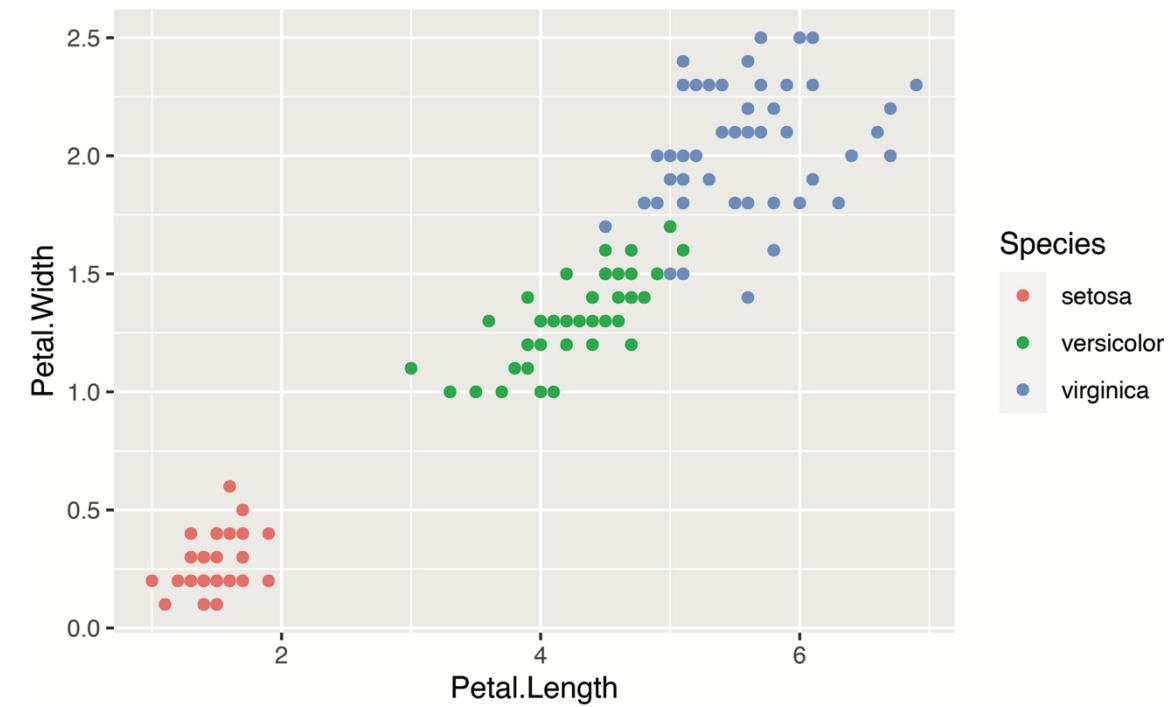
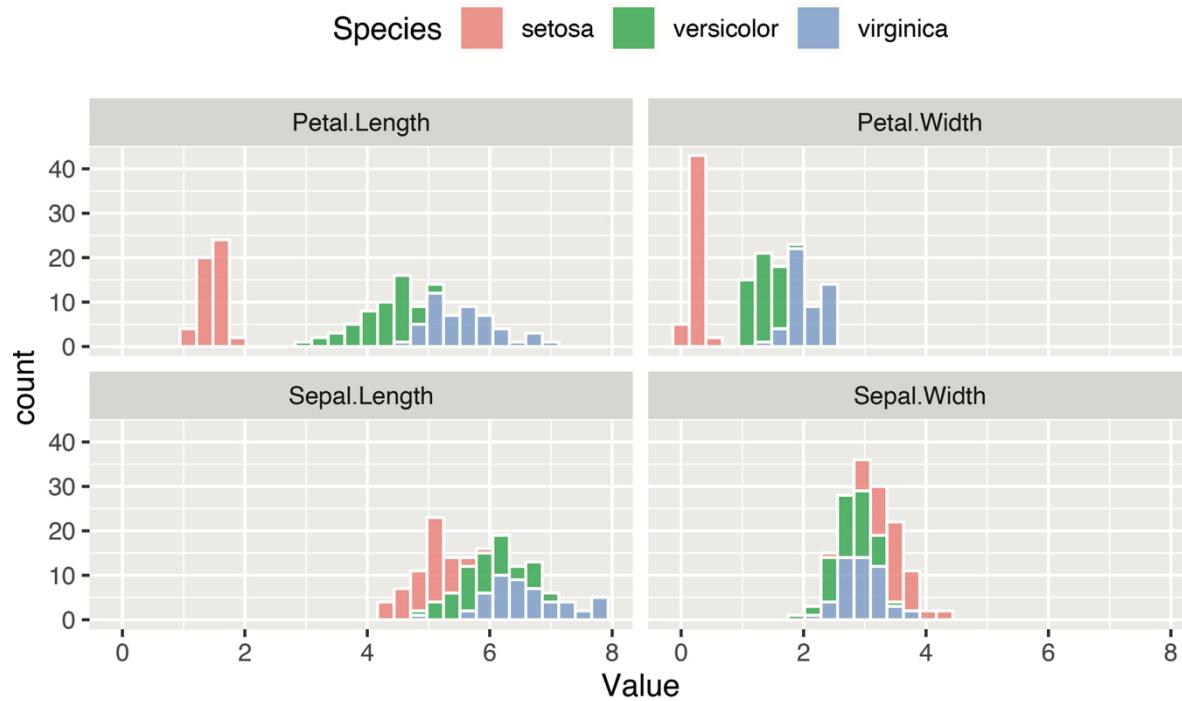
1. Exploratory Data Analysis



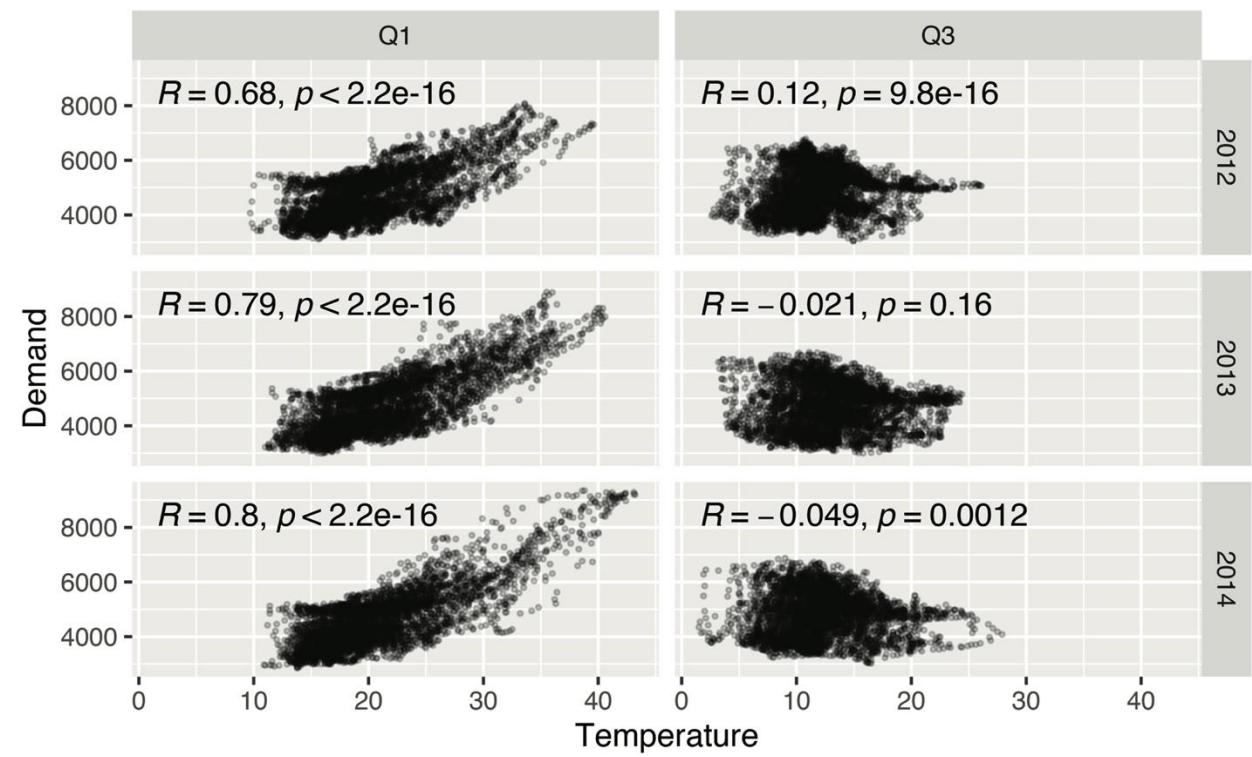
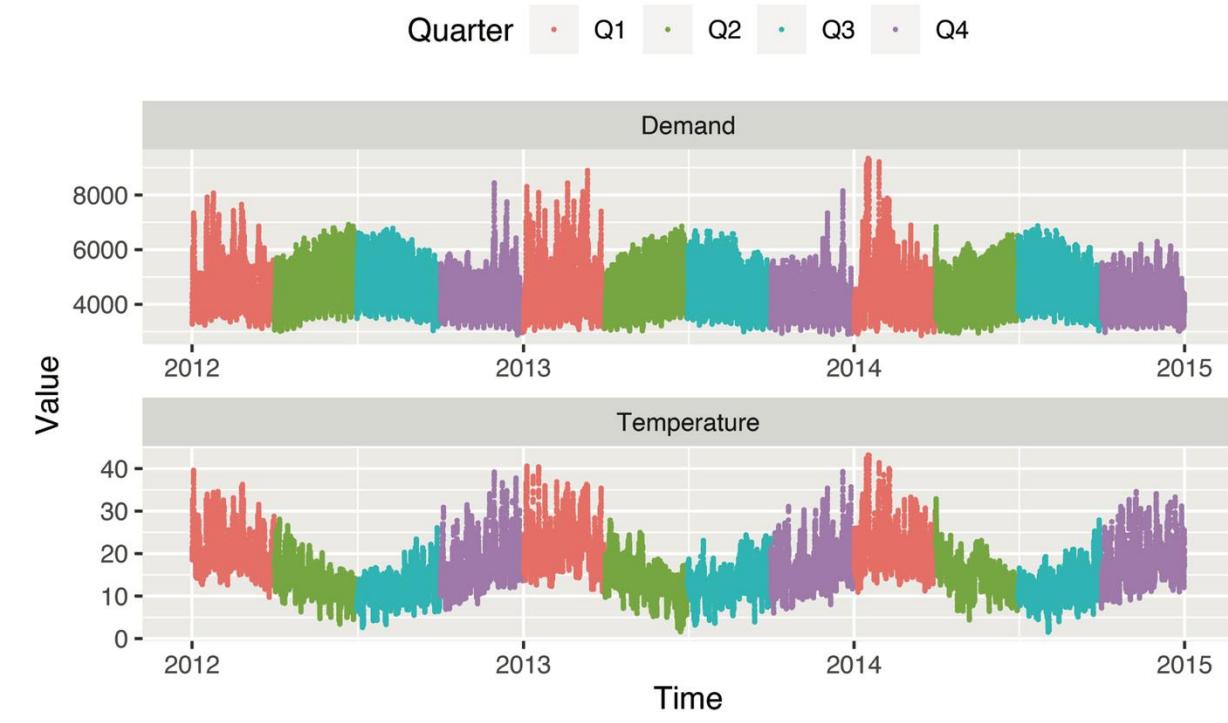
Questions posed...

- Can a plant's dimensions help us uniquely identify a species?
- Based on observations from 2012, 2013, and 2104, is there a potential association between temperature and energy demand in the state of Victoria, Australia?
- For a Boston dataset from the 1970s, can we find possible relationships between house value and pupil–teacher ratios across different suburbs?
- What passengers had the greatest chance of survival on the Titanic?
- During the winter season in Ireland, is there initial evidence to suggest that wind direction has an impact on temperature?

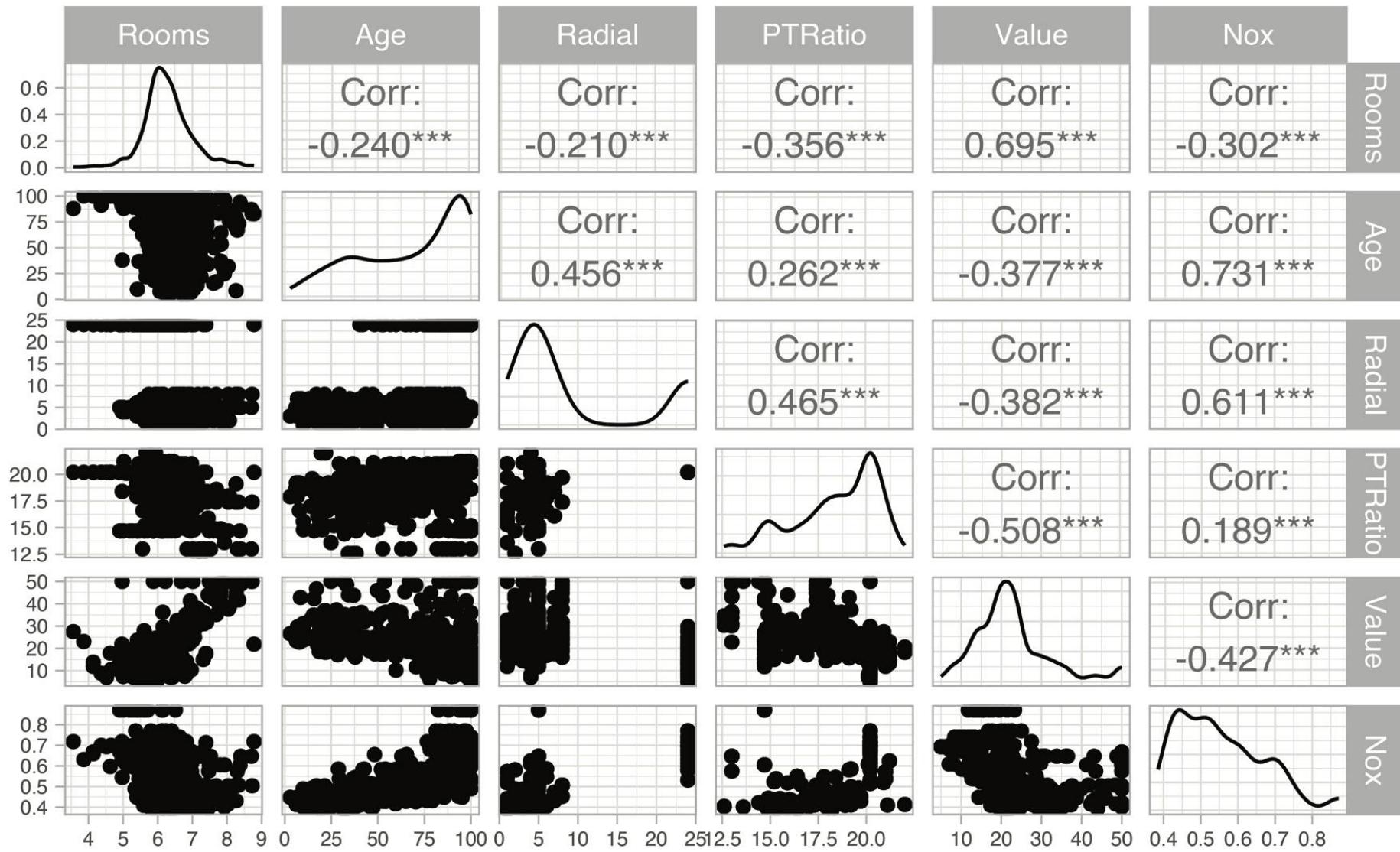
(1) Iris Data Set



(2) Electricity Demand (Victoria)



(3) Boston Data



(4) Titanic Survival Data

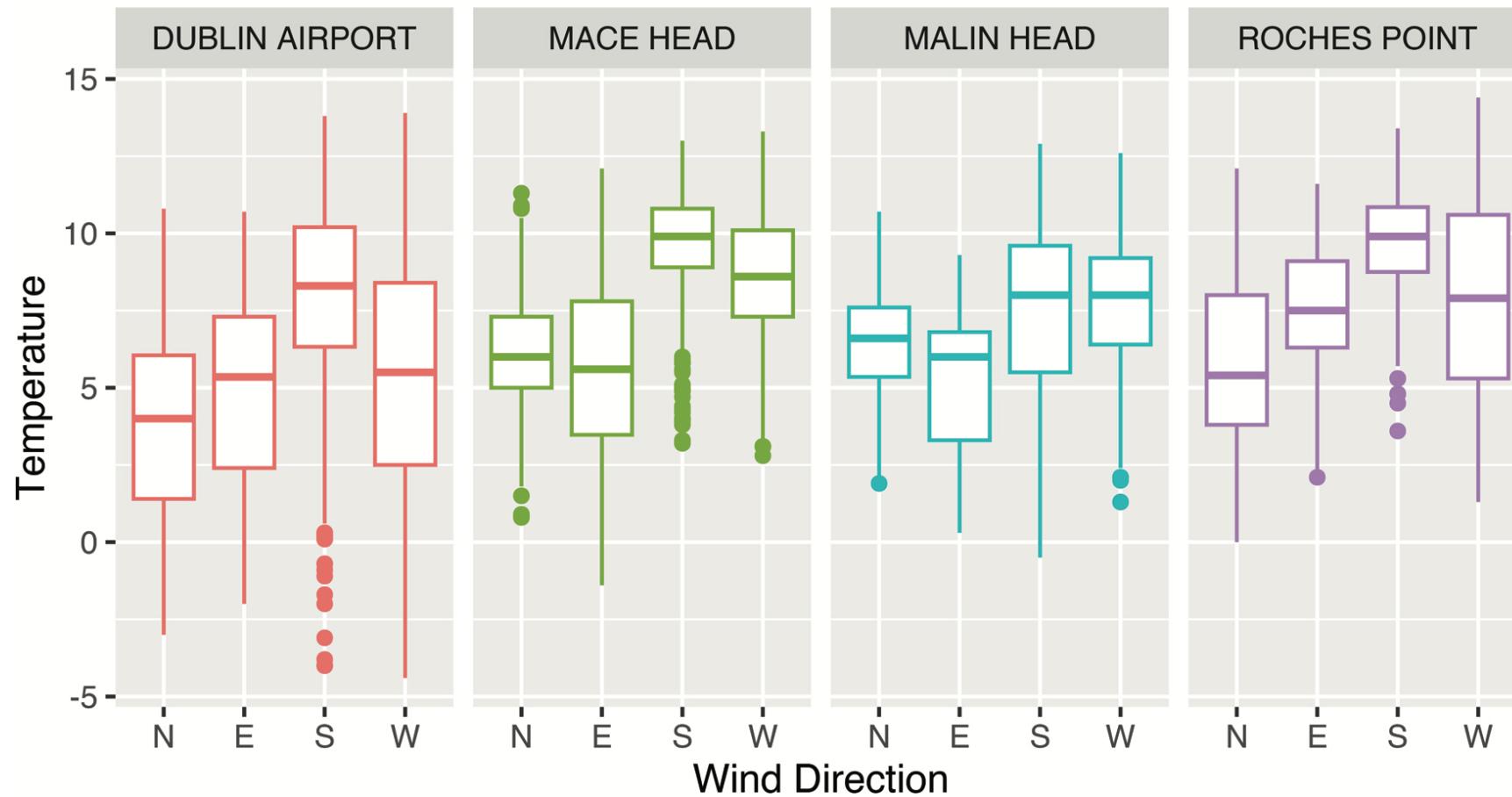
Survival outcomes on the Titanic



(5) Irish Weather Analysis (Winter)

Winter temperatures at weather stations

Data summarized by wind direction



(2) Linear Programming

“The success of an OR technique is ultimately measured by the spread of its use as a decision making tool. Ever since its introduction in the late 1940s, linear programming (LP) has proven to be one of the most effective operations research tools.”

Hamdy A. Taha ([Taha, 1992](#))

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$

⋮

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

and where

$$x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0$$

The Reddy Mikks example (Taha 1995)

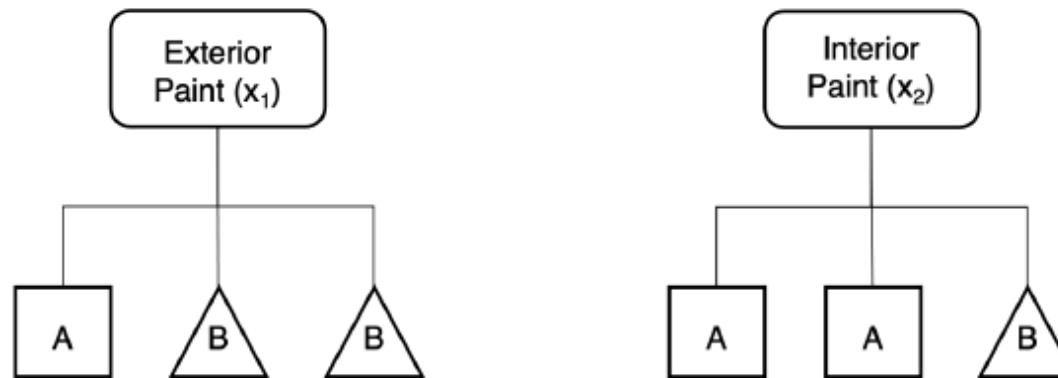


FIGURE 13.1 Raw materials needed (ton) for one ton of interior and exterior paints

Further to these manufacturing details, additional information is available through a market survey:

- The daily demand for interior paint cannot be greater than demand for exterior paint by more than one ton
- The maximum demand for interior paint is limited to two tons per day
- The wholesale price for exterior paint is \$3000 per ton, while the price for interior paint is \$2000 per ton.

In summary, the full mathematical model for this optimisation problem can be defined as follows:

To determine the tons of exterior and interior paint to be produced, expressed as the objective function

$$z = 3x_1 + 2x_2$$

Subject to the following constraints:

$$x_1 + 2x_2 \leq 6$$

$$2x_1 + x_2 \leq 8$$

$$x_2 + x_1 \leq 1$$

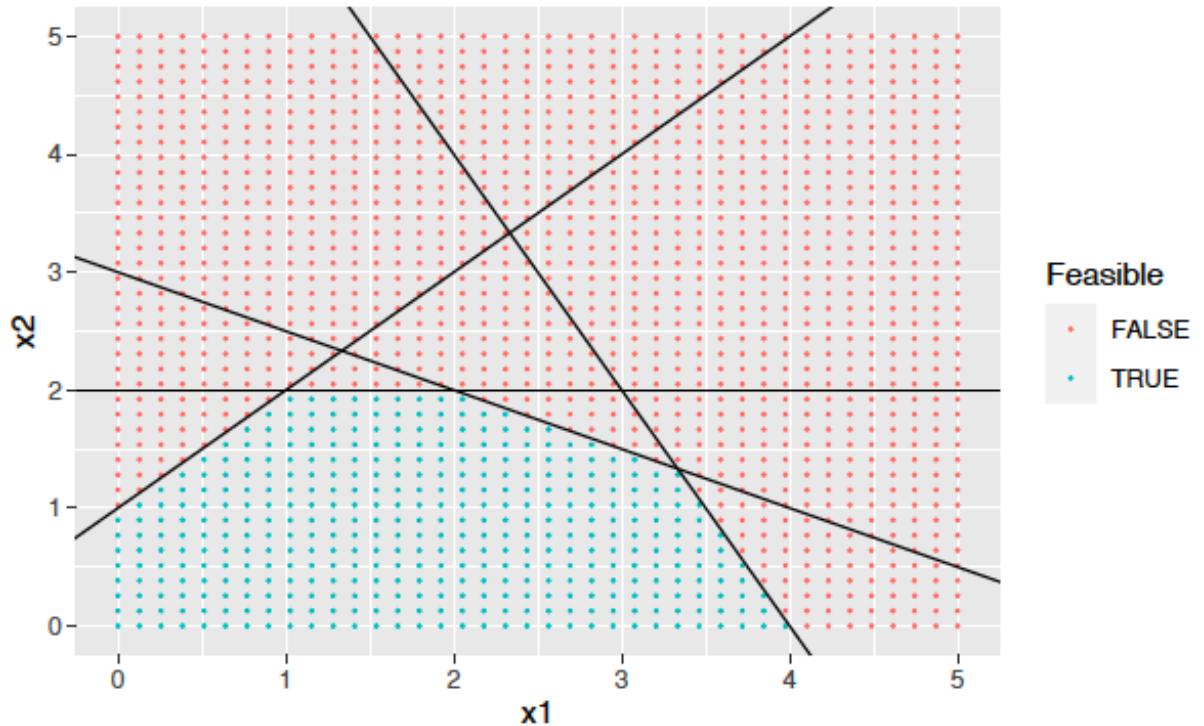
$$x_2 \leq 2$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

We will now focus on solving this problem, and the first step is to explore a range of feasible solutions using R.

13.4 Exploring a two-variable decision space using R



```
eql <- c("<=", "<=", "<=", "<=")
eql
#> [1] "<=" "<=" "<=" "<="
```

Finally, the right hand side values for the constraints are specified, with four values provided, one for each constraint.

```
rhs <- c(6,8,1,2)
rhs
#> [1] 6 8 1 2
```

$$z = 3x_1 + 2x_2$$

Subject to the following constraints:

$$x_1 + 2x_2 \leq 6$$

$$2x_1 + x_2 \leq 8$$

$$x_2 + x_1 \leq 1$$

$$x_2 \leq 2$$

With these values specified, we can now call the function `lp()` to generate an optimal solution.

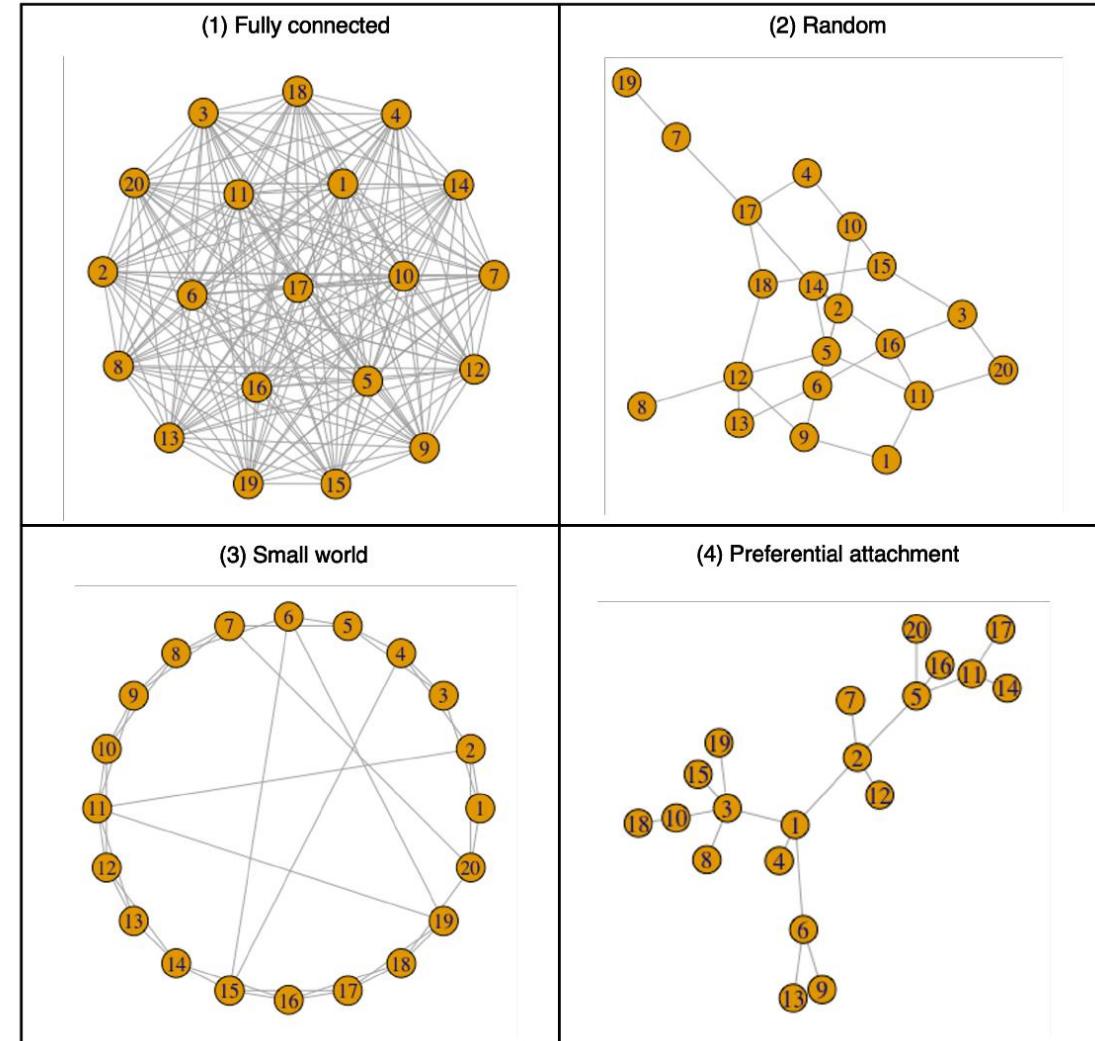
```
opt <- lp("max", z, cons, eql, rhs)
# Show the status
opt$status
#> [1] 0
# Show the objective function value
opt$objval
#> [1] 12.67
# Show the solution points

opt$solution
#> [1] 3.333 1.333
```

(3) Agent Based Simulation

Situate an initial population of autonomous heterogeneous agents in a relevant spatial environment; allow them to interact according to simple local rules, and thereby generate – or “grow” - the macroscopic regularity from the bottom up.

Joshua M. Epstein ([Epstein, 2012](#))



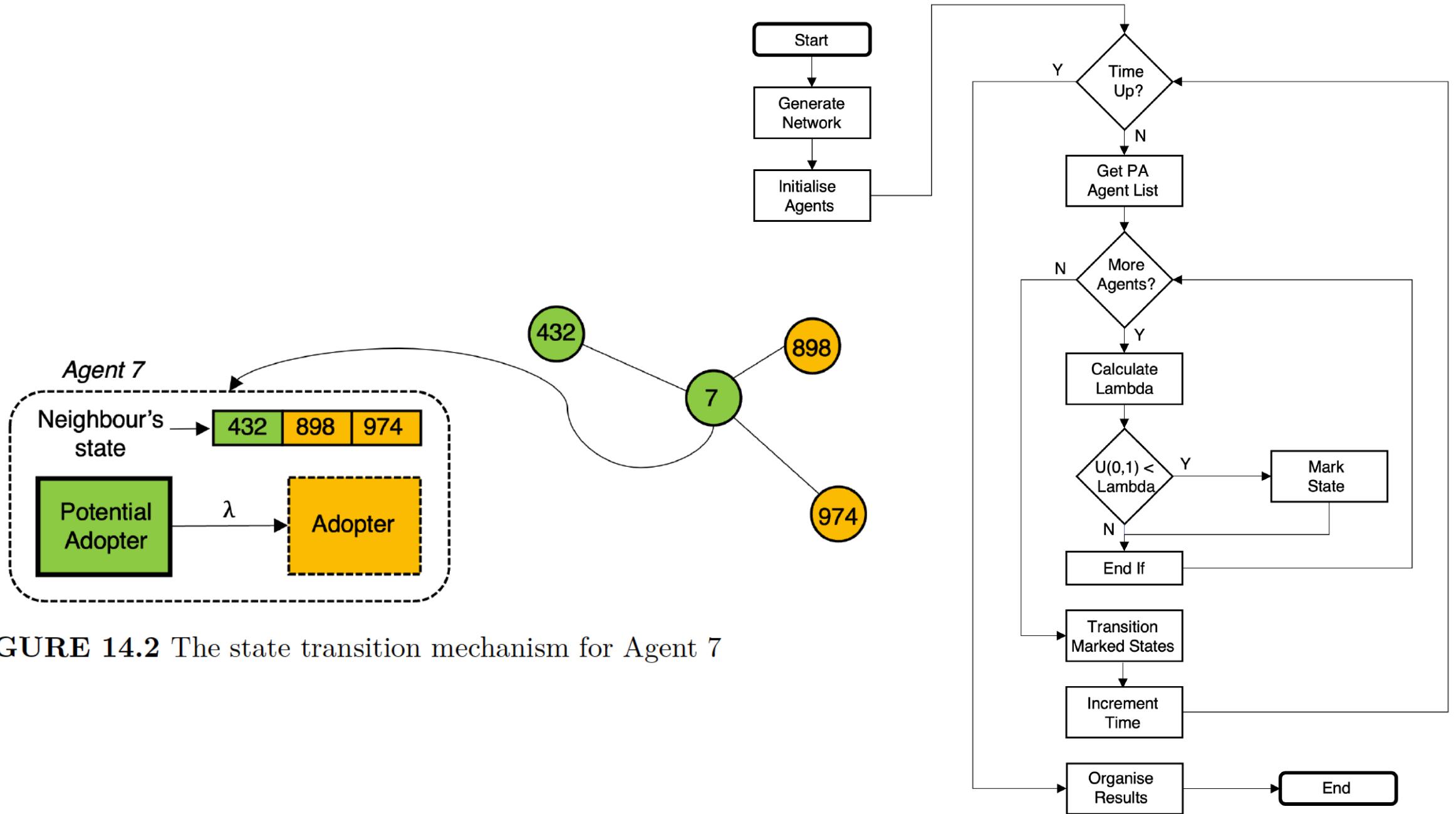
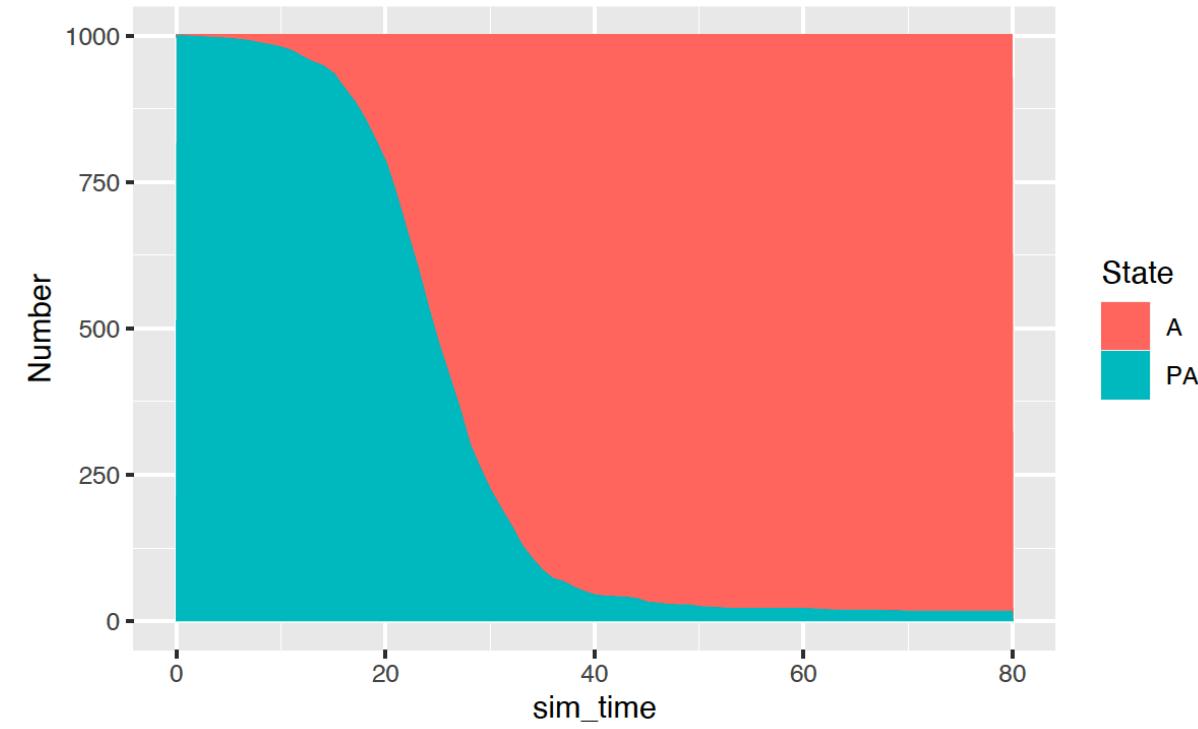
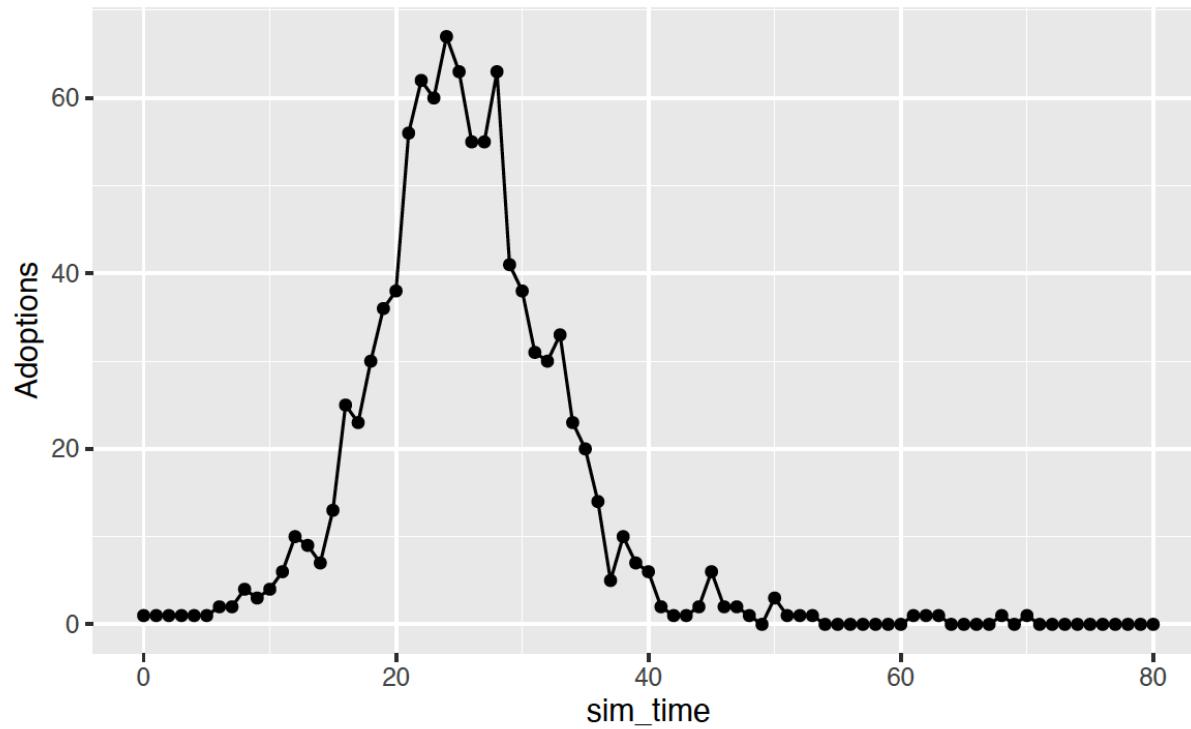
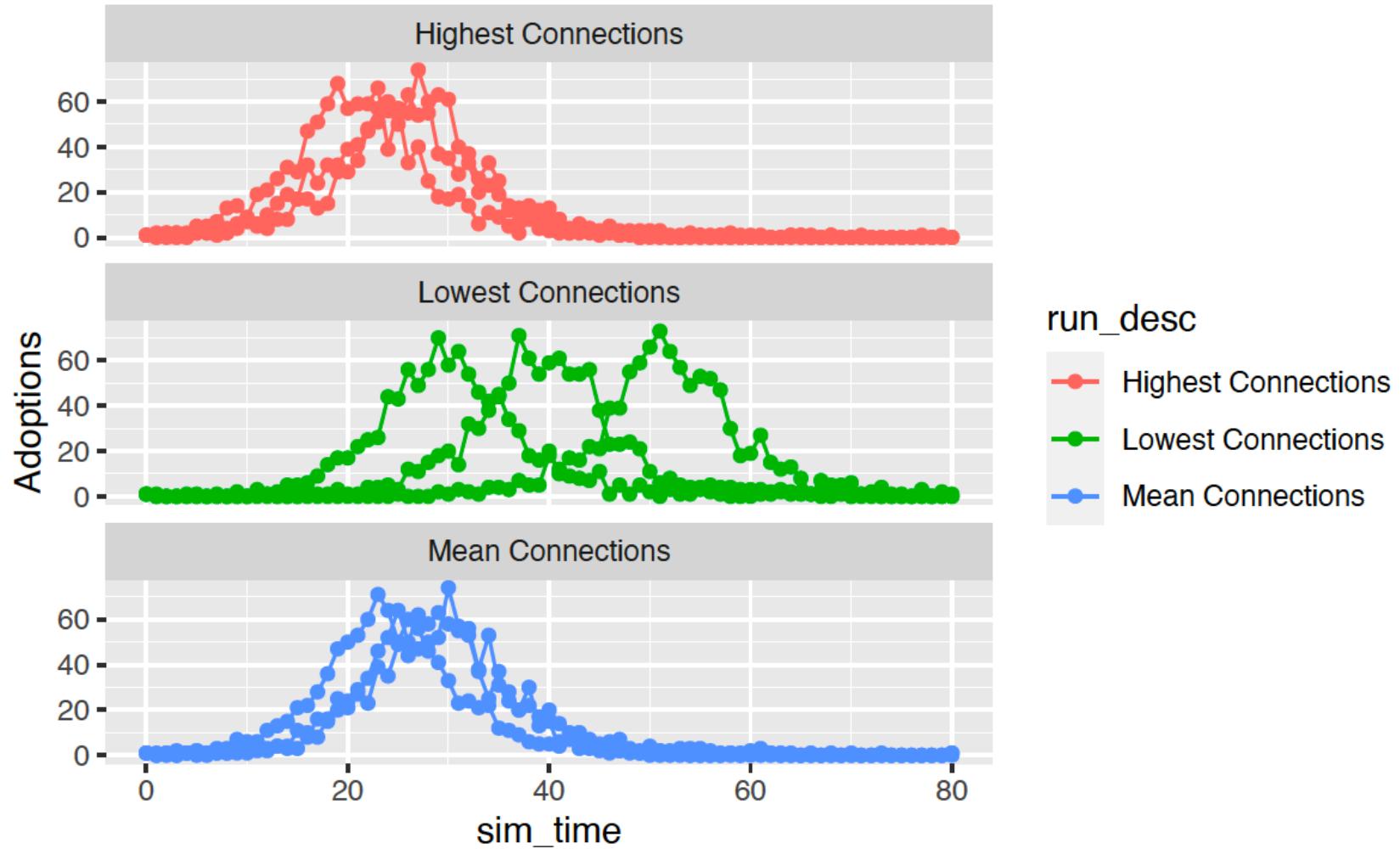


FIGURE 14.2 The state transition mechanism for Agent 7

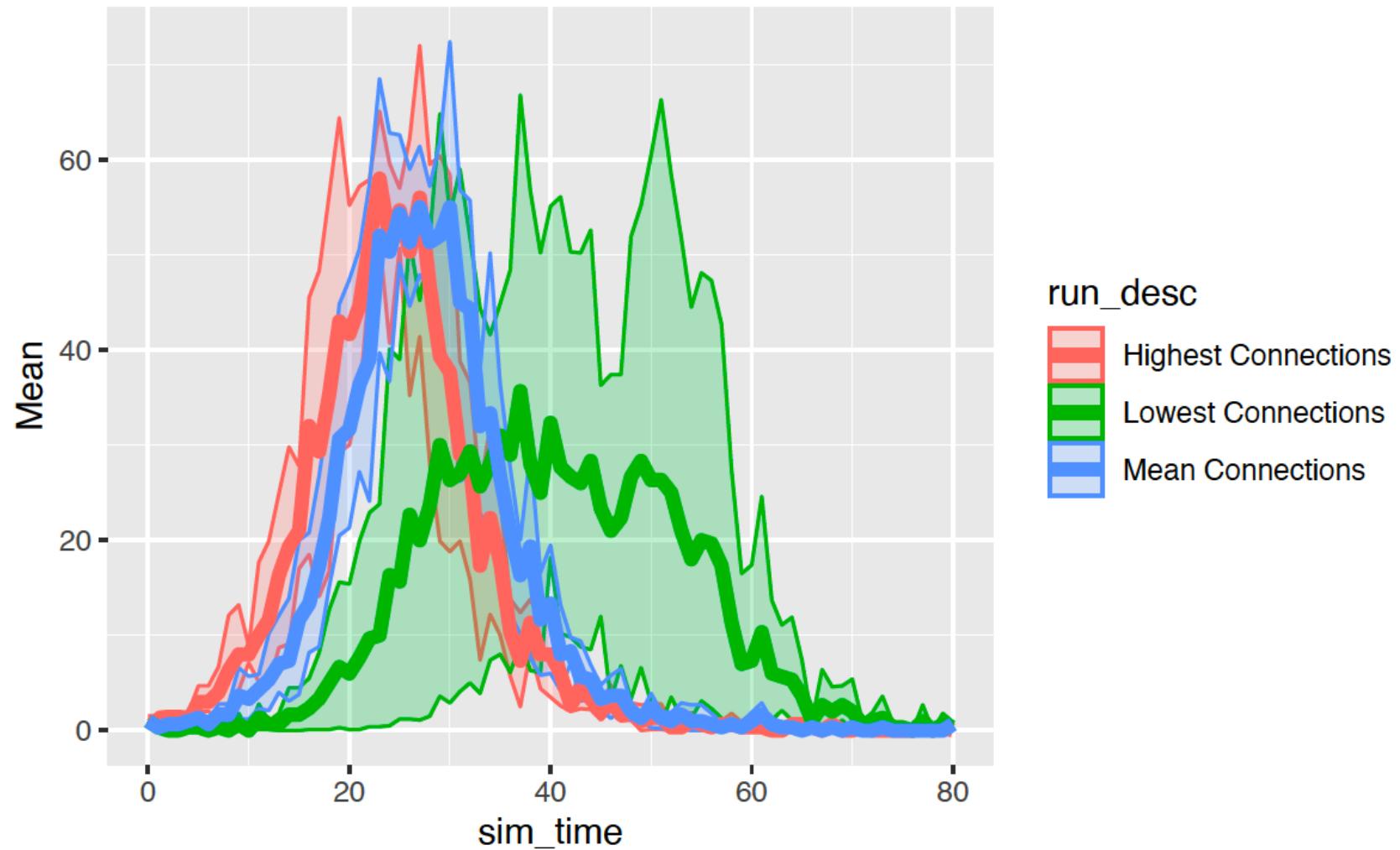
A single run...



Three Scenarios (Different seeded individual)



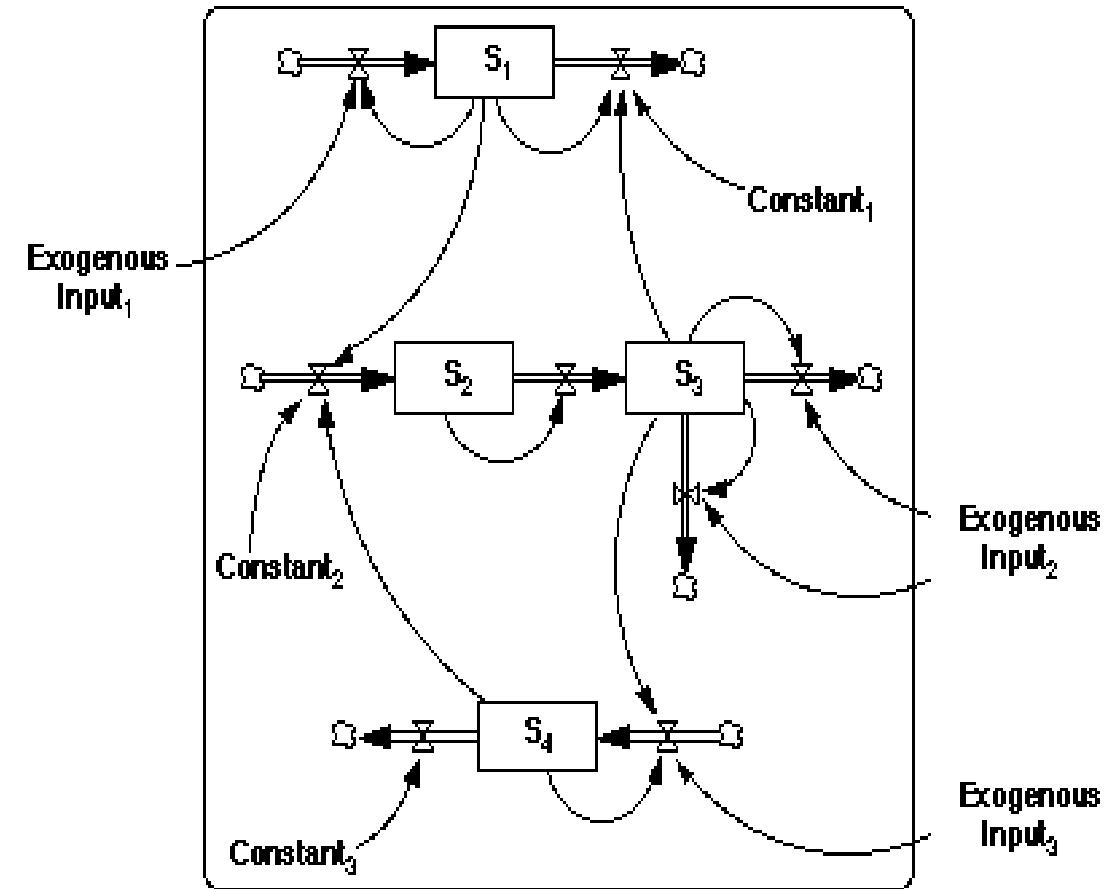
Quantile analysis



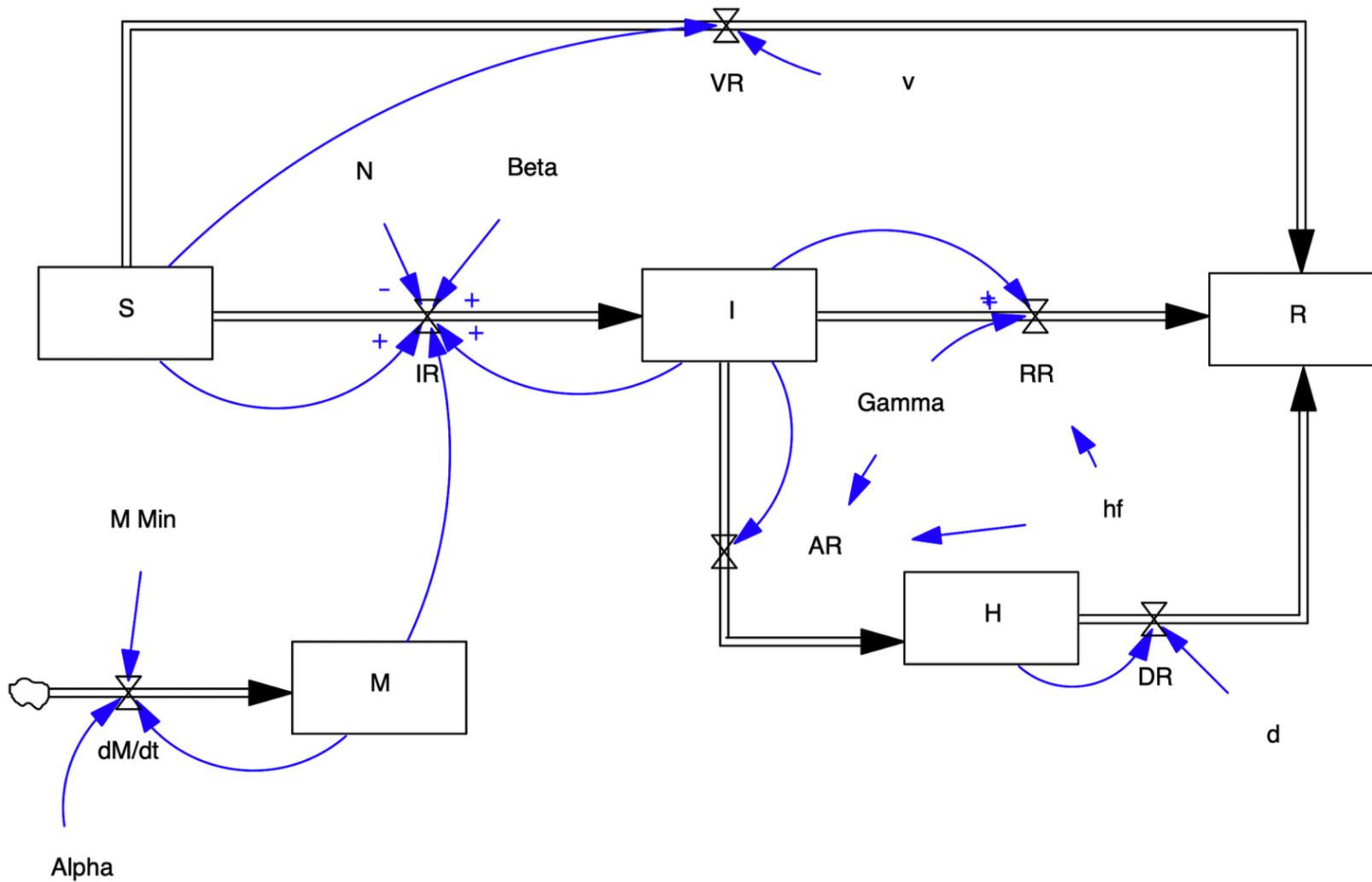
(4) System Dynamics

“Everything we do as individuals, as an industry, or as a society is done in the context of an information-feedback system.”

- Jay W. Forrester ([Forrester, 1961](#))



SIRH Model



$$\frac{dS}{dt} = -IR - VR \quad (17)$$

$$\frac{dI}{dt} = IR - RR - AR \quad (18)$$

$$\frac{dH}{dt} = AR - DR \quad (19)$$

$$\frac{dR}{dt} = RR + DR + VR \quad (20)$$

$$\frac{dM}{dt} = \alpha (M_{min} - M) \quad (21)$$

$$IR = \beta M I \frac{S}{N} \quad (22)$$

$$VR = \nu S \quad (23)$$

$$AR = h_f I \gamma \quad (24)$$

$$RR = (1 - h_f) I \gamma \quad (25)$$

$$DR = H d \quad (26)$$

$$\beta = 1.0 \quad (27)$$

$$\gamma = 0.25 \quad (28)$$

$$\nu = 0.1 \quad (29)$$

$$h_f = 0.1 \quad (30)$$

$$N = 10,000 \quad (31)$$

$$d = 0.10 \quad (32)$$

$$\alpha = 0.5 \quad (33)$$

$$S_{INIT} = 9999 \quad (34)$$

$$I_{INIT} = 1 \quad (35)$$

$$H_{INIT} = 0 \quad (36)$$

$$R_{INIT} = 0 \quad (37)$$

$$M_{INIT} = 1 \quad (38)$$

$$M_{MIN} = 0.3 \quad (39)$$

```

sirh <- function(time, stocks, auxs){
  with(as.list(c(stocks, auxs)),{
    N <- S + I + R + H      # Eq (31)
    IR <- beta*I*S/N*M      # Eq (22)
    VR <- v*S                # Eq (23)
    AR <- hf*gamma*I          # Eq (24)
    RR <- (1-hf)*gamma*I     # Eq (25)
    DR <- d*H                  # Eq (26)

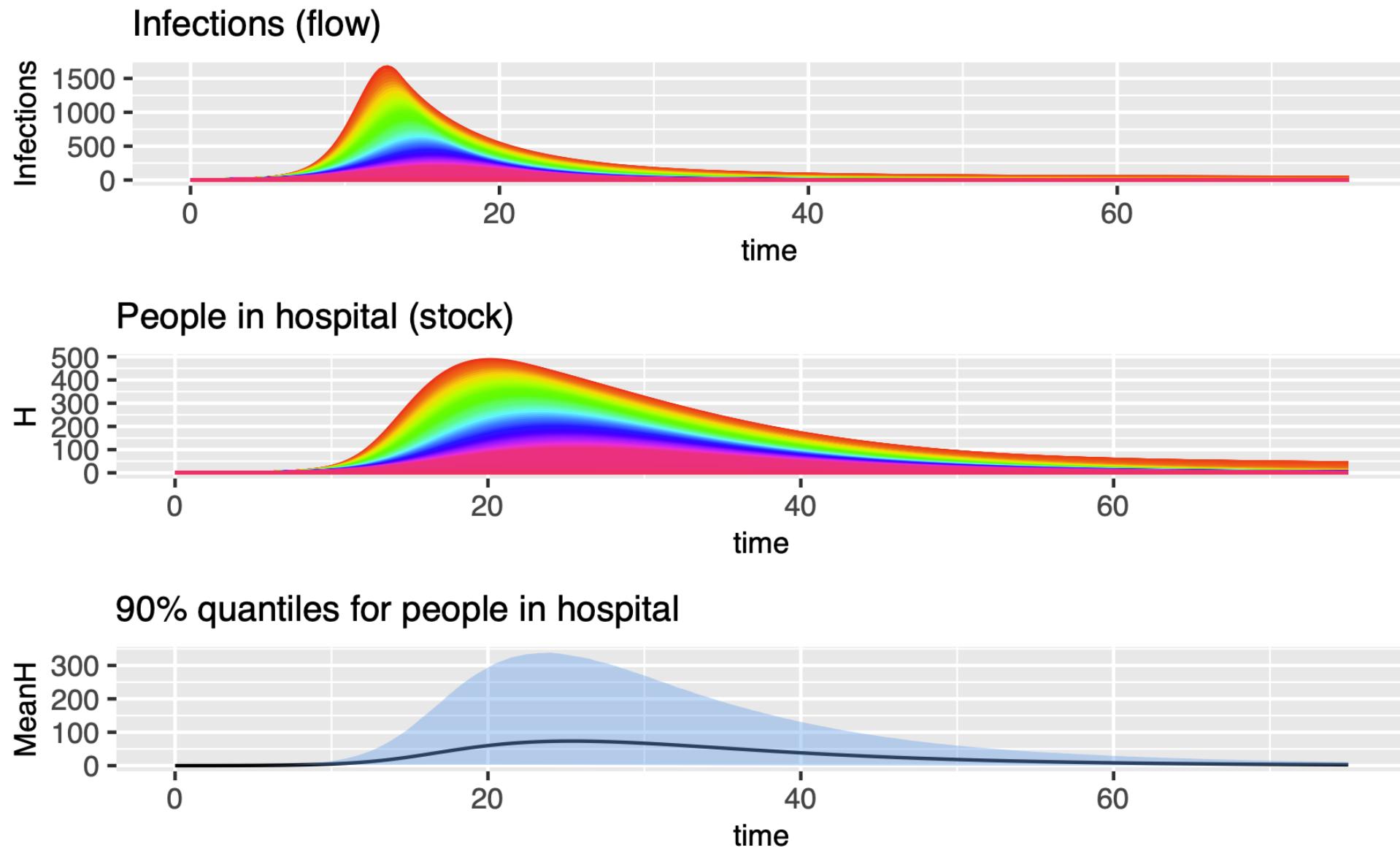
    dS_dt  <- -IR -VR        # Eq (17)
    dI_dt  <- IR - AR - RR   # Eq (18)
    dH_dt  <- AR - DR        # Eq (19)
  })
}

```

```

dR_dt  <- RR + DR + VR  # Eq (20)
dM_dt  <- (M_min - M) *
          alpha           # Eq (21)
return (list(c(dS_dt,dI_dt,dH_dt,dR_dt,dM_dt),
            Beta=beta,
            Gamma=gamma,
            HF=hf,
            V=v,
            Alpha=alpha,
            M_Min=M_min,
            Infections=IR,
            Recovering=RR,
            Vaccinated=VR,
            Hospitalised=AR,
            Discharged=DR,
            CheckSum=S + I + R + H))
}

```

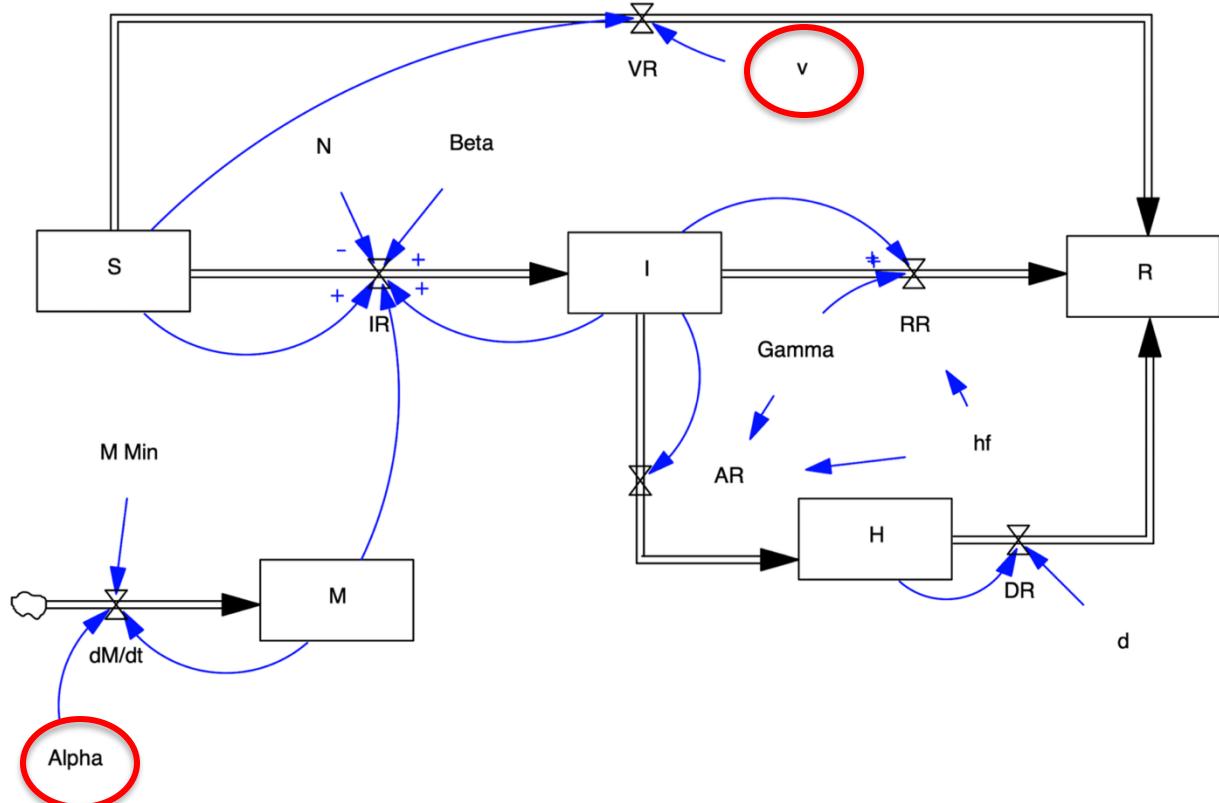


Exploring scenarios: 2 policy levers

The advantage of creating the function `run_scenario()` is that we can now call this for a range of parameter values. Here, we are going to sample two policy variables:

- α , which models the speed of mobility restriction implementations. For example, a higher value of α would mean that the population responds quickly to the request for social mobility reductions. In our simulations $0 \leq \alpha \leq 0.20$.
- v , which models the speed of vaccination. A higher value of v means that people transfer more quickly from S to R , and therefore the burden on the hospital sector should be reduced. In our simulations $0 \leq v \leq 0.05$, which indicates that the minimum vaccination duration is 20 days ($1/0.05$).

Speed of vaccination

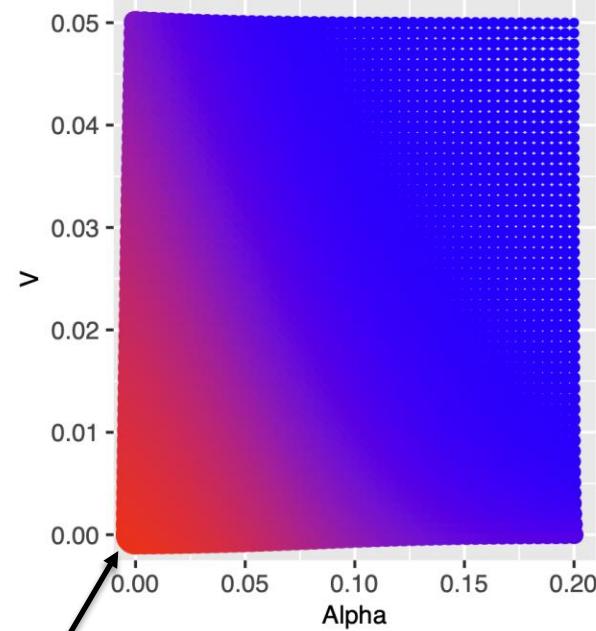


Speed of mobility reduction

*No vaccination,
no change in mobility = Max point*

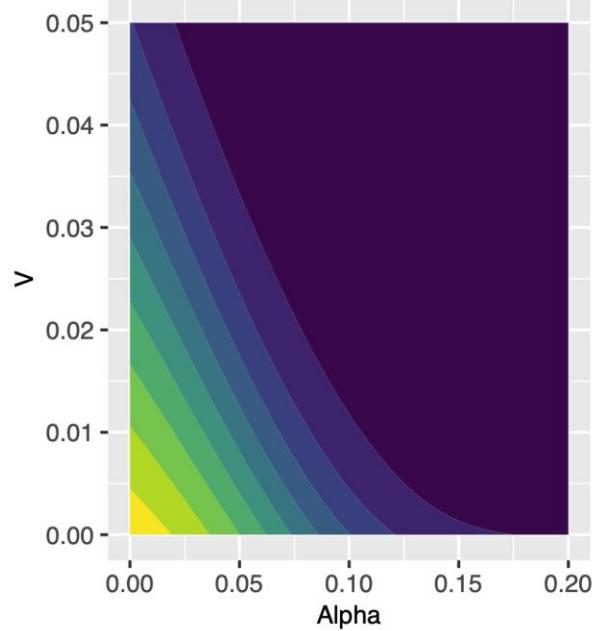
Parameter Analysis

Max peak = 487 at point (0,0)



Contour plot

Yellow band range (450,500]



Recap.

- R programming language can be a valuable tool – and way of thinking – which can be successfully applied to the field of operations research (OR).
- Application areas to OR:
 - Exploratory Data Analysis
 - Linear Programming
 - Agent-Based Simulation
 - System Dynamics

