Growth Arithmetic -- "hat rules" --

^ over a variable denotes the growth of the variable (continuous compound growth1);

1. $\hat{X}$ = Ln(Xend/Xstart)

from 100 to 102 ? ln(1.02)= ~ .0198 or 1.98% – or about 2%

100 to 105? 4.88% or 4.9% or 5%
* when growth is small, it is about the same as percentage change

100 to 150?	150 to 100	
+40.55%	-40.55%	total growth 0

% changes? +50% -33.3% total change is zero, so +50% - 33.3% = 0 yech!

2. average annual growth rate agr = X^ /#years -- simply growth averaged over the number of years.

pcGDP 1900 to 2000 (2009$) $6,004 $ 44,475 7.4 times as much in 100 years …
LN(44,475/6004)=2.003 growth of 2.003 in 100 years, average of 2.00 % per year ….

3. doubling is **69.3%** growth – Ln(2)~0.693 Re-arrange X^ = aag*t And time to double = 69.3%/agr

1% vs 3% vs 5% : 5 times as fast, takes 1/5 as long to achieve a growth ... doubling is 69.3% growth

Growth at 5% per year doubles in just under 14 years (5*14=70, just over 69.3).

20th C US pcGDP averaged 2 % per year, so doubled every 35 … quadrupled in a lifetime ...

4. growth of per capita output if output and pop both grow 2% ? 0 … substract

if c=a*b then c^=a^+b^ and c^-a^=b^ and c/a=b

thus pcGDP^ = GDP^ - Pop^

productivity (ρ) is defined as the ratio of output to input (y/z).
Measured productivity TFP=GDP/Z (total factor productivity)

Re-arrange to show that we can think of output as a product of two things GDP = ? Z * TFP ....
Next step: pcGDP = ? (Z/Pop) * TFP Growths ? pcGDP^ = (Z/Pop)^ + TFP^ Finally, growths ? pcy^ = (z/Pop)^ + TFP^

use model to interpret data on output and inputs – pcGDP^ = pcZ^ + TFP^

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1 the underlying model is $X_t = X_0 e^{rt}$