#### CSCI 5451: Introduction to Parallel Computing

**Lecture 12: MPI Collective Communications** 



#### Announcements (10/13)

- HW1
  - o Updated due date to Oct 15
  - o Logging is rather verbose. Make sure to scroll to the bottom.
  - o Only resubmissions are graded when the autograder is run
- ☐ HWs 2-5 are pushed back one week
- Group Formation due Oct 19 (<u>Canvas</u>)



#### Lecture Overview

- Non-Blocking MPI Communications
- Review of Collective Communication Patterns
- MPI Collective Communications
- ☐ Groups + Communicators
- Questions



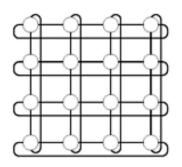
#### Lecture Overview

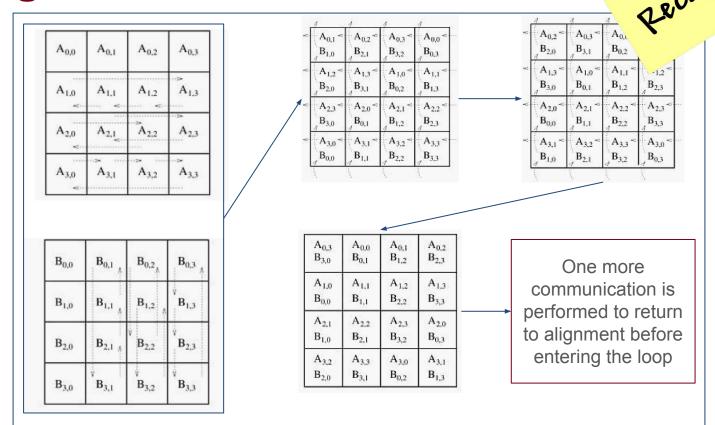
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## Cannon's Algorithm

2-d mesh with wraparound







# What is a downside of the current project to our implementation of Cannon's Algorithm?



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The CPU will idle during communication steps.



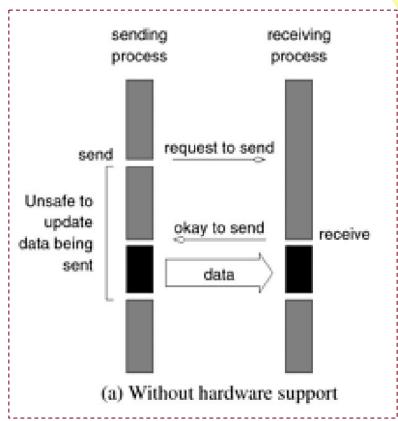
# What is a downside of the current project to our implementation of Cannon's Algorithm?

The CPU will idle during communication steps.

We can resolve this by overlapping communication + computation with non-blocking MPI calls.



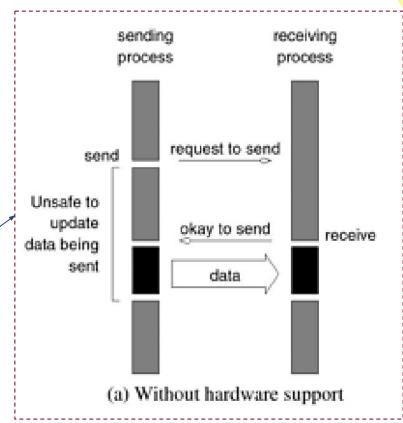
- A program using non-blocking Sends & Receives will immediately continue after finishing the Send or Receive function
- We do not have any guarantees that our program will not overwrite the data being sent
- This is dependent on whether or not the data will be buffered, and how that buffering occurs





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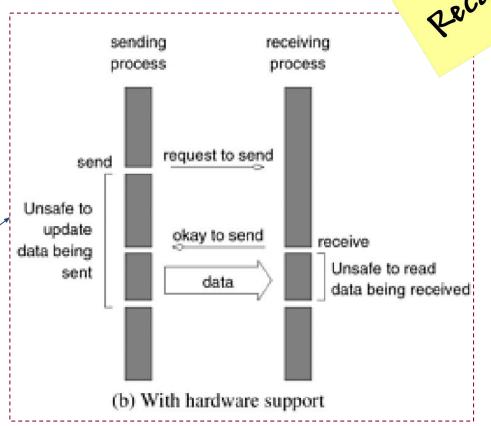
Non-Buffered





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Non-Buffered





- MPI\_Isend & MPI\_Irecv enable non-blocking communication with MPI
- With hardware support, these operations enable us to perfectly overlap communication and computation
- Without hardware support, we remove any idling, but the CPU must still participate in the network communication
- MPI\_Isend may be buffered, MPI\_Irecv is not
- As such, we must be careful when structuring program execution to not overwrite the buffers

```
int MPI_Isend(void *buf, int count,

MPI_Datatype datatype,

int dest, int tag, MPI_Comm comm,

MPI_Request *request)

int MPI_Irecv(void *buf, int count,

MPI_Datatype datatype,

int source, int tag, MPI_Comm comm,

MPI_Request *request)
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Same arguments as used in MPI\_Send & MPI\_Recv



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MPI_Datatype datatype,

int source, int tag, MPI_Comm comm,

MPI_Request *request)
```

The *request* object allows us to monitor for when the corresponding send or receive has completed (we will explore this more in upcoming slides)



```
if (myrank == 0) {
    MPI_Send(a, 10, MPI_INT, 1, 1, MPI_COMM_WORLD);
    MPI_Send(b, 10, MPI_INT, 1, 2, MPI_COMM_WORLD);
}
else if (myrank == 1) {
    MPI_Recv(b, 10, MPI_INT, 0, 2, &status, MPI_COMM_WORLD);
    MPI_Recv(a, 10, MPI_INT, 0, 1, &status, MPI_COMM_WORLD);
}
...
```

#### Eliminates deadlocks

```
if (myrank == 0) {
    MPI_Send(a, 10, MPI_INT, 1, 1, MPI_COMM_WORLD);
    MPI_Send(b, 10, MPI_INT, 1, 2, MPI_COMM_WORLD);
}
else if (myrank == 1) {
    MPI_Irecv(b, 10, MPI_INT, 0, 2, &requests[0], MPI_COMM_WORLD);
    MPI_Irecv(a, 10, MPI_INT, 0, 1, &requests[1], MPI_COMM_WORLD);
}
...
```



- MPI\_Test & MPI\_Wait are helpful utilities to determine the status of a non-blocking MPI\_Isend or MPI\_Irecv call
- MPI\_Test is a non-blocking operation used to check whether the corresponding non-blocking communication has completed
- MPI\_Wait is a blocking operation used to halt further execution until the corresponding non-blocking communication has completed



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```
int MPI_Test (MPI_Request *request, int *flag,

MPI_Status *status)

int MPI_Wait (MPI_Request *request,

MPI_Status *status)
```

The **request** object - returned from the earlier **MPI\_ISend** or **MPI\_Irecv** calls



- MPI\_Test & MPI\_Wait are helpful utilities to determine the status of a non-blocking MPI\_Isend or MPI\_Irecv call
- MPI\_Test is a non-blocking operation used to check whether the corresponding non-blocking communication has completed
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A variable indicating whether or not the given communication operation has completed. This returns '1' if it has, '0' otherwise.



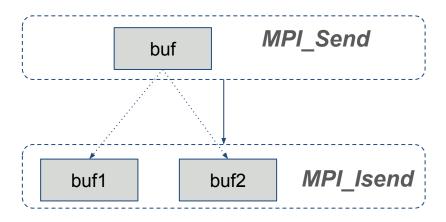
- MPI\_Test & MPI\_Wait are helpful utilities to determine the status of a non-blocking MPI\_Isend or MPI\_Irecv call
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The *status* of the communication operation. This is the same object returned by *MPI\_Revc*.



## Cannon's Algorithm in MPI

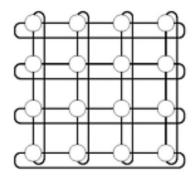
- We make the following changes to ensure that our MPI program works with non-blocking operations
  - Use MPI\_Isend, MPI\_Irecv, MPI\_Wait
  - Duplicate the local buffers a and b on each process
- In general, you will typically want to explicitly use duplicate buffers to ensure that one buffer is used for communication while another is used for computation



```
MatrixMatrixMultiply NonBlocking(int n, double *a, double *b,
                                 double *c, MPI Comm comm)
 int i, j, nlocal;
  double *a buffers[2], *b buffers[2];
  int npes, dims[2], periods[2];
  int myrank, my2drank, mycoords[2];
  int uprank, downrank, leftrank, rightrank, coords[2];
  int shiftsource, shiftdest;
 MPI Status status;
 MPI Comm comm 2d;
 MPI Request reqs[4];
 /* Get the communicator related information */
 MPI Comm size(comm, &npes);
 MPI Comm rank (comm, &myrank);
 /* Set up the Cartesian topology */
 dims[0] = dims[1] = sqrt(npes);
 /* Set the periods for wraparound connections */
 periods[0] = periods[1] = 1;
 /* Create the Cartesian topology, with rank reordering */
  MPI Cart create (comm, 2, dims, periods, 1, &comm 2d);
```



## 2-d mesh with wraparound



```
MatrixMatrixMultiply NonBlocking(int n, double *a, double *b,
                                 double *c, MPI Comm comm)
 int i, j, nlocal;
  double *a buffers[2], *b buffers[2];
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  int myrank, my2drank, mycoords[2];
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  MPI Cart create (comm, 2, dims, periods, 1, &comm 2d);
```



```
/* Get the rank and coordinates with respect to the new topology */
MPI Comm rank(comm 2d, &my2drank);
MPI Cart coords (comm 2d, my2drank, 2, mycoords);
/* Compute ranks of the up and left shifts */
MPI Cart shift (comm 2d, 0, -1, &rightrank, &leftrank);
MPI Cart shift(comm 2d, 1, -1, &downrank, &uprank);
/* Determine the dimension of the local matrix block */
nlocal = n/dims[0];
/* Setup the a buffers and b buffers arrays */
a buffers[0] = a;
a buffers[1] = (double *)malloc(nlocal*nlocal*sizeof(double));
b \text{ buffers}[0] = b;
b buffers[1] = (double *)malloc(nlocal*nlocal*sizeof(double));
/* Perform the initial matrix alignment. First for A and then for B */
MPI Cart shift(comm 2d, 0, -mycoords[0], &shiftsource, &shiftdest);
MPI Sendrecv replace(a buffers[0], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Cart shift(comm 2d, 1, -mycoords[1], &shiftsource, &shiftdest);
MPI Sendrecv replace(b buffers[0], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
```



$A_{0,0}$	A <sub>0,1</sub>	A <sub>0,2</sub>	A <sub>0,3</sub>
A <sub>1.0</sub>	A <sub>1,1</sub>	A <sub>1,2</sub>	A <sub>1,3</sub>
A <sub>2,0</sub>	A <sub>2,1,</sub>	A <sub>2,2</sub>	A <sub>2,3</sub>
A <sub>3,0</sub>	A <sub>3,1</sub>	A <sub>3,2</sub>	A <sub>3,3</sub>

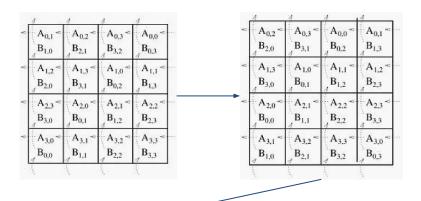
$\mathbf{B}_{0,0}$	B <sub>0,1</sub>	B <sub>0,2</sub>	B <sub>0,3</sub>
B <sub>1,0</sub>	B <sub>1,1</sub>	B <sub>1,2</sub>	$\mathbf{B}_{1,3}$
B <sub>2,0</sub>	В <sub>2,1 д</sub>	ў В <sub>2,2</sub>	B <sub>2,3</sub>
B <sub>3,0</sub>	B <sub>3,1</sub>	B <sub>3,2</sub>	B <sub>3,3</sub>

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/* Determine the dimension of the local matrix block */
nlocal = n/dims[0];
/* Setup the a buffers and b buffers arrays */
a buffers[0] = a;
a buffers[1] = (double *)malloc(nlocal*nlocal*sizeof(double));
b \text{ buffers}[0] = b;
b buffers[1] = (double *)malloc(nlocal*nlocal*sizeof(double));
/* Perform the initial matrix alignment. First for A and then for B */
MPI Cart shift(comm 2d, 0, -mycoords[0], &shiftsource, &shiftdest);
MPI Sendrecv replace(a buffers[0], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Cart shift(comm 2d, 1, -mycoords[1], &shiftsource, &shiftdest);
MPI Sendrecv replace(b buffers[0], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
```



```
/* Get into the main computation loop */
for (i=0; i<dims[0]; i++) {
  MPI Isend(a buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      leftrank, 1, comm 2d, &reqs[0]);
  MPI Isend(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      uprank, 1, comm 2d, &reqs[1]);
  MPI Irecv(a buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      rightrank, 1, comm 2d, &reqs[2]);
  MPI Irecv(b buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      downrank, 1, comm 2d, &regs[3]);
 /* c = c + a*b */
  MatrixMultiply(nlocal, a buffers[i%2], b buffers[i%2], c);
  for (j=0; j<4; j++)
    MPI Wait(&reqs[j], &status);
/* Restore the original distribution of a and b */
MPI Cart shift(comm 2d, 0, +mycoords[0], &shiftsource, &shiftdest);
MPI Sendrecv replace(a buffers[i%2], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Cart shift(comm 2d, 1, +mycoords[1], &shiftsource, &shiftdest);
MPI Sendrecv replace(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
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MPI Comm free (&comm 2d); /* Free up communicator */
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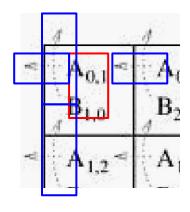


A <sub>0,3</sub>	$A_{0,0} \\ B_{0,1}$	A <sub>0,1</sub>	A <sub>0,2</sub>	
B <sub>3,0</sub>		B <sub>1,2</sub>	B <sub>2,3</sub>	
A <sub>1,0</sub>	A <sub>1,1</sub>	A <sub>1,2</sub>	A <sub>1,3</sub>	
B <sub>0,0</sub>	B <sub>1,1</sub>	B <sub>2,2</sub>	B <sub>3,3</sub>	
A <sub>2,1</sub>	A <sub>2,2</sub>	A <sub>2,3</sub>	A <sub>2,0</sub>	
B <sub>1,0</sub>	B <sub>2,1</sub>	B <sub>3,2</sub>	B <sub>0,3</sub>	
A <sub>3,2</sub>	A <sub>3,3</sub>	A <sub>3,0</sub>	A <sub>3,1</sub> B <sub>1,3</sub>	
B <sub>2,0</sub>	B <sub>3,1</sub>	B <sub>0,2</sub>		

One more communication is performed to return to alignment before entering the loop

```
/* Get into the main computation loop */
for (i=0; i<dims[0]; i++)
  MPI Isend(a buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      leftrank, 1, comm 2d, &reqs[0]);
  MPI Isend(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      uprank, 1, comm 2d, &reqs[1]);
  MPI Irecv(a buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      rightrank, 1, comm 2d, &reqs[2]);
  MPI Irecv(b buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      downrank, 1, comm 2d, &regs[3]);
  /* c = c + a*b */
  MatrixMultiply(nlocal, a buffers[i%2], b buffers[i%2], c);
  for (j=0; j<4; j++)
    MPI Wait(&reqs[j], &status);
/* Restore the original distribution of a and b */
MPI Cart shift(comm 2d, 0, +mycoords[0], &shiftsource, &shiftdest);
MPI Sendrecv replace(a buffers[i%2], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Cart shift(comm 2d, 1, +mycoords[1], &shiftsource, &shiftdest);
MPI Sendrecv replace(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Comm free (&comm 2d); /* Free up communicator */
free(a buffers[1]);
```

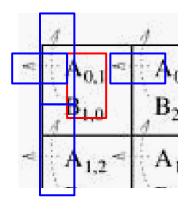




On each process, computations (red) + communications (blue) are done at the same time.

```
/* Get into the main computation loop */
for (i=0; i<dims[0]; i++)
  MPI_Isend(a_buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      leftrank, 1, comm 2d, &reqs[0]);
  MPI Isend(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      uprank, 1, comm 2d, &reqs[1]);
  MPI Irecv(a buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
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  /* c = c + a*b */
  MatrixMultiply(nlocal, a buffers[i%2], b buffers[i%2], c);
  for (j=0; j<4; j++)
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MPI Sendrecv replace(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Comm free (&comm 2d); /* Free up communicator */
free(a buffers[1]);
```





then the program will *poll* at regular intervals to see if it may communicate & the CPU will pause computation to carry out the necessary communication

```
/* Get into the main computation loop */
for (i=0; i<dims[0]; i++)
  MPI_Isend(a_buffers[i%2], nlocal*nlocal, MPI DOUBLE,
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      uprank, 1, comm 2d, &reqs[1]);
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  /* c = c + a*b */
  MatrixMultiply(nlocal, a buffers[i%2], b buffers[i%2], c);
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free(a buffers[1]);
```



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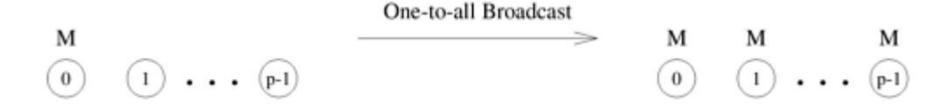
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#### One-to All Broadcast



A message **M** exists on one processor which we want to send to all *p* other processors



#### All-to-One Reduction



Messages **M** exist on each process i, and we want to combine these messages onto a single processor.

All-to-one Reduction

M













#### All-Reduce



Messages *M<sup>i</sup>* exist on each process *i*, and we want to combine these messages in the same way on each processor.











#### **Prefix Sum**



An array where each element stores the cumulative sum of all previous elements (useful for histograms, radix sort, etc.)

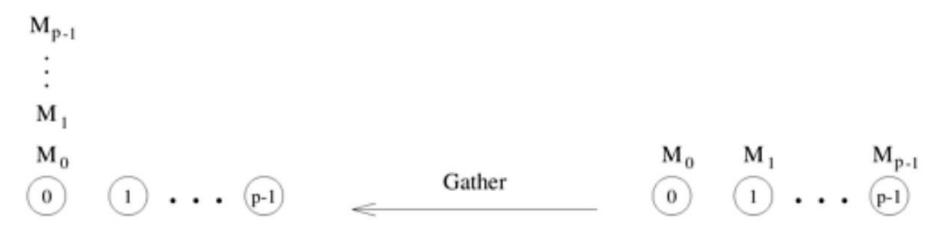
Input array  $[2, 4, 6, 8] \rightarrow \text{Prefix sums } [2, 6, 12, 20]$ 



#### Gather

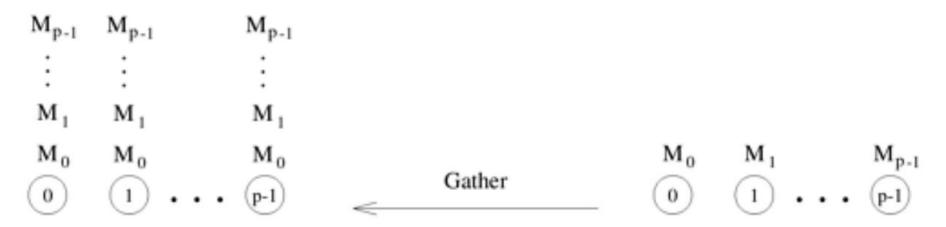


Each processor has a message *m*, all of which must be collected onto a single processor



#### All-Gather

Each processor has a message *m*, all of which must be collected onto *each* processor

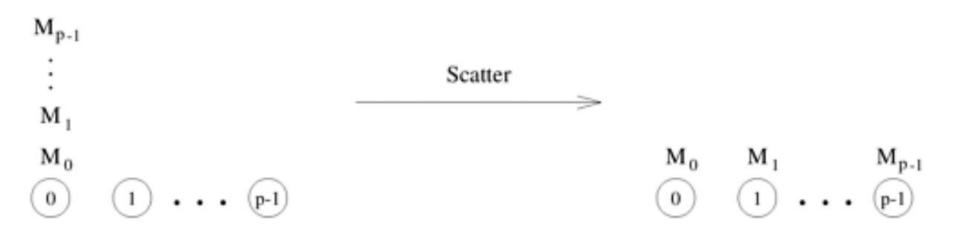




#### Scatter



A single processor has a separate message *m* for each other processor in the network



## All-to-All Personalized Communication (AAPC)

All processors contain a unique message for each other processor.

$M_{0,p-1}$	$M_{1,p-1}$	$M_{p-1, p-1}$		$M_{p-1,0}$	$M_{p-1,1}$	$M_{p-1, p-1}$
÷	:	:		÷	÷	÷
$M_{0,1}$	$M_{1,1}$	$M_{p-1,1}$		$M_{1,0}$	M 1,1	$M_{1,p-1}$
M <sub>0,0</sub>	M <sub>1,0</sub>	M <sub>p-1,0</sub>	All-to-all personalized communication	M <sub>0,0</sub>	M <sub>0,1</sub>	M <sub>0,p-1</sub>



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## MPI\_Barrier

int MPI\_Barrier(MPI\_Comm comm);

Synchronizes all processes within a communicator; no process exits until all have entered.

 comm: The communicator containing all participating processes.



#### MPI\_Barrier

```
// Assume 4 processes printf("Rank %d reached barrier\n", rank); MPI_Barrier(MPI_COMM_WORLD); printf("Rank %d passed barrier\n", rank);
```

```
int MPI_Barrier(MPI_Comm comm);
```

Synchronizes all processes within a communicator; no process exits until all have entered.



## MPI\_Barrier

int MPI\_Barrier(MPI\_Comm comm);

Synchronizes all processes within a communicator; no process exits until all have entered.

```
// Assume 4 processes printf("Rank %d reached barrier\n", rank); MPI_Barrier(MPI_COMM_WORLD); printf("Rank %d passed barrier\n", rank);
```

Rank 0 reached barrier Rank 1 reached barrier Rank 2 reached barrier Rank 3 reached barrier -- synchronization --Rank 0 passed barrier Rank 1 passed barrier Rank 2 passed barrier Rank 3 passed barrier



#### MPI Bcast

Broadcasts data from the root process to all other processes in the communicator.

- buffer: Starting address of data (send buffer for root, receive buffer for others).
- count: Number of elements in the buffer.
- datatype: Type of each buffer element (e.g., MPI\_INT)
- root: Rank of the sending process.
- comm: Communicator handle



#### MPI\_Bcast

Broadcasts data from the root process to all other processes in the communicator.

// Assume 4 processes, rank 0 initializes the value if (rank == 0) value = 42; MPI\_Bcast(&value, 1, MPI\_INT, 0, MPI\_COMM\_WORLD); printf("Rank %d received value %d\n", rank, value);



#### MPI\_Bcast

Broadcasts data from the root process to all other processes in the communicator.

```
// Assume 4 processes, rank 0 initializes the value if (rank == 0) value = 42; MPI_Bcast(&value, 1, MPI_INT, 0, MPI_COMM_WORLD); printf("Rank %d received value %d\n", rank, value);
```

Rank 0 received value 42 Rank 1 received value 42 Rank 2 received value 42 Rank 3 received value 42



#### MPI\_Reduce

Applies a reduction operation (e.g., sum, max) across all processes and delivers the result to the root.

- sendbuf: Starting address of send buffer.
- recvbuf: Starting address of receive buffer (used only by root)
- count: Number of elements in each buffer
- datatype: Data type of elements.
- op: Operation (e.g., MPI\_SUM, MPI\_MAX).
- root: Rank of root process.
- comm: Communicator handle.



#### MPI\_Reduce

Applies a reduction operation (e.g., sum, max) across all processes and delivers the result to the root.



## MPI\_Reduce

Applies a reduction operation (e.g., sum, max) across all processes and delivers the result to the root.



MPI Operation	Meaning	Supported Datatypes
MPI_MAX	Returns the <b>maximum</b> value across all processes	C integers and floating-point types (MPI_INT, MPI_FLOAT, MPI_DOUBLE, etc.)
MPI_MIN	Returns the <b>minimum</b> value across all processes	C integers and floating-point types
MPI_SUM	Computes the <b>sum</b> of all elements	C integers and floating-point types
MPI_PROD	Computes the <b>product</b> of all elements	C integers and floating-point types
MPI_LAND	Performs logical AND (element-wise)	C integer and logical types (MPI_INT, MPI_C_BOOL, etc.)
MPI_BAND	Performs bitwise AND	C integer types
MPI_LOR	Performs logical OR (element-wise)	C integer and logical types
MPI_BOR	Performs bitwise OR	C integer types
MPI_LXOR	Performs logical XOR (exclusive OR, element-wise)	C integer and logical types
MPI_BXOR	Performs bitwise XOR	C integer types
MPI_MAXLOC	Returns the (value, index) pair where value is maximum; index ties broken by smallest index	Pairs of (value, index) using special datatypes (MPI_FLOAT_INT, MPI_DOUBLE_INT, MPI_LONG_INT, MPI_2INT, etc.)
MPI_MINLOC	Returns the <b>(value, index)</b> pair where value is minimum; index ties broken by smallest index	Same as MPI_MAXLOC (paired value-index types)



#### MPI\_Allreduce

```
int MPI_Allreduce(const void *sendbuf,
     void *recvbuf, int count,
     MPI_Datatype datatype, MPI_Op op,
     MPI_Comm comm);
```

Performs a reduction operation across all processes and distributes the result to all.

- sendbuf: Address of input data.
- recvbuf: Address to store the result for each process.
- count: Number of elements in each buffer.
- datatype: Data type of elements.
- op: Reduction operation (sum, max, etc.).
- comm: Communicator handle.



## MPI\_Allreduce

```
int MPI_Allreduce(const void *sendbuf,
     void *recvbuf, int count,
     MPI_Datatype datatype, MPI_Op op,
     MPI_Comm comm);
```

Performs a reduction operation across all processes and distributes the result to all.



## MPI\_Allreduce

```
int MPI_Allreduce(const void *sendbuf,
     void *recvbuf, int count,
     MPI_Datatype datatype, MPI_Op op,
     MPI_Comm comm);
```

Performs a reduction operation across all processes and distributes the result to all.

```
int local = rank + 1;
int result:
MPI Allreduce(&local, &result, 1, MPI INT, MPI SUM,
              MPI COMM WORLD);
printf("Rank %d: global sum = %d\n", rank, result);
           Rank 0: global sum = 10
           Rank 1: global sum = 10
           Rank 2: global sum = 10
           Rank 3: global sum = 10
```



#### MPI\_Scan

```
int MPI_Scan(const void *sendbuf,
     void *recvbuf, int count,
     MPI_Datatype datatype, MPI_Op op,
     MPI_Comm comm);
```

Performs a prefix reduction (partial sum) such that process *i* receives the reduction of ranks ≤ *i*.

- sendbuf: Input buffer.
- recvbuf: Output buffer (prefix result).
- count: Number of elements in each buffer.
- datatype: Data type of elements.
- op: Reduction operation (sum, max, etc.).
- comm: Communicator handle.



#### MPI\_Scan

```
int MPI_Scan(const void *sendbuf,
    void *recvbuf, int count,
    MPI_Datatype datatype, MPI_Op op,
    MPI_Comm comm);
```

Performs a prefix reduction (partial sum) such that process *i* receives the reduction of ranks ≤ *i*.

#### MPI\_Scan

```
int MPI_Scan(const void *sendbuf,
    void *recvbuf, int count,
    MPI_Datatype datatype, MPI_Op op,
    MPI_Comm comm);
```

Performs a prefix reduction (partial sum) such that process i receives the reduction of ranks  $\leq i$ .

```
int local = rank + 1; // \{1,2,3,4\}
int prefix;
MPI Scan(&local, &prefix, 1, MPI INT, MPI SUM,
           MPI COMM WORLD);
printf("Rank %d: prefix sum = %d\n", rank, prefix);
           Rank 0: prefix sum = 1
           Rank 1: prefix sum = 3
           Rank 2: prefix sum = 6
           Rank 3: prefix sum = 10
```



## MPI\_Gather

```
int MPI_Gather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

from all processes and concatenates them at the root.

- sendbuf: Buffer with data to send.
- sendcount: Number of elements sent from each process.
- sendtype: Type of each send element.
- recvbuf: Buffer to store gathered data (valid on root).
- recvcount: Number of elements received from each process.
- recvtype: Type of each received element.
- root: Rank of root process.
- comm: Communicator handle



## MPI\_Gather

```
int MPI_Gather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

from all processes and concatenates them at the root.



## MPI\_Gather

```
int MPI_Gather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

from all processes and concatenates them at the root.

Rank 0 prints: Gathered: [10 11 12 13]



#### MPI\_Gatherv

```
int MPI_Gatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Similar to MPI\_Gather, but allows variable amounts of data to be received from each process.

- sendbuf: Buffer with data to send.
- sendcount: Number of elements sent by this process.
- sendtype: Type of send data.
- recvbuf: Buffer to store gathered data (root only).
- recvcounts: Array specifying number of elements received from each rank.
- displs: Array specifying offsets (displacements) in recybuf.
- recvtype: Type of received data.
- root: Root process rank.
- comm: Communicator handle.



# MPI\_Gatherv

```
int MPI_Gatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Similar to MPI\_Gather, but allows variable amounts of data to be received from each process.

```
int sendcount = rank + 1; // \{1,2,3,4\}
int sendbuf[4] = {rank, rank, rank, rank};
int recvbuf[10];
int counts[4] = \{1,2,3,4\};
int displs[4] = \{0,1,3,6\};
MPI Gatherv(sendbuf, sendcount, MPI INT,
        recvbuf, counts, displs, MPI INT,
        0, MPI COMM WORLD);
if (rank == 0) {
  printf("Rank 0 gathered: ");
  for (int i = 0; i < displs[3] + counts[3]; i++)
     printf("%d ", recvbuf[i]);
  printf("\n");
```



## MPI\_Gatherv

```
int MPI_Gatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Similar to MPI\_Gather, but allows variable amounts of data to be received from each process.

```
int sendcount = rank + 1; // \{1,2,3,4\}
int sendbuf[4] = {rank, rank, rank, rank};
int recvbuf[10];
int counts[4] = \{1,2,3,4\};
int displs[4] = \{0,1,3,6\};
MPI Gatherv(sendbuf, sendcount, MPI INT,
        recvbuf, counts, displs, MPI INT,
        0, MPI COMM WORLD);
if (rank == 0) {
  printf("Rank 0 gathered: ");
  for (int i = 0; i < displs[3] + counts[3]; i++)
     printf("%d ", recvbuf[i]);
  printf("\n");
```

Rank 0 gathered: 0 1 1 2 2 2 3 3 3 3



# MPI\_Allgather

```
int MPI_Allgather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Collects equal-sized data from all processes and distributes the full concatenated result to everyone.

- sendbuf: Local data to send.
- sendcount: Number of elements sent per process.
- sendtype: Data type of send buffer.
- recvbuf: Buffer to store gathered results on all processes.
- recvcount: Number of elements received from each process.
- recvtype: Type of received data.
- comm: Communicator handle.



# MPI\_Allgather

```
int MPI_Allgather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Collects equal-sized data from all processes and distributes the full concatenated result to everyone.



# MPI\_Allgather

```
int MPI_Allgather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Collects equal-sized data from all processes and distributes the full concatenated result to everyone.

Rank 0: received [10 11 12 13]
Rank 1: received [10 11 12 13]
Rank 2: received [10 11 12 13]
Rank 3: received [10 11 12 13]



# MPI\_Allgatherv

```
int MPI_Allgatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Gathers variable-sized data from all processes and distributes the complete result to everyone.

- sendbuf: Local data to send.
- sendcount: Number of elements sent by this process.
- sendtype: Type of send data.
- recvbuf: Buffer to store gathered results.
- recvcounts: Array specifying number of elements received from each rank.
- displs: Array specifying displacement offsets in recvbuf.
- recvtype: Type of received data.
- comm: Communicator handle.



# MPI\_Allgatherv

```
int MPI_Allgatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Gathers variable-sized data from all processes and distributes the complete result to everyone.

```
int sendcount = rank + 1; // \{1,2,3,4\}
int sendbuf[4] = {rank, rank, rank, rank};
int recvbuf[10];
int counts[4] = \{1,2,3,4\};
int displs[4] = \{0,1,3,6\};
MPI Allgatherv(sendbuf, sendcount, MPI INT,
          recybuf, counts, displs, MPI INT,
          MPI COMM WORLD);
printf("Rank %d received: ", rank);
for (int i = 0; i < displs[3] + counts[3]; i++)
  printf("%d ", recvbuf[i]);
printf("\n");
```



# MPI\_Allgatherv

```
int MPI_Allgatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Gathers variable-sized data from all processes and distributes the complete result to everyone.

```
int sendcount = rank + 1; // \{1,2,3,4\}
int sendbuf[4] = {rank, rank, rank, rank};
int recvbuf[10];
int counts[4] = \{1,2,3,4\};
int displs[4] = \{0,1,3,6\};
MPI Allgatherv(sendbuf, sendcount, MPI INT,
          recybuf, counts, displs, MPI INT,
          MPI COMM WORLD);
printf("Rank %d received: ", rank);
for (int i = 0; i < displs[3] + counts[3]; i++)
  printf("%d ", recvbuf[i]);
printf("\n");
```

Rank 0 received: 0 1 1 2 2 2 3 3 3 3 Rank 1 received: 0 1 1 2 2 2 3 3 3 3 Rank 2 received: 0 1 1 2 2 2 3 3 3 3 Rank 3 received: 0 1 1 2 2 2 3 3 3 3



#### MPI\_Scatter

```
int MPI_Scatter(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Distributes equal-sized blocks of data from the root to all processes.

- sendbuf: Root's buffer containing the data to scatter.
- sendcount: Number of elements sent to each process.
- sendtype: Type of send data.
   recvbuf: Buffer to store received data.
- recvcount: Number of elements received per process.
- recvtype: Type of received data.
- root: Rank of root process.
- comm: Communicator handle.



#### MPI\_Scatter

```
int MPI_Scatter(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Distributes equal-sized blocks of data from the root to all processes.

```
int send[4] = {10, 20, 30, 40};
int recv;
MPI_Scatter(send, 1, MPI_INT, &recv, 1, MPI_INT, 0,
MPI_COMM_WORLD);
printf("Rank %d received %d\n", rank, recv);
```



#### MPI\_Scatter

```
int MPI_Scatter(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Distributes equal-sized blocks of data from the root to all processes.

```
int send[4] = \{10, 20, 30, 40\};
int recv:
MPI Scatter(send, 1, MPI INT, &recv, 1, MPI INT, 0,
MPI COMM WORLD);
printf("Rank %d received %d\n", rank, recv);
           Rank 0 received 10
           Rank 1 received 20
           Rank 2 received 30
           Rank 3 received 40
```



#### MPI Scatterv

Distributes variable-sized blocks of data from the root to all processes.

- sendbuf: Root's buffer containing data.
- sendcounts: Array specifying number of elements to send to each process.
- displs: Array specifying starting offsets of each block in sendbuf.
- sendtype: Type of send data.
- recvbuf: Buffer to receive data.
- recvcount: Number of elements received by this process.
- recvtype: Type of receive data.
- root: Rank of root process.
- comm: Communicator handle.



## MPI\_Scatterv

Distributes variable-sized blocks of data from the root to all processes.



## MPI\_Scatterv

Distributes variable-sized blocks of data from the root to all processes.

```
Rank 0 received: 0
Rank 1 received: 1 1
Rank 2 received: 2 2 2
Rank 3 received: 3 3 3 3
```



# MPI\_Alltoall

```
int MPI_Alltoall(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Each process sends equal-sized data to all others and receives equal-sized data from all others.

- sendbuf: Starting address of send buffer.
- sendcount: Number of elements sent to each process.
- sendtype: Type of send data.
   recvbuf: Starting address of receive buffer.
- recvcount: Number of elements received from each process.
- recvtype: Type of receive data.
- comm: Communicator handle.



### MPI\_Alltoall

```
int MPI_Alltoall(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Each process sends equal-sized data to all others and receives equal-sized data from all others.



#### MPI\_Alltoall

```
int MPI_Alltoall(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Each process sends equal-sized data to all others and receives equal-sized data from all others. Rank 0 received [0 10 20 30] Rank 1 received [1 11 21 31] Rank 2 received [2 12 22 32] Rank 3 received [3 13 23 33]



# MPI\_Alltoallv

Each process sends variable-sized data to all others and receives variable-sized data from all others.

- sendbuf: Starting address of send buffer.
- sendcounts: Array specifying number of elements sent to each rank.
- sdispls: Array of offsets in sendbuf.
- sendtype: Type of send data.
- recvbuf: Starting address of receive buffer.
- recvcounts: Array specifying number of elements received from each rank.
- rdispls: Array of offsets in recvbuf.
- recvtype: Type of receive data.
- comm: Communicator handle.



# MPI\_Alltoallv

Each process sends variable-sized data to all others and receives variable-sized data from all others.

```
int sendbuf[10];
for (int i=0; i<10; i++) sendbuf[i] = rank*10 + i;
int sendcounts[4] = \{1,2,3,4\};
int sdispls[4] = \{0,1,3,6\};
int recvcounts[4] = {rank+1, rank+1, rank+1};
int rdispls[4] = \{0, (rank+1), (rank+1)^2, (rank+1)^3\};
int recvbuf[16];
MPI Alltoally(sendbuf, sendcounts, sdispls, MPI INT, recybuf,
              recvcounts, rdispls, MPI INT, MPI COMM WORLD);
printf("Rank %d received:", rank);
for (int i=0; i<(rank+1)*4; i++)
  printf(" %d", recvbuf[i]);
printf("\n");
```



# MPI\_Alltoallv

Each process sends variable-sized data to all others and receives variable-sized data from all others.

```
int sendbuf[10];
for (int i=0; i<10; i++) sendbuf[i] = rank*10 + i;
int sendcounts[4] = \{1,2,3,4\};
int sdispls[4] = \{0,1,3,6\};
int recvcounts[4] = {rank+1, rank+1, rank+1};
int rdispls[4] = \{0, (rank+1), (rank+1)^2, (rank+1)^3\};
int recvbuf[16];
MPI Alltoally(sendbuf, sendcounts, sdispls, MPI INT, recybuf,
              recvcounts, rdispls, MPI INT, MPI COMM WORLD);
printf("Rank %d received:", rank);
for (int i=0; i<(rank+1)*4; i++)
  printf(" %d", recvbuf[i]);
printf("\n");
```

Rank 0 received: 0 10 20 30

Rank 1 received: 1 2 11 12 21 22 31 32

Rank 2 received: 3 4 5 13 14 15 23 24 25 33 34 35

Rank 3 received: 6 7 8 9 16 17 18 19 26 27 28 29 36 37 38 39



# LECTURE ENDED HERE. THE REMAINDER OF THE LECTURE WILL BE COVERED ON 10/15.



#### **Lecture Overview**

- Non-Blocking MPI Communications
- Review of Collective Communication Patterns
- MPI Collective Communications
- Groups + Communicators
- Questions

```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```

Creates new communicators by splitting an existing communicator into subgroups based on color.

Processes with the same color are grouped together; within each group, ranks are ordered by key



```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```

Creates new communicators by splitting an existing communicator into subgroups based on color.

Processes with the same color are grouped together; within each group, ranks are ordered by key

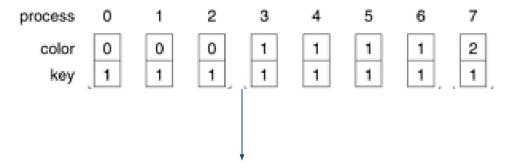
- comm: Existing communicator to be split (e.g., MPI\_COMM\_WORLD).
- color: Integer identifier for subgroup membership.
  - All processes with the same color form a new communicator.
  - A process can set color =
     MPI\_UNDEFINED to be excluded entirely.
- key: Determines ordering of ranks within the new communicator.
  - Lower keys get lower ranks.
  - Equal keys are ordered according to ordering in previous communicator
- newcomm: Output handle to the newly created communicator.



```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```



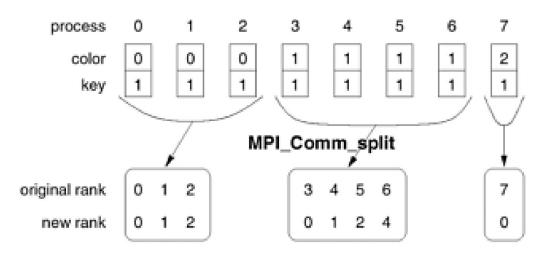
```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```



Which processes will be grouped together?



```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```





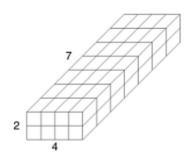
Creates a lower-dimensional subcommunicator from an existing Cartesian topology communicator by selecting which dimensions to keep.



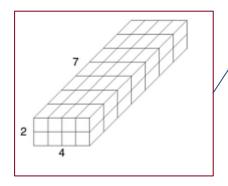
Creates a lower-dimensional subcommunicator from an existing Cartesian topology communicator by selecting which dimensions to keep.

- comm: Input communicator with Cartesian topology (created via MPI\_Cart\_create).
- remain\_dims: Logical array of length equal to the number of dimensions.
  - remain\_dims[i] = 1 → keep this dimension.
  - remain\_dims[i] = 0 → collapse (drop) this dimension.
- newcomm: Output communicator corresponding to the subgrid of the original topology.



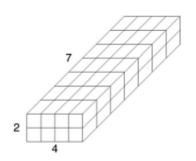


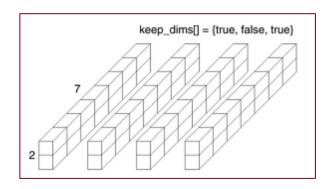




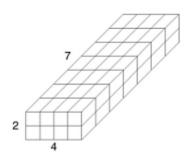
```
int dims[3] = \{2, 4, 7\}; // grid dimensions
int periods[3] = \{0, 0, 0\};
                        // non-periodic in all dimensions
int reorder = 0:
                        // keep original ranks
MPI Comm cart comm;
MPI Cart create(MPI COMM WORLD, 3, dims, periods,
     reorder, &cart comm);
int keep dims[3] = \{1, 0, 1\};
MPI Comm sub comm;
MPI Cart_sub(cart_comm, keep_dims, &sub_comm);
```



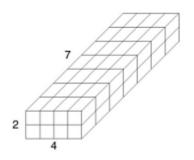


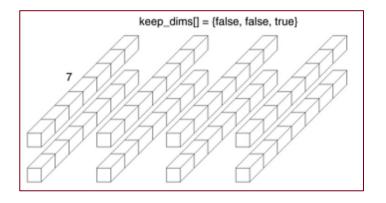














#### Lecture Overview

- Non-Blocking MPI Communications
- Review of Collective Communication Patterns
- MPI Collective Communications
- ☐ Groups + Communicators
- Questions