CSCI 5451: Introduction to Parallel Computing

Lecture 12: MPI Collective Communications



Announcements (10/13)

- HW1
 - o Updated due date to Oct 15
 - o Logging is rather verbose. Make sure to scroll to the bottom.
 - o Only resubmissions are graded when the autograder is run
- ☐ HWs 2-5 are pushed back one week
- Group Formation due Oct 19 (<u>Canvas</u>)



Lecture Overview

- Non-Blocking MPI Communications
- Review of Collective Communication Patterns
- MPI Collective Communications
- ☐ Groups + Communicators
- Questions



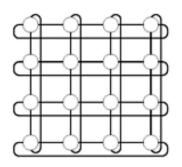
Lecture Overview

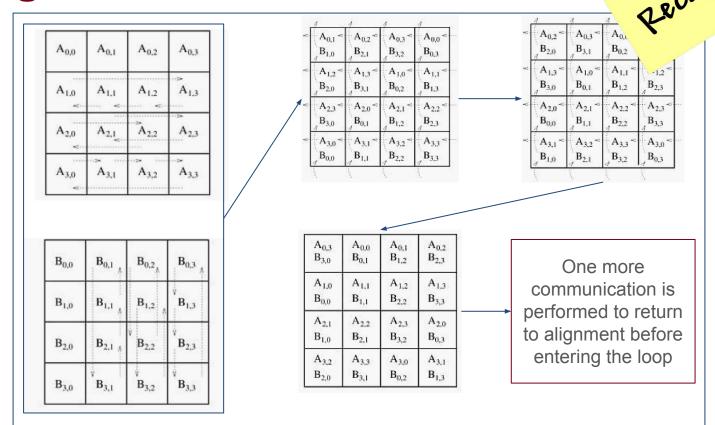
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Cannon's Algorithm

2-d mesh with wraparound







What is a downside of the current project to our implementation of Cannon's Algorithm?



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The CPU will idle during communication steps.



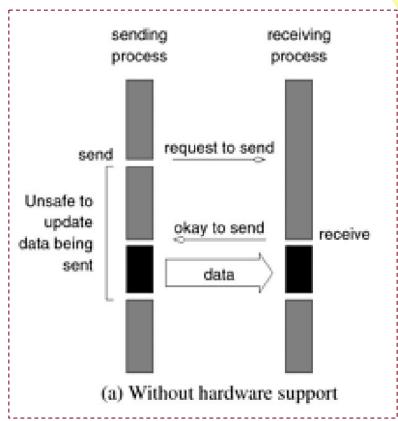
What is a downside of the current project to our implementation of Cannon's Algorithm?

The CPU will idle during communication steps.

We can resolve this by overlapping communication + computation with non-blocking MPI calls.



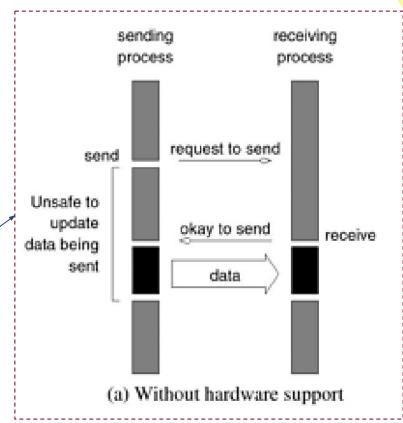
- A program using non-blocking Sends & Receives will immediately continue after finishing the Send or Receive function
- We do not have any guarantees that our program will not overwrite the data being sent
- This is dependent on whether or not the data will be buffered, and how that buffering occurs





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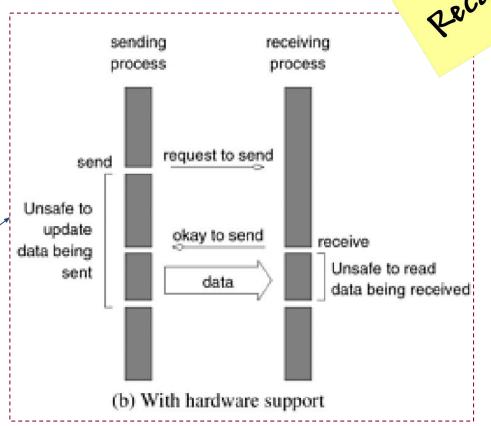
Non-Buffered





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Non-Buffered





- MPI_Isend & MPI_Irecv enable non-blocking communication with MPI
- With hardware support, these operations enable us to perfectly overlap communication and computation
- Without hardware support, we remove any idling, but the CPU must still participate in the network communication
- MPI_Isend may be buffered, MPI_Irecv is not
- As such, we must be careful when structuring program execution to not overwrite the buffers

```
int MPI_Isend(void *buf, int count,

MPI_Datatype datatype,

int dest, int tag, MPI_Comm comm,

MPI_Request *request)

int MPI_Irecv(void *buf, int count,

MPI_Datatype datatype,

int source, int tag, MPI_Comm comm,

MPI_Request *request)
```



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Same arguments as used in MPI_Send & MPI_Recv



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int dest, int tag, MPI_Comm comm,

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int MPI_Irecv(void *buf, int count,

MPI_Datatype datatype,

int source, int tag, MPI_Comm comm,

MPI_Request *request)
```

The *request* object allows us to monitor for when the corresponding send or receive has completed (we will explore this more in upcoming slides)



```
if (myrank == 0) {
    MPI_Send(a, 10, MPI_INT, 1, 1, MPI_COMM_WORLD);
    MPI_Send(b, 10, MPI_INT, 1, 2, MPI_COMM_WORLD);
}
else if (myrank == 1) {
    MPI_Recv(b, 10, MPI_INT, 0, 2, &status, MPI_COMM_WORLD);
    MPI_Recv(a, 10, MPI_INT, 0, 1, &status, MPI_COMM_WORLD);
}
...
```

Eliminates deadlocks

```
if (myrank == 0) {
    MPI_Send(a, 10, MPI_INT, 1, 1, MPI_COMM_WORLD);
    MPI_Send(b, 10, MPI_INT, 1, 2, MPI_COMM_WORLD);
}
else if (myrank == 1) {
    MPI_Irecv(b, 10, MPI_INT, 0, 2, &requests[0], MPI_COMM_WORLD);
    MPI_Irecv(a, 10, MPI_INT, 0, 1, &requests[1], MPI_COMM_WORLD);
}
...
```



- MPI_Test & MPI_Wait are helpful utilities to determine the status of a non-blocking MPI_Isend or MPI_Irecv call
- MPI_Test is a non-blocking operation used to check whether the corresponding non-blocking communication has completed
- MPI_Wait is a blocking operation used to halt further execution until the corresponding non-blocking communication has completed



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```
int MPI_Test (MPI_Request *request, int *flag,

MPI_Status *status)

int MPI_Wait (MPI_Request *request,

MPI_Status *status)
```

The **request** object - returned from the earlier **MPI_ISend** or **MPI_Irecv** calls



- MPI_Test & MPI_Wait are helpful utilities to determine the status of a non-blocking MPI_Isend or MPI_Irecv call
- MPI_Test is a non-blocking operation used to check whether the corresponding non-blocking communication has completed
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A variable indicating whether or not the given communication operation has completed. This returns '1' if it has, '0' otherwise.



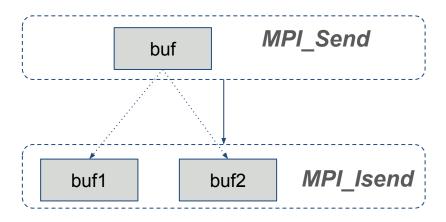
- MPI_Test & MPI_Wait are helpful utilities to determine the status of a non-blocking MPI_Isend or MPI_Irecv call
- MPI_Test is a non-blocking operation used to check whether the corresponding non-blocking communication has completed
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The *status* of the communication operation. This is the same object returned by *MPI_Revc*.



Cannon's Algorithm in MPI

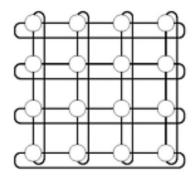
- We make the following changes to ensure that our MPI program works with non-blocking operations
 - Use MPI_Isend, MPI_Irecv, MPI_Wait
 - Duplicate the local buffers a and b on each process
- In general, you will typically want to explicitly use duplicate buffers to ensure that one buffer is used for communication while another is used for computation



```
MatrixMatrixMultiply NonBlocking(int n, double *a, double *b,
                                 double *c, MPI Comm comm)
 int i, j, nlocal;
  double *a buffers[2], *b buffers[2];
  int npes, dims[2], periods[2];
  int myrank, my2drank, mycoords[2];
  int uprank, downrank, leftrank, rightrank, coords[2];
  int shiftsource, shiftdest;
 MPI Status status;
 MPI Comm comm 2d;
 MPI Request reqs[4];
 /* Get the communicator related information */
 MPI Comm size(comm, &npes);
 MPI Comm rank (comm, &myrank);
 /* Set up the Cartesian topology */
 dims[0] = dims[1] = sqrt(npes);
 /* Set the periods for wraparound connections */
 periods[0] = periods[1] = 1;
 /* Create the Cartesian topology, with rank reordering */
  MPI Cart create (comm, 2, dims, periods, 1, &comm 2d);
```



2-d mesh with wraparound



```
MatrixMatrixMultiply NonBlocking(int n, double *a, double *b,
                                 double *c, MPI Comm comm)
 int i, j, nlocal;
  double *a buffers[2], *b buffers[2];
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 periods[0] = periods[1] = 1;
 /* Create the Cartesian topology, with rank reordering */
  MPI Cart create (comm, 2, dims, periods, 1, &comm 2d);
```



```
/* Get the rank and coordinates with respect to the new topology */
MPI Comm rank(comm 2d, &my2drank);
MPI Cart coords (comm 2d, my2drank, 2, mycoords);
/* Compute ranks of the up and left shifts */
MPI Cart shift (comm 2d, 0, -1, &rightrank, &leftrank);
MPI Cart shift(comm 2d, 1, -1, &downrank, &uprank);
/* Determine the dimension of the local matrix block */
nlocal = n/dims[0];
/* Setup the a buffers and b buffers arrays */
a buffers[0] = a;
a buffers[1] = (double *)malloc(nlocal*nlocal*sizeof(double));
b \text{ buffers}[0] = b;
b buffers[1] = (double *)malloc(nlocal*nlocal*sizeof(double));
/* Perform the initial matrix alignment. First for A and then for B */
MPI Cart shift(comm 2d, 0, -mycoords[0], &shiftsource, &shiftdest);
MPI Sendrecv replace(a buffers[0], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Cart shift(comm 2d, 1, -mycoords[1], &shiftsource, &shiftdest);
MPI Sendrecv replace(b buffers[0], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
```



| $A_{0,0}$ | A _{0,1} | A _{0,2} | A _{0,3} |
|------------------|-------------------|------------------|------------------|
| A _{1.0} | A _{1,1} | A _{1,2} | A _{1,3} |
| A _{2,0} | A _{2,1,} | A _{2,2} | A _{2,3} |
| A _{3,0} | A _{3,1} | A _{3,2} | A _{3,3} |

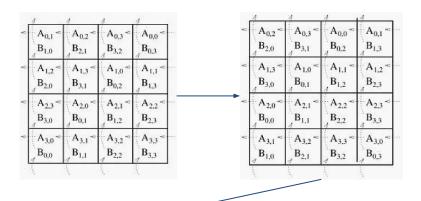
| $\mathbf{B}_{0,0}$ | B _{0,1} | B _{0,2} | B _{0,3} |
|--------------------|--------------------|-----------------------|--------------------|
| B _{1,0} | B _{1,1} | B _{1,2} | $\mathbf{B}_{1,3}$ |
| B _{2,0} | В _{2,1 д} | ў В _{2,2} | B _{2,3} |
| B _{3,0} | B _{3,1} | B _{3,2} | B _{3,3} |

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/* Get the rank and coordinates with respect to the new topology */
MPI Comm rank(comm 2d, &my2drank);
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MPI Cart shift(comm 2d, 1, -1, &downrank, &uprank);
/* Determine the dimension of the local matrix block */
nlocal = n/dims[0];
/* Setup the a buffers and b buffers arrays */
a buffers[0] = a;
a buffers[1] = (double *)malloc(nlocal*nlocal*sizeof(double));
b \text{ buffers}[0] = b;
b buffers[1] = (double *)malloc(nlocal*nlocal*sizeof(double));
/* Perform the initial matrix alignment. First for A and then for B */
MPI Cart shift(comm 2d, 0, -mycoords[0], &shiftsource, &shiftdest);
MPI Sendrecv replace(a buffers[0], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Cart shift(comm 2d, 1, -mycoords[1], &shiftsource, &shiftdest);
MPI Sendrecv replace(b buffers[0], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
```



```
/* Get into the main computation loop */
for (i=0; i<dims[0]; i++) {
  MPI Isend(a buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      leftrank, 1, comm 2d, &reqs[0]);
  MPI Isend(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      uprank, 1, comm 2d, &reqs[1]);
  MPI Irecv(a buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      rightrank, 1, comm 2d, &reqs[2]);
  MPI Irecv(b buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      downrank, 1, comm 2d, &regs[3]);
 /* c = c + a*b */
  MatrixMultiply(nlocal, a buffers[i%2], b buffers[i%2], c);
  for (j=0; j<4; j++)
    MPI Wait(&reqs[j], &status);
/* Restore the original distribution of a and b */
MPI Cart shift(comm 2d, 0, +mycoords[0], &shiftsource, &shiftdest);
MPI Sendrecv replace(a buffers[i%2], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Cart shift(comm 2d, 1, +mycoords[1], &shiftsource, &shiftdest);
MPI Sendrecv replace(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Comm free (&comm 2d); /* Free up communicator */
free(a buffers[1]);
```



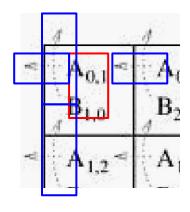


| A _{0,3} | $A_{0,0} \\ B_{0,1}$ | A _{0,1} | A _{0,2} | |
|------------------|----------------------|------------------|-----------------------------------|--|
| B _{3,0} | | B _{1,2} | B _{2,3} | |
| A _{1,0} | A _{1,1} | A _{1,2} | A _{1,3} | |
| B _{0,0} | B _{1,1} | B _{2,2} | B _{3,3} | |
| A _{2,1} | A _{2,2} | A _{2,3} | A _{2,0} | |
| B _{1,0} | B _{2,1} | B _{3,2} | B _{0,3} | |
| A _{3,2} | A _{3,3} | A _{3,0} | A _{3,1} B _{1,3} | |
| B _{2,0} | B _{3,1} | B _{0,2} | | |

One more communication is performed to return to alignment before entering the loop

```
/* Get into the main computation loop */
for (i=0; i<dims[0]; i++)
  MPI Isend(a buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      leftrank, 1, comm 2d, &reqs[0]);
  MPI Isend(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      uprank, 1, comm 2d, &reqs[1]);
  MPI Irecv(a buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      rightrank, 1, comm 2d, &reqs[2]);
  MPI Irecv(b buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      downrank, 1, comm 2d, &regs[3]);
  /* c = c + a*b */
  MatrixMultiply(nlocal, a buffers[i%2], b buffers[i%2], c);
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MPI Sendrecv replace(a buffers[i%2], nlocal*nlocal, MPI DOUBLE,
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MPI Cart shift(comm 2d, 1, +mycoords[1], &shiftsource, &shiftdest);
MPI Sendrecv replace(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Comm free (&comm 2d); /* Free up communicator */
free(a buffers[1]);
```

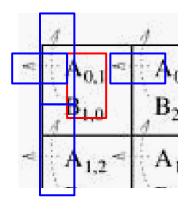




On each process, computations (red) + communications (blue) are done at the same time.

```
/* Get into the main computation loop */
for (i=0; i<dims[0]; i++)
  MPI_Isend(a_buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      leftrank, 1, comm 2d, &reqs[0]);
  MPI Isend(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      uprank, 1, comm 2d, &reqs[1]);
  MPI Irecv(a buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      rightrank, 1, comm 2d, &reqs[2]);
  MPI Irecv(b buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
      downrank, 1, comm 2d, &regs[3]);
  /* c = c + a*b */
  MatrixMultiply(nlocal, a buffers[i%2], b buffers[i%2], c);
  for (j=0; j<4; j++)
    MPI Wait(&reqs[j], &status);
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MPI Sendrecv replace(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
    shiftdest, 1, shiftsource, 1, comm 2d, &status);
MPI Comm free (&comm 2d); /* Free up communicator */
free(a buffers[1]);
```





then the program will *poll* at regular intervals to see if it may communicate & the CPU will pause computation to carry out the necessary communication

```
/* Get into the main computation loop */
for (i=0; i<dims[0]; i++)
  MPI_Isend(a_buffers[i%2], nlocal*nlocal, MPI DOUBLE,
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  MPI Isend(b buffers[i%2], nlocal*nlocal, MPI DOUBLE,
      uprank, 1, comm 2d, &reqs[1]);
  MPI Irecv(a buffers[(i+1)%2], nlocal*nlocal, MPI DOUBLE,
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MPI Comm free (&comm 2d); /* Free up communicator */
free(a buffers[1]);
```



Lecture Overview

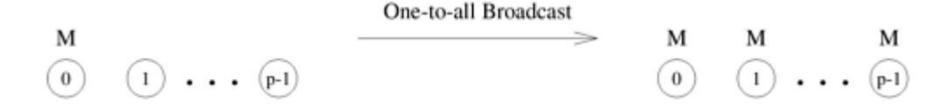
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One-to All Broadcast



A message **M** exists on one processor which we want to send to all *p* other processors



All-to-One Reduction



Messages **M** exist on each process i, and we want to combine these messages onto a single processor.

All-to-one Reduction

M













All-Reduce



Messages *Mⁱ* exist on each process *i*, and we want to combine these messages in the same way on each processor.











Prefix Sum



An array where each element stores the cumulative sum of all previous elements (useful for histograms, radix sort, etc.)

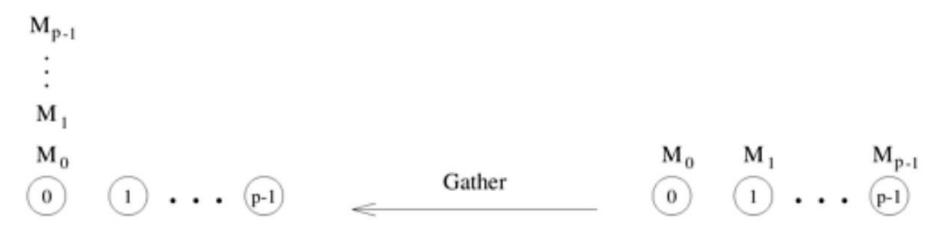
Input array $[2, 4, 6, 8] \rightarrow \text{Prefix sums } [2, 6, 12, 20]$



Gather

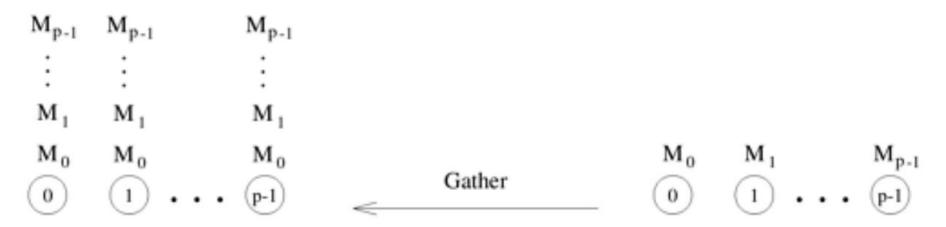


Each processor has a message *m*, all of which must be collected onto a single processor



All-Gather

Each processor has a message *m*, all of which must be collected onto *each* processor

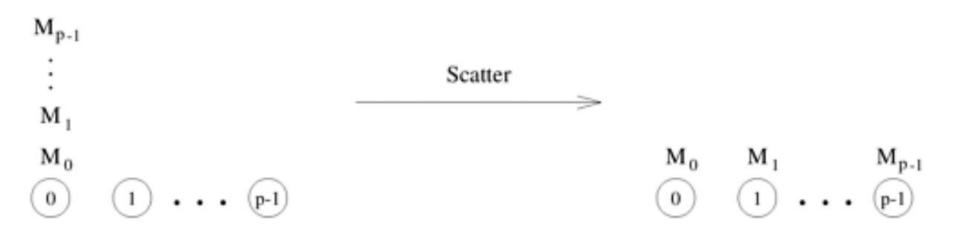




Scatter



A single processor has a separate message *m* for each other processor in the network



All-to-All Personalized Communication (AAPC)

All processors contain a unique message for each other processor.

| $M_{0,p-1}$ | $M_{1,p-1}$ | $M_{p-1, p-1}$ | | $M_{p-1,0}$ | $M_{p-1,1}$ | $M_{p-1, p-1}$ |
|------------------|------------------|--------------------|---------------------------------------|------------------|------------------|--------------------|
| ÷ | : | : | | ÷ | ÷ | ÷ |
| $M_{0,1}$ | $M_{1,1}$ | $M_{p-1,1}$ | | $M_{1,0}$ | M 1,1 | $M_{1,p-1}$ |
| M _{0,0} | M _{1,0} | M _{p-1,0} | All-to-all personalized communication | M _{0,0} | M _{0,1} | M _{0,p-1} |



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MPI_Barrier

int MPI_Barrier(MPI_Comm comm);

Synchronizes all processes within a communicator; no process exits until all have entered.

 comm: The communicator containing all participating processes.



MPI_Barrier

```
// Assume 4 processes printf("Rank %d reached barrier\n", rank); MPI_Barrier(MPI_COMM_WORLD); printf("Rank %d passed barrier\n", rank);
```

```
int MPI_Barrier(MPI_Comm comm);
```

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MPI_Barrier

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Synchronizes all processes within a communicator; no process exits until all have entered.

```
// Assume 4 processes printf("Rank %d reached barrier\n", rank); MPI_Barrier(MPI_COMM_WORLD); printf("Rank %d passed barrier\n", rank);
```

Rank 0 reached barrier Rank 1 reached barrier Rank 2 reached barrier Rank 3 reached barrier -- synchronization --Rank 0 passed barrier Rank 1 passed barrier Rank 2 passed barrier Rank 3 passed barrier



MPI Bcast

Broadcasts data from the root process to all other processes in the communicator.

- buffer: Starting address of data (send buffer for root, receive buffer for others).
- count: Number of elements in the buffer.
- datatype: Type of each buffer element (e.g., MPI_INT)
- root: Rank of the sending process.
- comm: Communicator handle



MPI_Bcast

Broadcasts data from the root process to all other processes in the communicator.

// Assume 4 processes, rank 0 initializes the value if (rank == 0) value = 42; MPI_Bcast(&value, 1, MPI_INT, 0, MPI_COMM_WORLD); printf("Rank %d received value %d\n", rank, value);



MPI_Bcast

Broadcasts data from the root process to all other processes in the communicator.

```
// Assume 4 processes, rank 0 initializes the value if (rank == 0) value = 42; MPI_Bcast(&value, 1, MPI_INT, 0, MPI_COMM_WORLD); printf("Rank %d received value %d\n", rank, value);
```

Rank 0 received value 42 Rank 1 received value 42 Rank 2 received value 42 Rank 3 received value 42



MPI_Reduce

Applies a reduction operation (e.g., sum, max) across all processes and delivers the result to the root.

- sendbuf: Starting address of send buffer.
- recvbuf: Starting address of receive buffer (used only by root)
- count: Number of elements in each buffer
- datatype: Data type of elements.
- op: Operation (e.g., MPI_SUM, MPI_MAX).
- root: Rank of root process.
 comm: Communicator handle.



MPI_Reduce

Applies a reduction operation (e.g., sum, max) across all processes and delivers the result to the root.



MPI_Reduce

Applies a reduction operation (e.g., sum, max) across all processes and delivers the result to the root.



| MPI Operation | Meaning | Supported Datatypes |
|---------------|--|---|
| MPI_MAX | Returns the maximum value across all processes | C integers and floating-point types (MPI_INT, MPI_FLOAT, MPI_DOUBLE, etc.) |
| MPI_MIN | Returns the minimum value across all processes | C integers and floating-point types |
| MPI_SUM | Computes the sum of all elements | C integers and floating-point types |
| MPI_PROD | Computes the product of all elements | C integers and floating-point types |
| MPI_LAND | Performs logical AND (element-wise) | C integer and logical types (MPI_INT, MPI_C_BOOL, etc.) |
| MPI_BAND | Performs bitwise AND | C integer types |
| MPI_LOR | Performs logical OR (element-wise) | C integer and logical types |
| MPI_BOR | Performs bitwise OR | C integer types |
| MPI_LXOR | Performs logical XOR (exclusive OR, element-wise) | C integer and logical types |
| MPI_BXOR | Performs bitwise XOR | C integer types |
| MPI_MAXLOC | Returns the (value, index) pair where value is maximum; index ties broken by smallest index | Pairs of (value, index) using special datatypes (MPI_FLOAT_INT, MPI_DOUBLE_INT, MPI_LONG_INT, MPI_2INT, etc.) |
| MPI_MINLOC | Returns the (value, index) pair where value is minimum; index ties broken by smallest index | Same as MPI_MAXLOC (paired value-index types) |



MPI_Allreduce

```
int MPI_Allreduce(const void *sendbuf,
     void *recvbuf, int count,
     MPI_Datatype datatype, MPI_Op op,
     MPI_Comm comm);
```

Performs a reduction operation across all processes and distributes the result to all.

- sendbuf: Address of input data.
- recvbuf: Address to store the result for each process.
- count: Number of elements in each buffer.
- datatype: Data type of elements.
- op: Reduction operation (sum, max, etc.).
- comm: Communicator handle.



MPI_Allreduce

```
int MPI_Allreduce(const void *sendbuf,
     void *recvbuf, int count,
     MPI_Datatype datatype, MPI_Op op,
     MPI_Comm comm);
```

Performs a reduction operation across all processes and distributes the result to all.



MPI_Allreduce

```
int MPI_Allreduce(const void *sendbuf,
     void *recvbuf, int count,
     MPI_Datatype datatype, MPI_Op op,
     MPI_Comm comm);
```

Performs a reduction operation across all processes and distributes the result to all.

```
int local = rank + 1;
int result:
MPI Allreduce(&local, &result, 1, MPI INT, MPI SUM,
              MPI COMM WORLD);
printf("Rank %d: global sum = %d\n", rank, result);
           Rank 0: global sum = 10
           Rank 1: global sum = 10
           Rank 2: global sum = 10
           Rank 3: global sum = 10
```



MPI_Scan

```
int MPI_Scan(const void *sendbuf,
     void *recvbuf, int count,
     MPI_Datatype datatype, MPI_Op op,
     MPI_Comm comm);
```

Performs a prefix reduction (partial sum) such that process *i* receives the reduction of ranks ≤ *i*.

- sendbuf: Input buffer.
- recvbuf: Output buffer (prefix result).
- count: Number of elements in each buffer.
- datatype: Data type of elements.
- op: Reduction operation (sum, max, etc.).
- comm: Communicator handle.



MPI_Scan

```
int MPI_Scan(const void *sendbuf,
    void *recvbuf, int count,
    MPI_Datatype datatype, MPI_Op op,
    MPI_Comm comm);
```

Performs a prefix reduction (partial sum) such that process *i* receives the reduction of ranks ≤ *i*.

MPI_Scan

```
int MPI_Scan(const void *sendbuf,
    void *recvbuf, int count,
    MPI_Datatype datatype, MPI_Op op,
    MPI_Comm comm);
```

Performs a prefix reduction (partial sum) such that process i receives the reduction of ranks $\leq i$.

```
int local = rank + 1; // \{1,2,3,4\}
int prefix;
MPI Scan(&local, &prefix, 1, MPI INT, MPI SUM,
           MPI COMM WORLD);
printf("Rank %d: prefix sum = %d\n", rank, prefix);
           Rank 0: prefix sum = 1
           Rank 1: prefix sum = 3
           Rank 2: prefix sum = 6
           Rank 3: prefix sum = 10
```



MPI_Gather

```
int MPI_Gather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

from all processes and concatenates them at the root.

- sendbuf: Buffer with data to send.
- sendcount: Number of elements sent from each process.
- sendtype: Type of each send element.
- recvbuf: Buffer to store gathered data (valid on root).
- recvcount: Number of elements received from each process.
- recvtype: Type of each received element.
- root: Rank of root process.
- comm: Communicator handle



MPI_Gather

```
int MPI_Gather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

from all processes and concatenates them at the root.



MPI_Gather

```
int MPI_Gather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

from all processes and concatenates them at the root.

Rank 0 prints: Gathered: [10 11 12 13]



MPI_Gatherv

```
int MPI_Gatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Similar to MPI_Gather, but allows variable amounts of data to be received from each process.

- sendbuf: Buffer with data to send.
- sendcount: Number of elements sent by this process.
- sendtype: Type of send data.
- recvbuf: Buffer to store gathered data (root only).
- recvcounts: Array specifying number of elements received from each rank.
- displs: Array specifying offsets (displacements) in recybuf.
- recvtype: Type of received data.
- root: Root process rank.
- comm: Communicator handle.



MPI_Gatherv

```
int MPI_Gatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Similar to MPI_Gather, but allows variable amounts of data to be received from each process.

```
int sendcount = rank + 1; // \{1,2,3,4\}
int sendbuf[4] = {rank, rank, rank, rank};
int recvbuf[10];
int counts[4] = \{1,2,3,4\};
int displs[4] = \{0,1,3,6\};
MPI Gatherv(sendbuf, sendcount, MPI INT,
        recvbuf, counts, displs, MPI INT,
        0, MPI COMM WORLD);
if (rank == 0) {
  printf("Rank 0 gathered: ");
  for (int i = 0; i < displs[3] + counts[3]; i++)
     printf("%d ", recvbuf[i]);
  printf("\n");
```



MPI_Gatherv

```
int MPI_Gatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Similar to MPI_Gather, but allows variable amounts of data to be received from each process.

```
int sendcount = rank + 1; // \{1,2,3,4\}
int sendbuf[4] = {rank, rank, rank, rank};
int recvbuf[10];
int counts[4] = \{1,2,3,4\};
int displs[4] = \{0,1,3,6\};
MPI Gatherv(sendbuf, sendcount, MPI INT,
        recvbuf, counts, displs, MPI INT,
        0, MPI COMM WORLD);
if (rank == 0) {
  printf("Rank 0 gathered: ");
  for (int i = 0; i < displs[3] + counts[3]; i++)
     printf("%d ", recvbuf[i]);
  printf("\n");
```

Rank 0 gathered: 0 1 1 2 2 2 3 3 3 3



MPI_Allgather

```
int MPI_Allgather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Collects equal-sized data from all processes and distributes the full concatenated result to everyone.

- sendbuf: Local data to send.
- sendcount: Number of elements sent per process.
- sendtype: Data type of send buffer.
- recvbuf: Buffer to store gathered results on all processes.
- recvcount: Number of elements received from each process.
- recvtype: Type of received data.
- comm: Communicator handle.



MPI_Allgather

```
int MPI_Allgather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Collects equal-sized data from all processes and distributes the full concatenated result to everyone.



MPI_Allgather

```
int MPI_Allgather(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Collects equal-sized data from all processes and distributes the full concatenated result to everyone.

Rank 0: received [10 11 12 13]
Rank 1: received [10 11 12 13]
Rank 2: received [10 11 12 13]
Rank 3: received [10 11 12 13]



MPI_Allgatherv

```
int MPI_Allgatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Gathers variable-sized data from all processes and distributes the complete result to everyone.

- sendbuf: Local data to send.
- sendcount: Number of elements sent by this process.
- sendtype: Type of send data.
- recvbuf: Buffer to store gathered results.
- recvcounts: Array specifying number of elements received from each rank.
- displs: Array specifying displacement offsets in recybuf.
- recvtype: Type of received data.
- comm: Communicator handle.



MPI_Allgatherv

```
int MPI_Allgatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Gathers variable-sized data from all processes and distributes the complete result to everyone.

```
int sendcount = rank + 1; // \{1,2,3,4\}
int sendbuf[4] = {rank, rank, rank, rank};
int recvbuf[10];
int counts[4] = \{1,2,3,4\};
int displs[4] = \{0,1,3,6\};
MPI Allgatherv(sendbuf, sendcount, MPI INT,
          recybuf, counts, displs, MPI INT,
          MPI COMM WORLD);
printf("Rank %d received: ", rank);
for (int i = 0; i < displs[3] + counts[3]; i++)
  printf("%d ", recvbuf[i]);
printf("\n");
```



MPI_Allgatherv

```
int MPI_Allgatherv(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, const int *recvcounts,
    const int *displs,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Gathers variable-sized data from all processes and distributes the complete result to everyone.

```
int sendcount = rank + 1; // \{1,2,3,4\}
int sendbuf[4] = {rank, rank, rank, rank};
int recvbuf[10];
int counts[4] = \{1,2,3,4\};
int displs[4] = \{0,1,3,6\};
MPI Allgatherv(sendbuf, sendcount, MPI INT,
          recybuf, counts, displs, MPI INT,
          MPI COMM WORLD);
printf("Rank %d received: ", rank);
for (int i = 0; i < displs[3] + counts[3]; i++)
  printf("%d ", recvbuf[i]);
printf("\n");
```

Rank 0 received: 0 1 1 2 2 2 3 3 3 3 Rank 1 received: 0 1 1 2 2 2 3 3 3 3 Rank 2 received: 0 1 1 2 2 2 3 3 3 3 Rank 3 received: 0 1 1 2 2 2 3 3 3 3



MPI Scatter

```
int MPI_Scatter(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Distributes equal-sized blocks of data from the root to all processes.

- sendbuf: Root's buffer containing the data to scatter.
- sendcount: Number of elements sent to each process.
- sendtype: Type of send data.
 recvbuf: Buffer to store received data.
- recvcount: Number of elements received per process.
- recvtype: Type of received data.
- root: Rank of root process.
- comm: Communicator handle.



MPI_Scatter

```
int MPI_Scatter(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Distributes equal-sized blocks of data from the root to all processes.

```
int send[4] = {10, 20, 30, 40};
int recv;
MPI_Scatter(send, 1, MPI_INT, &recv, 1, MPI_INT, 0,
MPI_COMM_WORLD);
printf("Rank %d received %d\n", rank, recv);
```



MPI_Scatter

```
int MPI_Scatter(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype, int root,
    MPI_Comm comm);
```

Distributes equal-sized blocks of data from the root to all processes.

```
int send[4] = \{10, 20, 30, 40\};
int recv:
MPI Scatter(send, 1, MPI INT, &recv, 1, MPI INT, 0,
MPI COMM WORLD);
printf("Rank %d received %d\n", rank, recv);
           Rank 0 received 10
           Rank 1 received 20
           Rank 2 received 30
           Rank 3 received 40
```



MPI_Scatterv

Distributes variable-sized blocks of data from the root to all processes.

- sendbuf: Root's buffer containing data.
- sendcounts: Array specifying number of elements to send to each process.
- displs: Array specifying starting offsets of each block in sendbuf.
- sendtype: Type of send data.
- recvbuf: Buffer to receive data.
- recvcount: Number of elements received by this process.
- recvtype: Type of receive data.
- root: Rank of root process.
- comm: Communicator handle.



MPI_Scatterv

Distributes variable-sized blocks of data from the root to all processes.



MPI_Scatterv

Distributes variable-sized blocks of data from the root to all processes.

```
Rank 0 received: 0
Rank 1 received: 1 1
Rank 2 received: 2 2 2
Rank 3 received: 3 3 3 3
```



MPI_Alltoall

```
int MPI_Alltoall(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Each process sends equal-sized data to all others and receives equal-sized data from all others.

- sendbuf: Starting address of send buffer.
 sendcount: Number of elements sent to each process.
- sendtype: Type of send data.
 recvbuf: Starting address of receive buffer.
- recvcount: Number of elements received from each process.
 - recvtype: Type of receive data.
- comm: Communicator handle.



MPI_Alltoall

```
int MPI_Alltoall(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Each process sends equal-sized data to all others and receives equal-sized data from all others.



MPI_Alltoall

```
int MPI_Alltoall(const void *sendbuf,
    int sendcount, MPI_Datatype sendtype,
    void *recvbuf, int recvcount,
    MPI_Datatype recvtype,
    MPI_Comm comm);
```

Each process sends equal-sized data to all others and receives equal-sized data from all others. Rank 0 received [0 10 20 30] Rank 1 received [1 11 21 31] Rank 2 received [2 12 22 32] Rank 3 received [3 13 23 33]



MPI_Alltoallv

Each process sends variable-sized data to all others and receives variable-sized data from all others.

- sendbuf: Starting address of send buffer.
- sendcounts: Array specifying number of elements sent to each rank.
- sdispls: Array of offsets in sendbuf.
- sendtype: Type of send data.
- recvbuf: Starting address of receive buffer.
- recvcounts: Array specifying number of elements received from each rank.
- rdispls: Array of offsets in recvbuf.
- recvtype: Type of receive data.
- comm: Communicator handle.



MPI_Alltoallv

Each process sends variable-sized data to all others and receives variable-sized data from all others.

```
int sendbuf[16];
for (int i=0; i<16; i++) sendbuf[i] = rank*10 + i;
int sendcounts[4] = \{1,2,3,4\};
int sdispls[4] = \{0,1,3,6\};
int recvcounts[4] = \{1,2,3,4\};
int rdispls[4] = \{0,1,3,6\};
int recvbuf[10];
MPI Alltoally(sendbuf, sendcounts, sdispls, MPI INT, recybuf,
              recvcounts, rdispls, MPI INT, MPI COMM WORLD);
printf("Rank %d received:", rank);
for (int i=0; i<recvcounts[rank]; i++)</pre>
  printf(" %d", recvbuf[i]);
printf("\n");
```



MPI_Alltoallv

Each process sends variable-sized data to all others and receives variable-sized data from all others.

```
int sendbuf[16];
for (int i=0; i<16; i++) sendbuf[i] = rank*10 + i;
int sendcounts[4] = \{1,2,3,4\};
int sdispls[4] = \{0,1,3,6\};
int recvcounts[4] = \{1,2,3,4\};
int rdispls[4] = \{0,1,3,6\};
int recvbuf[10];
MPI Alltoally(sendbuf, sendcounts, sdispls, MPI INT, recybuf,
              recvcounts, rdispls, MPI INT, MPI COMM WORLD);
printf("Rank %d received:", rank);
for (int i=0; i<recvcounts[rank]; i++)</pre>
  printf(" %d", recvbuf[i]);
printf("\n");
```

Rank 0 received: 0
Rank 1 received: 10 11

Rank 2 received: 20 21 22

Rank 3 received: 30 31 32 33



LECTURE ENDED HERE. THE REMAINDER OF THE LECTURE WILL BE COVERED ON 10/15.



Lecture Overview

- Non-Blocking MPI Communications
- Review of Collective Communication Patterns
- MPI Collective Communications
- Groups + Communicators
- Questions

```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```

Creates new communicators by splitting an existing communicator into subgroups based on color.

Processes with the same color are grouped together; within each group, ranks are ordered by key



```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```

Creates new communicators by splitting an existing communicator into subgroups based on color.

Processes with the same color are grouped together; within each group, ranks are ordered by key

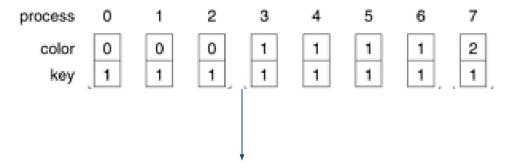
- comm: Existing communicator to be split (e.g., MPI_COMM_WORLD).
 color: Integer identifier for subgroup membership.
 - All processes with the same color form a new communicator.
 - A process can set color =
 MPI_UNDEFINED to be excluded entirely.
- key: Determines ordering of ranks within the new communicator.
 - Lower keys get lower ranks.
 - Equal keys are ordered according to ordering in previous communicator
- newcomm: Output handle to the newly created communicator.



```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```



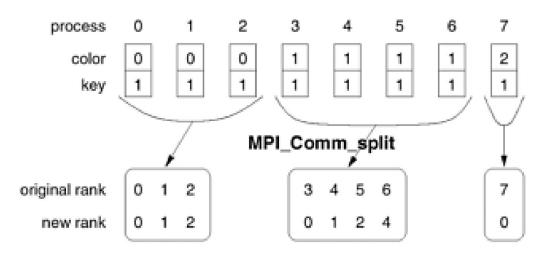
```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```



Which processes will be grouped together?



```
int MPI_Comm_split(MPI_Comm comm,
    int color, int key,
    MPI_Comm *newcomm);
```





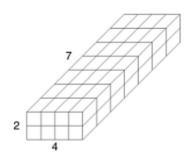
Creates a lower-dimensional subcommunicator from an existing Cartesian topology communicator by selecting which dimensions to keep.



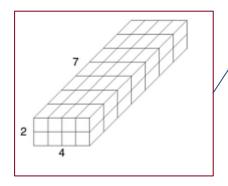
Creates a lower-dimensional subcommunicator from an existing Cartesian topology communicator by selecting which dimensions to keep.

- comm: Input communicator with Cartesian topology (created via MPI_Cart_create).
- remain_dims: Logical array of length equal to the number of dimensions.
 - remain_dims[i] = 1 → keep this dimension.
 - remain_dims[i] = 0 → collapse (drop) this dimension.
- newcomm: Output communicator corresponding to the subgrid of the original topology.



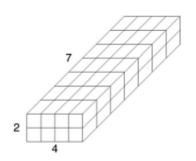


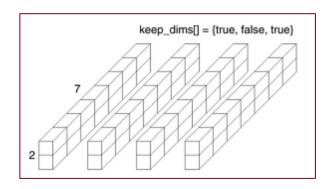




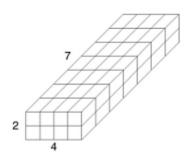
```
int dims[3] = \{2, 4, 7\}; // grid dimensions
int periods[3] = \{0, 0, 0\};
                        // non-periodic in all dimensions
int reorder = 0:
                        // keep original ranks
MPI Comm cart comm;
MPI Cart create(MPI COMM WORLD, 3, dims, periods,
     reorder, &cart comm);
int keep dims[3] = \{1, 0, 1\};
MPI Comm sub comm;
MPI Cart_sub(cart_comm, keep_dims, &sub_comm);
```



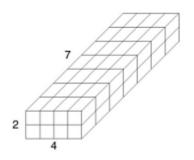


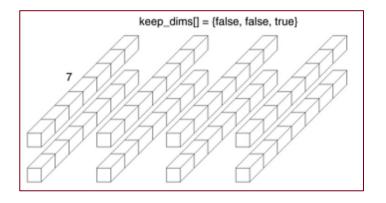














Lecture Overview

- Non-Blocking MPI Communications
- Review of Collective Communication Patterns
- MPI Collective Communications
- ☐ Groups + Communicators
- Questions