CSCI 5451: Introduction to Parallel Computing

Lecture 4: Parallel Algorithm Design

computer science
& engineering

University of Minnesota
Driven to Discover®

Announcement (09/15)

- Office Hours are posted to the course site
- Homework Testing
  - We will be using two clusters (plate and cuda) for all homework assignments in this course
  - Examine this pdf and corresponding code to test that you are able to use these machines going forward (these are simple ‘hello world’ style tests)
  - Doing this in advance gives us the chance to fix any problems you may have with connecting/running programs before we get closer to homework deadlines

Lecture Overview

- Recap
- Task Decomposition
  - Background
  - MatVec Example Decompositions
  - Metrics & Definitions
- Decompositions (Recursive, Exploratory, Data, Speculative, Hybrid)
- Classifying Task Interaction & Generation

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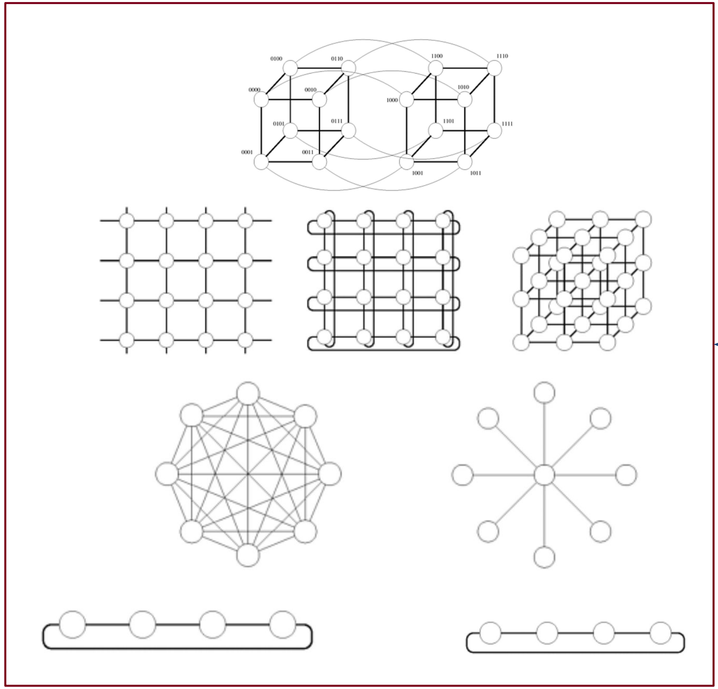
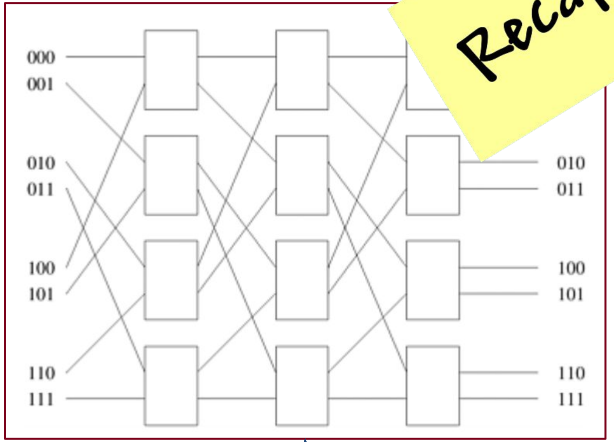
Recap



Topologies



Omega Networks

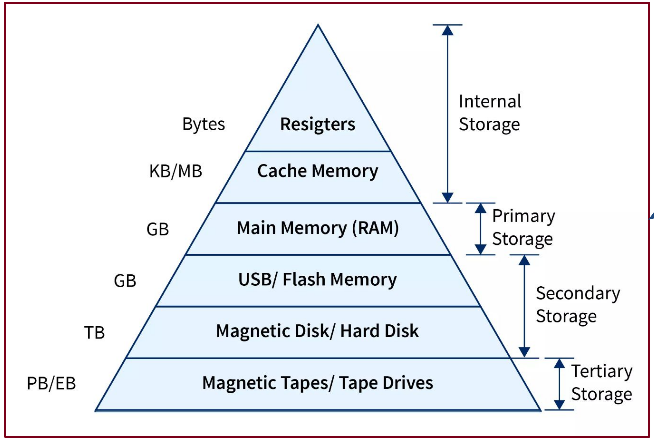
Recap

Topology Metrics

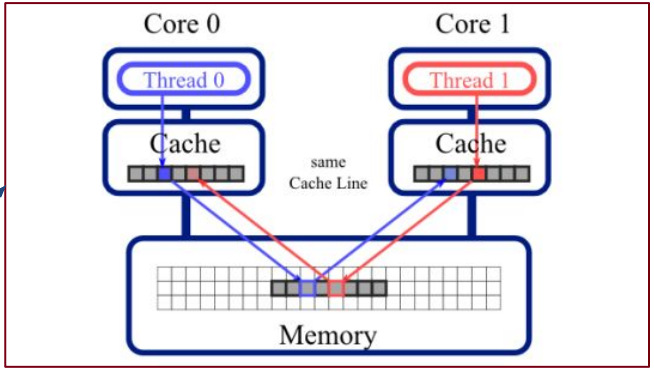
Table 2.1 A summary of the characteristics of various static network to[...] nodes.

Network	Diameter	Bisection Width	Arc Connectivity	(... of links)
Completely-connected	1	$p^2/4$	$p-1$	$p(p-1)/2$
Star	2	1	1	$p-1$
Complete binary tree	$2\log((p+1)/2)$	1	1	$p-1$
Linear array	$p-1$	1	1	$p-1$
2-D mesh, no wraparound	$2(\sqrt{p}-1)$	$\sqrt{p}$	2	$2(p-\sqrt{p})$
2-D wraparound mesh	$2\lfloor\sqrt{p}/2\rfloor$	$2\sqrt{p}$	4	$2p$
Hypercube	$\log p$	$p/2$	$\log p$	$(p\log p)/2$
Wraparound k-ary d-cube	$d\lfloor k/2\rfloor$	$2k^{d-1}$	$2d$	dp

Memory Basics



False Sharing



Recap

Topology Metrics

Table 2.1 A summary of the characteristics of various static network to[illegible] nodes.

Network	Diameter	Bisection Width	Arc Connectivity	(N[illegible] of links)
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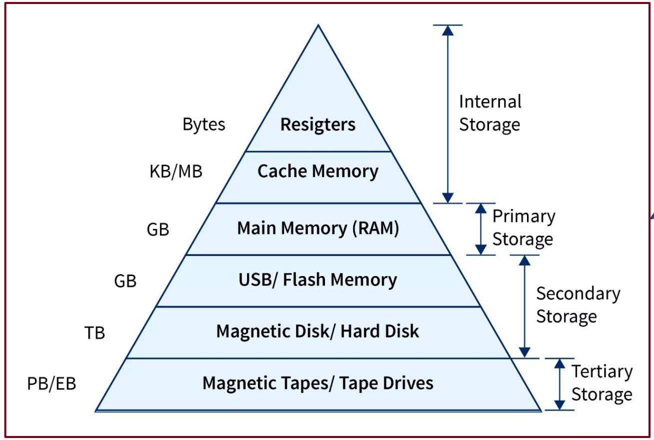

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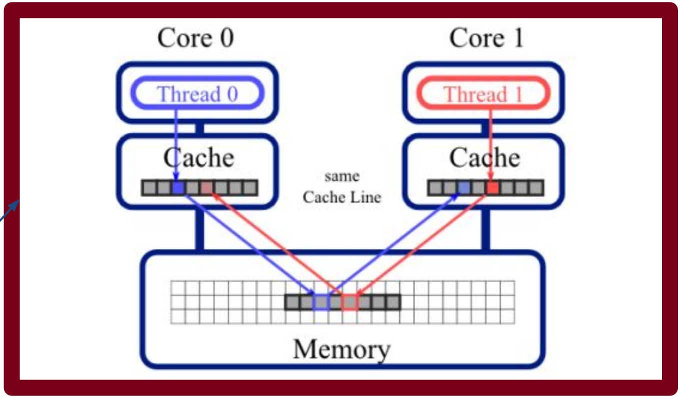
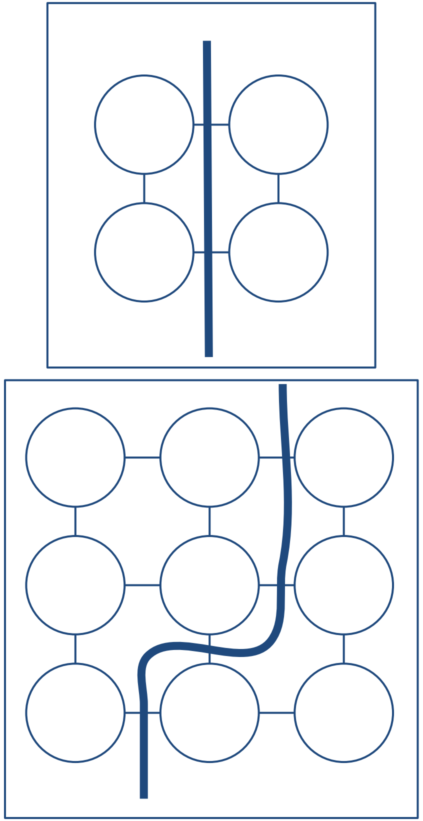


Recap (Revisiting Mesh Bisection Width)



General Case

- $(\sqrt{p}) \bmod 2 == 0$
  - Bisection width = $\sqrt{p}$
- $(\sqrt{p}) \bmod 2 == 1$
  - Bisection width = $\sqrt{p} + 1$

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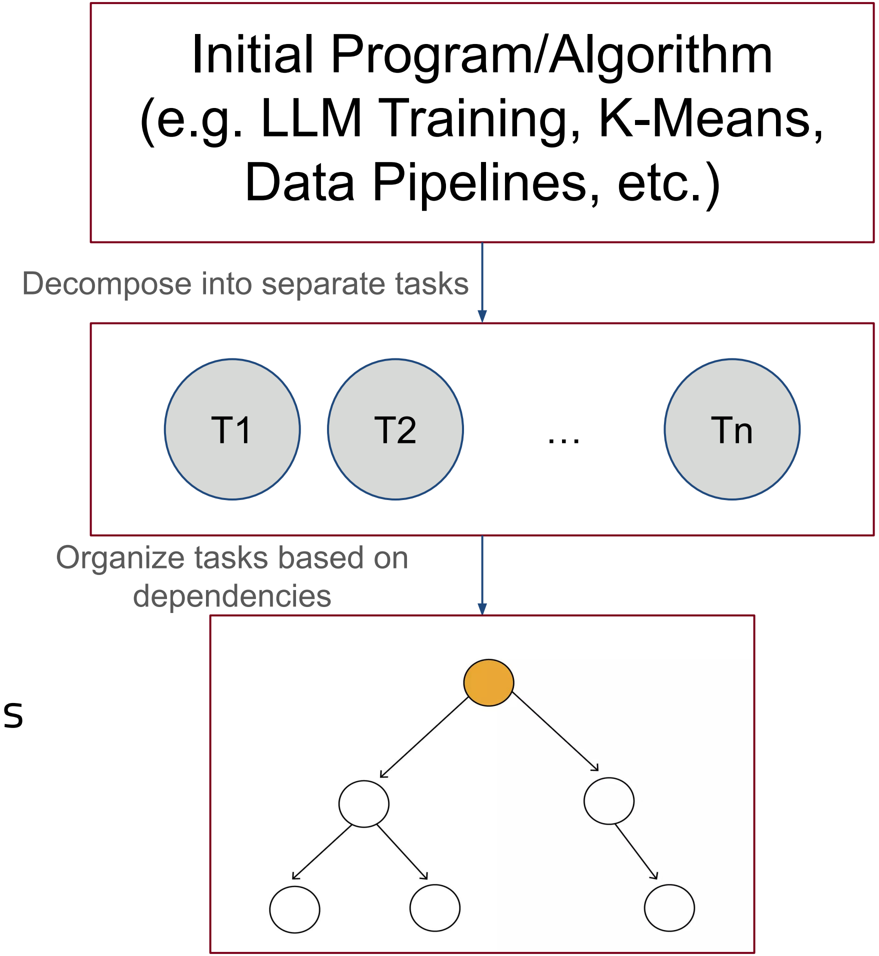
How do we map from a serial program into some parallel program?

How do we map from a serial program into some parallel program?

Decompose the program into smaller tasks, then find out how to organize them.

Task Decomposition

- We have to break down the tasks individually and determine their dependencies
- What are tasks?
  - Atomic units of computation within our program
  - We as programmers decide what these tasks are
  - Once we define a task, we assume that it is both serial and indivisible



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Matrix Vector Multiplication

$$A * x = b$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{bmatrix}$$

Matrix Vector Multiplication

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Simplest decomposition: Don't decompose any tasks at all. This is the same as serial execution. No parallel speedups = bad task decomposition.

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Other Decomposition Ideas?

Matrix Vector Multiplication

$$A * x = b$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \boxed{ax + by + cz} \\ \boxed{dx + ey + fz} \\ \boxed{gx + hy + iz} \end{bmatrix}$$

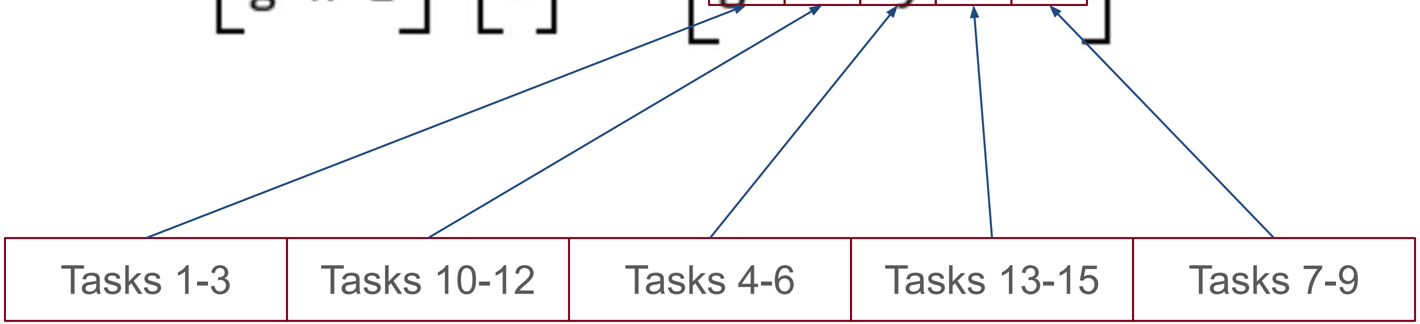
Task 1
Task 2
Task 3

Matrix Vector Multiplication

Most Granular*

$$A * x = b$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax & + & by & + & cz \\ dx & + & ey & + & fz \\ gx & + & hy & + & iz \end{bmatrix}$$



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Tasks 1-3	Tasks 7-9	Tasks 4-6

Task Dependency Graph (TDG)

- Once we break things down into smaller tasks, we need to build out how we should execute them
- There can be *more than one* way to combine the tasks
- Each node is a task, arrows represent dependencies
- Each node will oftentimes have the amount of work required attached to them (typically represented by the number of atomic operations performed within that task - add, subtract, multiply, divide)
- We assume, within each task, that work is serial

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Why might this be a poor way of measuring work?

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For our purposes, we will assume all work is equal until later on in the course.

Matrix Vector Multiplication (TDG)

$$\boxed{A * x = b}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{bmatrix}$$

How to represent this decomposition as a TDG?

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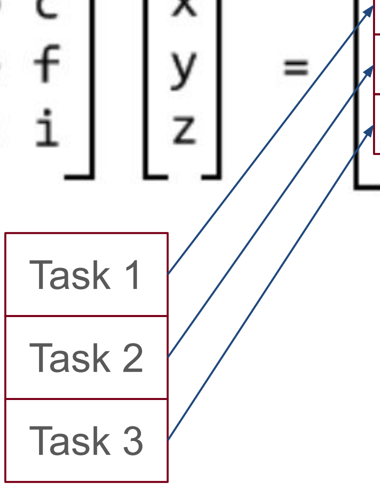

How to represent this decomposition as a TDG?

Matrix Vector Multiplication (TDG)

No edges because there are no dependencies between tasks

$$A * x = b$$

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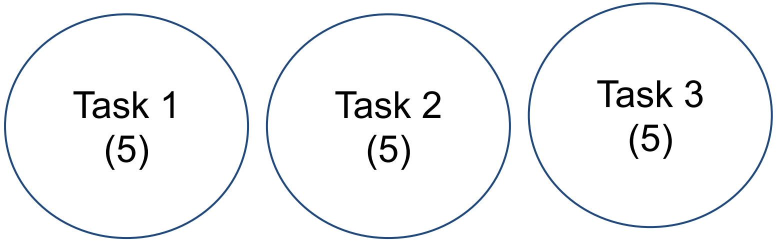
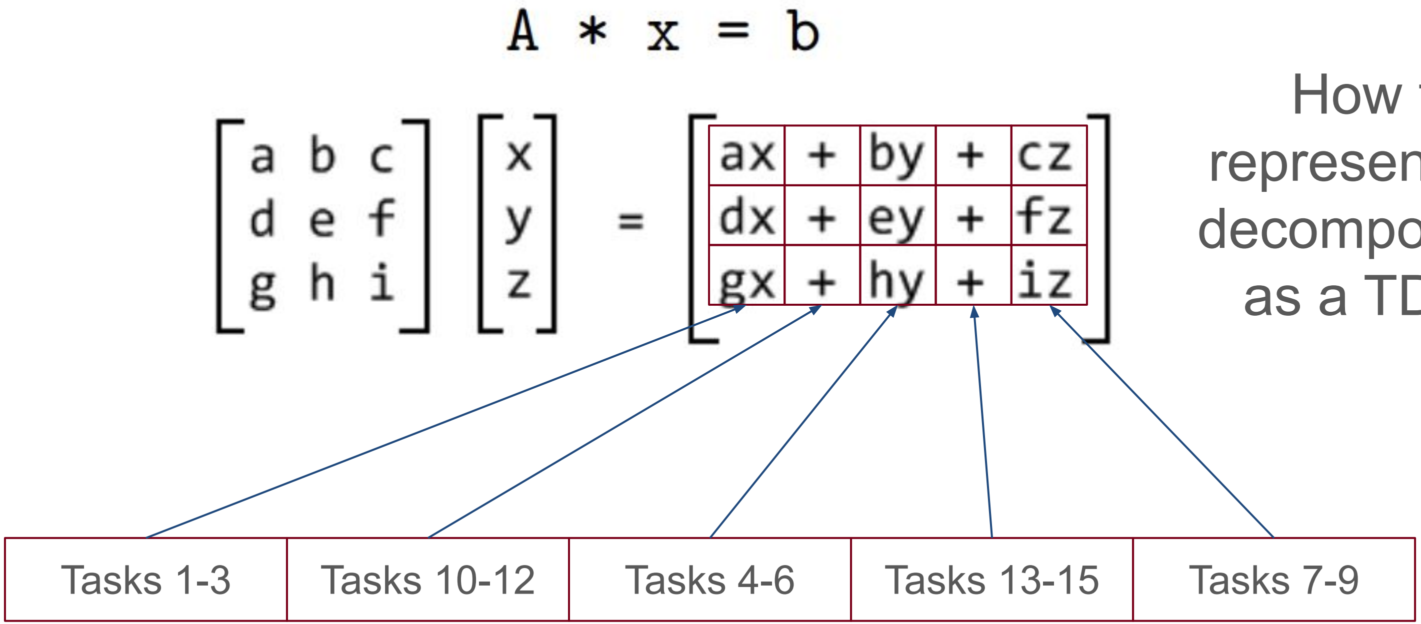
Task 1
Task 2
Task 3

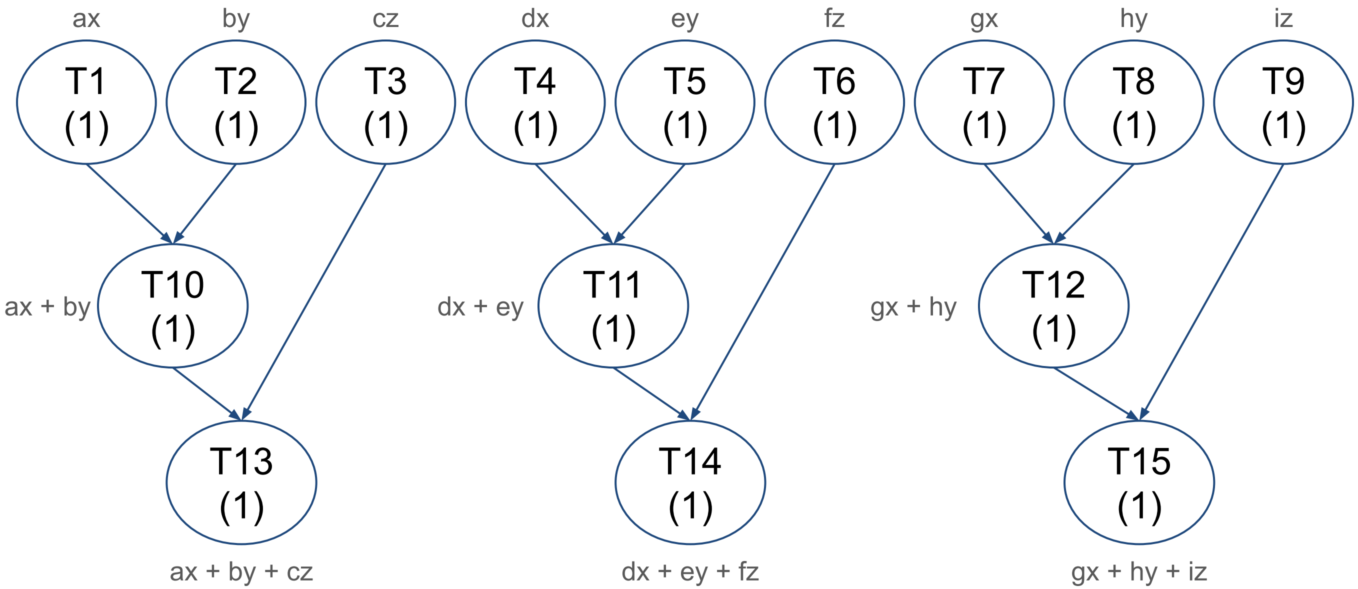


Matrix Vector Multiplication

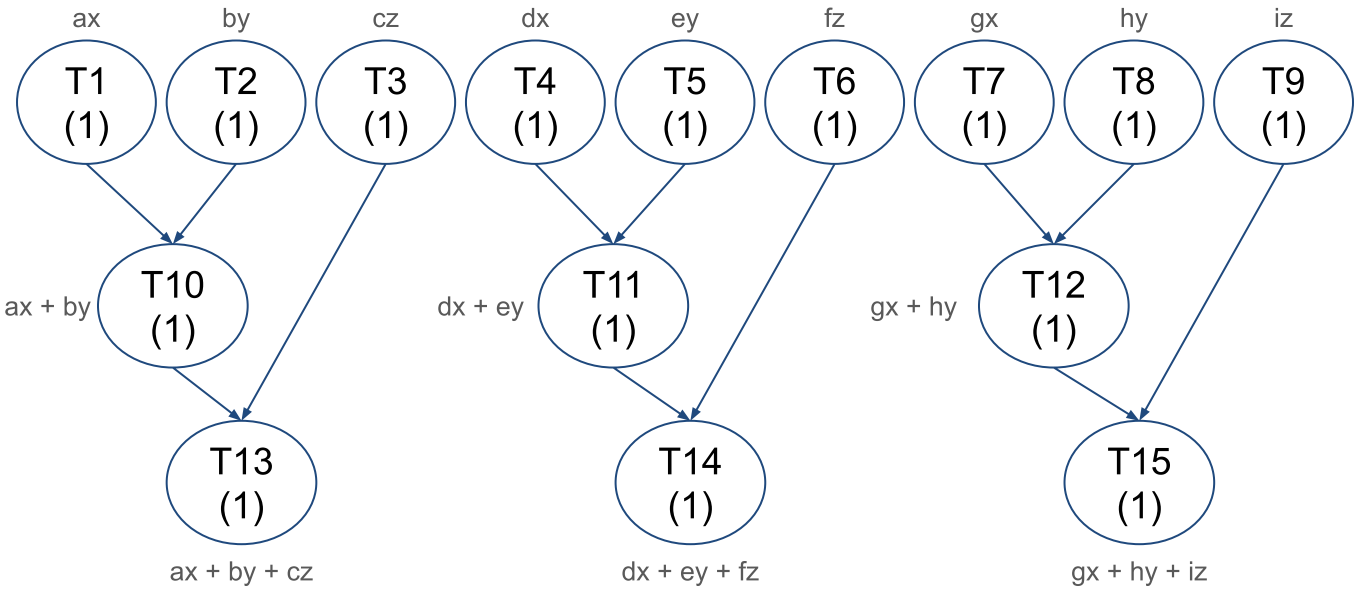


How to represent this decomposition as a TDG?

Matrix Vector Multiplication



Matrix Vector Multiplication



Alternative Decompositions?

Matrix Vector Multiplication

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Tasks 1-3	Tasks 7-9	Tasks 4-6

How to represent this decomposition as a TDG?

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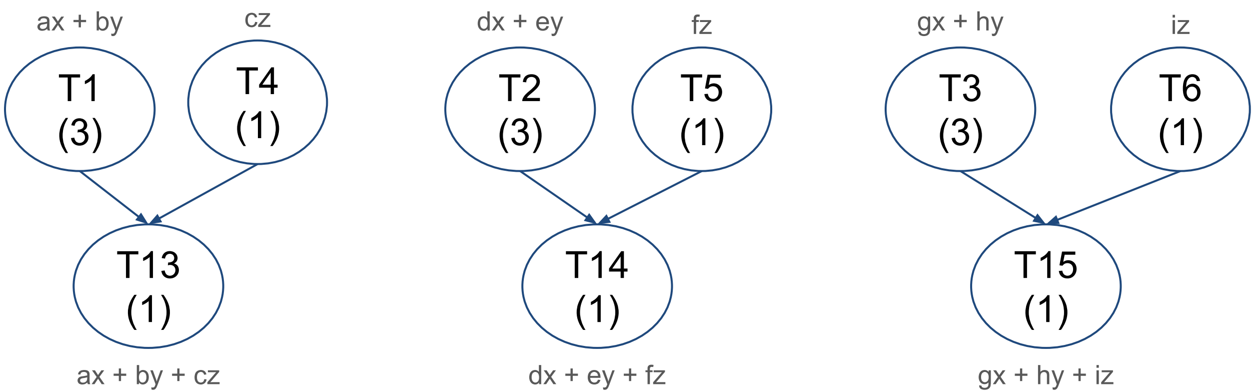

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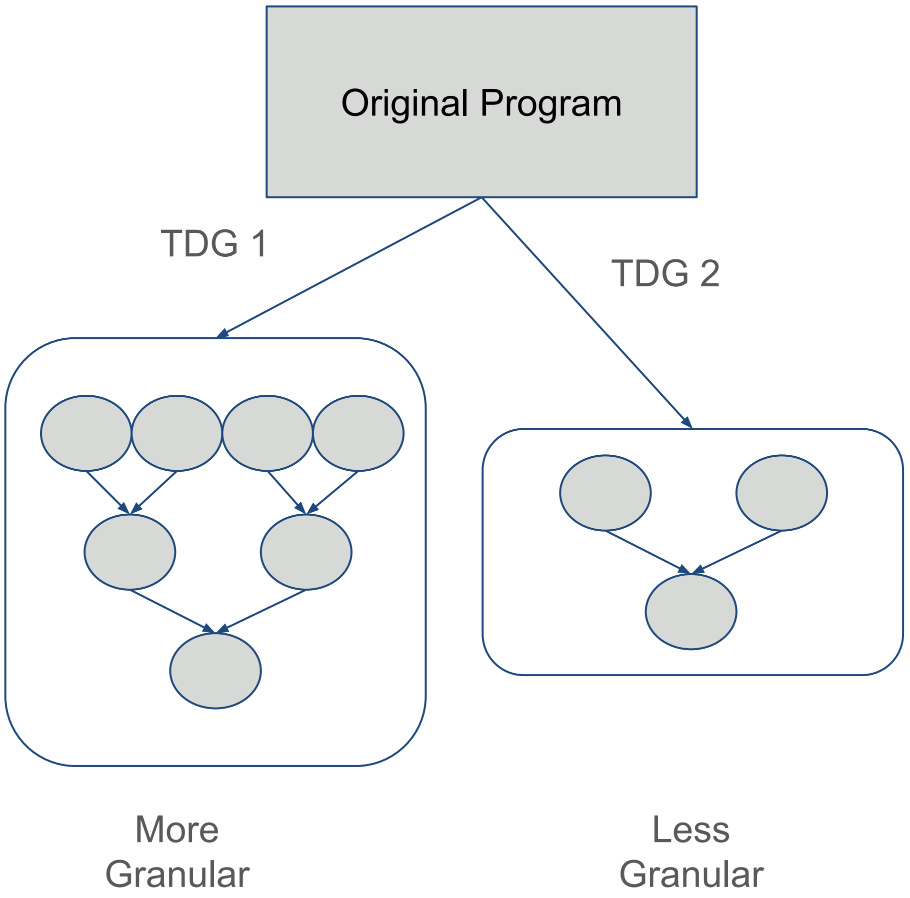
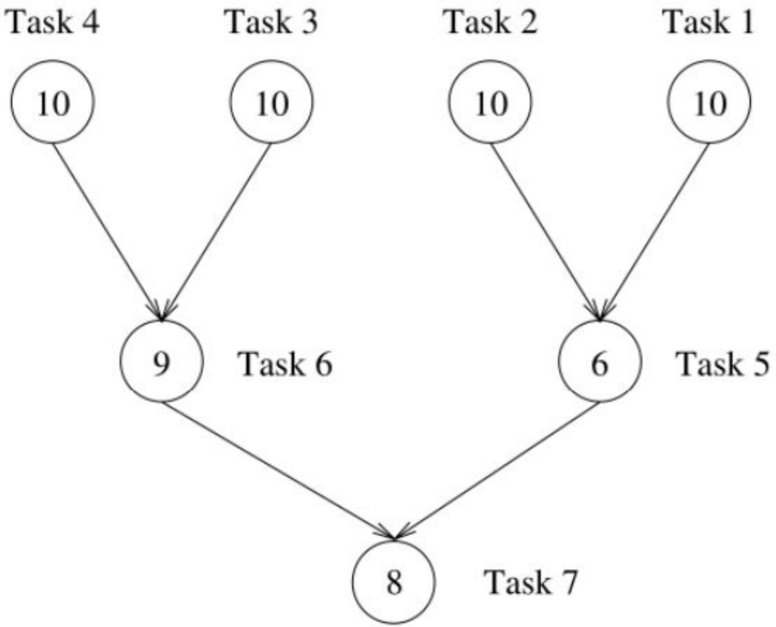
Granularity

- Defines how fine-grain the parallel program has been decomposed
- More granular = smaller tasks with less work per task
- Less granular = larger tasks with more work per task



Total Work

Sum of all work completed over all tasks



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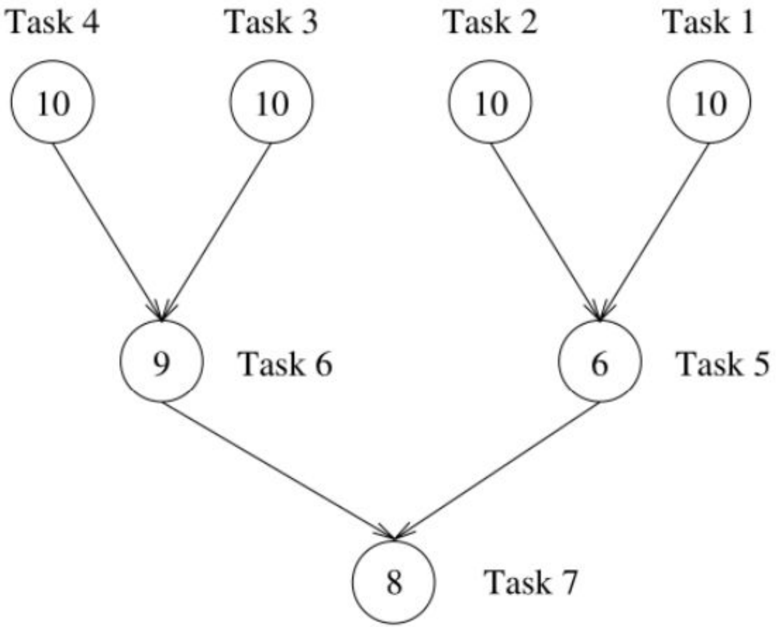
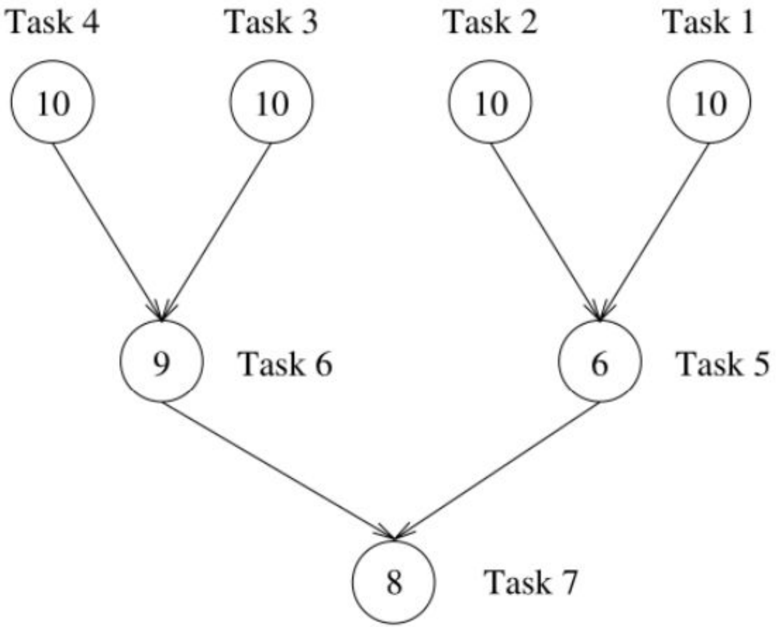


Total Work = 63

Critical Path Length

Largest amount of *sequential* work which must be performed in order to complete program execution



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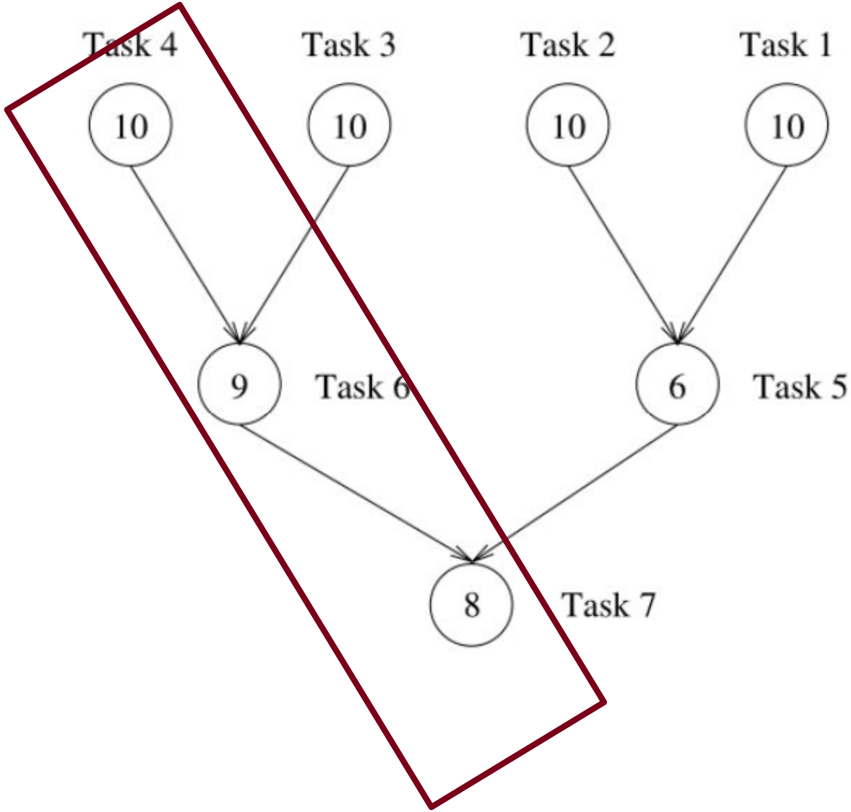
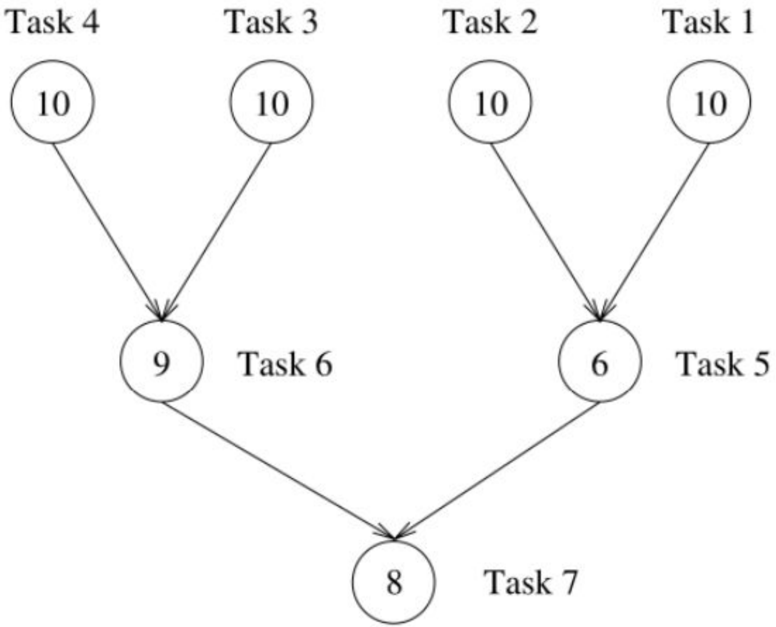


Critical Path Length = 63

Concurrency

- Maximum Concurrency
  - Largest number of tasks which can be completed concurrently
  - The largest *width* of the TDG
- Average Concurrency
  - Total Work/Critical Path Length
  - This term usually sets an upper bound on parallel speedups



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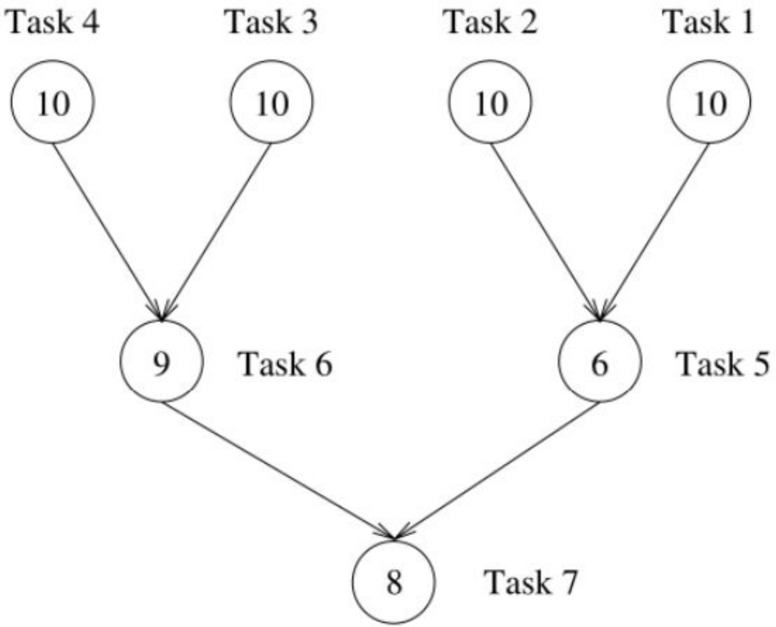
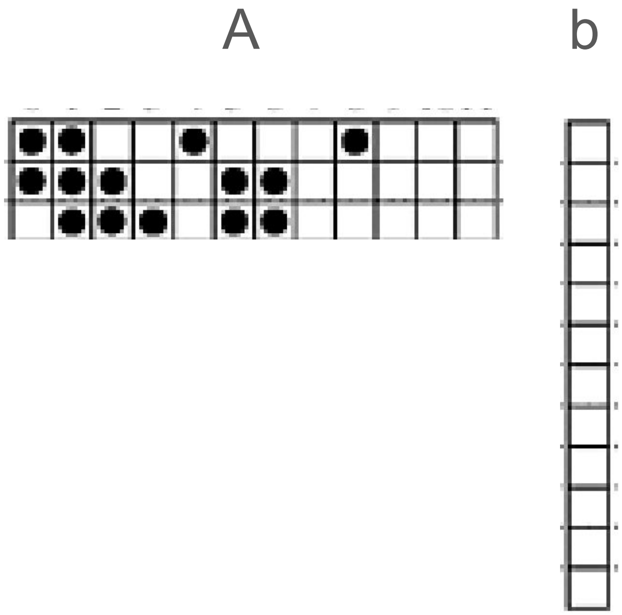


Maximum Concurrency = 4
Average Concurrency = 2.33

Sparse MatVec Example TDG + Metrics

- Assume
  - *A* is sparse
  - *b* is dense
- In Class Example
  - Decompose this into tasks
  - Create a Task Decomposition Graph (TDG) for these tasks
  - Define the following metrics for this TDG
    - ✔ Total Work
    - ✔ Critical Path Length
    - ✔ Average + Maximum Currency



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What are some general strategies we can use for decomposing algorithms?

Recursive Decomposition

- Problems which can be solved via some divide + conquer strategies
- Subproblems can be recursively parallelized
- Partition tasks to each recursive call
- Examples
  - Finding the minimum element of an array
  - Quicksort Example

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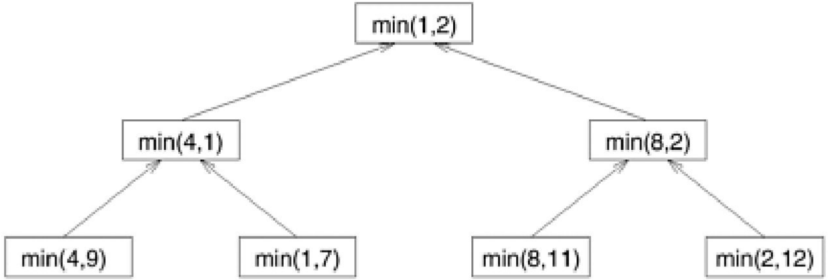
{4, 9, 1, 7, 8, 11, 2, 12}



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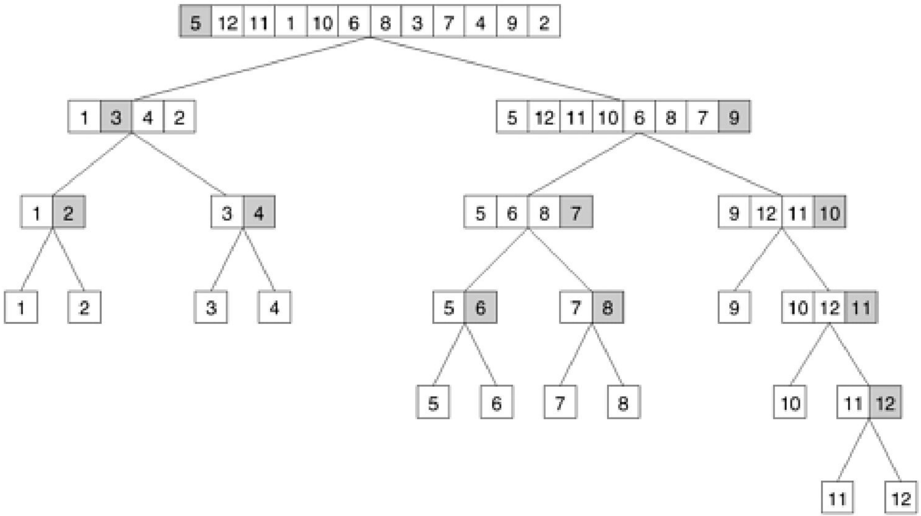

Exploratory Decomposition

- Use this for problems involving some kind of search
- Partition the search space among different processes
- Can lead to much anomalous speedups (slower or faster)
- Example
  - 15-puzzle search

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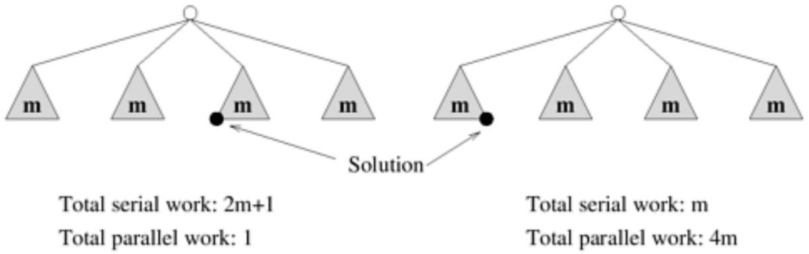

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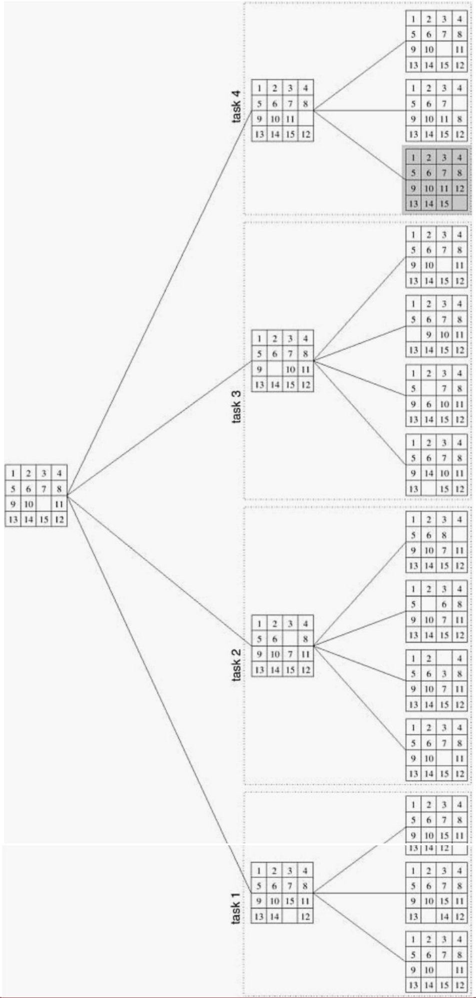


Data Decomposition (Output)

- Take the expected outputs of your program, then split them among your processes
- Assign tasks to each of these output elements

Task 1: $C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1}$
Task 2: $C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2}$
Task 3: $C_{1,1} = A_{2,1}B_{1,1} + A_{2,2}B_{2,1}$
Task 4: $C_{1,1} = A_{2,1}B_{1,2} + A_{2,2}B_{2,2}$

$$\begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \cdot \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix} \rightarrow \begin{pmatrix} \boxed{C_{1,1}} & \boxed{C_{1,2}} \\ \boxed{C_{2,1}} & \boxed{C_{2,2}} \end{pmatrix}$$

Data Decomposition (Intermediate)

- Many algorithms have multiple, sequential stages of computation
- First partition the problem into these intermediate computations
- Then assign tasks based on which processes *own* which computation

Stage I

$$\begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \cdot \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix} \rightarrow \left(\begin{matrix} \begin{pmatrix} D_{1,1,1} & D_{1,1,2} \\ D_{1,2,2} & D_{1,2,2} \end{pmatrix} \\ \begin{pmatrix} D_{2,1,1} & D_{2,1,2} \\ D_{2,2,2} & D_{2,2,2} \end{pmatrix} \end{matrix} \right)$$

Stage II

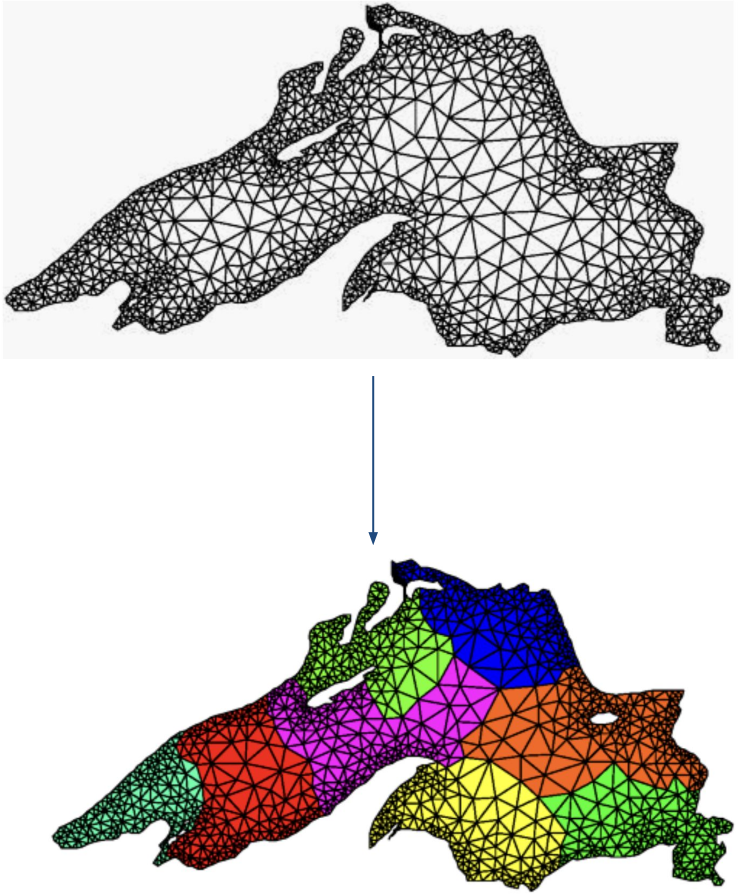
$$\begin{pmatrix} D_{1,1,1} & D_{1,1,2} \\ D_{1,2,2} & D_{1,2,2} \end{pmatrix} + \begin{pmatrix} D_{2,1,1} & D_{2,1,2} \\ D_{2,2,2} & D_{2,2,2} \end{pmatrix} \rightarrow \begin{pmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{pmatrix}$$

A decomposition induced by a partitioning of D

Task 01:	$D_{1,1,1} = A_{1,1}B_{1,1}$
Task 02:	$D_{2,1,1} = A_{1,2}B_{2,1}$
Task 03:	$D_{1,1,2} = A_{1,1}B_{1,2}$
Task 04:	$D_{2,1,2} = A_{1,2}B_{2,2}$
Task 05:	$D_{1,2,1} = A_{2,1}B_{1,1}$
Task 06:	$D_{2,2,1} = A_{2,2}B_{2,1}$
Task 07:	$D_{1,2,2} = A_{2,1}B_{1,2}$
Task 08:	$D_{2,2,2} = A_{2,2}B_{2,2}$
Task 09:	$C_{1,1} = D_{1,1,1} + D_{2,1,1}$
Task 10:	$C_{1,2} = D_{1,1,2} + D_{2,1,2}$
Task 11:	$C_{2,1} = D_{1,2,1} + D_{2,2,1}$
Task 12:	$C_{2,2} = D_{1,2,2} + D_{2,2,2}$

Data Decomposition (Input)

- First split the inputs of the problem into separate pieces so that each process will only have some subset of data
- Then assign tasks based on what data each of your processes stores
- Example:
  - Measuring temperatures at various positions in Lake Superior
  - Want to predict temperatures in the future



Speculative Decomposition

- Useful when a program has many conditional branches & computationally intensive statements to execute to determine the appropriate branch
- Examples
  - Simple Switch Statement
  - Discrete Event Simulation

```
int main() {
    int choice = long_running_program();

    switch (choice) {
        case 1:
            f_1();
            break;
        case 2:
            f_2();
            break;
        case 3:
            f_3();
            break;
        case 4:
            f_4();
            break;
        default:
            printf("Invalid choice.\n");
    }
}
```

Speculative Decomposition

- Useful when a program has many conditional branches & computationally intensive statements to execute to determine the appropriate branch
- Examples
  - Simple Switch Statement
  - Discrete Event Simulation

```
int main() {
    int choice = long_running_program();

    switch (choice) {
        case 1:
            f_1();
            break;
        case 2:
            f_2();
            break;
        case 3:
            f_3();
            break;
        case 4:
            f_4();
            break;
        default:
            printf("Invalid choice.\n");
    }
}
```

Run all in parallel

Speculative Decomposition

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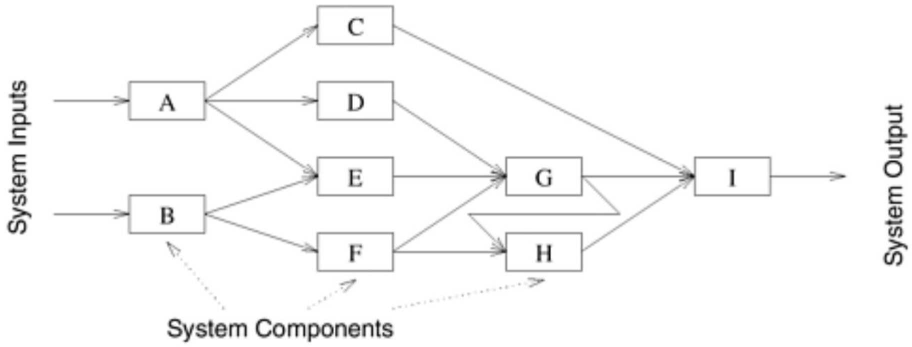

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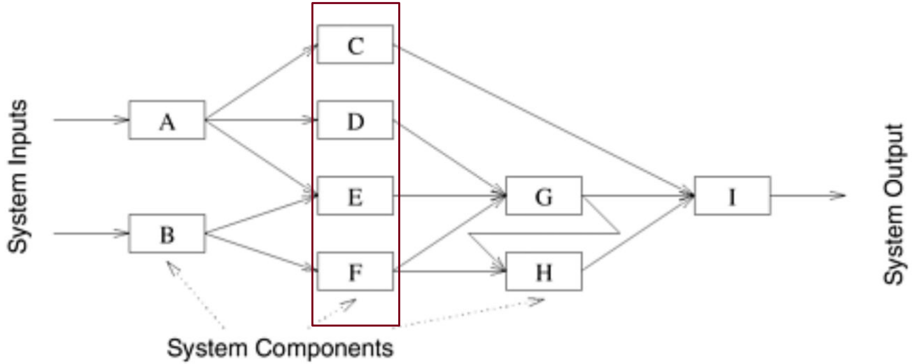
Run multiple conditional branches at a time



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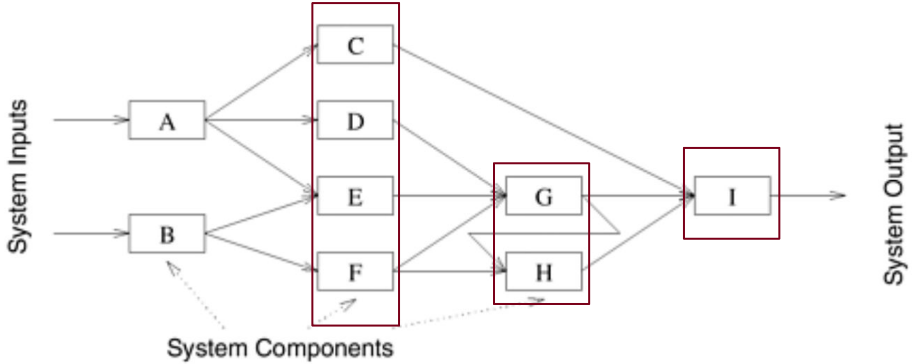

…we can also run computations at greater depth in parallel

Speculative Decomposition

Under what conditions, can we not run each of the following tasks at the same time?

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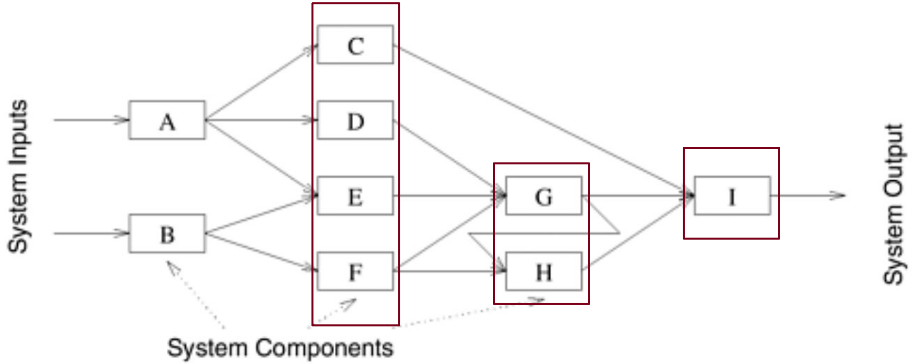

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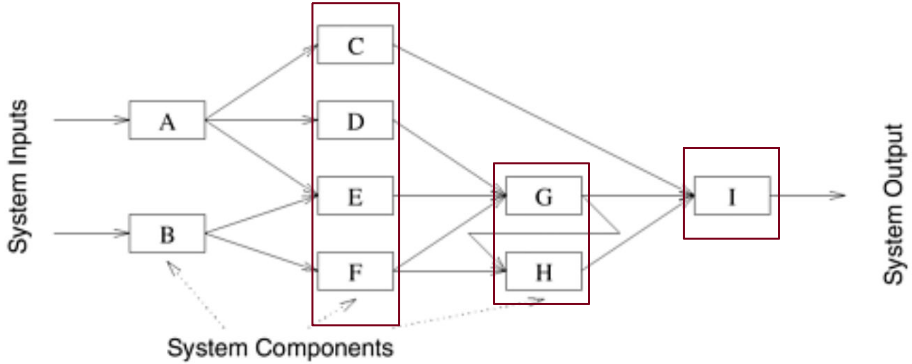
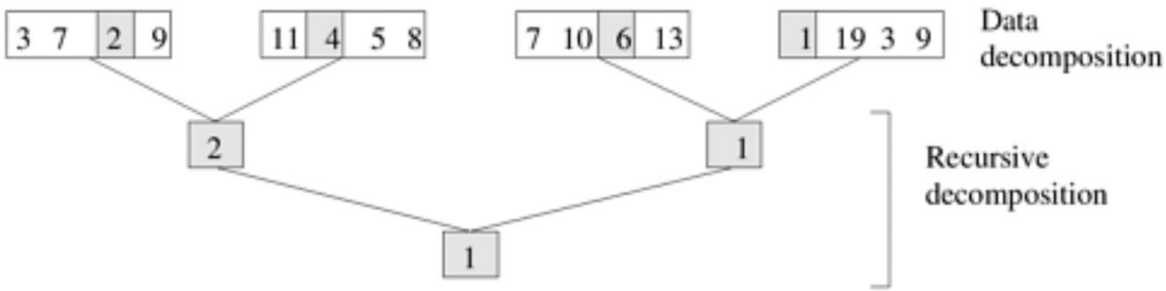


…when the data produced from earlier tasks is required for later ones (i.e. computing ***G*** is dependent on ***A***)

Hybrid Decomposition

- Combine multiple decomposition techniques at the same time
- More typical in practical settings
- Example (Min-Array)



Lecture Overview

- Recap
- Task Decomposition
  - Background
  - MatVec Example Decompositions
  - Metrics & Definitions
- Decompositions (Recursive, Exploratory, Data, Speculative, Hybrid)
- **Classifying Task Interaction & Generation**

How can we classify task generation & interaction?

Static vs. Dynamic Task Generation

- Static Task Generation: All tasks are known *exactly* before program execution
- Dynamic Task Generation: Tasks and the Task Dependency Graph are not known exactly before program execution

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(a)

Task 1: $C_{1,1} = A_{1,1}B_{1,1} + A_{1,2}B_{2,1}$
Task 2: $C_{1,2} = A_{1,1}B_{1,2} + A_{1,2}B_{2,2}$
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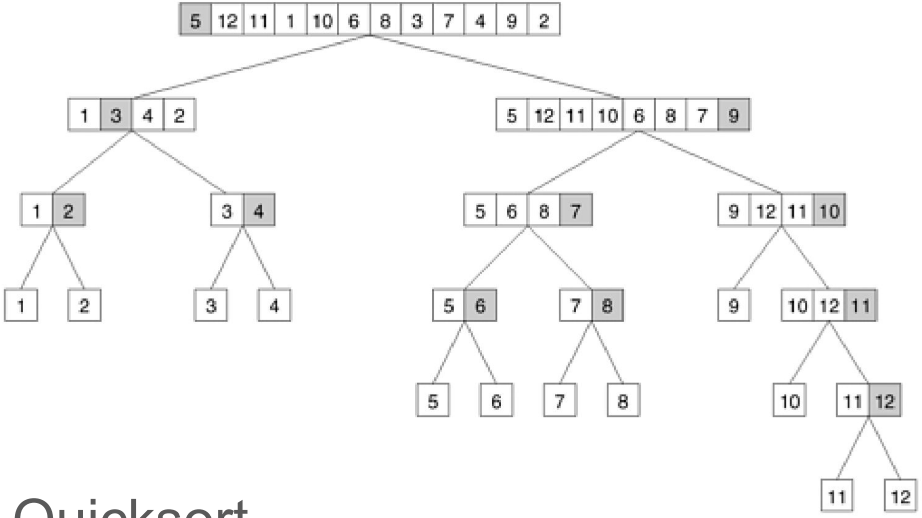

Quicksort

Uniform vs. Non-Uniform Task Sizes

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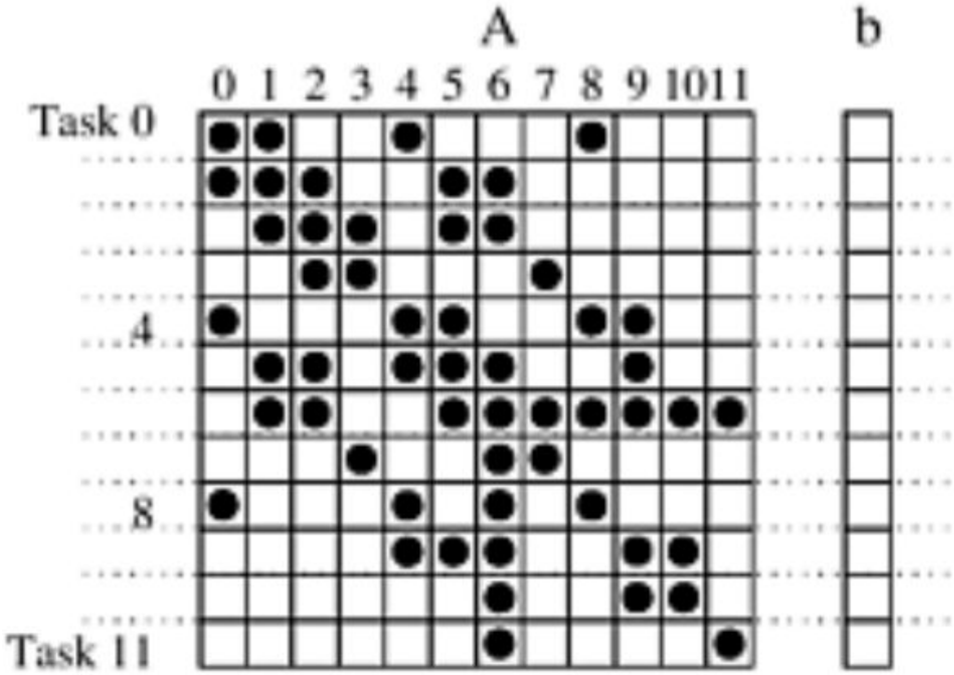

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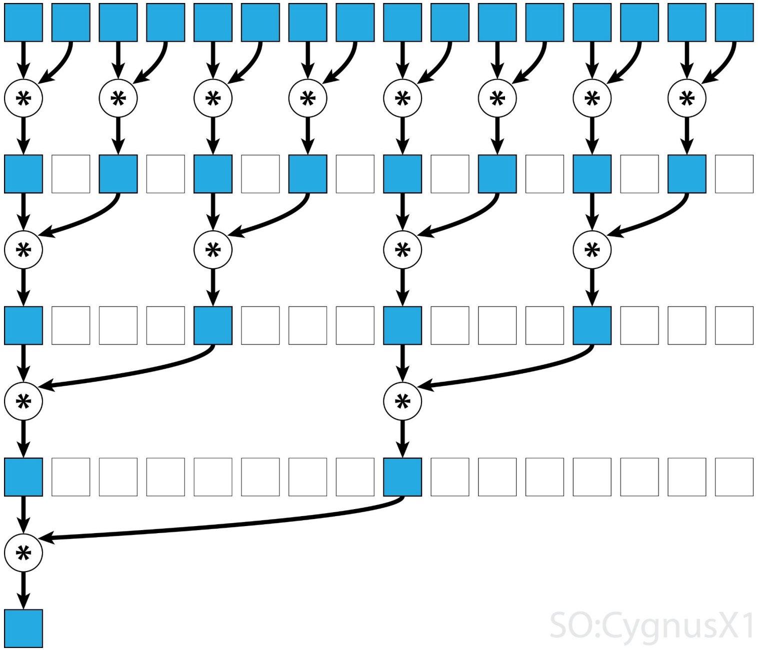

Parallel Array Sum
Interactions are fixed

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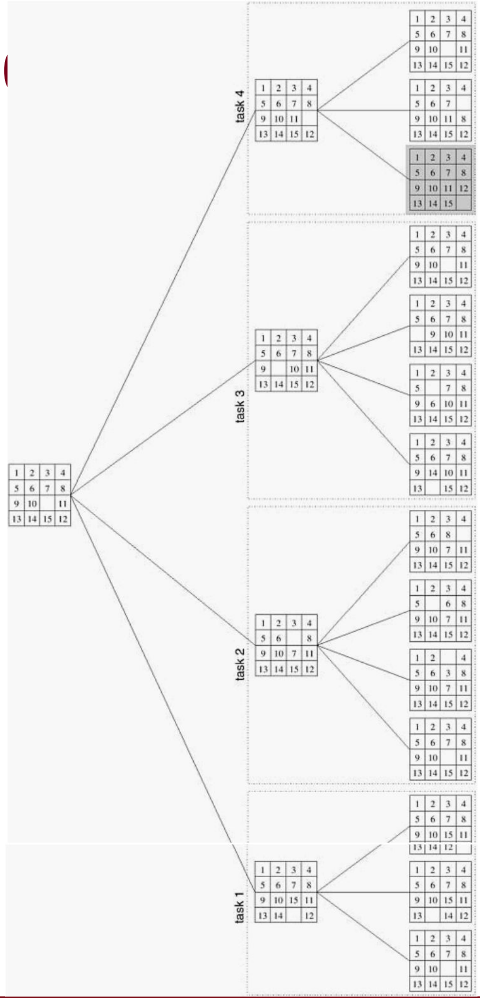
Search Problems require communication to prevent redundant rollouts



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