Final Project Abstract
MTH 610
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The aim of this project is to compare two methods of obtaining solutions to Lorenz '63 model differential equation system

$$x'(t) = \sigma(y - x)$$
$$y'(t) = \rho x - y - xz$$
$$z'(t) = xy - \beta z,$$

where I will denote $\mathbf{X}(t) = [x(t), y(t), z(t)]^\top$ to represent a solution to the system compactly. I will start by computing a numerical solution using a 4th order Runge-Kutta Method (RK4) with 10,000 time steps. Afterwards, I will treat the RK4 solution as the "true state" and sample points of this solution at every tenth time step, with which I will use as 'observational data' to compute another numerical solution to the Lorenz model using an extended Kalman filter (EKF) on a sparser time scale (1,000 time steps). I will use the EKF method three times, changing how I create observations. These observations will be taken as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \varepsilon_0$$

where $\mathbf{y} \in \mathbb{R}^m$, $m = 3, 2, 1$ in cases 1, 2, and 3 respectively, and $\varepsilon_0 \in \mathbb{R}^m$ is a random vector of observational errors. In the first case ($m = 3$), $\mathbf{H} \in \mathbb{R}^{3\times3}$ will observe all three components of $\mathbf{X}$; in the second, $\mathbf{H} \in \mathbb{R}^{2\times3}$ will observe the first two only; in the third, $\mathbf{H} \in \mathbb{R}^{1\times3}$ will observe the first only. Finally, I will compare each of these three EKF solutions to the RK4 solution of the Lorenz model.