MTH 410/510: HOMEWORK 4

PART I: Use the data set "LogisticData.txt" from d2l to implement the gradient decent method and the Newton method for the Logistic Regression. Find the coefficients w and b using the stopping criterion $\|\nabla L(x)\| < \epsilon$, where $\epsilon = 10^{-4}$. How many iterations are needed for each method?

PART II:

Problem 1. Consider the following optimization problem:

$$\begin{aligned}&\text{minimize } f(x_1, x_2, x_3) = x_1^2 + 2x_1x_2 + 3x_2^2 + 4x_1 + 5x_2 + 6x_3\\&\text{subject to } x_1 + 2x_2 = 3, 4x_1 + 5x_3 = 6.\end{aligned}$$

(a) Show that the function $f : I\!R^3 \to I\!R$ is convex.
(b) Find an absolute minimizer of the problem.
(Hint: Show that the Hessian of the function f is positive semidefinite and then use the method of Lagrange multipliers).

Problem 2. Consider the problem

$$\begin{aligned}&\text{minimize } \frac{1}{2}x^TQx - c^Tx + d\\&\text{subject to } Ax = b,\end{aligned}$$

where $Q^T = Q$ is an $n \times n$ positive definite matrix, A is an $m \times n$ matrix with rank $A = m$, $c \in I\!R^n$, and d is a constant. Derive a closed-form solution to the problem in terms of Q, c, d, A, b.

Problem 3. Find the absolute maximum of the problem

$$\begin{aligned}&\text{maximize } x + 2y\\&\text{subject to } x^2 + y^2 \le 1.\end{aligned}$$

(Hint: Convert the problem to a minimization problem and then apply the method of Lagrange multipliers).

Problem 4. Find the absolute minimum of the problem

$$\begin{aligned}&\text{mminimize } x_1^2 + x_2^2\\&\text{subject to } x_1 \ge 0, x_2 \ge 0, x_1 + x_2 \ge 5.\end{aligned}$$

(Hint: Convert the constraint to the right form and then apply the method of Lagrange multipliers).

Problem 5(bonus). Let $X_1, X_2, \ldots, X_n$ be independent random variables with $E(X_i) = \mu$ and $\text{Var } X_i = \sigma_i^2$. Find the coefficients a_i that minimize

$$\begin{aligned}&\text{Var } (\sum_{i=1}^n a_iX_i)\\&\text{subject to } E(\sum_{i=1}^n a_iX_i) = \mu.\end{aligned}$$

(Hint: Use $\text{Var } (aX) = a^2\text{Var } X$, $\text{Var } (X + Y) = \text{Var } (X) + \text{Var } (Y)$ for independent random variables X and Y.)