MTH 410/510 HOMEWORK 2, SET 2

1. Consider the Rosenbrock function given by

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

(a) Find the gradient and the Hessian of the function at any point $x = (x_1, x_2)$.
(b) Find the absolute minimizer of the function and the associated optimal value.
(c) write a MATLAB code to implement the gradient method with backtracking line search with $s = 2, \alpha = 0.25, \beta = 0.5, x_0 = [2; 5]$ and $\epsilon = 10^{-4}$ to minimize this function. How many iterations does this run require?

2. Let $f(x) = \sqrt{1 + x^2}$. Prove that f is a $C^{1,1}$ function. Hint: Find f'' and show that it is bounded.

3. Suppose that $f \in C_\ell^{1,1}$.
(a) Prove that for any $x \in \mathbb{R}^n$ and $t > 0$,

$$f(x) - f(x - t\nabla f(x)) \geq t(1 - \frac{t\ell}{2})\|\nabla f(x)\|^2.$$

(b) Let $\{x_k\}$ be the sequence generated by the gradient method for minimizing the function f with constant stepsize $0 < t < \frac{2}{\ell}$. Prove that $\{f(x_k\}$ is monotone decreasing. In addition, $f(x_{k+1}) < f(x_k)$ unless $\nabla f(x_k) = 0$.

MTH 410/510 HOMEWORK 2, SET 1

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1. Write a MATLAB codes to implement the gradient method to minimize the function $f(x_1, x_2) = x_1^2 + x_1x_2 + x_2^2 + x_1 - x_2$. Run the following methods and compare the number of iterations required by each method when the initial vectors is $x_0 = (5, 10)$ to obtain a solution x with $\|\nabla f(x)\| \leq 10^{-4}$.
(a) The gradient method with exact line search.
(b) The gradient method with constant stepsize $t = 0.1$.
(c) The gradient method with backtracking stepsize rule and parameters $\alpha = 0.5, \beta = 0.5, s = 1$.

Hint: Convert the function to the matrix form $f(x) = x^T Ax + bx$, where $A = [1, 1/2; 1/2, 1], b = [1; -1]$.

2. Consider the minimization problem

$$\text{minimize } x^T Ax, x \in \mathbb{R}^5,$$

where A is the 5×5 Hilbert matrix defined by

$$A_{i,j} = \frac{1}{i + j - 1}, i, j = 1, 2, 3, 4, 5.$$

The matrix can be constructed via MATLAB command $A = hilb(5)$. Run the following methods and compare the number of iterations required by each method when the initial vectors is $x_0 = (1, 2, 3, 4, 5)$ to obtain a solution x with $\|\nabla f(x)\| \leq 10^{-2}$.
(a) The gradient method with exact line search.
(b) The gradient method with constant stepsize $t = 0.1$.
(c) The gradient method with backtracking stepsize rule and parameters $\alpha = 0.5, \beta = 0.5, s = 1$.

3. (Bonus) (a) Use the data set "housing.txt" from d2l and the gradient method to find w_0, w_1, w_2 in the linear function $f(x_1, x_2) = w_0 + w_1x_1 + w_2x_2$ in the linear regression model. Then make prediction for the price of a house with a living area of $2080ft^2$ and 4 bedrooms.
(b) Use the data set "TwinCityHomes.csv" from d2l and the gradient descent for the linear regression model to predict the price of a house with 4 bedrooms, 2.5 bathrooms, $2080ft^2$, lot size $5600ft^2$, year built 2007, and 2 parking spots.