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Some convergence histories for Problem 4

NOTE: In all tables, the 'SuccessiveRatioSquared' columns are the current error divided by the square of the previous error.

Problem 4 a):

A brief convergence history for the first 10 iterations of 1998

T =

10×3 table

Errors	SuccessiveRatios	SuccessiveRatioSquared
5.000000000000000e-01	NaN	NaN
3.333333333333333e-01	6.666666666666667e-01	1.333333333333333e+00
2.500000000000000e-01	7.500000000000000e-01	2.250000000000000e+00
2.000000000000000e-01	8.000000000000000e-01	3.200000000000000e+00
1.666666666666667e-01	8.333333333333333e-01	4.166666666666667e+00
1.42857142857143e-01	8.57142857142857e-01	5.14285714285714e+00
1.250000000000000e-01	8.750000000000000e-01	6.125000000000000e+00
1.111111111111111e-01	8.888888888888889e-01	7.111111111111111e+00
1.000000000000000e-01	9.000000000000000e-01	8.100000000000000e+00
9.09090909090909e-02	9.09090909090909e-01	9.09090909090909e+00

The successive ratio of errors is increasing and approaching 1, which suggests convergence is sublinear.

Problem 4 b):

A brief convergence history for the first 10 iterations of 25

T =

10×3 table

Errors	SuccessiveRatios	SuccessiveRatioSquared
3.333333333333333e-01	NaN	NaN
1.111111111111111e-01	3.333333333333333e-01	1.000000000000000e+00
3.70370370370370e-02	3.333333333333333e-01	3.000000000000000e+00
1.23456790123457e-02	3.333333333333333e-01	9.000000000000000e+00
4.11522633744856e-03	3.333333333333333e-01	2.700000000000000e+01
1.37174211248285e-03	3.333333333333333e-01	8.100000000000000e+01
4.57247370827618e-04	3.333333333333333e-01	2.430000000000000e+02
1.52415790275873e-04	3.333333333333333e-01	7.290000000000000e+02
5.08052634252909e-05	3.333333333333333e-01	2.187000000000000e+03
1.69350878084303e-05	3.333333333333333e-01	6.561000000000000e+03

The successive ratio of errors is constant throughout all iterations, so convergence is linear.

Problem 4 c):

A brief convergence history for the first 4 iterations of 4

T =

4×3 table

Errors	SuccessiveRatios	SuccessiveRatioSquared
1.000000000000002e-03	NaN	NaN
1.000000000000006e-06	1.000000000000004e-03	1.000000000000002e+00
1.000000000000026e-09	1.000000000000020e-03	1.000000000000015e+03
1.000000000000013e-12	9.99999999999864e-04	9.99999999999601e+05

This one converged right away and the errors are hopping down constantly by a factor of about 10^{-3} , so convergence is linear.

Problem 4 d):

A brief convergence history for the first 6 iterations of 6

T =

6×3 table

Errors	SuccessiveRatios	SuccessiveRatioSquared
5.000000000000000e-01	NaN	NaN
1.250000000000000e-01	2.500000000000000e-01	5.000000000000000e-01
7.812500000000000e-03	6.250000000000000e-02	5.000000000000000e-01
3.051757812500000e-05	3.906250000000000e-03	5.000000000000000e-01
4.65661287307746e-10	1.52587890625002e-05	5.000000000000007e-01
1.08420217198063e-19	2.32830643545446e-10	4.99999999767155e-01

The ratio the current error the the previous squared error is about constant for each iteration, suggesting convergence here is quadratic.

Problem 4 e):

A brief convergence history for the first 4 iterations of 4

T =

4×3 table

Errors	SuccessiveRatios	SuccessiveRatioSquared
5.000000000000000e-01	NaN	NaN
6.250000000000001e-02	1.250000000000000e-01	2.500000000000000e-01
1.220703125000000e-04	1.953125000000000e-03	3.124999999999999e-02
9.09494701773755e-13	7.45058059693061e-09	6.10351562500555e-05

Apparently convergence again happened really fast, so it's hard to say at what rate it happened. Based on my figures I can only say that the convergence is at least super-linear.

Problem 4 f):

A brief convergence history for the first 10 iterations of 10

T =

10×3 table

Errors	SuccessiveRatios	SuccessiveRatioSquared
5.000000000000000e-01	NaN	NaN
1.76776695296637e-01	3.53553390593274e-01	7.07106781186547e-01
3.71627223438351e-02	2.10224103813429e-01	1.18920711500272e+00
3.58204704376825e-03	9.63881765879962e-02	2.59367910930201e+00
1.07193125022784e-04	2.99251025218304e-02	8.35419025941930e+00
5.54907254589841e-07	5.17670563734283e-03	4.82932616829905e+01
2.06681025741479e-10	3.72460486021663e-04	6.71212140300754e+02
1.48566508397415e-15	7.18820258726825e-06	3.47792089838930e+04
2.86319798953002e-23	1.92721631571967e-08	1.29720778694237e+07
7.66010893713344e-35	2.67536822991092e-12	9.34398612912571e+10

Convergence here is not quadratic, as the ratio of successive squared errors gets large. On the otherhand, the linear ratio appears to be going down by non-constant amounts, so convergence seems at least super-linear.

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