AB 5: Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

AB 6: For $0 \le t \le 24$ hours, the temperature inside a refrigerator in a kitchen is given by the function W that satisfies the differential equation $\frac{dW}{dt} = \frac{3\cos t}{2W}$. $W(t)$ is measured in degrees Celsius (°C), and t is measured in hours. At time $t = 0$ hours, the temperature inside the refrigerator is 3°C.

(a) Write an equation for the line tangent to the graph of $y = W(t)$ at the point where $t = 0$. Use the equation to approximate the temperature inside the refrigerator at $t = 0.4$ hour.

(b) Find $y = W(t)$, the particular solution to the differential equation with initial condition $W(0) = 3$.

(c) The temperature in the kitchen remains constant at 20°C for $0 \le t \le 24$. The cost of operating the refrigerator accumulates at the rate of \$0.001 per hour for each degree that the temperature in the kitchen exceeds the temperature inside the refrigerator. Write, but do not evaluate, an expression involving an integral that can be used to find the cost of operating the refrigerator for the 24 hour interval.

AB6: The rate of change of the population $P(t)$ of a herd of deer is given by:

$$\frac{dP}{dt} = \frac{1}{4}(1200 - P),$$

where t is measured in years. When $t = 0,$ the population P is 200.

(a) Write an equation of the line tangent to the graph of P at $t = 0$. Use the tangent line to P in order to approximate the population of the herd after 2 years.

(b) Find $\frac{d^2P}{dt^2}$ in terms of P

(c) Find $y = P(t),$ the particular solution to $\frac{dP}{dt} = \frac{1}{4}(1200 - P),$ with initial condition $P(0) = 200.$

AB6: The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

2017 AB4: At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use the equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

AB6: Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f, the slope is given by $\frac{3x^2}{y}$.

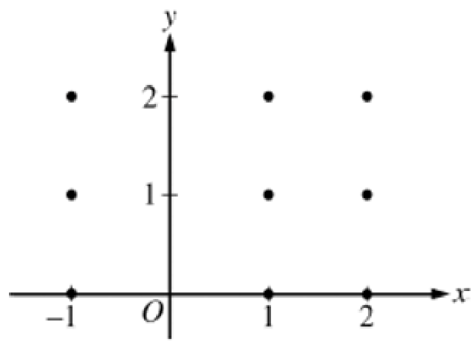
(a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

(b) Find an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = \frac{3x^2}{y}$ with the initial condition $f(2) = -8$.

AB6: Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$, where $x \neq 0$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(1) = 2$.

(a) Find the slope of the line tangent to the graph of f at the point $(1, 2)$.

(b) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$ with initial condition $f(1) = 2$.