Learning from Data Through Models

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Abstract

TBW

1 Introduction

2 Framework

REWRITE IN GENERAL TERMS WITH THE PROBABILITY BEING GIVEN BY A FUNCTION THAT SATIEFIES ALL THE PROPERTIES IN BA SO THAT I CAN REFER TO HER RESULTS. THEN EXPAND ON THE EXAMPLE FOR THE LAB AND SAY WHY IT CAPTURES ALL THE FORCES WE WANT.

An agent is of type $\theta \in \{\theta_L, \theta_M, \theta_H\}$ with $\theta_H > \theta_M > \theta_L$ and they face an unknown exogenous state $\omega \in \{\omega_L, \omega_M, \omega_H\}$ with $\omega_H > \omega_M > \omega_L$. Each of the values of ω is realized with equal probability. The agent knows the distribution of ω but not its value. They also hold some prior belief about θ which is potentially misspecified.[1] The agent chooses a binary gamble $ein\{e_L, e_M, e_H\}$ and observes whether the gamble is a success or a failure. ' If it is a success, the agent gets a payoff of 1 and if it is a failure, the agent gets a payoff of 0. The probability of success is increasing in both θ and ω and is fully described in the following table:

[1] The particular types of misspecification that are allowed are discussed below.

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_L

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_M

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

θ_H

Conditional on a type, the agent's flow payoff is maximized by choosing the gamble that matches the state. For example, if the value of ω is ω_H, the agent's flow payoff is maximized by choosing e_H and if the state is ω_L the flow payoff is maximized by choosing gamble e_L. The agent myopically chooses gambles every period to maximize the flow payoff for $T < \infty$ periods, and the agent's payoff is the sum of the payoffs from each period.

After observing the outcome of each gamble, the agent updates their beliefs of both theta and omega in one of 4 different ways: As a *dogmatic* as in Heidhues et al. [2018] ; as a *switcher* as in Ba [2023] ; as a *self-attribution* updater as in Coutts et al. [2020]; or as a fully Bayesian agent. Each of these updating rules is described in detail below.

Notice that both θ and ω can be identified from the outcomes if enough variation in the effort choices exists. This can be seen by confirming that no two columns are the same. Therefore, the agent can learn both their type and the state by observing enough outcomes of the gambles for different effort choices.

2.1 The Bayesian Benchmark

A Bayesian agent simultaneously updates their beliefs about θ and ω by using Bayes' rule. The posterior odds at period t about θ after observing an outcome are given by:

$$\frac{p_t[\theta_H|\text{outcome}]}{p_t[\theta_M|\text{outcome}]} = \frac{p[\text{outcome}|\theta_H]p_{t-1}[\theta_H]}{p[\text{outcome}|\theta_M]p_{t-1}[\theta_M]} \tag{1}$$

and

$$\frac{p_t[\theta_M|\text{outcome}]}{p_t[\theta_L|\text{outcome}]} = \frac{p[\text{outcome}|\theta_M]p_{t-1}[\theta_M]}{p[\text{outcome}|\theta_L]p_{t-1}[\theta_L]} \tag{2}$$

Where p_{t-1} is the prior at period t and $p[\text{outcome}|\theta] = \sum_{\omega} p[\text{outcome}|\theta, \omega, e] p_{t-1}(\omega)$ is the probability of observing the outcome given the agent's type and the effort chosen. The update is symmetric for ω.

Bayesian agents always choose the effort level that maximizes their flow payoff by taking expectations over their prior beliefs about θ and ω. Since agents are myopic, even though all the parameters could be identified with enough variation in choices, a fully Bayesian agent might not learn their true type. This happens because they do not internalize the tradeoff between flow payoff and learning and thus might not experiment enough to learn their type. An alternative to this approach is given by Hestermann and Yaouanq [2021] and is discussed with the results.

2.2 A Biased Agent

An agent that updates their beliefs with self-attribution bias will update their beliefs about the state ω and their type θ by over-attributing successes to a high value of θ and under-estimating the role of higher ω. Similarly, they will attribute failure to a low state more than an unbiased agent would. To model the self-serving attribution bias, I take the approach of Benjamin [2019], where a generalization of the Bayes rule above gives the update.

$$\frac{p_t[\theta_H|\text{outcome}]}{p_t[\theta_M|\text{outcome}]} = \left(\frac{p[\text{outcome}|\theta_H]}{p[\text{outcome}|\theta_M]}\right)^{c_s^\theta \mathbb{I}\{\text{success}\} + c_f^\theta \mathbb{I}\{\text{failure}\}} \frac{p_{t-1}[\theta_H]}{p_{t-1}[\theta_M]} \tag{3}$$

and

$$\frac{p_t[\theta_M|\text{outcome}]}{p_t[\theta_L|\text{outcome}]} = \left(\frac{p[\text{outcome}|\theta_M]}{p[\text{outcome}|\theta_L]}\right)^{c_s^\theta \mathbb{I}\{\text{success}\} + c_f^\theta \mathbb{I}\{\text{failure}\}} \frac{p_{t-1}[\theta_M]}{p_{t-1}[\theta_L]} \tag{4}$$

c_s^θ and c_f^θ are the self-serving attribution bias parameters for the agent's type θ. If $c_s^\theta = c_f^\theta = 1$, the agent is unbiased and the update is the same as the Bayesian update. On the other hand, if $c_s^\theta > c_f \theta$ the agent over-attributes success to their type and under-attributes failure to their type[2].

[2]notice that the values of c_s^θ and c_f^θ are not restricted to be greater than 1. If they are both equal to each other but less (more) than one, then the bias is simply underinference (overinference).

The update for ω is analogous but with c_f^ω and c_s^ω instead of c_f^θ and c_s^θ and the bias is present whenever $c_f^\omega > c_s^\omega$. That is, the agent over-attributes failure to a low state relative the higher states and under-attributes success to a low state relative to the higher states.

2.3 The Dogmatic Agent

A dogmatic agent does not update their beliefs about θ; instead, they hold a degenerate belief that places probability one on being a particular type, $\hat{\theta}$. For instance, the dogmatic agent might be of type $\hat{\theta} = \theta_M$ but holds a belief that places probability one on being of type θ_H. In this case, no matter how much information he gathers against being of type θ_H, he will not update his beliefs. Any discrepancies between the observed outcomes are incorporated using the Bayes rule to update their beliefs about ω. This results in the following posterior odds:

$$\frac{p_t[\omega_H|\text{outcome}]}{p_t[\omega_M|\text{outcome}]} = \frac{p[\text{outcome}|\omega_H, \hat{\theta}, e]p_{t-1}[\omega_H]}{p[\text{outcome}|\omega_M, \hat{\theta}, e]p_{t-1}[\omega_M]} \tag{5}$$

and

$$\frac{p_t[\omega_M|\text{outcome}]}{p_t[\omega_L|\text{outcome}]} = \frac{p[\text{outcome}|\omega_M, \hat{\theta}, e]p_{t-1}[\omega_M]}{p[\text{outcome}|\omega_L, \hat{\theta}, e]p_{t-1}[\omega_L]} \tag{6}$$

A crucial difference between the dogmatic agent and the Bayesian agent is that the dogmatic agent does not aggregate across types when updating their beliefs about ω. This means the dogmatic agent will never learn their true type if they are misspecified.

Heidhues et al. [2018] show that in a setting such as ours, a dogmatic modeler will inevitably fall into a self-confirming equilibrium where the outcomes they observe reinforce their belief on ω in such a way that as $t \to \infty$ the agent will be certain that the state is some ω^* consistent with their believed type and the observed data.

2.4 The Switcher

An agent is a *switcher* if they behave as a dogmatic but is willing to entertain the possibility that they are of a different type. In particular, when they start off as a misspecified dogmatic, they are willing to switch to a different dogmatic belief if the data is convincing enough.

In order to abandon their initial dogmatic belief, the agent needs to observe a sequence of outcomes that are sufficiently unlikely to have happened if they were of the type they initially believed. They do so by keeping track of the likelihood that each of the possible types generated the data. If the likelihood ratio is sufficiently large, the agent will switch to the alternative and behave as if they are dogmatic about the new type.

In particular, for an agent that starts off with a dogmatic belief that they are of type $\hat{\theta}$ but is willing to consider the alternative explanation that they are of type $\tilde{\theta}$, the agent will switch to the alternative if:

$$\frac{p[h^t|\tilde{\theta}]}{p[h^t|\hat{\theta}]} > \alpha$$

Where h^t is the history of outcomes up to time t and $\alpha \geq 1$ is a threshold that determines how convincing the data needs to be for the agent to change their belief. Notice that if $\alpha \to \infty$, we get the dogmatic agent. In this sense, the switcher is a generalization of the dogmatic type, just as the self-attribution agent is a generalization of the Bayesian agent.

By allowing the agent to keep track of the likelihoods and switching to an alternative type, the switcher can avoid the self-confirming equilibrium the dogmatic agent falls into. However, if the prior belief on *theta* is sufficiently tight around a self-confirming equilibrium, the switcher might look identical to the dogmatic even in a case where α is not too large.

3 Experimental Design

4 Analysis

Conclusion

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