Learning with Misspecified Models: The case of overconfidence

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Overconfidence

OVERCONFIDENCE: Belief that type is higher than it truly is (“overestimation” as in Moore and Healy (2008))

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Seems to be persistent in various settings.

- Excess entry of entrepreneurs (Camerer and Lovallo, 1999)
- Suboptimal genetic testing and savings (Oster et al. 2013)
- Workers overestimate their productivity (Hoffman and Burks, 2020)

Ultimately it leads to sub-optimal choices

Models of Learning

Focus on setting with 2 parameters:

- An **Ego-Relevant** parameter
- An **Exogenous** parameter

Some of the assumptions that theory has incorporated to rationalize overconfidence are:

- Dogmatism
- Paradigm shifts
- Motivated beliefs
- Myopic optimiztion

Four Theories of Misspecified Learning

1. **Self-defeating equilibrium** (Heidhues et al. (2018))
   - Bayesian about exogenous parameters
   - Dogmatic about ego-relevant parameters
2. **Bayesian hypothesis testing** (Schwarstein and Sunderam (2021), Ba (2022))
   - Bayesian about exogenous parameters
   - Paradigm shift for ego-relevant parameters
3. **Motivated Beliefs / Self-Attribution Bias** (Brunnermeier and Parker (2005), Bracha and Brown (2012))
   - Optimally biased updating
   - Utility from held beliefs
4. **Myopic Bayesian** (Hestermann and Le Yaouanq, (2021))
   - Bayesian about both
   - Maximizes flow utility only

Questions

Which of the proposed theories gives a better explanation of behavior?

Do the theories apply only to misspecifications about ego-relevant parameters?

- Can the same theories explain the prevalence of stereotypes?

An Example (from Heidhues et al. (2018))

A student has unknown **intrinsic ability** θ^* (ego-relevant parameter)

They choose a level of **effort** $e \geq 0$ (choice)

Effort and ability are evaluated by a **grading system** ω (exogenous parameter)

The student wants to maximize:

$$u(e) = (\theta^* + e)\omega - \frac{1}{2}e^2 + \varepsilon$$

Regardless of their own type and of their beliefs about it, they should choose $e^*(\omega) = \omega$

Learning is Possible

This exercise is repeated for $t = 0, 1, ...$

$$y_t = (\theta^* + e_t)\omega - \frac{1}{2}e_t^2 + \varepsilon_t$$

Note that both parameters are identified in this setting:

- Choosing $\hat{e}$ and $\hat{e} + 1$ over multiple periods allows identification of ω
- Once ω is known, θ can be backed out

Why do people not learn the true values of the parameters?

Road-map

1. Unifying Framework
2. Mechanisms and Predictions
3. Experimental Design
4. The Data
5. Parameter Estimation
6. Results

Framework

A Unifying Framework

Ego-relevant paremeter: $\theta \in \{\theta_H, \theta_M, \theta_L\}$

Exogenous parameter: $\omega \in \{\omega_H, \omega_M, \omega_L\}$ with $p(\omega_k) = 1/3$

Choices: $e \in \{e_H, e_M, e_L\}$

Binary Outcomes: $s_t \in \{\text{success, failure}\}$ with $p\,[success|e, \omega, \theta]$ and p is an order-preserving transformation of $u(x)$

The Data Generating Process

The probability of success is given by:

θ_L

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

The Data Generating Process

θ_L

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

The Data Generating Process

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_L

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_M

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

θ_H

A Stable Misspecified Belief

θ_L

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

The Stable Beliefs

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_L

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_M

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

θ_H

Mechanisms and Predictions

An Example

- True type is θ_M
- True parameter is $\omega_M \rightarrow$ the student believes it is uniformly distributed

θ_L:

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M:

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H:

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

The Dogmatic Modeler

Holds a degenerate belief: type is $\hat{\theta}$ with probability 1

Their belief is potentially misspecified:

- Overconfident if $\hat{\theta} > \theta^*$
- Underconfident if $\hat{\theta} < \theta^*$

Updates $p_t(\omega)$ using Bayes Rule

$$p_{t+1}(\omega|s, \hat{\theta}) = \frac{p_t(s_t|\omega, \hat{\theta}) p_t(\omega)}{\sum_{\omega'} p_t(s_t|\omega', \hat{\theta}) p_t(\omega')}$$

The Dogmatic Modeler: Mechanism

A student who dogmatically believes he is θ_H but truly is θ_M

The exogenos parameter is ω_M

1. Chooses e_H and is disappointed $\rightarrow$ adjust belief about ω downward
2. Eventually chooses e_M and is disappointed as well $\rightarrow$ adjust belief about ω
3. Eventually chooses e_L and falls into a self-confirming equilibrium

θ_L

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

Dogmatic Overconfident: Simulated

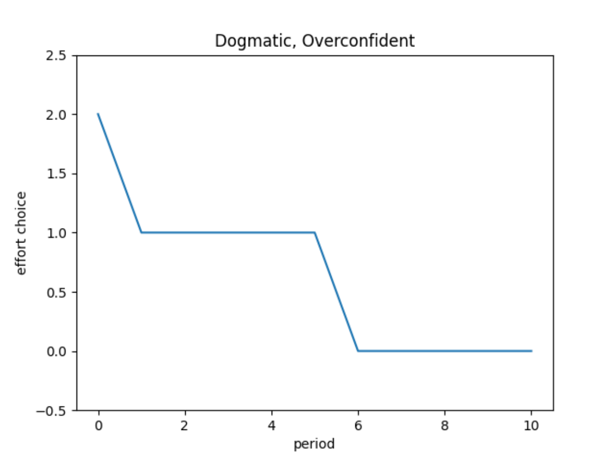


Figure 1: $\theta^* = \theta_M$, $\hat{\theta} = \theta_H$, $\omega^* = \omega_M$

The Switcher (paradigm shifts)

Same initial belief as the Dogmatic, but is willing to consider and alternative paradigm θ'

Keeps track of the likelihoods of the two possible paradigms:

- $p_t(s_t|\cdot)$ for $\hat{\theta}$ and θ'

They switch to whichever paradigm is morelikely to have generated the signals

$$\frac{p_t(s_t|\theta')}{p_t(s_t|\hat{\theta})} > \alpha \geq 1$$

The Switcher: Mechanism

1. Chooses e_H and is disappointed $\rightarrow$ adjust belief about ω downward
2. Eventually chooses e_M and is disappointed as well $\rightarrow$ adjust belief about ω
3. Avoids the self-defeating equilibrium if the likelihood of θ_M becomes larger than that of θ_H

Switcher Overconfident: Simulation

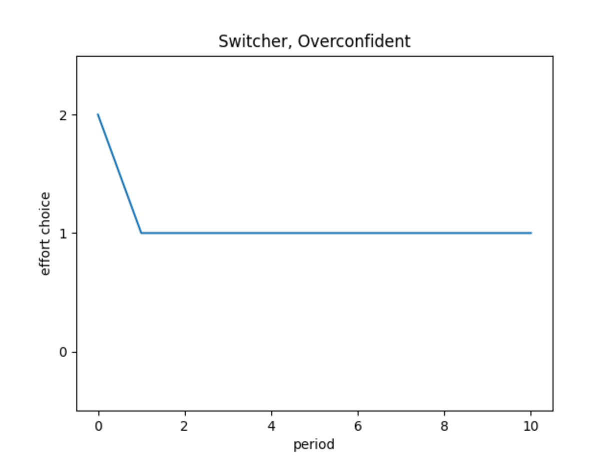


Figure 2: $\theta^* = \theta_M$, $\hat{\theta} = \theta_H$, $\omega^* = \omega_M$, $\alpha = 1.1$

Self-Attribution Bias / Optimal Expectations

Start with a diffused prior over (θ, ω) but updates with a bias

$$p_{t+1}(\theta, \omega | s_t) = \frac{p_t(s_t | \theta, \omega)^{c(\theta, \omega, s_t)} p_t(\theta, \omega)}{\sum_{(\theta', \omega')} p_t(s_t | \theta', \omega')^{c(\theta', \omega', s_t)} p_t(\theta', \omega')}$$

Bias is such that

$$c(\theta_H, \omega, \text{good news}) \leq c(\theta_M, \omega, \text{good news}) \leq c(\theta_L, \omega, \text{good news}) \leq 1 \quad \forall \omega$$

And

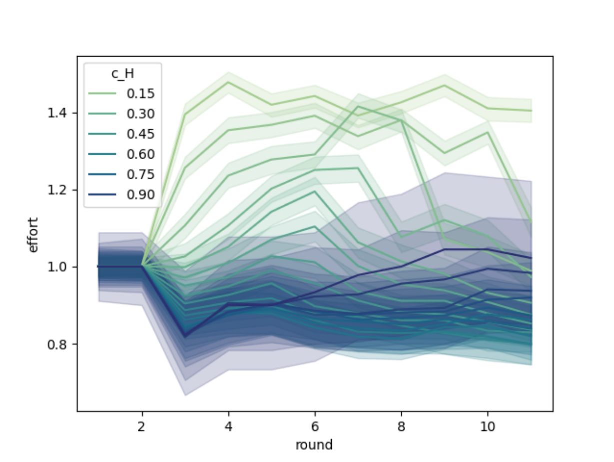
$$c(\theta, \omega_L, \text{bad news}) \leq c(\theta, \omega_M, \text{bad news}) \leq c(\theta, \omega_H, \text{bad news}) \leq 1 \quad \forall \theta$$

Self-Attribution: Mechanism

1. Chooses e that maximizes utility according to priors

   - Belief on $\mathbb{E}[\omega]$ deteriorates a lot after bad news $\rightarrow$ big change in effort
   - Belief on $\mathbb{E}[\theta]$ increases a lot after good news $\rightarrow$ small positive (or negative) change in effort

Self-Attribution: Simulation



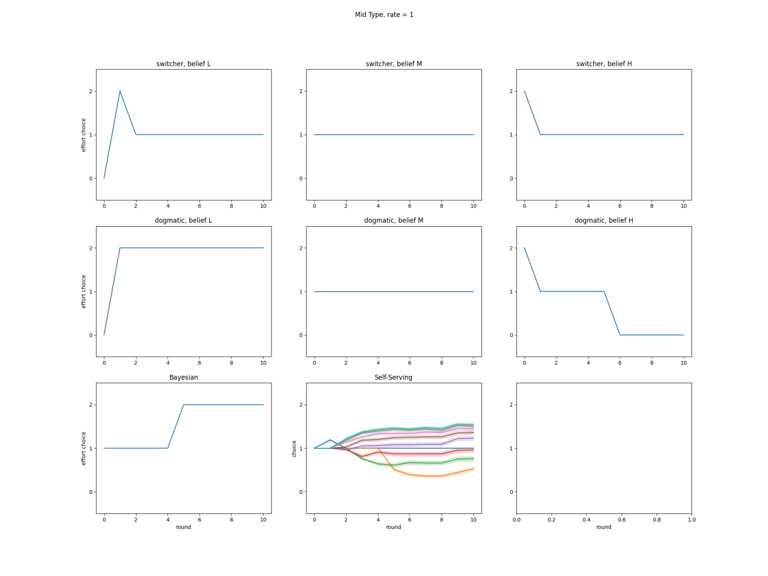
Myopic Bayesian

Start with a diffused prior over (θ, ω) and updates correctly

$$p_{t+1}(\theta, \omega | s_t) = \frac{p_t(s_t|\theta, \omega) p_t(\theta, \omega)}{\sum_{(\theta', \omega')} p_t(s_t|\theta', \omega') p_t(\theta', \omega')}$$

But if they start with a prior that is "tight" around a self-defeating equilibrium they will never learn

All Models


Mid Type, rate = 1
switcher, belief L
switcher, belief M
switcher, belief H
dogmatic, belief L
dogmatic, belief M
dogmatic, belief H
Bayesian
Self-Serving
effort choice
choice
round
0
1
2
4
6
8
10
0.0
0.2
0.4
0.6
0.8
1.0

Experimental Design

The Experiment

Two parts:

1. Setting the types
2. updating

Two treatments:

1. Ego
2. Stereotype

Set the Types

- Quiz: Answer as many questions as you can in 2 minutes
  - Math, Verbal, Pop-Culture, Science, Us Geography, Sports and Video games

- How many questions do you think you answered correctly in each quiz?
  - 0 to 5 (θ_L)
  - 6 to 15 (θ_M)
  - 16 or more (θ_H)
- How sure are you about your guess?
  - Random guess $\rightarrow 1/3$
  - Another is equally likely $\rightarrow 1/2$
  - Fairly certain $\rightarrow 3/4$
  - Completely sure $\rightarrow 1$

Choice and Update

"Effort" choice and feedback (One topic at a time)

- A success rate is drawn at random (A, B or C)
- Choose a gamble: A, B or C (effort)
- Receive a sample of 10 signal realizations

x 11 per topic

Stereotype condition

Observe the characteristics of a participant

- Gender,
- US National or not

Answer the same questions about slef and other

Belief updating and effort choice:

- The DGP depends on the θ the other participant

x 11 per topic

Eliciting Beliefs?

- Track their belief about ω with their choices
- Eliciting beliefs for θ can incentivize learning in a way that is not consistent with the theory

Allow them to see the success rate matrix for only one type.

- Track the matrices they choose to see in each round

Screen

Based on the other participant's Science and Technology Quiz results

Which probability matrix would you like to see?

Low Score | Mid Score | High Score

Your Previous Outcomes

Choice	Successes	Failures
You have no data for this task yet		

See History

Next

Screen

Based on the other participant's Science and Technology Quiz results

Which probability matrix would you like to see?

Low Score | Mid Score | High Score

Choose a gamble	:	Rate A	Rate B	Rate C
A	○	40	45	65
B	○	30	65	69
C	○	5	50	80

Your Previous Outcomes

Choice	Successes	Failures
You have no data for this task yet		

See History

Next

The Data

The Data

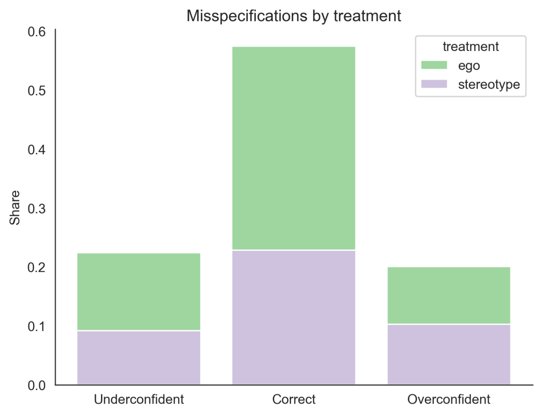
Subject pool:

- Run at the CESS lab in person
- 45 subjects in Ego
- 33 subjects in Stereotype

The Sessions:

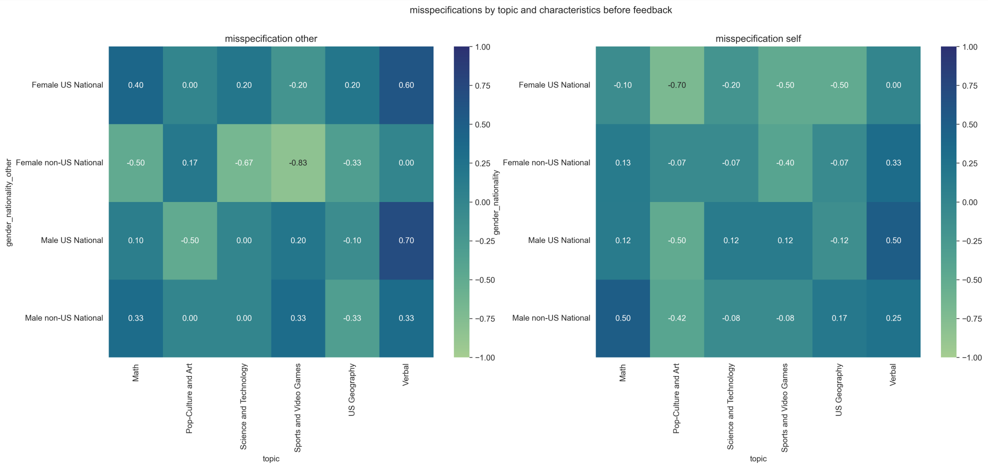
- 8 sessions
- 45 minutes on average
- Average payment: $23
    - $10 show-up fee
    - $0.20 per correct answer
    - $0.20 per success
    - Paid one topic at random

Initial Misspecifications



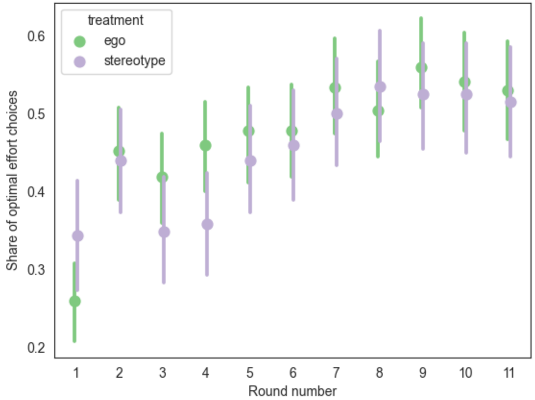
certainties

The Stereotypes

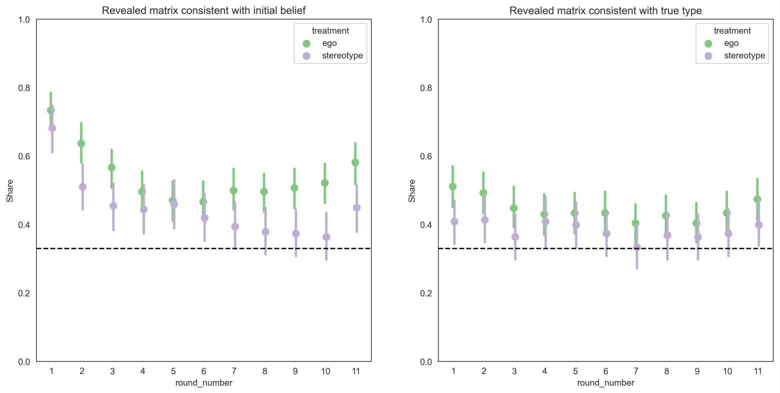


types

Learning ω

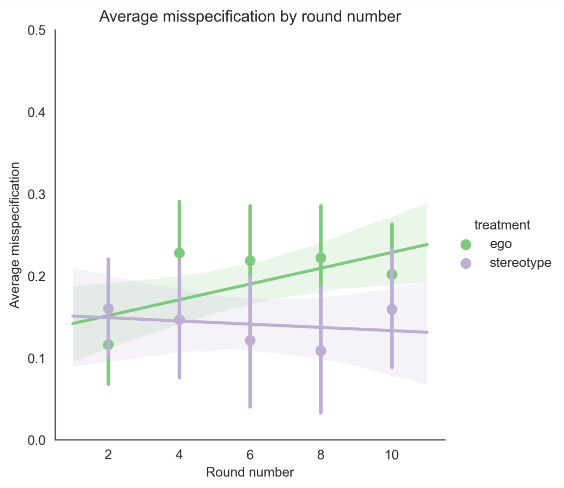


Learning ⊖



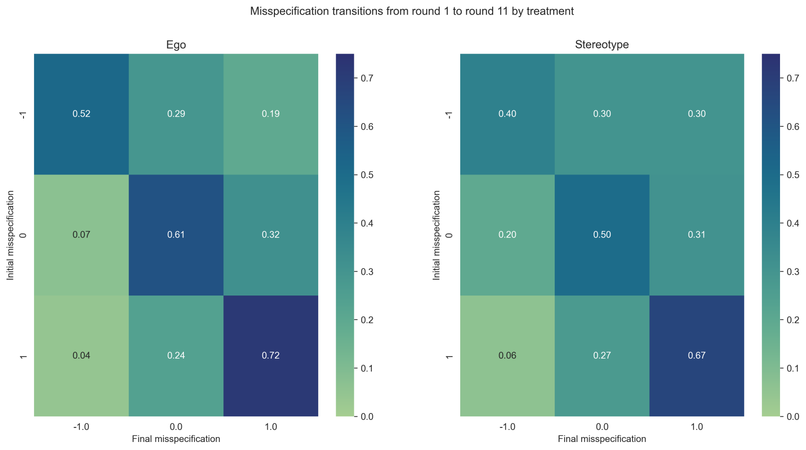
Revealed matrix consistent with initial belief
Revealed matrix consistent with true type
treatment
ego
stereotype
Share
round_number
1.0
0.8
0.6
0.4
0.2
0.0
1 2 3 4 5 6 7 8 9 10 11

Changes in Misspecifications

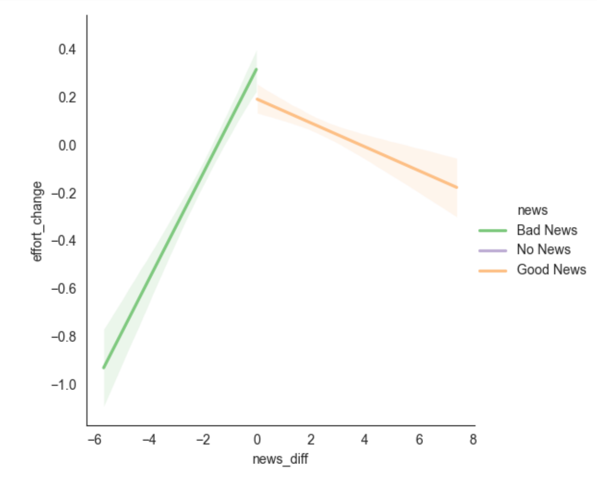


overall

Transitions



Good News v. Bad News



other

Parameters

Calibration of α

Whenever the agent switches from one paradigm to another, they are revealing that

$$\frac{p_t(s^t|\theta')}{p_t(s^t|\hat{\theta})} = \alpha$$

Notice that this identifies an upper bound for α

I take the average value of the likelihood ratio when the agent changes their choice of θ to be α

I find $\alpha = 1.48$ and no difference across treatments

Calibration of Bias

Simulation on a grid of parameters

For each task take the parameters that minimize the distance between the simulated and the actual effort

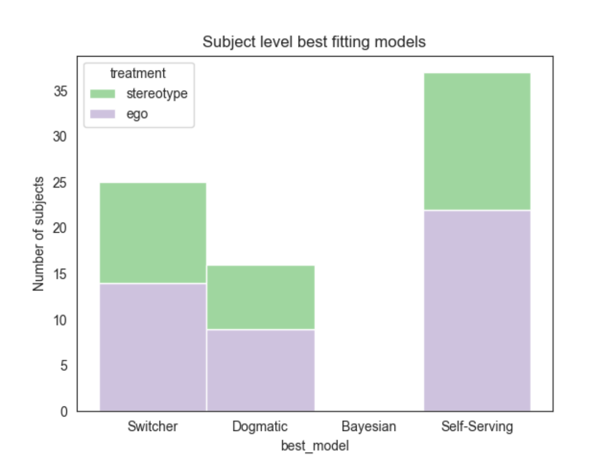
Average for each subject

Average across subjects

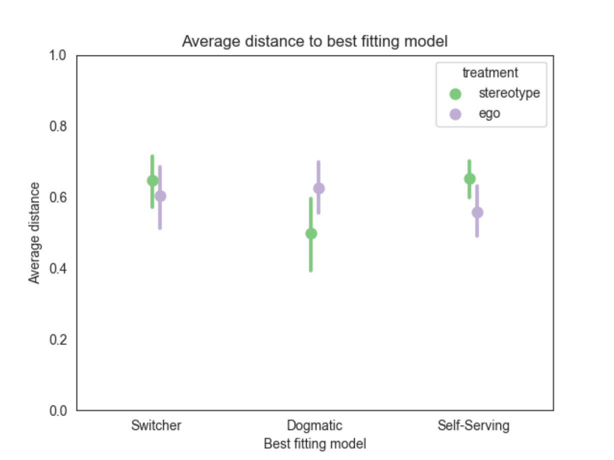
$$c(\theta_H, \omega, \text{good news}) = c(\theta, \omega_L, \text{bad news}) = 0.137$$
$$c(\theta_M, \omega, \text{good news}) = c(\theta, \omega_M, \text{bad news}) = 0.36$$
$$c(\theta_L, \omega, \text{good news}) = c(\theta, \omega_H, \text{bad news}) = 1$$

Heterogeneity

Model Fit: Distributions



Model Fit: Distance



paths

Concluding Remarks

Summary

I develop a framework that nests predictions from several models of overconfidence

I compare the fit of the predictions of these models to behavior in a laboratory experiment

I find that the data is best explained by a model of self-attribution bias or paradigm shifts

The models seem to be able to explain the prevalence of stereotypes as well as overconfidence

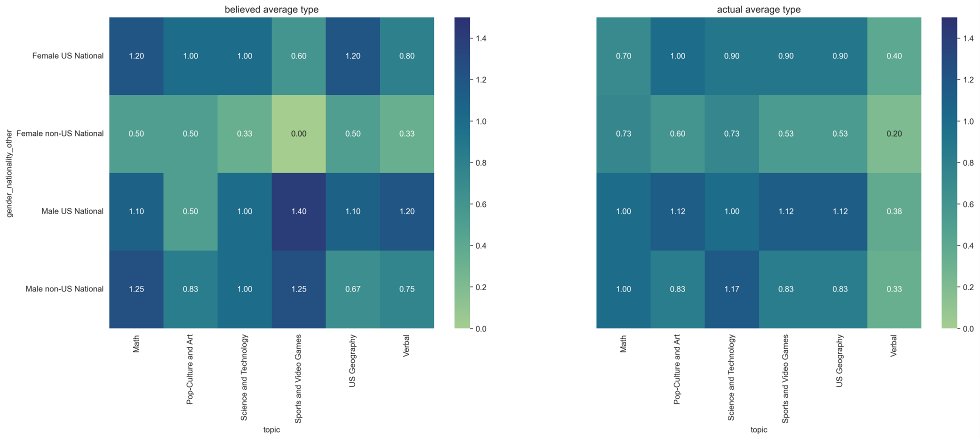
What is Next

1. Have a better estimation of the attribution bias parameters
    - Estimate using SMM
    - Elicit beliefs within this framework
2. Can dynamic learning explain the data better?
    - Hestermann and Le Yaouanq (2019)

The end

Thank you!

Misspecifications



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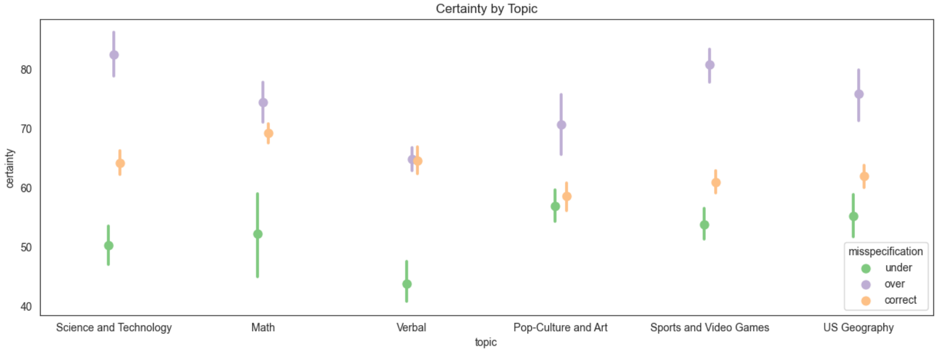
Certainties



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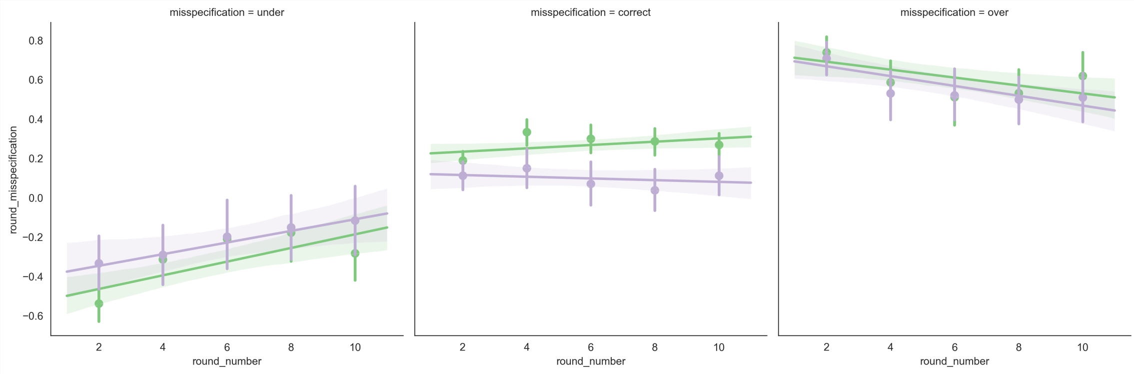
Misspecification changes by treatment



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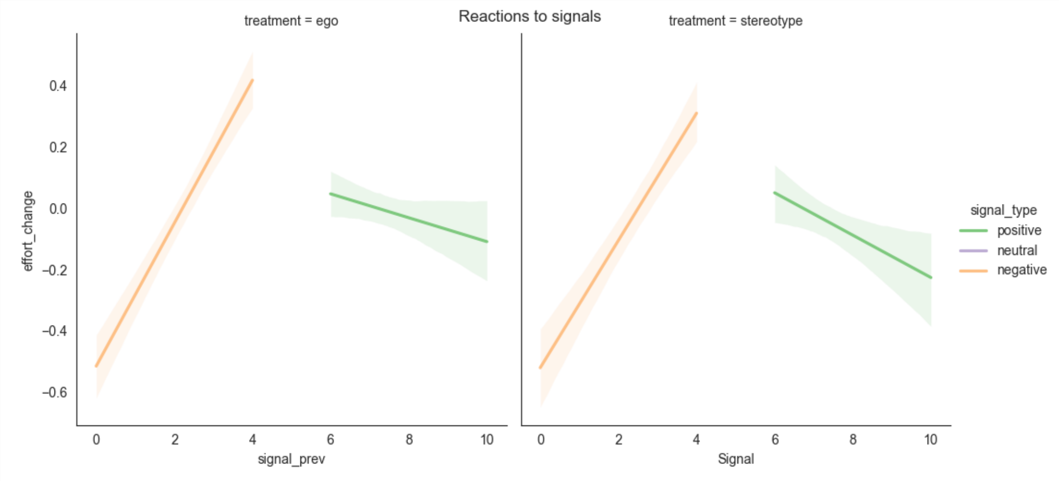
Positive Signals v. Negative Signals



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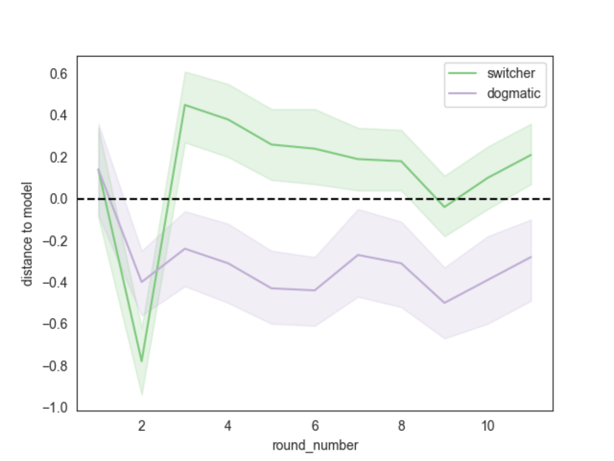
Dogmatic v. Switcher



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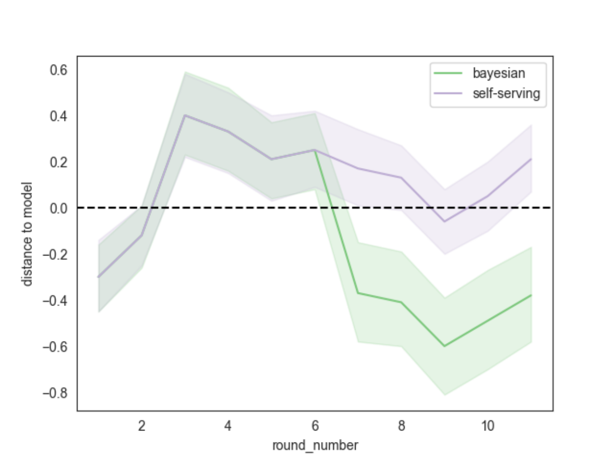
Bayesian v. Self-Attribution



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