Learning with Misspecified Models: The case of overconfidence

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Overconfidence

Overestimation: Belief that type is higher than it truly is

- *e.g. Believing you have an IQ of 150 when it is actually 100*

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Seems to be persistent in various settings.

- Excess entry of entrepreneurs (Camerer and Lovallo, 1999)
- Suboptimal genetic testing and savings (Oster et al. 2013)
- Workers overestimate their productivity (Hoffman and Burks, 2020)

Ultimately it leads to costly choices

Models of Learning

Focus on setting with 2 parameters:

- An **Ego-Relevant** parameter
- An **Exogenous** parameter

For instance skill and luck when playing a game

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Some of the assumptions that theory has incorporated to rationalize overconfidence are:

- Dogmatism
- Paradigm shifts
- Motivated beliefs
- Myopic Bayesian

Four Theories of Misspecified Learning

1. **Self-defeating equilibrium** (Heidhues et al. (2018))
   - Bayesian about exogenous parameters
   - Dogmatic about ego-relevant parameters
2. **Bayesian hypothesis testing** (Schwarstein and Sunderam (2021), Ba (2022))
   - Bayesian about exogenous parameters
   - Paradigm shift for ego-relevant parameters
3. **Motivated Beliefs / Self-Attribution Bias** (Brunnermeier and Parker (2005), Bracha and Brown (2012))
   - Optimally biased updating
   - Utility from held beliefs
4. **Myopic Bayesian** (Hestermann and Le Yaouanq, (2021))
   - Bayesian about both
   - Maximizes flow utility only

Questions

Which of the proposed theories gives a better explanation of behavior?

Do the theories apply only to misspecifications about ego-relevant parameters?

- Can the same theories explain the prevalence of stereotypes?

An Example (from Heidhues et al. (2018))

A student has unknown **intrinsic ability** θ^* (ego-relevant parameter)

They choose a level of **effort** $e \geq 0$ (choice)

Effort and ability are evaluated by a **grading system** ω (exogenous parameter)

The student wants to maximize:

$$u(e) = (\theta^* + e)\omega - \frac{1}{2}e^2 + \varepsilon$$

Regardless of their own type and of their beliefs about it, they should choose $e^*(\omega) = \omega$

Learning is Possible

This exercise is repeated for $t = 0, 1, \ldots$

$$y_t = (\theta^* + e_t)\omega - \frac{1}{2}e_t^2 + \varepsilon_t$$

Note that both parameters are identified in this setting:

- Choosing $\hat{e}$ and $\hat{e} + 1$ over multiple periods allows identification of ω
- Once ω is known, θ can be backed out

Why do people not learn the true values of the parameters?

Preview of Results

From the proposed mechanisms:

- Dogmatism and Bayesian Updating do not seem to explain the behavior
- Some evidence supporting Hypothesis Testing
- Most evidence supporting Motivated Beliefs
- Biased updating about others (but for potentially different reasons)

Roadmap

1. Unifying Framework
2. Mechanisms and Predictions
3. Experimental Design
4. The Data
5. Results

Framework

A Unifying Framework

Ego-relevant paremeter: $\theta \in \{\theta_H, \theta_M, \theta_L\}$

Exogenous parameter: $\omega \in \{\omega_H, \omega_M, \omega_L\}$ with $p(\omega_k) = 1/3$

Choices: $e \in \{e_H, e_M, e_L\}$

Binary Outcomes: $s_t \in \{\text{success, failure}\}$ with $p\left[success | e, \omega, \theta\right]$ and p is an order-preserving transformation of $u(x)$

The Data Generating Process

The probability of success is given by:

θ_L

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

The Data Generating Process

θ_L

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

The Data Generating Process

	ω_H	ω_M	ω_L
e_H	50	20	2
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e_L	40	25	20

θ_L

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_M

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

θ_H

A Stable Misspecified Belief

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_L

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_M

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

θ_H

The Stable Beliefs

θ_L

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

Mechanisms and Predictions

An Example

- True type is θ_M
- True parameter is $\omega_M \rightarrow$ the student believes it is uniformly distributed

θ_L:

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M:

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H:

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

The Dogmatic Modeler

Holds a degenerate belief: type is $\hat{\theta}$ with probability 1

Their belief is potentially misspecified:

- Overconfident if $\hat{\theta} > \theta^*$
- Underconfident if $\hat{\theta} < \theta^*$

Updates $p_t(\omega)$ using Bayes Rule

$$p_{t+1}(\omega|s, \hat{\theta}) = \frac{p_t(s_t|\omega, \hat{\theta})p_t(\omega)}{\sum_{\omega'} p_t(s_t|\omega', \hat{\theta})p_t(\omega')}$$

The Dogmatic Modeler: Mechanism

A student who dogmatically believes he is θ_H

1. Chooses e_H and is disappointed $\rightarrow$ adjust belief about ω downward

2. Eventually chooses e_M and is disappointed as well $\rightarrow$ adjust belief about ω

3. Eventually chooses e_L and falls into a self-confirming equilibrium

θ_L

	ω_H	ω_M	ω_L
e_H	50	20	2
e_M	45	30	7
e_L	40	25	20

θ_M

	ω_H	ω_M	ω_L
e_H	80	50	5
e_M	69	65	30
e_L	65	45	40

θ_H

	ω_H	ω_M	ω_L
e_H	98	65	25
e_M	80	69	35
e_L	75	55	45

path

The Switcher (paradigm shifts)

Same initial belief as the Dogmatic, but is willing to consider and alternative paradigm θ'

Keeps track of the likelihoods of the two possible paradigms:

- $p_t(s_t|\cdot)$ for $\hat{\theta}$ and θ'

They switch to whichever paradigm is more likely to have generated the signals

$$\frac{p_t(s_t|\theta')}{p_t(s_t|\hat{\theta})} > \alpha \geq 1$$

The Switcher: Mechanism

1. Chooses e_H and is disappointed $\rightarrow$ adjust belief about ω downward
2. Eventually chooses e_M and is disappointed as well $\rightarrow$ adjust belief about ω
3. Avoids the self-defeating equilibrium if the likelihood of θ_M becomes larger than that of θ_H

A change in paradigm will often be accompanied with a change in effort in the opposite direction of the signal

path

Self-Attribution Bias / Optimal Expectations

Start with a diffused prior over (θ, ω) but updates with a bias

$$p_{t+1}(\theta, \omega | s_t) = \frac{p_t(s_t|\theta, \omega)^{c(\theta,\omega,s_t)} p_t(\theta, \omega)}{\sum_{(\theta',\omega')} p_t(s_t|\theta', \omega')^{c(\theta',\omega',s_t)} p_t(\theta', \omega')}$$

Bias is such that

$$c(\theta_H, \omega, \text{good news}) \leq c(\theta_M, \omega, \text{good news}) \leq c(\theta_L, \omega, \text{good news}) \leq 1 \quad \forall \omega$$

And

$$c(\theta, \omega_L, \text{bad news}) \leq c(\theta, \omega_M, \text{bad news}) \leq c(\theta, \omega_H, \text{bad news}) \leq 1 \quad \forall \theta$$

Self-Attribution: Mechanism

1. Chooses e that maximizes utility according to priors

    - Belief on $\mathbb{E}[\omega]$ deteriorates a lot after bad news $\rightarrow$ overreaction in effort
    - Belief on $\mathbb{E}[\theta]$ increases a lot after good news $\rightarrow$ underreaction in effort (or in opposite direction)

path

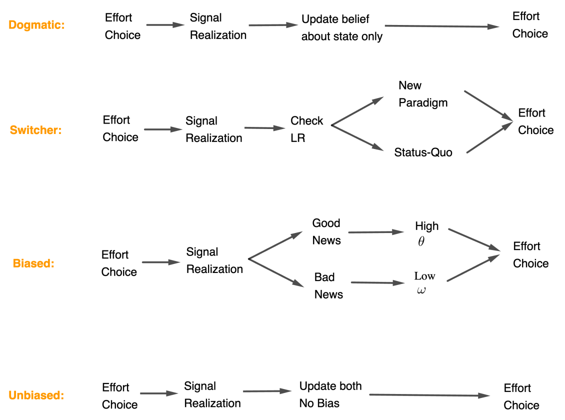
Myopic Bayesian

Start with a diffused prior over (θ, ω) and updates correctly

$$p_{t+1}(\theta, \omega | s_t) = \frac{p_t(s_t|\theta, \omega) p_t(\theta, \omega)}{\sum_{(\theta', \omega')} p_t(s_t|\theta', \omega') p_t(\theta', \omega')}$$

But if they start with a prior that is "tight" around a self-defeating equilibrium they will never learn

All Models


Dogmatic:
Effort Choice
Signal Realization
Update belief about state only
Effort Choice
Switcher:
Effort Choice
Signal Realization
Check LR
New Paradigm
Status-Quo
Effort Choice
Biased:
Effort Choice
Signal Realization
Good News
Bad News
High θ
Low ω
Effort Choice
Unbiased:
Effort Choice
Signal Realization
Update both No Bias
Effort Choice

Predictions

	Good News	Bad News	
Dogmatic:	Increase Effort	Decrease Effort	Reacts more than Bayesian
Switcher:	Decrease Effort ← Paradigm Shift	→ Increase Effort	Depends on hypothesis test
	Increase Effort ← Status-Quo	→ Decrease Effort	
Biased:	Small Increase in Effort or Decrease Effort	Decrease Effort	Reacts more than Bayesian to bad news Reacts less than Bayesian to good news
Unbiased:	Increase Effort	Decrease Effort	Benchmark

Experimental Design

The Experiment

Two parts:

1. Setting the types
2. Updating

Two treatments:

1. Ego
2. Stereotype

Set the Types

- Quiz: Answer as many questions as you can in 2 minutes
  - Math, Verbal, Pop-Culture, Science, Us Geography, Sports and Video games

- How many questions do you think you answered correctly in each quiz?
  - 0 to 5 (θ_L)
  - 6 to 15 (θ_M)
  - 16 or more (θ_H)
- How sure are you about your guess?
  - Random guess $\rightarrow 1/3$
  - Another is equally likely $\rightarrow 1/2$
  - Fairly certain $\rightarrow 3/4$
  - Completely sure $\rightarrow 1$

Choice and Update

"Effort" choice and feedback (One topic at a time)

- A success rate is drawn at random (A, B or C)
- Choose a gamble: A, B or C (effort)
- Receive a sample of 10 signal realizations

x 11 per topic

Stereotype condition

Observe the characteristics of a participant

- Gender
- US National or not

Answer the same questions about self and other

Belief updating and effort choice:

- The DGP depends on the θ the other participant

x 11 per topic

Eliciting Beliefs?

- Track their belief about ω with their choices
- Eliciting beliefs for θ can incentivize learning in a way that is not consistent with the theory

Allow them to see the probability matrix for only one type

- Track the matrix they choose to see in each round

Screen

Based on the other participant's Science and Technology Quiz results

Which probability matrix would you like to see?

Low Score | Mid Score | High Score

Your Previous Outcomes

Choice	Successes	Failures
You have no data for this task yet		

See History

Next

Screen

Based on the other participant's Science and Technology Quiz results

Which probability matrix would you like to see?

Low Score | Mid Score | High Score

Choose a gamble	:	Rate A	Rate B	Rate C
A	◯	40	45	65
B	◯	30	65	69
C	◯	5	50	80

Your Previous Outcomes

Choice	Successes	Failures
You have no data for this task yet		

See History

Next

The Data

The Data

Subject pool:

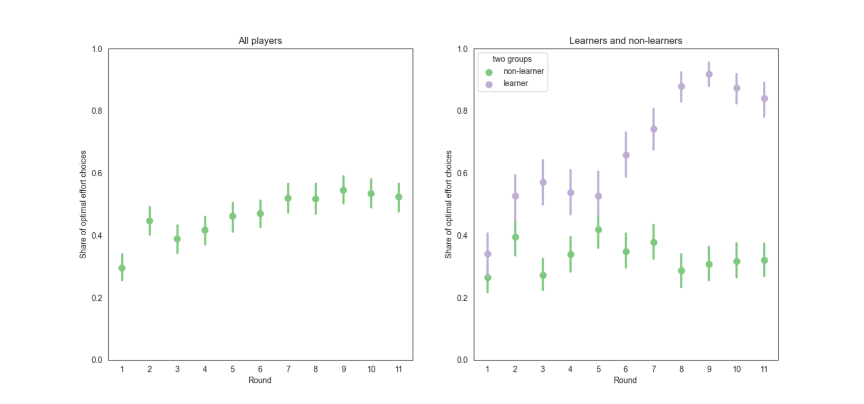
- Run at the CESS lab in person
- 45 subjects in Ego
- 33 subjects in Stereotype

The Sessions:

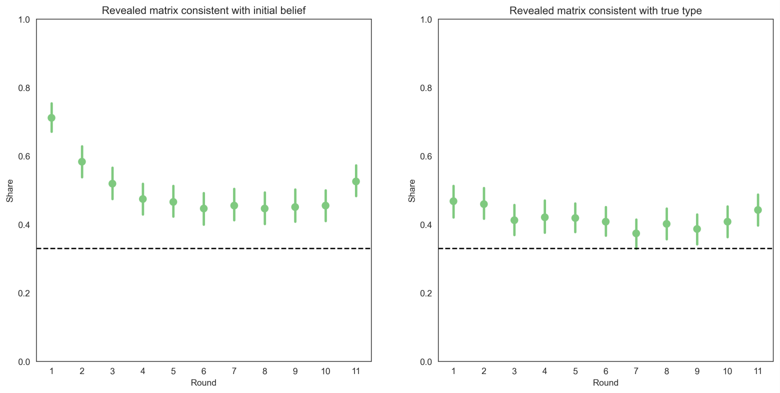
- 8 sessions
- About 45 minutes long
- Average payment: $23
  - $10 show-up fee
  - $0.20 per correct answer
  - $0.20 per success
  - Paid one topic at random

Learning

Are they learning ω?



Are they learning ⊖

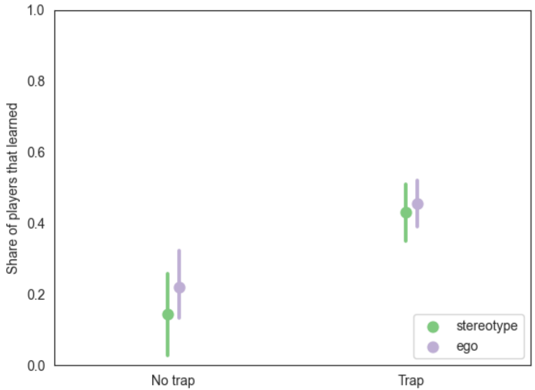


Reasons for lack of learning

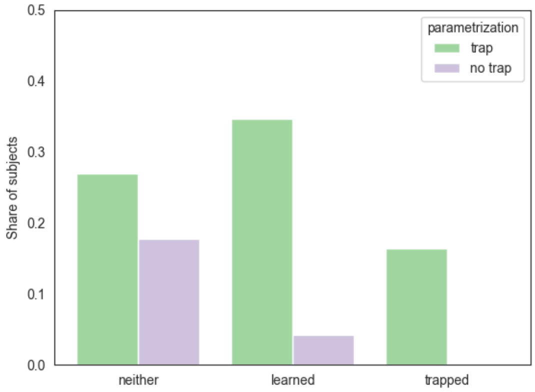
- Learning traps (self-defeating equilibria)
- Misattributions
- Others
  - Considering the wrong paradigms
  - Learning is too costly

Learning Traps

Learning when there are traps



Are people falling into traps?



paths

Learners, Trapped and Others

So far we have seen that:

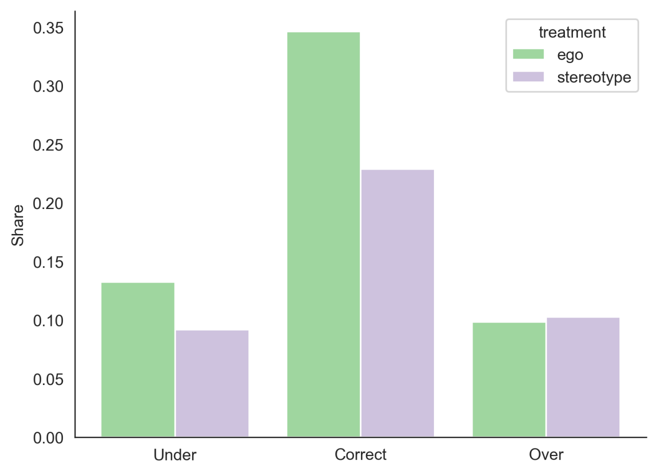
- 40% of the subjects learn the true state
- About 16% of the subjects fall into self-defeating equilibria
- 44% of the subjects don't learn correctly and don't fall into traps
    - From these 60% were facing parameters for which there were traps

How did the learners escape the traps?

What is the remaining 44% doing?

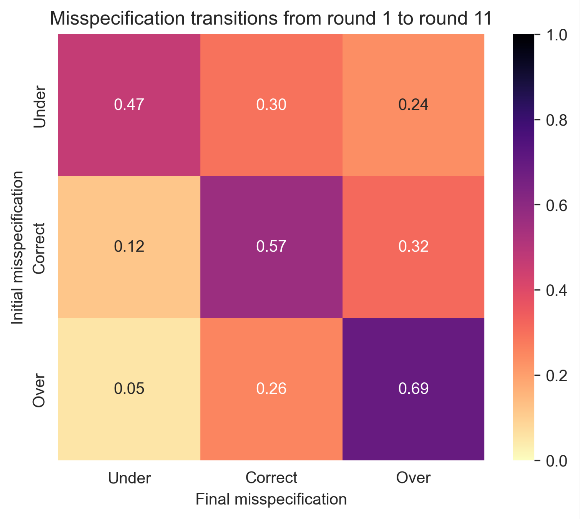
Misattributions

Initial Misspecifications

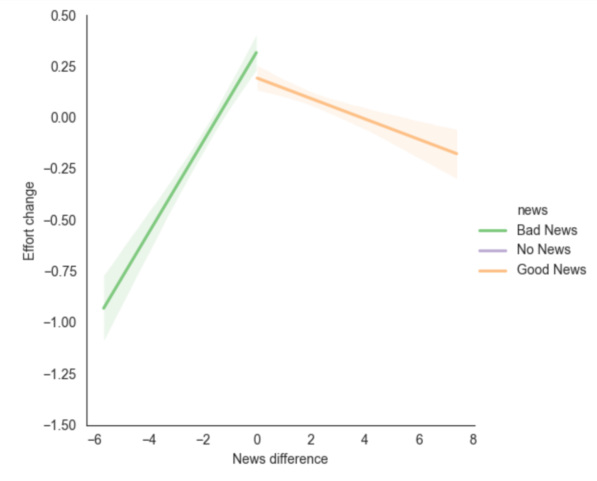


certainties seterotypes

Transition Matrix



Good News v. Bad News



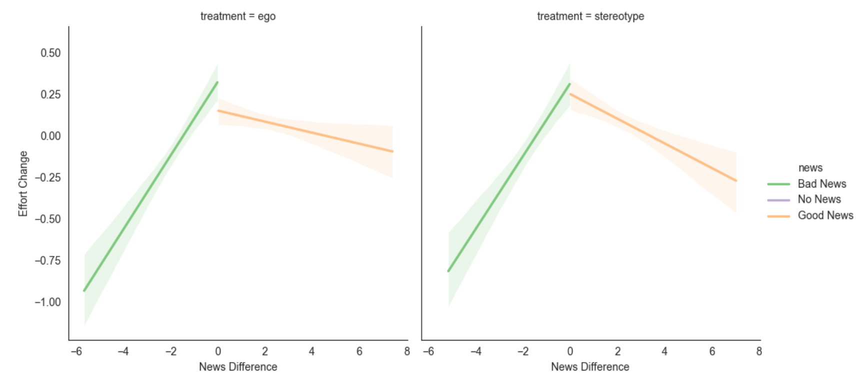
other

Regression Results

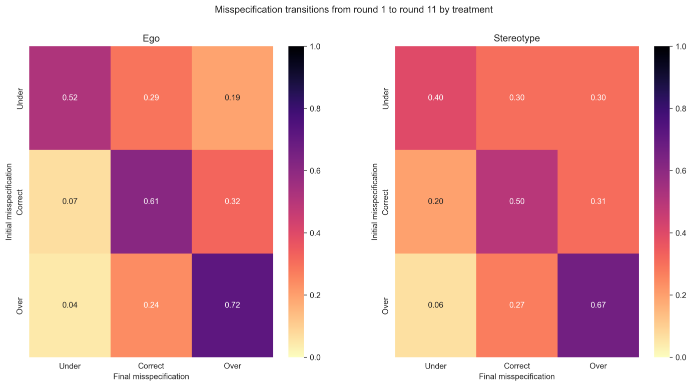
	Dependent variable:			
	Change in effort			
	All	Ego-relevant	Stereotype	Bayesian Simulation
	(1)	(2)	(3)	(4)
Good News	−0.12**	−0.16***	−0.05	0.08
	(0.05)	(0.05)	(0.05)	(0.05)
News differential	0.22***	0.22***	0.21***	0.06***
	(0.02)	(0.02)	(0.02)	(0.02)
News Differential * Good News	−0.27***	−0.25***	−0.29***	−0.04
	(0.02)	(0.02)	(0.02)	(0.02)
Constant	0.31***	0.31***	0.30***	−0.08*
	(0.04)	(0.04)	(0.04)	(0.04)
Observations	4,680	2,700	1,980	4,680
R^2	0.04	0.04	0.04	0.05
Adjusted R^2	0.04	0.04	0.04	0.05
Note:				*p<0.1; **p<0.05; ***p<0.01

Stereotypes

Asymmetric Updating in the Stereotype Condition



Do misspecifications persist more often in the Ego condition?



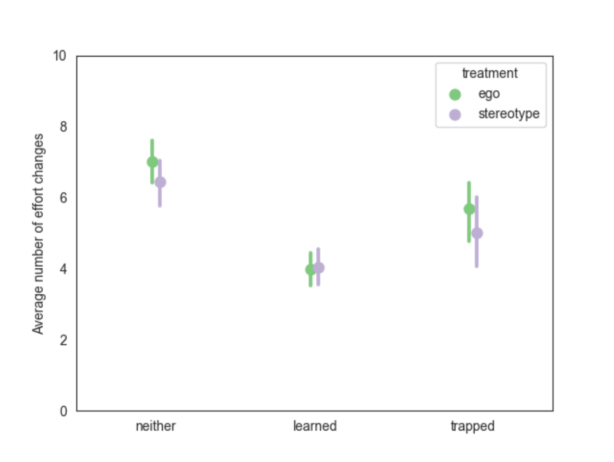
Differences across treatments

Very slight differences across treatments

- Less stickiness in initial beliefs in Stereotype
- Attribution bias in Ego condition
- Possible self-censoring in Stereotype

Other Explanations

Excessive Switching



Concluding Remarks

Summary

Overall:

- Traps don't seem to be the main reason for lack of learning
- Evidence pointing to misattributions
- Ego-relevance seems to play a minor role

In the presence of traps:

- 44% of subjects learn the true state
- About 20% of the subjects fall into self-defeating equilibria when they exist
- 36% of the subjects don't learn correctly and don't fall into traps

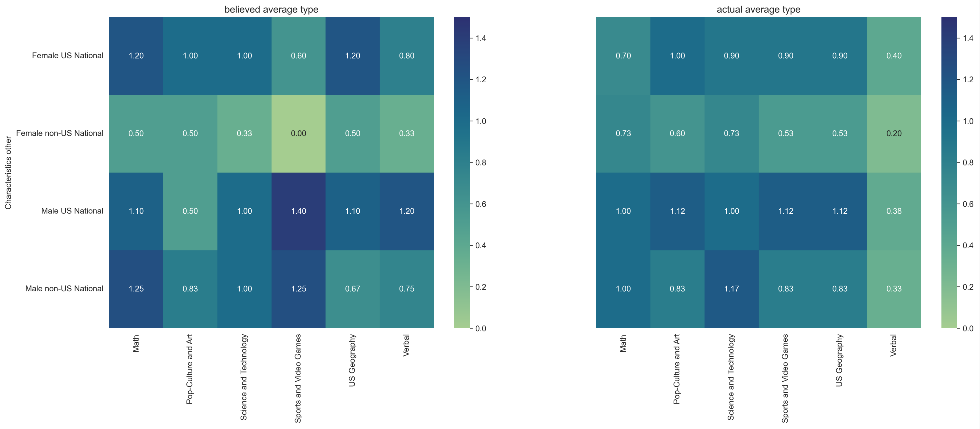
Stereotypes:

- Subjects might be self-censoring their beliefs
- Trying to correct initial biases can look like missatribution bias
- No confirmation bias

The end

Thank you!

Misspecifications



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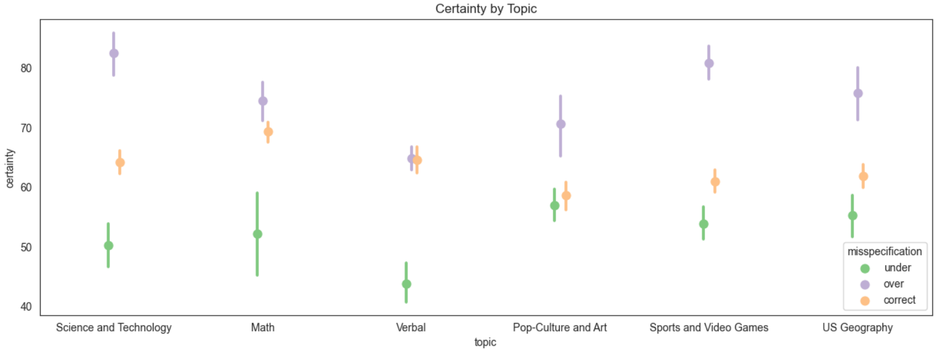
Certainties



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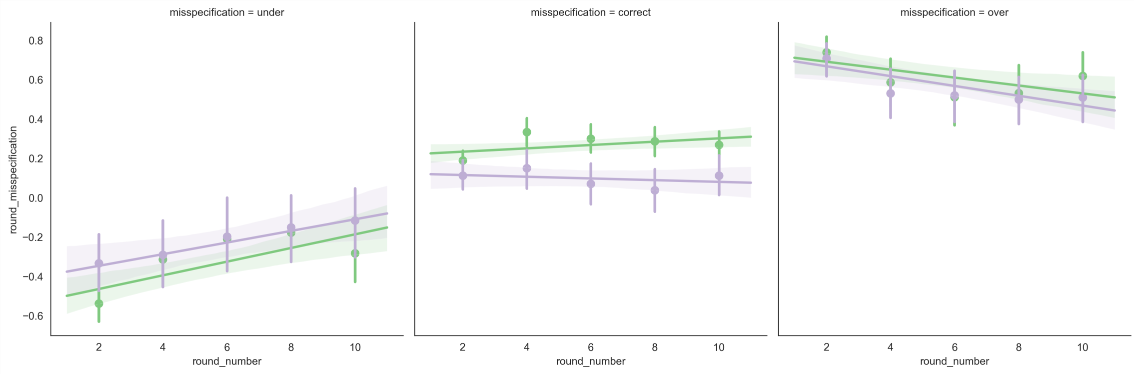
Misspecification changes by treatment



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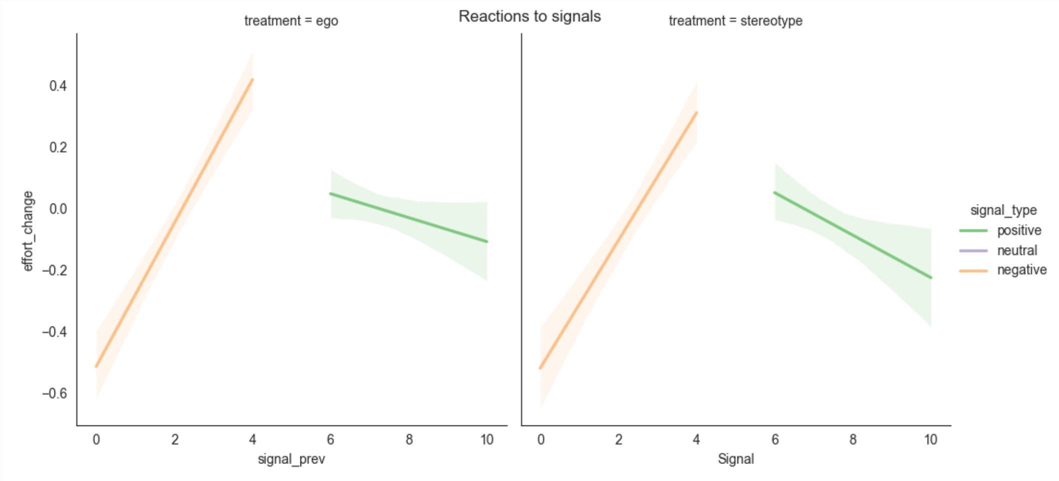
Positive Signals v. Negative Signals



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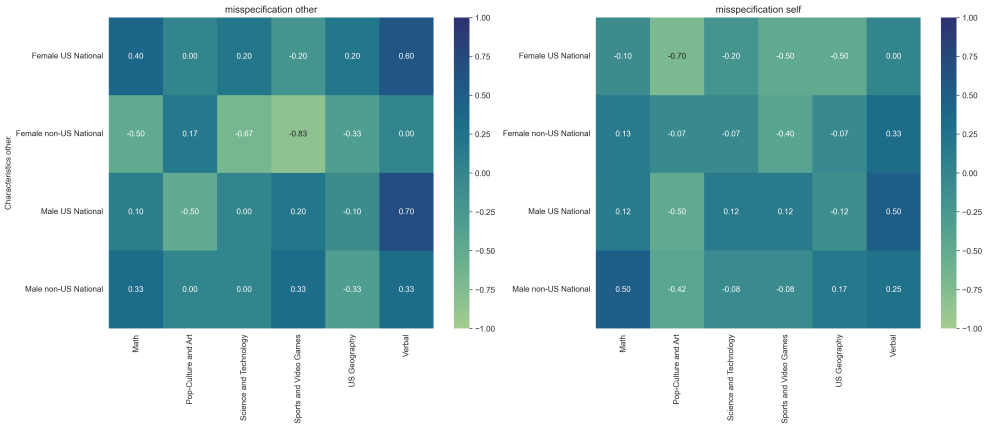
The Stereotypes



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Dogmatic Overconfident: Simulated

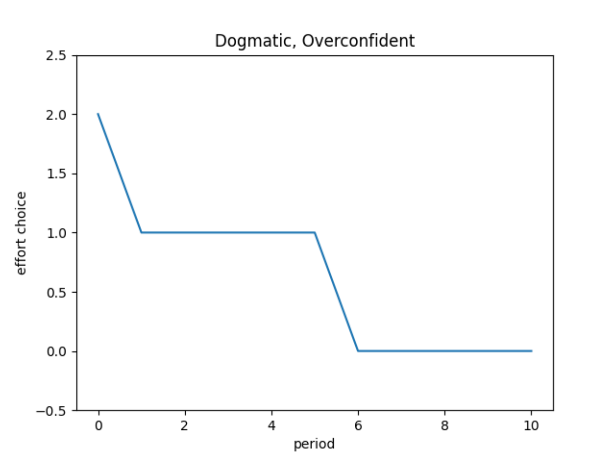


Figure 1: $\theta^* = \theta_M$, $\hat{\theta} = \theta_H$, $\omega^* = \omega_M$

Switcher Overconfident: Simulation

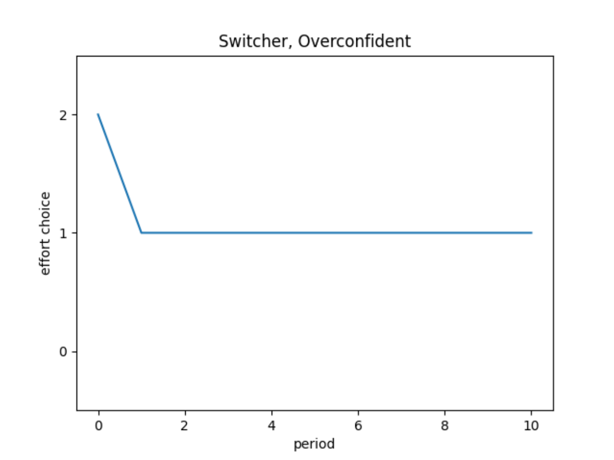
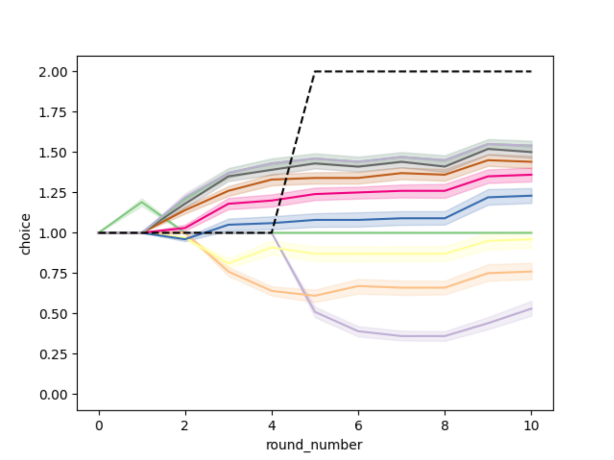


Figure 2: $\theta^* = \theta_M$, $\hat{\theta} = \theta_H$, $\omega^* = \omega_M$, $\alpha = 1.1$

Self-Attribution: Simulation



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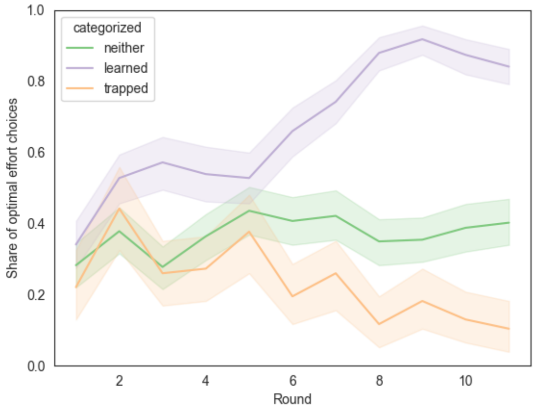
Subject categorization



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