Análisis Factorial en diferentes escalas

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Análisi Factorial

1- Lectura de la matriz de datos

```
x<-as.data.frame(state.x77)
```

2- Quitar los espacios de los nombres

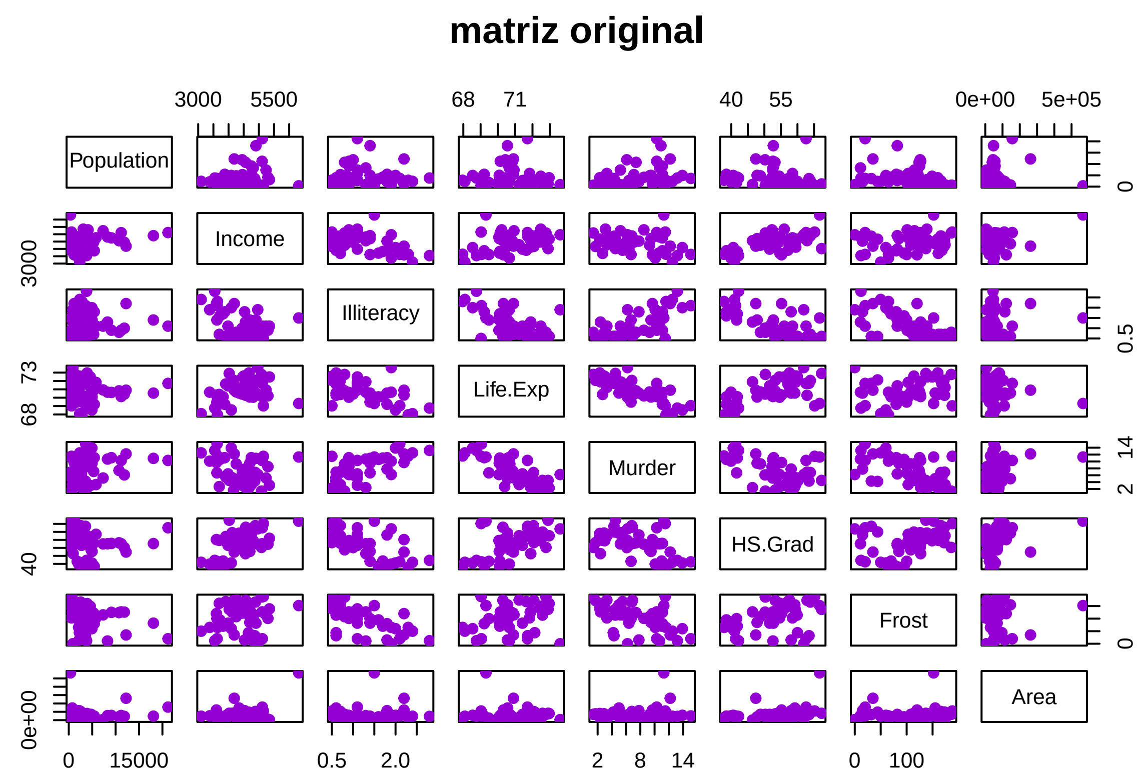
```
colnames(x)[4]="Life.Exp"
colnames(x)[6]= "HS.Grad"
```

3- Separa n (estados) y p (variables)

```
n<-dim(x)[1]
p<-dim(x)[2]
```

4- Generacion de un scater plot para la visualización de variables originales

```
pairs(x, col="darkviolet", pch=19, main="matriz original")
```



Transformación de alguna varibles

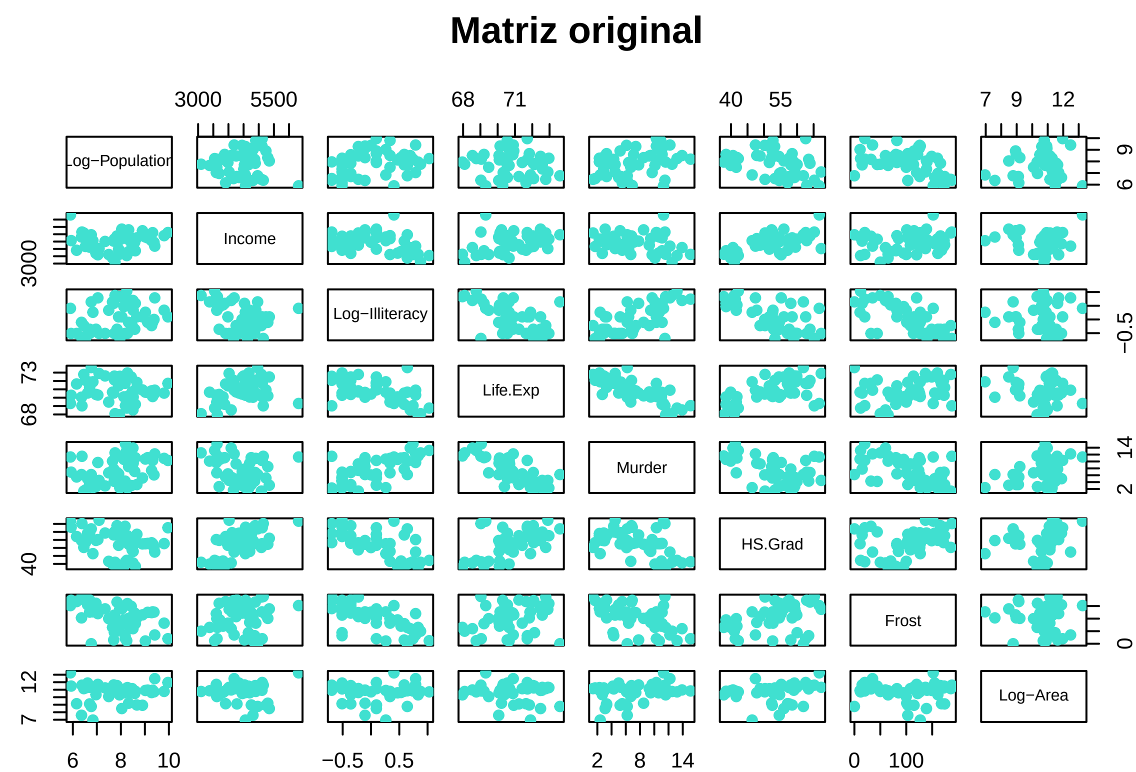
1- Aplicamos logaritmo para las columnas 1,3 y 8

```
x[,1]<-log(x[,1])
colnames(x)[1]<-"Log-Population"

x[,3]<-log(x[,3])
colnames(x)[3]<-"Log-Illiteracy"

x[,8]<-log(x[,8])
colnames(x)[8]<-"Log-Area"
```

Grafico scater para la visualizacion de la

matriz original con 3 variables que se incluyeron

```
pairs(x,col="turquoise", pch=19, main="Matriz original")
```



Nota: Como las variables tiene diferentes unidades de medida, se va a implementar la matriz de correlaciones para estimar la matriz de carga

Reduccion de la dimensionalidad

Análsis Factorial de componentes principales (PCFA)

1- Calcular la matriz de medias y de correlaciones # Matriz de medias

```
mu<-colMeans(x)
mu
```

```
## Log-Population          Income Log-Illiteracy        Life.Exp          Murder
##    7.863443e+00    4.435800e+03    3.128251e-02    7.087860e+01    7.378000e+00
##         HS.Grad           Frost        Log-Area
##    5.310800e+01    1.044600e+02    1.066237e+01
```

#Matriz de correlaciones

```r
R<-cor(x)
R
```

```
##                 Log-Population       Income Log-Illiteracy   Life.Exp     Murder
## Log-Population      1.00000000  0.034963788     0.28371749 -0.1092630  0.3596542
## Income              0.03496379  1.000000000    -0.35147773  0.3402553 -0.2300776
## Log-Illiteracy      0.28371749 -0.351477726     1.00000000 -0.5699943  0.6947320
## Life.Exp           -0.10926301  0.340255339    -0.56999432  1.0000000 -0.7808458
## Murder              0.35965424 -0.230077610     0.69473198 -0.7808458  1.0000000
## HS.Grad            -0.32211720  0.619932323    -0.66880911  0.5822162 -0.4879710
## Frost              -0.45809012  0.226282179    -0.67656232  0.2620680 -0.5388834
## Log-Area            0.08541473 -0.007462068    -0.05830524 -0.1086351  0.2963133
##                    HS.Grad       Frost     Log-Area
## Log-Population -0.3221172 -0.45809012  0.085414734
## Income          0.6199323  0.22628218 -0.007462068
## Log-Illiteracy -0.6688091 -0.67656232 -0.058305240
## Life.Exp        0.5822162  0.26206801 -0.108635052
## Murder         -0.4879710 -0.53888344  0.296313252
## HS.Grad         1.0000000  0.36677970  0.196743429
## Frost           0.3667797  1.00000000 -0.021211992
## Log-Area        0.1967434 -0.02121199  1.000000000
```

2- Reducción de la dimensionalidad mediante # Análisis factorial de componentes principales (PCFA).

1- Calcular los valores y vectores propios.

```r
eR<-eigen(R)
```

2- Valores propios

```r
eigen.val<-eR$values
eigen.val
```

```
## [1] 3.6796976 1.3201021 1.1357357 0.7517550 0.6168266 0.2578511 0.1366186
## [8] 0.1014132
```

3- Vectores propios

```r
eigen.vec<-eR$vectors
eigen.vec
```

```
##             [,1]        [,2]        [,3]       [,4]        [,5]       [,6]
## [1,] -0.23393451 -0.41410075  0.50100922  0.2983839  0.58048485  0.0969034
## [2,]  0.27298977 -0.47608715  0.24689968 -0.6449631  0.09036625 -0.3002708
## [3,] -0.45555443  0.04116196  0.12258370 -0.1824471 -0.32684654 -0.6084112
## [4,]  0.39805075 -0.04655529  0.38842376  0.4191134 -0.26287696 -0.3565095
## [5,] -0.44229774 -0.27640285 -0.21639177 -0.2610739  0.02383706  0.1803894
## [6,]  0.41916283 -0.36311753 -0.06807465 -0.1363534 -0.34015424  0.3960855
## [7,]  0.36358674  0.21893783 -0.37542494 -0.1299519  0.59896253 -0.3507630
## [8,] -0.03545293 -0.58464797 -0.57421867  0.4270918 -0.06252285 -0.3012063
##            [,7]        [,8]
```

```
## [1,] -0.1777562 -0.23622413
## [2,]  0.3285840  0.12483849
## [3,] -0.3268997 -0.39825363
## [4,] -0.3013983  0.47519991
## [5,] -0.4562245  0.60970476
## [6,] -0.4808140 -0.40675672
## [7,] -0.4202943 -0.06001175
## [8,]  0.2162424 -0.05831177
```

4- Calcular la proporcion de variabilidad

```
prop.var<-eigen.val/sum(eigen.val)
prop.var
```

```
## [1] 0.45996220 0.16501277 0.14196697 0.09396938 0.07710332 0.03223139 0.01707733
## [8] 0.01267665
```

5- Calcular la proporcion de variabilidad acumulada

```
prop.var.acum<-cumsum(eigen.val)/sum(eigen.val)
prop.var.acum
```

```
## [1] 0.4599622 0.6249750 0.7669419 0.8609113 0.9380146 0.9702460 0.9873233
## [8] 1.0000000
```

Estimacion de la matriz de carga

Nota: se estima la matriz de carga usando los

autovalores y autovectores.

se aplica la rotación varimax

Primera estimación de Lamda mayuscula

se calcula multiplicando la matriz de los

3 primeros autovectores por la matriz diagonal

formada por la raiz cuadrada de los primeros

3 autovalores.

```r
L.est.1<-eigen.vec[,1:3] %*% diag(sqrt(eigen.val[1:3]))
L.est.1
```

```
##             [,1]        [,2]        [,3]
## [1,] -0.44874575 -0.47578394  0.53393005
## [2,]  0.52366367 -0.54700365  0.26312322
## [3,] -0.87386900  0.04729332  0.13063856
## [4,]  0.76356236 -0.05349003  0.41394671
## [5,] -0.84843932 -0.31757498 -0.23061066
## [6,]  0.80406070 -0.41720642 -0.07254777
## [7,]  0.69745163  0.25155014 -0.40009375
## [8,] -0.06800771 -0.67173536 -0.61195003
```

Rotación varimax

```r
L.est.1.var<-varimax(L.est.1)
L.est.1.var
```

```
## $loadings
##
## Loadings:
##      [,1]   [,2]   [,3]
## [1,]                0.840
## [2,]  0.785 -0.106  0.121
## [3,] -0.665         0.583
## [4,]  0.763  0.384 -0.168
## [5,] -0.573 -0.528  0.517
## [6,]  0.825 -0.202 -0.323
## [7,]  0.281        -0.794
## [8,]        -0.906
##
##                 [,1]  [,2]  [,3]
## SS loadings    2.744 1.300 2.091
## Proportion Var 0.343 0.163 0.261
## Cumulative Var 0.343 0.506 0.767
##
## $rotmat
##            [,1]      [,2]       [,3]
## [1,]  0.7824398 0.1724744 -0.5983649
## [2,] -0.5274231 0.6944049 -0.4895169
## [3,]  0.3310784 0.6986089  0.6342970
```

Estimación de la matriz de los errores

1- Estimación de la matriz de perturbaciones

```r
Psi.est.1<-diag(diag(R-as.matrix(L.est.1.var$loadings)%*% t(as.matrix(L.est.1.var$loadings))))
Psi.est.1
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]      [,7]
## [1,] 0.2871756 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
## [2,] 0.0000000 0.3573295 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
## [3,] 0.0000000 0.0000000 0.2170499 0.0000000 0.0000000 0.000000 0.0000000
## [4,] 0.0000000 0.0000000 0.0000000 0.2427595 0.0000000 0.000000 0.0000000
## [5,] 0.0000000 0.0000000 0.0000000 0.0000000 0.1261156 0.000000 0.0000000
## [6,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.174162 0.0000000
## [7,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.2902087
## [8,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.000000 0.0000000
##           [,8]
## [1,] 0.0000000
## [2,] 0.0000000
## [3,] 0.0000000
## [4,] 0.0000000
## [5,] 0.0000000
## [6,] 0.0000000
## [7,] 0.0000000
## [8,] 0.1696637
```

2- Se utiliza el método Análisis de factor principal (PFA) Para estimación de autovalores y autovectores

```r
RP<-R-Psi.est.1
RP
```

```
##                Log-Population       Income Log-Illiteracy   Life.Exp     Murder
## Log-Population     0.71282441  0.034963788     0.28371749 -0.1092630  0.3596542
## Income             0.03496379  0.642670461    -0.35147773  0.3402553 -0.2300776
## Log-Illiteracy     0.28371749 -0.351477726     0.78295012 -0.5699943  0.6947320
## Life.Exp          -0.10926301  0.340255339    -0.56999432  0.7572405 -0.7808458
## Murder             0.35965424 -0.230077610     0.69473198 -0.7808458  0.8738844
## HS.Grad           -0.32211720  0.619932323    -0.66880911  0.5822162 -0.4879710
## Frost             -0.45809012  0.226282179    -0.67656232  0.2620680 -0.5388834
## Log-Area           0.08541473 -0.007462068    -0.05830524 -0.1086351  0.2963133
##                   HS.Grad       Frost     Log-Area
## Log-Population -0.3221172 -0.45809012  0.085414734
## Income          0.6199323  0.22628218 -0.007462068
## Log-Illiteracy -0.6688091 -0.67656232 -0.058305240
## Life.Exp        0.5822162  0.26206801 -0.108635052
## Murder         -0.4879710 -0.53888344  0.296313252
## HS.Grad         0.8258380  0.36677970  0.196743429
## Frost           0.3667797  0.70979126 -0.021211992
## Log-Area        0.1967434 -0.02121199  0.830336270
```

3-Calculo de la matriz de autovalores y autovectores

```r
eRP<-eigen(RP)
```

4-Autovalores

```r
eigen.val.RP<-eRP$values
eigen.val.RP
```

```
## [1]  3.46137648  1.10522195  0.88152416  0.48705680  0.35360597  0.02813553
## [7] -0.06758176 -0.11380367
```

5-Autovectores

```r
eigen.vec.RP<-eRP$vectors
eigen.val.RP
```

```
## [1]  3.46137648  1.10522195  0.88152416  0.48705680  0.35360597  0.02813553
## [7] -0.06758176 -0.11380367
```

6-Proporción de variabilidad

```r
prop.var.RP<-eigen.val.RP/ sum(eigen.val.RP)
prop.var.RP
```

```
## [1]  0.564152306  0.180134556  0.143675179  0.079382934  0.057632455
## [6]  0.004585668 -0.011014811 -0.018548286
```

7-Proporción de variabilidad acumulada

```r
prop.var.RP.acum<-cumsum(eigen.val.RP)/ sum(eigen.val.RP)
prop.var.RP.acum
```

```
## [1] 0.5641523 0.7442869 0.8879620 0.9673450 1.0249774 1.0295631 1.0185483
## [8] 1.0000000
```

8-Estimación de la matriz de cargas Con rotación varimax

```r
L.est.2<-eigen.vec.RP[,1:3] %*% diag(sqrt(eigen.val.RP[1:3]))
L.est.2
```

```
##             [,1]        [,2]        [,3]
## [1,] -0.42621819 -0.27609775  0.56228420
## [2,]  0.48528446 -0.36092954  0.32467098
## [3,] -0.84791581  0.08163995  0.10816670
## [4,]  0.73812189  0.02688907  0.36866093
## [5,] -0.84699944 -0.34227865 -0.12211117
## [6,]  0.78817342 -0.40399024  0.04935203
## [7,]  0.66112453  0.12457105 -0.40191996
## [8,] -0.06868291 -0.77165602 -0.36531090
```

Rotacion varimax

```r
L.est.2.var<-varimax(L.est.2)
```

1-Estimación de la matriz de covarianzas de los errores.

```r
Psi.est.2<-diag(diag(R-as.matrix(L.est.2.var$loadings)%*% t(as.matrix(L.est.2.var$loadings))))
Psi.est.2
```

```
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
## [1,] 0.4259446 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [2,] 0.0000000 0.5288176 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [3,] 0.0000000 0.0000000 0.2626737 0.0000000 0.0000000 0.0000000 0.0000000
## [4,] 0.0000000 0.0000000 0.0000000 0.3185422 0.0000000 0.0000000 0.0000000
## [5,] 0.0000000 0.0000000 0.0000000 0.0000000 0.1505261 0.0000000 0.0000000
## [6,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.2131389 0.0000000
## [7,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.3858568
## [8,] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
##           [,8]
## [1,] 0.0000000
## [2,] 0.0000000
## [3,] 0.0000000
## [4,] 0.0000000
## [5,] 0.0000000
## [6,] 0.0000000
## [7,] 0.0000000
## [8,] 0.2663776
```

2-Obtención de los scores de ambos métodos

PCFA

```r
FS.est.1<-scale(x)%*% as.matrix(L.est.1.var$loadings)
FS.est.1
```

```
##                       [,1]          [,2]       [,3]
## Alabama        -5.84072356 -1.3993671511  4.0008109
## Alaska          2.12443806 -3.6163397014 -1.3435941
## Arizona        -0.77245459 -1.1030150088  1.7864181
## Arkansas       -4.26961555 -0.1287634469  1.8680205
## California      1.57843978 -1.6386262821  3.0959757
## Colorado        3.35619481 -0.5747409714 -1.9955520
## Connecticut     2.96609993  2.5265114588 -1.0120520
## Delaware        0.15111765  2.2707877284 -1.3473631
## Florida        -0.91278118 -0.8518787165  3.2141818
## Georgia        -5.10406769 -1.5374188978  3.5972606
## Hawaii          1.68679592  2.0782245763  0.6972161
## Idaho           1.93931571  0.0374520725 -2.6403015
## Illinois        0.36572803 -0.9730363911  1.3246992
## Indiana         0.69870165  0.1740586327 -0.1660034
## Iowa            3.77325852  0.8634090197 -2.4308546
## Kansas          3.22079390  0.2206198504 -1.7333568
## Kentucky       -3.97957229 -0.1711842990  1.8581455
## Louisiana      -6.15095874 -1.1449716511  4.2193388
## Maine           0.38912287  0.9352663421 -2.8385772
## Maryland        0.54556931  0.6481615589  0.7313943
```

```
## Massachusetts   1.95531363  1.9508870989 -0.0699601
## Michigan        0.06109118 -0.8995742724  1.1610156
## Minnesota       3.83625590  0.7199310360 -2.2609012
## Mississippi    -6.73875213 -1.1336057288  3.0124928
## Missouri       -0.63621057 -0.5673516660  0.5606479
## Montana         1.70022911 -0.7530855537 -2.9827203
## Nebraska        3.31393569  0.5702899251 -2.6630094
## Nevada          1.83953234 -2.1624547546 -2.8632403
## New Hampshire   1.76672303  1.8835104424 -3.2522623
## New Jersey      1.23076573  1.5154423999  0.6483326
## New Mexico     -2.42369795 -1.2184859435  0.1095350
## New York       -0.55160991 -0.8431042602  2.9025469
## North Carolina -4.53932589 -0.7126552652  2.8168209
## North Dakota    3.26810535  1.0664889529 -3.5180166
## Ohio            0.67643704 -0.0394642439  0.5816740
## Oklahoma       -0.43628926  0.0293430043  0.2108486
## Oregon          2.64633236 -0.0126633017 -0.6563722
## Pennsylvania   -0.06313819  0.0425262164  0.8538298
## Rhode Island    0.25059508  4.0533333045 -1.3779994
## South Carolina -6.20030464 -0.7067780563  3.0142562
## South Dakota    2.51505516  0.8539599931 -3.9694575
## Tennessee      -3.75602365 -0.3764569265  2.4225536
## Texas          -2.74825842 -2.0176142597  4.0126966
## Utah            3.40911641  0.2638533973 -3.0642167
## Vermont         1.26368503  1.7670538099 -3.5748058
## Virginia       -1.45435214 -0.4332714574  1.8388594
## Washington      2.95298764  0.0002978623 -0.1436737
## West Virginia  -3.41599674  0.5649932020  0.5132111
## Wisconsin       2.58972274  0.8701285803 -1.5397225
## Wyoming         1.92267355 -0.8906222579 -3.6087703
```

PFA

```
FS.est.2<-scale(x)%*% as.matrix (L.est.2.var$loadings)
FS.est.2
```

```
##                       [,1]         [,2]       [,3]
## Alabama        -5.69766092 -1.133005866  3.9030908
## Alaska          1.77921500 -3.310049553 -1.2425530
## Arizona        -0.80948635 -1.007423566  1.6833688
## Arkansas       -4.04451164 -0.036340306  1.8899610
## California      1.28900772 -1.589528660  2.7938220
## Colorado        3.21256763 -0.645092519 -1.9103448
## Connecticut     2.85639977  2.291700954 -1.1152442
## Delaware        0.22491218  2.168332191 -1.3109174
## Florida        -1.04778981 -0.760012075  2.9630979
## Georgia        -5.04193484 -1.243399542  3.4848855
## Hawaii          1.64548810  1.848120424  0.5487863
## Idaho           1.99602286 -0.067186945 -2.4442739
## Illinois        0.17329771 -0.870927790  1.1838509
## Indiana         0.66348403  0.140717116 -0.1900850
```

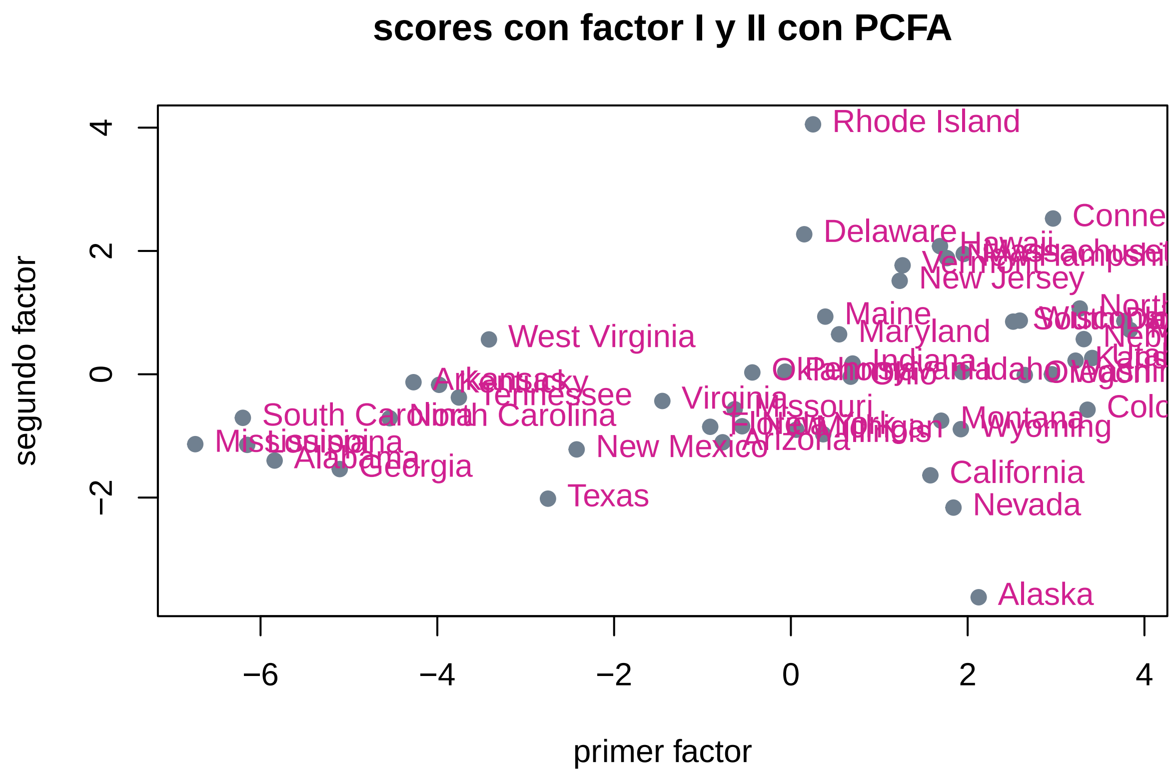
```
## Iowa            3.70915552  0.657976435 -2.3698485
## Kansas          3.13617617  0.071725764 -1.6894853
## Kentucky       -3.82119443 -0.051170443  1.8492550
## Louisiana      -5.97309240 -0.880509145  4.1021292
## Maine           0.58567717  0.845398887 -2.6098620
## Maryland        0.40855637  0.650876372  0.5867974
## Massachusetts   1.91021424  1.761365924 -0.1964750
## Michigan       -0.07208772 -0.823049544  1.0671998
## Minnesota       3.74953682  0.518054623 -2.2104937
## Mississippi    -6.45121865 -0.852611917  3.0320154
## Missouri       -0.64446964 -0.519762510  0.5472506
## Montana         1.72574501 -0.752576236 -2.7507980
## Nebraska        3.28773039  0.392513546 -2.5439122
## Nevada          1.69672312 -1.994626548 -2.6292009
## New Hampshire   1.87991014  1.704867403 -3.0632652
## New Jersey      1.10782292  1.425042094  0.4638907
## New Mexico     -2.26112419 -1.086582245  0.2653217
## New York       -0.72255151 -0.744949928  2.6624378
## North Carolina -4.42441540 -0.513264749  2.7372284
## North Dakota    3.22068093  0.897031063 -3.3556310
## Ohio            0.59453054 -0.051780182  0.4905274
## Oklahoma       -0.36512462  0.000708499  0.2244101
## Oregon          2.56050584 -0.129810062 -0.6934180
## Pennsylvania   -0.10451900  0.054229408  0.7553645
## Rhode Island    0.40356926  3.785456289 -1.3760426
## South Carolina -5.98815271 -0.435831413  2.9745853
## South Dakota    2.60764548  0.683975660 -3.7117087
## Tennessee      -3.63769564 -0.249263663  2.3593673
## Texas          -2.80670233 -1.827474308  3.8156526
## Utah            3.44131011  0.069209103 -2.8669774
## Vermont         1.44160727  1.580578146 -3.3086066
## Virginia       -1.50774364 -0.328200587  1.7151967
## Washington      2.81601549 -0.109025242 -0.2503494
## West Virginia  -3.18525955  0.632647668  0.5745805
## Wisconsin       2.55487697  0.699000994 -1.5141208
## Wyoming         1.92835024 -0.866073018 -3.3204601
```

Graficamos ambos scores

```
par(mfrow=c(2,1))
```

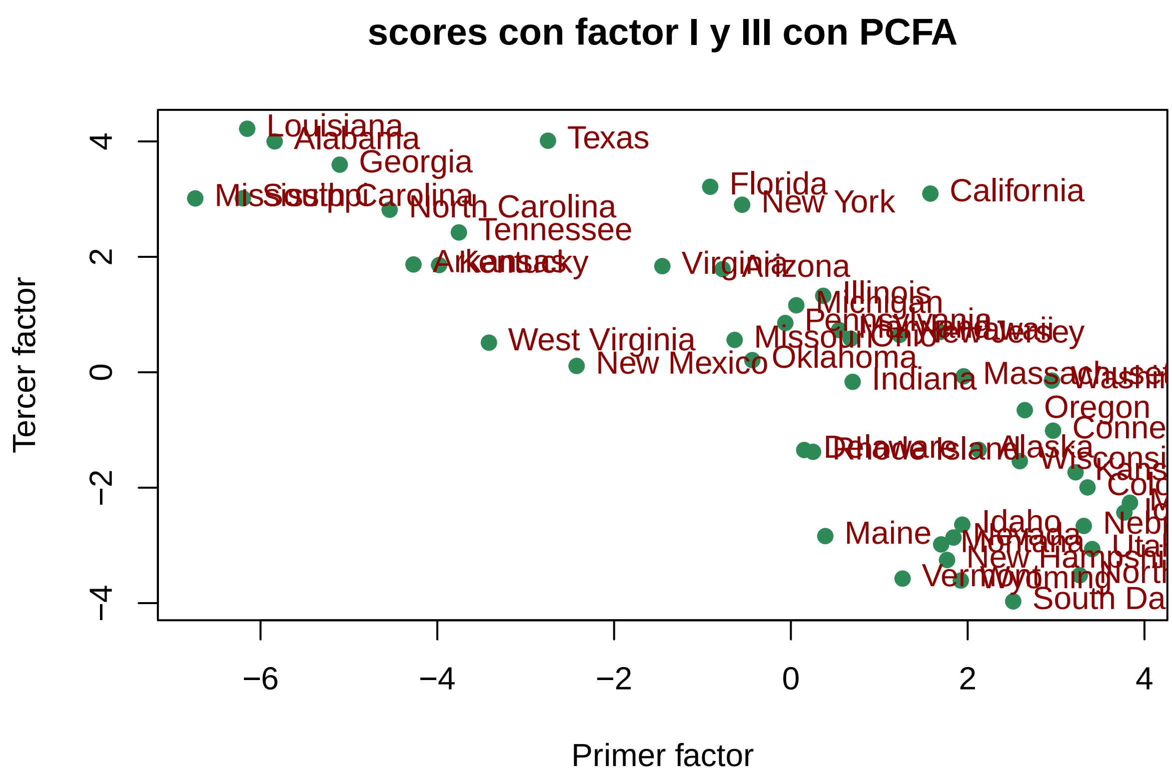
Factor I y II

```
pl1<-plot(FS.est.1[,1], FS.est.1[,2], xlab="primer factor",
          ylab="segundo factor", main="scores con factor I y II con PCFA",
          pch=19, col="slategrey")
text(FS.est.1[,1], FS.est.1[,2], labels = rownames(x), pos=4, col="violetred")
```



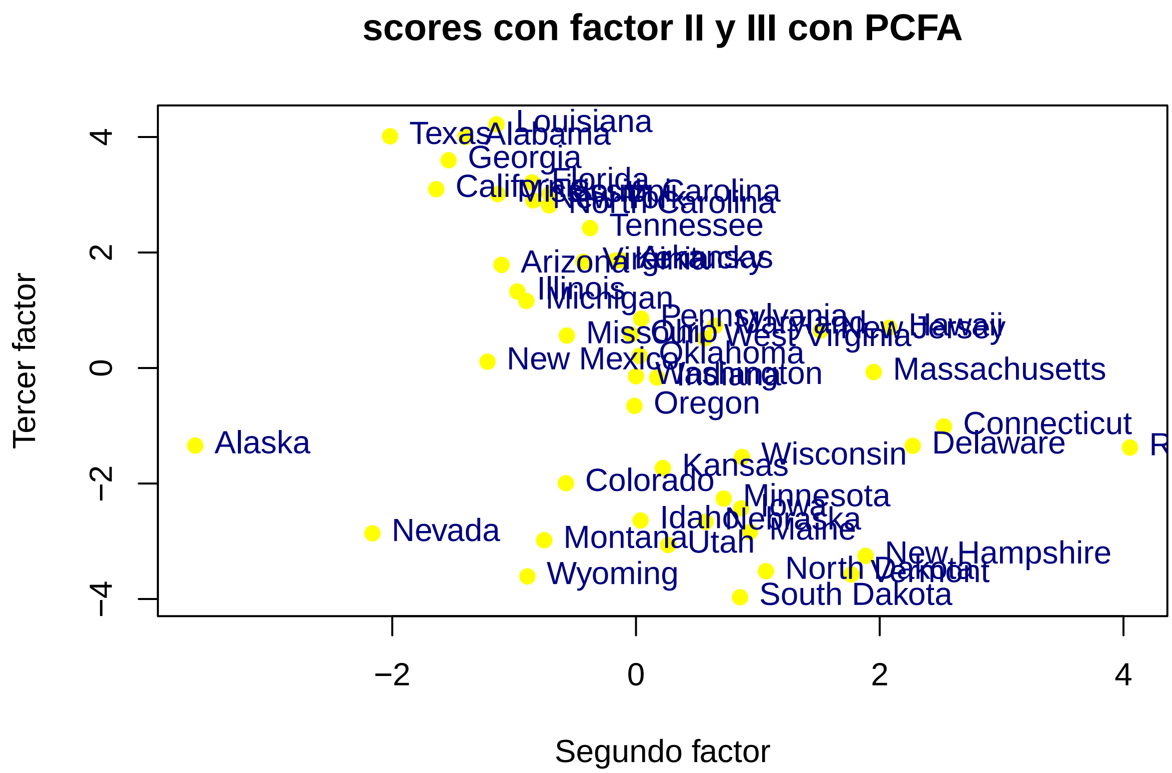
Factor I y III

```
pl2<-plot(FS.est.1[,1], FS.est.1[,3], xlab="Primer factor",
          ylab="Tercer factor", main="scores con factor I y III con PCFA",
          pch=19, col="seagreen")
text(FS.est.1[,1], FS.est.1[,3], labels = rownames(x), pos=4, col="darkred")
```



Factor II y III

```
pl3<-plot(FS.est.1[,2], FS.est.1[,3], xlab="Segundo factor",
          ylab="Tercer factor", main="scores con factor II y III con PCFA",
          pch=19, col="yellow")
text(FS.est.1[,2], FS.est.1[,3], labels = rownames(x), pos=4, col="navy")
```


scores con factor II y III con PCFA
Tercer factor
Segundo factor
4
2
0
−2
−4
−2
0
2
4
Louisiana
Texas
Alabama
Georgia
Florida
South Carolina
North Carolina
Tennessee
Arizona
Arkansas
Illinois
Michigan
Pennsylvania
Maryland
New Jersey
Hawaii
Missouri
West Virginia
New Mexico
Oklahoma
Washington
Massachusetts
Oregon
Connecticut
Alaska
Delaware
Wisconsin
Kansas
Colorado
Minnesota
Iowa
Nevada
Idaho
Nebraska
Maine
Montana
Utah
New Hampshire
North Dakota
Vermont
Wyoming
South Dakota