Universidad de los Andes

Phenomenological Study of Search of Heavy Neutrinos, with Displaced Vertices and Vector Boson Fusion

This dissertation is submitted for the degree of

Physicist

by

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Chapter 1

State of the Art

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As it was mentioned earlier, the SM does not explain the reason why the mass of neutrinos is a factor of almost 10^{-6} smaller that the mass of the other fermions. Moreover the SM predicts that the mass of the neutrinos is zero. Addiotionally, it does not provide an explanation to the fact that only left-handed neutrinos have been observed in nature. In this section we are going to work on possible solutions to these problems. [1]

1.2.1 Dirac Mass

First, we start by studying the Dirac mass term of a free fermion. The lagrangian equation for a fermion particle is given by the expression:

$$L = \overline{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi \tag{1.1}$$

Where ψ is the Dirac Spinor. From this Lagrangian expression it is possible to see that in the SM the mass is included through the second term in the equation which is called "Dirac mass term":

[1] The detailed calculations of the theory explained here are stated in A

$$m\overline{\psi}\psi \tag{1.2}$$

We can write the Dirac Spinor as a sum of its left- and right- chiral states:

$$m\overline{\psi}\psi = m\left(\overline{\psi_L + \psi_R}\right)(\psi_L + \psi_R) = m\overline{\psi_L}\psi_R + m\overline{\psi_R}\psi_L \tag{1.3}$$

Previously we have used the fact that: $\overline{\psi_L}\psi_L = \overline{\psi_R}\psi_R = 0$ which is proved in Appedix A. It can be seen from the lastest equation that a massive particle must have both quiral states: left and right. Thus, the Dirac Mass can be interpreted as the coupling constant between the two chiral states. Since right-handed neutrinos had been never observed in nature, it is expected that neutrinos have zero mass. Although, experiments of neutrino oscillations indicate that neutrinos have a small mass of the order of meV. The former implies either the existence of a right-handed neutrino which is responsible for the mass of the neutrino, or that there exists other sort of mass term.

1.2.2 Majorana Mass

The Majorana Mechanism is based on expressing the mass term in the Lagrangian in only the left-handed chiral state terms. To do this we start by decomposing the wavefunction into its left and right chiral states in the Dirac Lagrangian:

$$\begin{aligned} L &= \overline{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi \\ &= (\overline{\psi_L} + \overline{\psi_R})(i\gamma^\mu\partial_\mu - m)(\psi_L + \psi_R) \\ &= i\overline{\psi_L}\gamma^\mu\partial_\mu\psi_L - \overline{\psi_L}m\psi_R + i\overline{\psi_R}\gamma^\mu\partial_\mu\psi_R - \overline{\psi_R}m\psi_L \end{aligned} \tag{1.4}$$

Since $\overline{\psi_L}\psi_L = \overline{\psi_R}\psi_R = 0$ and $\overline{\psi_R}\gamma^\mu\partial_\mu\psi_L = \overline{\psi_L}\gamma^\mu\partial_\mu\psi_R = 0$ as it is explained in the Appendix A. Now, we can replace the expression of this Lagrangian in the Euler-Lagrange equation:

$$\frac{\partial L}{\partial(\partial\phi)} - \frac{\partial L}{\partial\phi} = 0 \tag{1.5}$$

By doing this we find that the two equations of motion for the fields are two coupled Dirac equations for the right- and left- handed fields:

$$i\gamma^\mu \partial_\mu \psi_L = m\psi_R \tag{1.6}$$

$$i\gamma^\mu \partial_\mu \psi_R = m\psi_L \tag{1.7}$$

In the formulation of the SM the mass of the neutrino is zero, in this case we obtain two equations which are called "Weyl equations":

$$i\gamma^\mu \partial_\mu \psi_L = 0 \tag{1.8}$$

$$i\gamma^\mu \partial_\mu \psi_R = 0 \tag{1.9}$$

The former means that neutrinos can be described using two two-component spinors that are helicity eigenstates. These eigenstates represent two states with definite and opposite helicity which correspond to the left- and right-handed neutrinos. However, since we have not observed a right-handed neutrino we just represent the neutrino as a single left-handed massless field.

Majorana worked out a way to describe a massive neutrino just in terms of it's left-handed field. This calculation is performed in the Appendix A. The objective of Majorana was to write the Equation 1.7 as 1.6 by finding an expression for ψ_R in terms of ψ_L. By doing some manipulations of the Equation 1.7 we find that it can be written as:

$$i\gamma^\mu \partial_\mu C\overline{\psi}_R^\intercal = mC\overline{\psi}_L^\intercal \tag{1.10}$$

Where C is the operator charge conjugation operator. This operator and its properties are explained in Appendix B. Now, the Equation 1.10 would have the same structure as Equation 1.6 if the right-handed term is imposed to be:

$$\psi_R = C\overline{\psi}_L^\intercal \tag{1.11}$$

The former assumption requires $C\overline{\psi}_L^\intercal$ to be right-handed, this is proved in the Appendix A. Thus, the complete Majorana field can be written as:

$$\psi = \psi_L + \psi_R = \psi_L + C\overline{\psi}_L^\intercal \tag{1.12}$$

Defining the charge-conjugate field as: $\psi_L^C = C\overline{\psi}_L^{\intercal}$. We get for the expression of the complete Majorana field:

$$\psi = \psi_L + \psi_L^C \tag{1.13}$$

The implications of requiring the right-handed component of ψ to satisfy the 1.13 expression can be studied by taking the charge conjugate of the complete Majorana field.

$$\psi^C = (\psi_L + \psi_L^C)^C = \psi_L^C + \psi_L = \psi \tag{1.14}$$

Having in mind that the charge conjugation operator turns a particle state into an antiparticle state, it can be deduced that a Majorana particle is its own antiparticle. Since the charge conjugation operator flips the sign of electric charge, a Majorana particle must be neutral. Thus, the neutrino is the only fermion that could be a Majorana particle.

Majorana Mass Term

Previously, we saw that the mass term in the Lagrangian couples the left and right chiral states of the neutrino (Equation 1.3). Replacing the expression we found for the right-handed component of the neutrino field in the mass term of the Lagrangian, we get (having in mind that its hermitian conjugate is identical):

$$L_{Maj}^L = m\overline{\nu_L}\nu_L^C + m\overline{\nu_L^C}\nu_L = \frac{1}{2}m\overline{\nu_L^C}\nu_L \tag{1.15}$$

1.3 Seesaw Mechanism

As it was mentioned before, in the case that the right-handed chiral field does not exist there can be no Dirac mass term. However we can have a Majarona mass term in the Lagrangian so the neutrino would be a Majorana particle:

$$L_{Maj}^L = \frac{1}{2}m_L\overline{\nu^C}_L\nu_L \tag{1.16}$$

The term m_L is forbidden by electroweak symmetry and it only can appears after its spontaneous breakdown, hence such a term can not exist.

In order to let the neutrino to have mass, a right-handed neutrino that interacts only with gravity and the Higgs mechanism must exist. If we consider that a right-handed chiral neutrino can exist, we would have to add different terms to the Lagrangian. First, if we assume that it is possible to write a left-handed Majorana field, we have for the first term:

$$L_L^M = m_L \overline{\nu_L} \nu_L^C + m_L \overline{\nu_L^C} \nu_L \tag{1.17}$$

Additionally, we have to include a similar term which is the right-handed Majorana field:

$$L_R^M = m_R \overline{\nu_R^C} \nu_R + m_R \overline{\nu_R} \nu_R^C \tag{1.18}$$

We also have to add Dirac mass terms: the first Dirac mass term we mentioned on this section (Equation 1.19) and another one that comes from the charge-conjugate fields (Equation 1.20):

$$L = m_D \overline{\nu_L} \nu_R + m_D \overline{\nu_R} \nu_L \tag{1.19}$$

$$L = m_D \overline{\nu_R^C} \nu_L^C + m_D \overline{\nu_L^C} \nu_R^C \tag{1.20}$$

Since the hermitian conjugate of each equation is identical, we can write the most general mass term as:

$$L = \frac{1}{2} \left(m_L \overline{\nu_L^C} \nu_L + m_R \overline{\nu_R^C} \nu_R + m_D \overline{\nu_R} \nu_L + m_D \overline{\nu_L^C} \nu_R^C \right) \tag{1.21}$$

The former equation can be written as a matrix equation:

$$L_{mass} \propto \begin{pmatrix} \overline{\nu_L^C} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} \tag{1.22}$$

The Equation 1.22 expresses the Lagrangian in terms of the left and right chiral states. These states do not have a definite mass because the matrix is not diagonal. Thus, the left and right chiral states do not correspond to the physical particles (which have a definite mass). Instead the real particles are a superposition of the mass eigenstates. In order to find the mass eigenvalues we need to diagonalize the M matrix (the one in the middle of the former equation). This calculation is explained in Appendix A. We find the mass eigenstates are given by the expression:

$$m_{1,2} = \frac{1}{2}\left[(m_L + m_R) \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right] \tag{1.23}$$

The fact that the SM does not allow a Majorana left-chiral mass term implies $m_L = 0$. Next, we are going to study the expression of the mass eigenstates m_1 and m_2. When we choose $m_R >> m_D$, we get for the mass eigenvalues:

$$m_1 = \frac{m_D^2}{m_R} \tag{1.24}$$

$$m_2 = m_R\left(1 + \frac{m_D^2}{m_R^2}\right) \approx m_R \tag{1.25}$$

From both equations above we can deduce that if there a neutrino with mass m_2 very large exists, then the other neutrino must have a small mass. The former fact is the reason why this mechanism is called "Seesaw": the mass of each physical neutrino is controlled by the mass eigenvalues in a way such that when one neutrino is light the other is heavier. Now, the neutrino mass eigenstates are given by the following expresions:

$$\nu_1 \propto \left(\nu_L + \nu_L^C\right) - \frac{m_D}{m_R^2}\left(\nu_R + \nu_R^C\right) \tag{1.26}$$

$$\nu_2 \propto \left(\nu_R + \nu_R^C\right) + \frac{m_D}{m_R^2}\left(\nu_L + \nu_L^C\right) \tag{1.27}$$

The former equations show that ν_1 is mostly the left-handed light Majorana neutrino while ν_2 is the heavy sterile right-handed neutrino. This is the explanation that the Seesaw Mechanism gives to the fact that the neutrino is much lighter than the other fermions.

Appendix A

Neutrinos and Seesaw Mechanism

First of all we are going to start by defining some fundamental concepts: helicity, quirality and projection operators. The helicity of a particle is defined as the projection of its spin onto the direction of its motion. It is said that a particle is right-handed when its spin is in the same direction as its motion and it is said a particle is left-handed when its spin is opposite in the opposite direction of its motion. In the case of massless particles the concept of quirality and helicity is equivalent. The quirality for a Dirac fermion is defined through the operator γ^5 with eigenvalues ± 1. Thus a Dirac field can be projected into its left or right component by acting the operators P_R and P_L upon it. The right- and left-handed projection operators are defined as:

$$P_R = \frac{1+\gamma^5}{2} \quad \text{and} \quad P_L = \frac{1-\gamma^5}{2} \tag{A.1}$$

A.0.1 Dirac Mass

In this Appendix we are going to perform with detail the calculations for neutrino physics which were mentioned in the State of the Art Chapter. We start here by studying the Dirac Mass, which is a term of the form:

$$m\overline{\psi}\psi = m\overline{(\psi_L + \psi_R)}(\psi_L + \psi_R) = m(\overline{\psi_L}\psi_L + \overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L + \overline{\psi_R}\psi_R) \tag{A.2}$$

Lets study the term $\overline{\psi_L}\psi_L$ and using $P_R P_L = 0$:

$$\overline{\psi_L}\psi_L = \overline{\psi}P_L^\dagger P_L\psi = \overline{\psi}P_R P_L\psi = 0 \tag{A.3}$$

Using an analogous reasoning we can find $\overline{\psi_R}\psi_R = 0$, too. Finally, we obtain the expresion:

$$m\overline{\psi}\psi = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L) \tag{A.4}$$

A.0.2 Majorana Mass

The expression we had for the Dirac Lagrangian was:

$$\begin{aligned} L &= \overline{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi \\ &= (\overline{\psi_L} + \overline{\psi_R})(i\gamma^\mu\partial_\mu - m)(\psi_L + \psi_R) \\ &= i\overline{\psi_L}\gamma^\mu\partial_\mu\psi_L + i\overline{\psi_L}\gamma^\mu\partial_\mu\psi_R - m\overline{\psi_L}\psi_L - m\overline{\psi_L}\psi_R \\ &\quad + i\overline{\psi_R}\gamma^\mu\partial_\mu\psi_L + i\overline{\psi_R}\gamma^\mu\partial_\mu\psi_R - m\overline{\psi_R}\psi_L - m\overline{\psi_R}\psi_R \end{aligned} \tag{A.5}$$

(A.6)

We already proved that $\overline{\psi_L}\psi_L = \overline{\psi_R}\psi_R = 0$. Now we are going to study the second term in the latest (REFERENCIAR) equation, which has a term of the form:

$$\begin{aligned} P_R\gamma^\mu &= \frac{1}{2}(1+\gamma^5)\gamma^\mu = \frac{1}{2}(\gamma^\mu + \gamma^5\gamma^\mu) \\ &= \frac{1}{2}(\gamma^\mu - \gamma^\mu\gamma^5) \qquad \text{Since } \{\gamma^5, \gamma^\mu\} = \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0 \\ &= \frac{1}{2}\gamma^\mu(1-\gamma^5) = \gamma^\mu P_L \end{aligned} \tag{A.7}$$

Using what we have found in the last expression, we get for the second term:

$$\begin{aligned} i\overline{\psi_L}\gamma^\mu\partial_\mu\psi_R &= i\overline{\psi}P_R\gamma^\mu\partial_\mu P_R\psi \\ &= i\overline{\psi}\gamma^\mu P_L\partial_\mu P_R\psi \\ &= i\overline{\psi}\gamma^\mu\partial_\mu P_L P_R\psi \quad \text{Since } P_L \text{ is a constant operator} \\ &= 0 \end{aligned} \tag{A.8}$$

Following a similar calculation we get: $i\overline{\psi_R}\gamma^\mu\partial_\mu\psi_L = 0$. Our next step is to find the two coupled Dirac equations using the Euler-Lagrange equation. We obtained for the Lagrangian:

$$L = i\overline{\psi_R}\gamma^\mu\partial_\mu\psi_R + i\overline{\psi_L}\gamma^\mu\psi_L - m\overline{\psi_R}\psi_L - m\psi_L\psi_R \tag{A.9}$$

Replacing in the Euler-Lagrange equation, we get for both states:

$$\begin{aligned} \frac{\partial L}{\partial(\partial\overline{\psi_R})} &= \frac{\partial L}{\partial\overline{\psi_R}} \quad \rightarrow \quad 0 = i\gamma^\mu\partial_\mu\psi_L - m\psi_R \\ \frac{\partial L}{\partial(\partial\overline{\psi_L})} &= \frac{\partial L}{\partial\overline{\psi_L}} \quad \rightarrow \quad 0 = i\gamma^\mu\partial_\mu\psi_R - m\psi_L \end{aligned} \tag{A.10}$$

Now, we are going to find an expression for ψ_R in terms of ψ_L. First, we take the hermitian conjugate of the bottom equation in A.10:

$$\begin{aligned} i\gamma^\mu\partial_\mu\psi_R &= m\psi_L \\ (i\gamma^\mu\partial_\mu\psi_R)^\dagger &= m\psi_L^\dagger && \text{Taking the hermitian conjugate} \\ -i\partial_\mu\psi_R^\dagger\gamma^{\mu\dagger} &= m\psi_L^\dagger \\ -i\partial_\mu\psi_R^\dagger\gamma^{\mu\dagger}\gamma^0 &= m\psi_L^\dagger\gamma^0 && \text{Multiplying on the right by } \gamma^0 \\ -i\partial_\mu\psi_R^\dagger\gamma^0\gamma^\mu &= m\psi_L^\dagger\gamma^0 && \text{Using } \gamma^{\mu\dagger}\gamma^0 = \gamma^0\gamma^\mu \\ -i\partial_\mu\overline{\psi_R}\gamma^\mu &= m\overline{\psi_L} && \text{We have } \overline{\psi} = \psi^\dagger\gamma^0 \\ -i(\partial_\mu\overline{\psi_R}\gamma^\mu)^\intercal &= m\overline{\psi_L}^\intercal && \text{Taking the transpose} \\ -i\gamma^{\mu\intercal}\partial_\mu\overline{\psi_R}^\intercal &= m\overline{\psi_L}^\intercal \\ -i(-C^{-1}\gamma^\mu C)\partial_\mu\overline{\psi_R}^\intercal &= m\overline{\psi_L}^\intercal && \text{Using } \gamma^{\mu\intercal} = -C^{-1}\gamma^\mu C \\ i\gamma^\mu\partial_\mu C\overline{\psi_R}^\intercal &= mC\overline{\psi_L}^\intercal && \text{Multiplying on the left by C} \end{aligned} \tag{A.11}$$

As we saw previously, for the lastest equation to have a similar structure as the top equation of A.10, the right-handed component of ψ must be:

$$\psi_R = C\overline{\psi_L}^\intercal \tag{A.12}$$

Now, we need to prove that $C\overline{\psi_L}^{\intercal}$ is actually right-handed. To do this we apply the left-handed chiral projection operator P_L on this state and the result must be zero.

$$\begin{aligned} P_L\left(C\overline{\psi_L}^{\intercal}\right) &= CP_L^{\intercal}\overline{\psi_L}^{\intercal} \qquad \text{Property of C: } P_LC = CP_L^{\intercal} \\ &= C\left(\overline{\psi_L}P_L\right)^{\intercal} \end{aligned} \tag{A.13}$$

Now, let us examine the term $\overline{\psi_L}P_L$:

$$\begin{aligned} \overline{\psi_L}P_L = (P_L\psi)^{\dagger}\gamma_0 P_L &= \psi^{\dagger}P_L\gamma_0 P_L \\ &= \psi^{\dagger}\gamma^0 P_R P_L = 0 \end{aligned} \tag{A.14}$$

Hence $C\overline{\psi_L}^{\intercal}$ is in fact a right-handed quiral state.

Appendix B

Charge Conjugation Operator