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Propositional Logic CNF Examples

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Procedure for converting to CNF

- **To eliminate** $\leftrightarrow$
  - $(a \leftrightarrow b) \equiv (a \rightarrow b) \wedge (b \rightarrow a)$
- **To eliminate** $\rightarrow$
  - $(a \rightarrow b) \equiv \neg a \vee b$
- **Double negation** $\neg$
  - $\neg(\neg a) \equiv a$
- **De Morgan**
  - $\neg(a \wedge b) \equiv (\neg a \vee \neg b)$
  - $\neg(a \vee b) \equiv (\neg a \wedge \neg b)$
- **Distributivity of** $\wedge$ **over** $\vee$
  - $(a \wedge (b \vee c)) \equiv ((a \wedge b) \vee (a \wedge c))$
- **Distributivity of** $\vee$ **over** $\wedge$
  - $(a \vee (b \wedge c)) \equiv ((a \vee b) \wedge (a \vee c))$

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Example #1

Reduce to Conjunctive Normal Form (CNF) the formula

$$\neg(\neg p \lor q) \lor (r \to \neg s)$$

1. $\neg(\neg p \lor q) \lor (\neg r \lor \neg s)$
2. $(\neg\neg p \land \neg q) \lor (\neg r \lor \neg s)$
3. $(p \land \neg q) \lor (\neg r \lor \neg s)$
4. $(p \lor \neg r \lor \neg s) \land (\neg q \lor \neg r \lor \neg s)$

Example #2

Reduce to Conjunctive Normal Form (CNF) the formula

$$(\neg p \to q) \to (q \to \neg r)$$

1. $\neg(\neg p \to q) \lor (q \to \neg r)$
2. $\neg(p \lor q) \lor (\neg q \lor \neg r)$
3. $(\neg p \land \neg q) \lor (\neg q \lor \neg r)$
4. $(\neg p \lor \neg q \lor \neg r) \land (\neg q \lor \neg r)$

Example #3

Reduce to Conjunctive Normal Form (CNF) the formula

$$\neg((\neg p \to \neg q) \wedge \neg r)$$

$$\begin{aligned}
\neg((\neg p \to \neg q) \wedge \neg r) &\equiv \neg((\neg\neg p \vee \neg q) \wedge \neg r) \\
&\equiv \neg((p \vee \neg q) \wedge \neg r) \\
&\equiv \neg(p \vee \neg q) \vee \neg\neg r \\
&\equiv \neg(p \vee \neg q) \vee r \\
&\equiv (\neg p \wedge \neg\neg q) \vee r \\
&\equiv (\neg p \wedge q) \vee r \\
&\equiv (\neg p \vee r) \wedge (q \vee r)
\end{aligned}$$

Example #4

Reduce to Conjunctive Normal Form (CNF) the formula

$$(p \to q) \to (\neg r \wedge q)$$

$$\begin{aligned}
(p \to q) \to (\neg r \wedge q) &\equiv \neg(p \to q) \vee (\neg r \wedge q) \\
&\equiv \neg(\neg p \vee q) \vee (\neg r \wedge q) \\
&\equiv (\neg\neg p \wedge \neg q) \vee (\neg r \wedge q) \\
&\equiv (p \wedge \neg q) \vee (\neg r \wedge q) \\
&\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee (\neg r \wedge q)) \\
&\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee \neg r) \wedge (\neg q \vee q) \\
&\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee \neg r) \wedge \mathbf{T} \\
&\equiv (p \vee (\neg r \wedge q)) \wedge (\neg q \vee \neg r) \\
&\equiv (p \vee \neg r) \wedge (p \vee q) \wedge (\neg q \vee \neg r)
\end{aligned}$$

Example #5

Reduce to Conjunctive Normal Form (CNF) the formula

$$(A \rightarrow B \vee C) \rightarrow (A \wedge D)$$

$$\begin{aligned}
&\equiv \neg(A \rightarrow B \vee C) \vee (A \wedge D)\\
&\equiv (A \wedge \neg(B \vee C)) \vee (A \wedge D)\\
&\equiv (A \wedge \neg B \wedge \neg C) \vee (A \wedge D)\\
&\equiv ((A \wedge \neg B \wedge \neg C) \vee A) \wedge ((A \wedge \neg B \wedge \neg C) \vee D)\\
&\equiv A \wedge ((A \wedge \neg B \wedge \neg C) \vee D)\\
&\equiv A \wedge (A \vee D) \wedge (\neg B \vee D) \wedge (\neg C \vee D)\\
&\equiv A \wedge (\neg B \vee D) \wedge (\neg C \vee D)
\end{aligned}$$

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Example #6

Reduce to Conjunctive Normal Form (CNF) the formula

$$((p \wedge q) \vee (r \wedge s)) \vee (\neg q \wedge (p \vee t))$$

$$\begin{aligned}
&\equiv (((p \wedge q) \vee r) \wedge ((p \wedge q) \vee s)) \vee (\neg q \wedge (p \vee t))\\
&\equiv ((p \vee r) \wedge (q \vee r) \wedge (p \vee s) \wedge (q \vee s)) \vee (\neg q \wedge (p \vee t))\\
&\equiv \quad ((p \vee r) \vee (\neg q \wedge (p \vee t))\wedge\\
&\qquad ((q \vee r) \vee (\neg q \wedge (p \vee t))\wedge\\
&\qquad ((p \vee s) \vee (\neg q \wedge (p \vee t))\wedge\\
&\qquad ((q \vee s) \vee (\neg q \wedge (p \vee t))\\
&\equiv \quad (p \vee r \vee \neg q) \wedge (p \vee r \vee p \vee t)\wedge\\
&\qquad (q \vee r \vee \neg q) \wedge (q \vee r \vee p \vee t)\wedge\\
&\qquad (p \vee s) \vee \neg q) \wedge (p \vee s \vee p \vee t)\wedge\\
&\qquad (q \vee s \vee \neg q) \wedge (q \vee s \vee p \vee t)
\end{aligned}$$

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