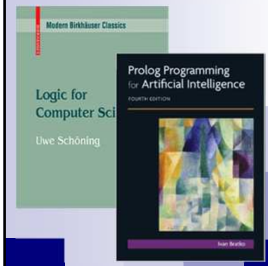

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First Order Logic (Examples #2)

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Skolemization. Example #1

$$\underline{\exists u}\,\forall v\,\exists w\,\exists x\,\forall y\,\exists z\,((P(h(\underline{u},v)) \lor Q(w)) \land R(x,h(y,z)))$$

Eliminate the $\exists u$ using the Skolem constant c:

$$\forall v\,\underline{\exists w}\,\exists x\,\forall y\,\exists z\,((P(h(c,v)) \lor Q(\underline{w})) \land R(x,h(y,z)))$$

Eliminate the $\exists w$ using the 1-place Skolem function f:

$$\forall v\,\underline{\exists x}\,\forall y\,\exists z\,((P(h(c,v)) \lor Q(f(v))) \land R(\underline{x},h(y,z)))$$

Eliminate the $\exists x$ using the 1-place Skolem function g:

$$\forall v\,\forall y\,\underline{\exists z}\,((P(h(c,v)) \lor Q(f(v))) \land R(g(v),h(y,\underline{z})))$$

Eliminate the $\exists z$ using the 2-place Skolem function j (note that function h is already used!):

$$\forall v\,\forall y\,((P(h(c,v)) \lor Q(f(v))) \land R(g(v),h(y,j(v,y))))$$

Finally drop the universal quantifiers, getting a set of clauses:

Skolemization. Example #2

- Every philosopher writes at least one book.
  $\forall x[Philo(x) \rightarrow \exists y[Book(y) \wedge Write(x, y)]]$
- Eliminate Implication:
  $\forall x[\neg Philo(x) \vee \exists y[Book(y) \wedge Write(x, y)]]$
- Skolemize: substitute y by $g(x)$
  $\forall x[\neg Philo(x) \vee [Book(g(x)) \wedge Write(x, g(x))]]$

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Skolemization. Example #3

- All students of a philosopher read one of their teacher's books.
  $\forall x \forall y[Philo(x) \wedge StudentOf(y, x) \rightarrow$
  $\exists z[Book(z) \wedge Write(x, z) \wedge Read(y, z)]]$
- Eliminate Implication:
  $\forall x \forall y[\neg Philo(x) \vee \neg StudentOf(y, x) \vee \exists z[Book(z) \wedge$
  $Write(x, z) \wedge Read(y, z)]]$
- Skolemize: substitute z by $h(x, y)$
  $\forall x \forall y[\neg Philo(x) \vee \neg StudentOf(y, x) \vee [Book(h(x, y)) \wedge$
  $Write(x, h(x, y)) \wedge Read(y, h(x, y))]]$

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Skholemization. Example #4

- There exists a philosopher with students.
  $\exists x \exists y [Philo(x) \wedge StudentOf(y, x)]$
- Skolemize: substitute x by a and y by b
  $Philo(a) \wedge StudentOf(b, a)$

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Substitutions. Example #1

P(x,f(y),B)	
P(z,f(w),B)	s={x/z,y/w}
P(x,f(A),B)	s={y/A}
P(g(z),f(A),B)	s={x/g(z),y/A}
P(C,f(A),A)	no substitution !

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Must General Unifier (MGU). Example #2

$$W = \{P(a, x, f(g(y))), P(z, f(z), f(u))\}$$

Sol. **MGU = {[u/g(y)], [x/f(a)], [z/a]}**

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Unification. Examples

{P(x,f(y),B),P(x,f(B),B)}

s={y/B,x/A} not the simplest unifier

s={y/B} most general unifier (mgu)

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Resolution principle. Example #1

Prove the **validity** of:

$F = \exists x[(P(x) \rightarrow P(a)) \wedge (P(x) \rightarrow P(b))]$

Resolution principle. Example #2

Show that $F_1 \wedge F_2 \models G$, where

- $F_1 \triangleq (\forall x)(C(x) \rightarrow (W(x) \wedge R(x)))$
- $F_2 \triangleq (\exists x)(C(x) \wedge O(x))$
- $G \triangleq (\exists x)(O(x) \wedge R(x))$

Resolution principle. Example #3

Prove that **F1** ∧ **F2** ∧ **F3** ∧ **F4** ⊨ **F5**, where

F1 = ∀x(allergies(x) → sneeze(x))

F2 = ∀y∀x(cat(y) ∧ allergicToCats(x) → allergies(x))

F3 = cat(felix)

F4 = allergicToCats(mary)

F5 = sneeze(mary)

Resolution principle. Example #3 (cont…)

¬allergies(w) v sneeze(w)   ¬cat(y) v ¬allergicToCats(z) v allergies(z)

w/z

¬cat(y) v sneeze(z) v ¬allergicToCats(z)   cat(felix)

y/felix

sneeze(z) v ¬allergicToCats(z)   allergicToCats(mary)

z/mary

sneeze(mary)   ¬sneeze(mary)

$\square$

negated query

Resolution principle. Exercises (HW)

Prove by resolution that:

(a) $\forall x(P(x) \to Q(x)) \models \forall y(\neg Q(y) \to \neg P(y))$

(b) $\forall x(P(x) \to Q(x)) \models \exists x P(x) \to \exists x Q(x)$

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Resolution principle. Example #3

Prove that $\exists y \forall x R(x,y) \models \forall x \exists y R(x,y)$

- The antecedent is $\exists y \forall x\, R(x, y)$; replacing y by the Skolem constant a yields the clause $\{R(x, a)\}$.
- In $\neg(\forall x\, \exists y\, R(x, y))$, pushing in the negation produces $\exists x\, \forall y\, \neg R(x, y)$. Replacing x by the Skolem constant b yields the clause $\{\neg R(b, y)\}$.

Unifying $R(x, a)$ with $R(b, y)$ detects the contradiction $R(b, a) \wedge \neg R(b, a)$.

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Resolution principle. Exercise (HW)

Doctors and Quacks

Show that $\mathbf{F_1} \wedge \mathbf{F_2} \vDash \mathbf{F_3}$, where

- Some patients like all doctors
- $F_1 \triangleq \exists x(P(x) \wedge \forall y(D(y) \to L(x,y))$
- No patient likes any quack
- $F_2 \triangleq \forall x(P(x) \to \forall y(Q(y) \to \neg L(x,y)))$
- No doctor is a quack
- $F_3 \triangleq \forall x D(x) \to \neg Q(x)$