Assignment 1

1) Convert the following sentences to Conjunctive Normal Form (CNF)

a) $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$

$\equiv (\neg P \lor Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$

$\equiv (\neg P \lor Q) \rightarrow ((\neg Q \lor R) \rightarrow (P \rightarrow R))$

$\equiv (\neg P \lor Q) \rightarrow ((\neg Q \lor R) \rightarrow (\neg P \lor R))$

$\equiv (\neg P \lor Q) \rightarrow (\neg (\neg Q \lor R) \lor (\neg P \lor R))$

$\equiv (\neg P \lor Q) \rightarrow ((Q \land \neg R) \lor (\neg P \lor R))$

$\equiv \neg (\neg P \lor Q) \lor ((Q \land \neg R) \lor (\neg P \lor R))$

$\equiv (P \land \neg Q) \lor ((Q \land \neg R) \lor (\neg P \lor R))$

$\equiv (P \land \neg Q) \lor ((Q \lor \neg P \lor R) \land (\neg R \lor P \lor R))$

$\equiv (P \land \neg Q) \lor Q \lor \neg P \lor R$

$\equiv (P \lor Q \lor \neg P \lor R) \land (\neg Q \lor Q \lor \neg P \lor R)$

$\equiv (1 \lor Q \lor R) \land (1 \lor \neg P \lor R)$

$\equiv (1) \land (1)$

$\equiv (1)$

b) $(P \rightarrow Q) \leftrightarrow (P \rightarrow R)$

$\equiv (\neg P \lor Q) \leftrightarrow (\neg P \lor R)$

$\equiv ((\neg P \lor Q) \rightarrow (\neg P \lor R)) \land ((\neg P \lor R) \rightarrow (\neg P \lor Q))$

$\equiv (\neg (\neg P \lor Q) \lor (\neg P \lor R)) \land (\neg (\neg P \lor R) \lor (\neg P \lor Q))$

$\equiv ((P \land \neg Q) \lor (\neg P \lor R)) \land ((P \land \neg R) \lor (\neg P \lor Q))$

$\equiv ((P \lor \neg P \lor R) \land (\neg Q \lor \neg P \lor R)) \land ((P \lor \neg P \lor Q) \land (\neg R \lor \neg P \lor Q))$

$\equiv (\neg Q \lor \neg P \lor R) \land (\neg R \lor \neg P \lor Q)$

c) $(P \land Q) \rightarrow (\neg P \leftrightarrow Q)$

$\equiv (P \land Q) \rightarrow ((\neg P \rightarrow Q) \land (Q \rightarrow \neg P))$

$\equiv (P \land Q) \rightarrow ((P \lor Q) \land (\neg Q \lor \neg P))$

$\equiv \neg (P \land Q) \lor ((P \lor Q) \land (\neg Q \lor \neg P))$

$\equiv (\neg P \lor \neg Q) \lor ((P \lor Q) \land (\neg Q \lor \neg P))$

$\equiv ((\neg P \lor \neg Q) \lor (P \lor Q)) \land ((\neg P \lor \neg Q) \lor (\neg Q \lor \neg P))$

$\equiv (\neg P \lor \neg Q \lor P \lor Q) \land (\neg P \lor \neg Q)$

$\equiv (1) \land (\neg P \lor \neg Q)$

2a.1) Translate the reasoning into propositional logic formulas. Use s, j, b, p for atomic propositions that are true if Sydney, Johnson, Benson, or Presley (respectively) have a dog, and write S, J, B, P for atomic propositions that are true if they have a cat.

F1: If Mr. Sydney has a dog, then Mrs. Benson has a cat.

F2: If Mr. Johnson has a dog, then he has a cat, too.

F3: If Mr. Sydney has a dog and Mr. Johnson has a cat, then Mrs. Presley has a dog.

F4: If Mrs. Benson and Mr. Johnson share a pet of the same species, then Mr. Sydney has a cat.

F5: Mr. Sydney and Mr. Johnson have dogs.

F1: $s \rightarrow B$

F2: $j \rightarrow J$

F3: $(s \wedge J) \rightarrow p$

F4: $((b \wedge j) \vee (B \wedge J)) \rightarrow S$

F5: $s \wedge j$

2a.2) Let F = F1 ∧ F2 ∧ F3 ∧ F4 ∧ F5. Check F for satisfiability using the Horn's formula satisfiability test. If you verify that F is satisfiable, then present a model for it. Justify.

$F = F1 \wedge F2 \wedge F3 \wedge F4 \wedge F5$

$\equiv (s \rightarrow B) \wedge (j \rightarrow J) \wedge ((s \wedge J) \rightarrow p) \wedge (((b \wedge j) \vee (B \wedge J)) \rightarrow S) \wedge (s \wedge j)$

$\equiv (s \rightarrow B) \wedge (j \rightarrow J) \wedge ((s \wedge J) \rightarrow p) \wedge (((b \rightarrow \neg j) \rightarrow (B \wedge J)) \rightarrow S) \wedge (1 \rightarrow s) \wedge (1 \rightarrow j)$

$\mathcal{A}(s) = \cancel{0}, 1$

$\mathcal{A}(j) = \cancel{0}, 1$

$\mathcal{A}(b) = 0$

$\mathcal{A}(p) = \cancel{0}, 1$

$\mathcal{A}(S) = \cancel{0}, 1$

$\mathcal{A}(J) = \cancel{0}, 1$

$\mathcal{A}(B) = \cancel{0}, 1$

Formula is satisfiable with the model $\mathcal{A}(b) = \mathcal{A}(P) = 0$ and $\mathcal{A}(s) = \mathcal{A}(j) = \mathcal{A}(p) = \mathcal{A}(S) = \mathcal{A}(J) = \mathcal{A}(B) = 1$.

2b) Check the following formula for satisfiability using the Horn's formula satisfiability test. If you verify that the formula is satisfiable, then present a model for it.

$(\neg A \vee E) \wedge \neg B \wedge (\neg C \vee (A \rightarrow B)) \wedge A \wedge (\neg E \vee C \vee \neg D) \wedge (D \wedge (D \vee F))$

$\equiv (A \rightarrow E) \wedge (B \rightarrow 0) \wedge (C \rightarrow (A \rightarrow B)) \wedge (1 \rightarrow A) \wedge ((E \wedge D) \rightarrow C) \wedge ((D \wedge D) \vee (D \wedge F))$

$\equiv (A \rightarrow E) \wedge (B \rightarrow 0) \wedge (C \rightarrow (A \rightarrow B)) \wedge (1 \rightarrow A) \wedge ((E \wedge D) \rightarrow C) \wedge (\neg (D) \rightarrow (D \wedge F))$

$\equiv (A \rightarrow E) \wedge (B \rightarrow 0) \wedge (C \rightarrow (A \rightarrow B)) \wedge (1 \rightarrow A) \wedge ((E \wedge D) \rightarrow C) \wedge (D \rightarrow 0) \rightarrow (D \wedge F))$

$\mathcal{A}(A) = \cancel{0}, 1$

$\mathcal{A}(B) = 0$

$\mathcal{A}(C) = 0$

$\mathcal{A}(D) = 0$

$\mathcal{A}(E) = \cancel{0}, 1$

$\mathcal{A}(F) = 0$

Formula is satisfiable with the model $\mathcal{A}(B) = \mathcal{A}(C) = \mathcal{A}(D) = \mathcal{A}(F) = 0$ and $\mathcal{A}(A) = \mathcal{A}(E) = 1$.

3a) Prove or disprove the following claim: (P → Q) ∧ ((Q ∧ R) → S) ∧ ¬ (P → ¬R) ⊨ S using the unit resolution strategy.

F = (P → Q) ∧ ((Q ∧ R) → S) ∧ ¬ (P → ¬R) ⊨ S

≡ (¬P ∨ Q) ∧ (¬Q ∨ ¬R ∨ S) ∧ ¬ (¬P ∨ ¬R) ⊨ S

≡ (¬P ∨ Q) ∧ (¬Q ∨ ¬R ∨ S) ∧ (P ∧ R) ⊨ S

≡ (¬P ∨ Q) ∧ (¬Q ∨ ¬R ∨ S) ∧ P ∧ R ⊨ S

Set of Clauses:

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}}

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}, {Q}} Clause 1 and 3

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}, {Q}, {¬R, S}} Clause 2 and 6

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}, {Q}, {¬R, S}, {S}} Clause 4 and 7

F = {{¬P, Q}, {¬Q, ¬R, S}, {P}, {R}, {¬S}, {Q}, {¬R, S}, {S}, □} Clause 5 and 8

Since the output is the empty clause and F is unsatisfiable, the claim is valid.

3b) "Sophia is either a college professor or a university professor. If Sophia is a college professor, then she has M.S (Master of Science) degree. If Sophia is a university professor and she has a M.S degree, then she is smart. Sophia is not smart, so (logical consequence) she is a college professor."

Is the argument logically correct? Justify your answer using a Davis-Putnam resolution strategy. Note: To model this problem you must use the following propositions:

P: Sophia is a college professor.

Q: Sophia is a university professor.

R: Sophia has a M.S. degree.

S: Sophia is smart.

Set of Clauses:

F = {{P, Q}, {¬Q, ¬R, S}, {¬S}, {¬P}}

By P:

F = {~~{P, Q}~~, {¬Q, ¬R, S}, {¬S}, ~~{¬P}~~, {Q}}

F = {{¬Q, ¬R, S}, {¬S}, {Q}}

By Q:

F = {~~{¬Q, ¬R, S}~~, {¬S}, ~~{Q}~~, {¬R, S}}

F = {{¬S}, {¬R, S}}

By R:

F = {{¬S}, ~~{¬R, S}~~}

F = {{¬S}}

By S:

F = {~~{¬S}~~}

F = {}

Since the output is the empty set of clauses and the formula is satisfiable, the argument is logically incorrect.