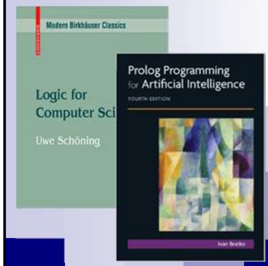

FLORIDA INTERNATIONAL UNIVERSITY

Propositional Logic (Examples)

Dr. Antonio L. Bajuelos

FIU School of Computing & Information Sciences

Example #1

Let's consider the interpretation v where $v(p) = F$, $v(q) = T$, $v(r) = T$. Does v satisfy the following propositional formulas?

1. $(p \rightarrow \neg q) \vee \neg(r \wedge q)$
2. $(\neg p \vee \neg q) \rightarrow (p \vee \neg r)$

- **v satisfy 1 but doesn't satisfy 2**

Example #2

Use the truth tables method to determine whether $(p \to q) \lor (p \to \neg q)$

p	q	$p \to q$	$\neg q$	$p \to \neg q$	$(p \to q) \lor (p \to \neg q)$
T	T	T	F	F	**T**
T	F	F	T	T	**T**
F	T	T	F	T	**T**
F	F	T	T	T	**T**

The formula is valid since it is satisfied by every interpretatio

Example #3

Use the truth tables method to determine whether $p \to (q \land \neg q)$ *and* $\neg p$ *are logically equivalent.*

p	q	$q \land \neg q$	$p \to (q \land \neg q)$	$\neg p$
T	T	F	**F**	**F**
T	F	F	**F**	**F**
F	T	F	**T**	**T**
F	F	F	**T**	**T**

The two formulas are **equivalent** since for every possible interpretation they evaluate to the same truth values.

Example #4

Let A =“Aldo is Italian” *and* B =“Bob is English”.

Formalize the following sentences:

1. “Aldo isn't Italian”
2. “Aldo is Italian while Bob is English”
3. “If Aldo is Italian then Bob is not English”
4. “Aldo is Italian or if Aldo isn't Italian then Bob is English”
5. “Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English”

1. $\neg A$
2. $A \wedge B$
3. $A \rightarrow \neg B$
4. $A \vee (\neg A \rightarrow B)$ *logically equivalent to* $A \vee B$
5. $(A \wedge B) \vee (\neg A \wedge \neg B)$ *logically equivalent to* $A \leftrightarrow B$

Example #5

Socrate says:

“If I'm guilty, I must be punished;
I'm guilty. Thus I must be punished.”

Is the argument logically correct?

Solution. *The argument is logically correct: if* p *means* “I'm guilty” *and* q *means* “I must be punished”, *then:*

$(p \rightarrow q) \wedge p \models q$ *(modus ponens)*

Example #6

"If I'm guilty, I must be punished;
I'm not guilty. Thus I must not be punished."

Is the argument logically correct?

Solution. *The argument is not logically correct:*

$(p \to q) \wedge \neg p \not\models \neg q$

☞ *consider for instance* $v(p) = F$ *and* $v(q) = T$

Example #7

Prove the following *equivalences* using truth tables.

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

P	Q	R	$(Q \wedge R)$	$P \vee (Q \wedge R)$	$(P \vee Q)$	$(P \vee R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

This equivalence is true!

Example #8

Show that $\neg(p \vee (\neg p \wedge q))$

is logically equivalent to $\neg p \wedge \neg q$

Solution:

$$
\begin{array}{llll}
\neg(p \vee (\neg p \wedge q)) & \equiv & \neg p \wedge \neg(\neg p \wedge q) & \text{by the second De Morgan law} \\
& \equiv & \neg p \wedge [\neg(\neg p) \vee \neg q] & \text{by the first De Morgan law} \\
& \equiv & \neg p \wedge (p \vee \neg q) & \text{by the double negation law} \\
& \equiv & (\neg p \wedge p) \vee (\neg p \wedge \neg q) & \text{by the second distributive law} \\
& \equiv & F \vee (\neg p \wedge \neg q) & \text{because } \neg p \wedge p \equiv F \\
& \equiv & (\neg p \wedge \neg q) \vee F & \text{by the commutative law} \\
& & & \text{for disjunction} \\
& \equiv & (\neg p \wedge \neg q) & \text{by the identity law for } \mathbf{F}
\end{array}
$$

Exercises:

For each of the following, use a truth table to establish if it is a correct logical consequence.

(i) $\{p \rightarrow q, \neg q\} \models \neg p$

(ii) $\{q, \neg p \rightarrow \neg q\} \models p$

(iii) $\{a \rightarrow b, b \rightarrow c\} \models a \rightarrow c$

(iv) $\{a \rightarrow \neg b, b \rightarrow \neg c\} \models \neg a \rightarrow \neg c$

Note: As it will be studied later, the "," should be interpreted as "$\wedge$".