Assignment 2

Define an appropriate language and formalize the negation of the following sentences using FOL formulas.

1.1a) Some people like Python.

FOL - $\exists x(People(x) \wedge Like(Python))$

Negation - $\forall x(\neg People(x) \vee \neg Like(Python))$

1.1b) Every box contains at least one coin.

FOL – $\forall x(Box(x) \rightarrow \exists y(Coin(y) \wedge Contains(x,y)))$

Negation - $\exists x(Box(x) \wedge \forall y(\neg Coin(y) \vee \neg Contains(x,y)))$

1.1c) All red objects are to the left of all green objects.

FOL – $\forall x\forall y(Red(x)\ Green(y) \rightarrow Left(x,y))$

Negation - $\exists x\exists y(Red(x) \wedge Green(y) \wedge \neg Left(x,y))$

Define an appropriate language and formalize the following sentences using FOL formulas

1.2a) Every cat loves anyone who gives the cat a good food.

FOL - $\forall x\exists y(Cat(x) \wedge Person(y) \wedge Loves\ (x,y) \rightarrow GivesFood(y,x))$

1.2b) There are at least two rooms.

FOL – $\exists x\exists y(Room(x) \wedge Room(y) \wedge (x \neq y))$

2.1) Give a suitable structure, $\mathcal{A} = (U_{\mathcal{A}}, I_{\mathcal{A}})$, where $U_{\mathcal{A}} = \{1, 2, 3\}$ such that under this structure $\mathcal{A}(F) =$ True. Justify

$F = F1 \wedge F2 \wedge F3$

$F1 = \exists x \exists y \exists z((P(x,y) \wedge P(y,z) \wedge \neg(x = y) \wedge \neg(y = z)) \rightarrow P(x,z))$

$F2 = \forall x \forall y((P(x,y) \wedge P(y,x)) \rightarrow x = y)$

$F3 = \forall x \forall y(P(a,y) \rightarrow P(x,b))$

A suitable structure is x = 1, y = 1 and z = 1 and P(x,y) = T if x = y and a = x and b = y.

Convert into equivalent Skolem normal form. Justify

2.2a) $\neg((\forall x)P(x) \rightarrow (\forall x)(\exists y)(\exists z)Q(x,y,z))$

$\equiv \neg(\neg ((\forall x)P(x)) \vee (\forall x)(\exists y)(\exists z)Q(x,y,z))$

$\equiv \neg(\neg(\forall x) \neg P(x) \vee (\forall x)(\exists y)(\exists z)Q(x,y,z))$

$\equiv (\forall x) P(x) \wedge \neg(\forall x) \neg(\exists y) \neg(\exists z) \neg Q(x,y,z)$

$\equiv (\forall x) P(x) \wedge (\exists x)(\forall y)(\forall z) \neg Q(x,y,z)$ – CNF

$\equiv (\forall x) P(x) \wedge (\exists w)(\forall y)(\forall z) \neg Q(w,y,z)$

$\equiv (\forall x)(\exists w)(\forall y)(\forall z)(P(x) \wedge \neg Q(w,y,z))$ – Prenex

$\equiv (\forall x)(\forall y)(\forall z)(P(x) \wedge \neg Q(f(x),y,z))$ - Skolem

2.2b) $\exists z(\exists x Q(x,z) \vee \exists x P(x)) \rightarrow \neg (\neg \exists x P(x) \wedge \forall x \exists z Q(z,x))$

$\equiv \neg \exists z(\neg(\exists x Q(x,z) \vee \exists x P(x))) \vee \neg (\neg \exists x P(x) \wedge \forall x \exists z Q(z,x))$

$\equiv \forall z(\forall x \neg Q(x,z) \wedge \forall x \neg P(x)) \vee \neg (\forall x \neg P(x) \wedge \forall x \exists z Q(z,x))$

$\equiv \forall z(\forall x \neg Q(x,z) \wedge \forall x \neg P(x)) \vee (\exists x P(x) \vee \exists x \forall z \neg Q(z,x))$

$\equiv \forall z \forall x(\neg Q(x,z) \wedge \neg P(x)) \vee (\exists x P(x) \vee \exists x \forall z \neg Q(z,x))$

$\equiv \forall z \forall x(\neg Q(x,z) \vee \exists x P(x) \vee \exists x \forall z \neg Q(z,x)) \wedge (\neg P(x) \vee \exists x P(x) \vee \exists x \forall z \neg Q(z,x))$ – CNF

$\equiv \forall z \forall x(\neg Q(x,z) \vee \exists y P(y) \vee \exists y \forall w \neg Q(w,y)) \wedge (\neg P(x) \vee \exists y P(y) \vee \exists y \forall w \neg Q(w,y))$

$\equiv \forall z \forall x \exists y \forall w(\neg Q(x,z) \vee P(y) \vee \neg Q(w,y)) \wedge (\neg P(x) \vee P(y) \vee \neg Q(w,y))$ – Prenex

$\equiv \forall z \forall x \forall w(\neg Q(x,z) \vee P(f(z,x)) \vee \neg Q(w,f(z,x))) \wedge (\neg P(x) \vee P(f(z,x)) \vee \neg Q(w,f(z,x)))$ - Skolem

Find the MGU if possible. Justify.

3a) {P(f(x,b),z), P(y,g(z))}

Not possible since cannot change z to g(z) since it is the same variable.

3b) {S(x,y,z), S(u,g(v,v),v)}

{S(u,y,z), S(u,g(v,v),v)}

{S(u,g(v,v),v), S(u,g(v,v),v)}

{S(u,g(v,v),v), S(u,g(v,v),v)}

MGU: [x/u, y/g(v,v), z/v]

3c) {Q(f(x),y,v), Q(z,g(w),h(z,y))}

{Q(f(x),y,v), Q(f(x),g(w),h(f(x),y))}

{Q(f(x),g(w),v), Q(f(x),g(w),h(f(x),g(w)))}

{Q(f(x),g(w),h(f(x),g(w))), Q(f(x),g(w),h(f(x),g(w)))}

MGU: [z/f(x), y/g(w), v/h(f(x),g(w))]

4.1) F = ∃x(¬P(x) ∧ ¬P(f(v)) ∧ ∃zQ(z)) ∨ ∃w(¬P(g(w,x)) ∧ ¬Q(x)) ∨ ∃yP(y)

¬F = ∀x(P(x) ∨ P(f(v)) ∨ ∀z¬Q(z)) ∧ ∀w(P(g(w,x)) ∨ Q(x)) ∧ ∀y¬P(y)

≡ ∀x∀z∀w∀y((P(x) ∨ P(f(v)) ∨ ¬Q(z)) ∧ (P(g(w,x)) ∨ Q(x)) ∧ ¬P(y))

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}} sub [y/g(w,x)] on clause 3

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}} use clause 2 and 4

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}} sub [x/z] 5

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}} use 1 and 6

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}} sub [y/x] on 3

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}, {¬P(x)}} use 7 and 8

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}, {¬P(x)}, {P(f(v))}} sub [y/f(v)] on 3

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}, {¬P(x)}, {P(f(v))}, {¬P(f(v))}} use 9 and 10

S = {{P(x), P(f(v)), ¬Q(z)}, {P(g(w,x), Q(x)}, {¬P(y)}, {¬P(g(w,x))}, {Q(x)}, {Q(z)}, {P(x), P(f(v))}, {¬P(x)}, {P(f(v))}, {¬P(f(v))}, {□}}

F is valid because S is unsatisfiable. S is unsatisfiable since the empty clause was found.

4.2) S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}} sub [y/x] on clause 3

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}} use clause 1 and 4

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}} sub [y/g(w,x)] on 3

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}} use 2 and 6

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}} sub [x/z] on 7

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}, {T(z)}} use 5 and 8

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}, {T(z)}, {R(f(a))}} sub [y/f(a)] on 3

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}, {T(z)}, {R(f(a))}, { ¬R(f(a))}} use 9 and 10

S = {{R(x),R(f(a)), ¬T(z)}, {R(g(w,x)),T(x)},{ ¬R(y)}, { ¬R(x)}, {R(f(a)), ¬T(z)}, { ¬R(g(w,x))}, {T(x)}, {T(z)}, {R(f(a))}, { ¬R(f(a))}, {□}}

S is unsatisfiable since the empty clause was found.