Assignment 3

1a) Prove that F is a valid formula:

F = ∃x∃y¬P(a,x,f(g(y))) ∨ ∃x∃y(P(x,f(x),f(y)) ∧ ¬Q(y,x)) ∨ ∃xQ(g(x),x)

¬F = ∀x∀yP(a,x,f(g(y))) ∧ ∀v∀w(¬P(v,f(v),f(w)) ∨ Q(w,v)) ∧ ∀z¬Q(g(z),z)

≡ ∀x∀y∀v∀w∀z(P(a,x,f(g(y))) ∧ (¬P(v,f(v),f(w)) ∨ Q(w,v)) ∧ ¬Q(g(z),z))

Clauses:

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}} sub [w/g(z)] and [v/z] on clause 2

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}} use clauses 3 and 4

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}, {¬P(z,f(z),f(g(z)))}} sub [z/a] on clause 5

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}, {¬P(z,f(z),f(g(z)))}, {¬P(a,f(a),f(g(a)))}} sub [x/f(a)] and [y/a] on clause 1

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}, {¬P(z,f(z),f(g(z)))}, {¬P(a,f(a),f(g(a)))}, {P(a,f(a),f(g(a)))}} use clauses 6 and 7

{{P(a,x,f(g(y)))}, {¬P(v,f(v),f(w)), Q(w,v)}, {¬Q(g(z),z)}, {¬P(z,f(z),f(g(z))), Q(g(z),z)}, {¬P(z,f(z),f(g(z)))}, {¬P(a,f(a),f(g(a)))}, {P(a,f(a),f(g(a)))}, {□}}

Since empty clause is found, the negation of the formula is unsatisfiable, meaning the formula F is a valid formula.

1b) Check if logical consequence is true:

P1: $\forall x(D(x) \rightarrow H(x)) \equiv \forall x(\neg D(x) \vee H(x))$

P2: $\forall x \forall y((I(x,y) \wedge C(y)) \rightarrow \neg \exists z(I(x,z) \wedge M(z))) \equiv \forall x \forall y \forall z((\neg I(x,y) \vee \neg C(y)) \vee (\neg I(x,z) \vee \neg M(z)))$

P3: $\forall x(L(x) \rightarrow \neg \exists y(I(x,y) \wedge H(y))) \equiv \forall x \forall y(\neg L(x) \vee (\neg I(x,y) \vee \neg H(y)))$

P4: $\exists x(I(c,x) \wedge (C(x) \vee D(x))) \equiv I(c,a) \wedge (C(a) \vee D(a))$

C: $L(c) \rightarrow \neg \exists z(I(c,z) \wedge M(z)) \equiv \neg L(c) \vee \forall z(\neg I(c,z) \vee \neg M(z))$

¬C: $L(c) \wedge (I(c,a) \wedge M(a))$

$F = P1 \wedge P2 \wedge P3 \wedge P4 \vDash C \equiv P1 \wedge P2 \wedge P3 \wedge P4 \wedge \neg C$

Clauses:

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}} substitute [z/a] on clause 2

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}} use clauses 8 and 9

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}} substitute [x/c] on clause 10

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}} use clauses 7 and 11

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}} substitute [y/a] on clause 12

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}} use clauses 7 and 13

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}} use clauses 5 and 14

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}} substitute [x/a] on clause 1

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}} use clauses 15 and 16

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}} substitute [y/a] on clause 3

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}} use clauses 17 and 18

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}, {¬L(x), ¬I(x,a)}} substitute [x/c] on clause 19

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}, {¬L(x), ¬I(x,a)}, {¬L(c), ¬I(c,a)}} use clauses 4 and 20

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}, {¬L(x), ¬I(x,a)}, {¬L(c), ¬I(c,a)}, {¬L(c)}} use clauses 6 and 21

{{¬D(x), H(x)}, {¬I(x,y), ¬C(y), ¬I(x,z), ¬M(z)}, {¬L(x), ¬I(x,y), ¬H(y)}, {I(c,a)} {C(a), D(a)}, {L(c)}, {I(c,a)}, {M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a), ¬M(a)}, {¬I(x,y), ¬C(y), ¬I(x,a)}, {¬I(c,y), ¬C(y), ¬I(c,a)}, {¬I(c,y), ¬C(y)}, {¬I(c,a), ¬C(a)}, {¬C(a)}, {D(a)}, {¬D(a), H(a)}, {H(a)}, {¬L(x), ¬I(x,a), ¬H(a)}, {¬L(x), ¬I(x,a)}, {¬L(c), ¬I(c,a)}, {¬L(c)}, {□}

Logical consequence is true since the formula f is unsatisfiable.

2a)

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starter(green_salad).
starter(melon).
starter(tomato_salad).
starter(rabbit_pate).
main(rock_salmon).
main(roast_beef).
main(pasta).
desert(cheese).
desert(yoghurt).
desert(montreal).
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2b)

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menu(hungry,X,Y,Z) :- starter(X), main(Y), desert(Z).
menu(not_so_hungry,X,Y) :- starter(X), main(Y).
menu(not_so_hungry,Y,Z) :- main(Y), desert(Z).
menu(on_diet,X) :- starter(X).
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2c)

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sits_left_of(X,Y) :- sits_right_of(Y,X).
are_neighbors_of(X,Y,Z) :- sits_left_of(X,Z), sits_right_of(Y,Z).
next_to_each_other(X,Y) :- sits_right_of(X,Y).
next_to_each_other(X,Y) :- sits_left_of(X,Y).
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3a)

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path(X,Y) :- connected(X,Y).

path(X,Y) :- connected(X,Z), path(Z,Y).
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3b)

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move(X,Y) :- by_Car(X,Y).

move(X,Y) :- by_Train(X,Y).

move(X,Y) :- by_Plane(X,Y).

travel(X,Y) :- move(X,Y).

travel(X,Y) :- move(X,Z), travel(Z,Y).
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