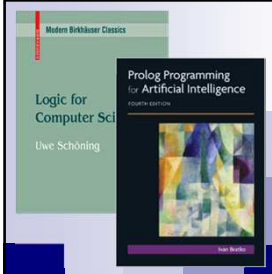

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Resolution Refutation (Example)

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Resolution Refutation Example

Given the following **set of axioms**:

- **A1.** If Bob has two days off, then if the weather is bad, Bob is not fishing.
- **A2.** If Bob is not fishing and is not meeting in a restaurant with his friends, then he is watching TV at home.
- **A3.** If Bob is working, then he is neither meeting in a restaurant with his friends nor watching TV at home.

Prove by **resolution refutation** that:

C. If Bob is not watching TV at home and he has two days off, then he is meeting in a restaurant with his friends provided that the weather is bad.

ATOMIC FORMULAS

p: Bob has two days off

q: weather is bad

r: Bob is fishing

s: Bob is meeting in a restaurant with his friends

t: Bob is watching TV at home

u: Bob is working

FORMALIZATION

A1. $(p \rightarrow (q \rightarrow \neg r)$

A2. $(\neg r \wedge \neg s) \rightarrow t$

A3. $u \rightarrow (\neg s \wedge \neg t)$

C. $(\neg t \wedge p) \rightarrow (q \rightarrow s)$

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Resolution Refutation Example

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C. $(\neg t \wedge p) \rightarrow (q \rightarrow s)$

CONVERSION to CNF and CLAUSE FORM

A1. $(p \rightarrow (q \rightarrow \neg r) \Rightarrow \neg p \vee \neg q \vee \neg r \Rightarrow$ **{¬p, ¬q, ¬r}**

A2. $(\neg r \wedge \neg s) \rightarrow t \Rightarrow \neg(\neg r \wedge \neg s) \vee t \Rightarrow r \vee s \vee t \Rightarrow$ **{r, s, t}**

A3. $u \rightarrow (\neg s \wedge \neg t) \Rightarrow \neg u \vee (\neg s \wedge \neg t) \Rightarrow (\neg u \vee \neg s) \wedge (\neg u \vee \neg t)$
$\Rightarrow$ **{¬u, ¬s}; {¬u, ¬t}**

¬C. $(\neg t \wedge p) \rightarrow (q \rightarrow s) \Rightarrow \neg((\neg t \wedge p) \rightarrow (q \rightarrow s))$
$\Rightarrow (\neg t \wedge p) \wedge \neg(\neg q \vee s) \Rightarrow \neg t \wedge p \wedge q \wedge \neg s \Rightarrow$ **{¬t}; {p}; {q}; {¬s}**

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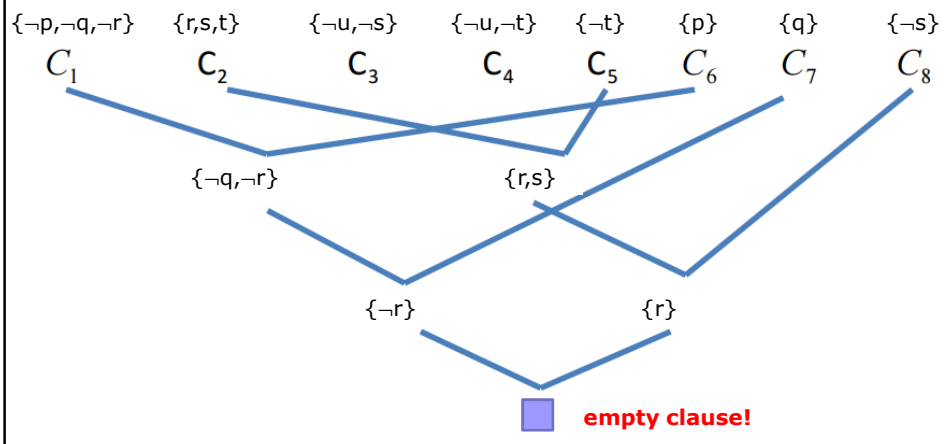
Resolution Refutation Example

F = {{¬p,¬q,¬r},{r,s,t},{¬u,¬s},{¬u,¬t},{¬t},{p},{q},{¬s}) is **unsatisfiable**???



F is **unsatisfiable** ⇒ **Claim is valid**

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