(from https://www.cs.utexas.edu/users/novak/reso.html)

Resolution Example #1:

The Barber Paradox:

> “The barber is a man in town who shaves all those, and only those, men in town who do not shave themselves.”

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Formalization using FOL:

Constants:

barber: b

Predicates:

Shaves(x,y) = S(x,y): x shaves y

Formalization:

$\forall x[S(b,x) \leftrightarrow \neg S(x,x)]$

Solution: ???

Resolution Example #2:

Consider the following axioms:

1. All hounds howl at night.
2. Anyone who has any cats will not have any mice.
3. Light sleepers do not have anything which howls at night.
4. John has either a cat or a hound.
5. (**Conclusion**) If John is a light sleeper, then John does not have any mice.

Formalization using FOL:

Constants:

John

Predicates:

HOUND(x) = H(x): x is a hound

HOWL(x) = O(x): x howls at night

HAVE(x,y) = A(x,y): x has y

CAT(x) = C(x): x is a cat

MOUSE(x) = M(x): x is a mouse

LightSleeper(x) = LS(x): x is a light sleeper

Constants:

John

Predicates:

HOUND(x) = H(x): x is a hound

HOWL(x) = O(x): x howls at night

HAVE(x,y) = A(x,y): x has y

CAT(x) = C(x): x is a cat

MOUSE(x) = M(x): x is a mouse

LightSleeper(x) = LS(x): x is a light sleeper

Formalization:

1. All hounds howl at night.

   $\forall x(H(x) \rightarrow O(x))$

2. Anyone who has any cats will not have any mice.

   $\forall x \forall y((A(x,y) \wedge C(y)) \rightarrow \neg \exists z(A(x,z) \wedge M(z)))$

3. Light sleepers do not have anything which howls at night.

   $\forall x(LS(x) \rightarrow \neg \exists y(A(x,y) \wedge O(y)))$

4. John has either a cat or a hound.

   $\exists x(A(John, x) \wedge (C(x) \vee H(x)))$

5. (**Conclusion**) If John is a light sleeper, then John does not have any mice.

   $LS(John) \rightarrow \neg \exists x(A(John,x) \wedge M(x))$

Skolemization

1. All hounds howl at night.

   $\forall x(H(x) \rightarrow O(x))$

   $\forall x(\neg H(x) \vee O(x))$

2. Anyone who has any cats will not have any mice.

   $\forall x \forall y((A(x,y) \wedge C(y)) \rightarrow \neg \exists z(A(x,z) \wedge M(z)))$

   $\forall x \forall y \forall z(\neg A(x,y) \vee \neg C(y) \vee \neg A(x,z) \vee \neg M(z))$

3. Light sleepers do not have anything which howls at night.

   $\forall x(LS(x) \rightarrow \neg \exists y(A(x,y) \wedge O(y)))$

   $\forall x \forall y(\neg LS(x) \vee \neg A(x,y) \vee \neg O(y))$

4. John has either a cat or a hound.

   $\exists x(A(John, x) \wedge (C(x) \vee H(x)))$

   $A(John,a) \wedge (C(a) \vee H(a))$

5. (**Conclusion**) If John is a light sleeper, then John does not have any mice.

   $LS(John) \rightarrow \neg \exists x(A(John,x) \wedge M(x))$

   Conclusion: $\forall x(\neg LS(John) \vee \neg A(John,x) \vee \neg M(x))$

   $\neg$(Conclusion): $\exists x(\neg LS(John) \wedge A(John,x) \wedge M(x))$

   $LS(John) \wedge A(John,b) \wedge M(b)$

Set of Clauses

1. All hounds howl at night.

   $\forall x(\neg H(x) \vee O(x))$
   C1 ={¬H(x),O(x)}

2. Anyone who has any cats will not have any mice.

   $\forall x \forall y \forall z(\neg A(x,y) \vee \neg C(y) \vee \neg A(x,z) \vee \neg M(z))$

   C2 ={¬A(x,y),¬C(y),¬A(x,z),¬M(z))}

3. Light sleepers do not have anything which howls at night.

   $\forall x \forall y(\neg LS(x) \vee \neg A(x,y) \vee \neg O(y))$

   C3 = {¬LS(x), ¬A(x,y), ¬O(y))}

4. John has either a cat or a hound.

   $A(John,a) \wedge (C(a) \vee H(a))$

   C4 = {A(John,a)}

   C5 = {C(a),H(a)}

5. (**Conclusion**) If John is a light sleeper, then John does not have any mice.
   ¬(Conclusion): $LS(John) \wedge A(John,b) \wedge M(b)$

   C6 = {LS(John)}

   C7 = {A(John,b)}

   C8 = {M(b)}

Resolution:

C1 ={¬H(x),O(x)}
C2 ={¬A(x,y),¬C(y),¬A(x,z),¬M(z))}
C3 = {¬LS(x), ¬A(x,y), ¬O(y))}
C4 = {A(John,a)}
C5 = {C(a),H(a)}
C6 = {LS(John)}
C7 = {A(John,b)}
C8 = {M(b)}

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C9 = C1 & C5, [x/a] = {O(a),C(a)}
C10 = C2 & C8, [z/b] = {¬A(x,y),¬C(y),¬A(x,b)},
[y/b] = {¬A(x,b),¬C(b)}
C11 = C7 & C10, [x/John] = {¬A(John,y),¬C(y)}
C12 = C9 & C11, [y/a] = {¬A(John,a),O(a)}
C13 = C4 & C12 = {O(a)}
C14 = C3 & C13, [y/a] = {¬LS(x), ¬A(x,a)}
C17 = C4 & C14, [x/John] = {¬LS(John)}
C18 = C6 & C17 = Empty Clause
Claim is valid