

Hashing (II)

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Note: The most of the information of these slides was extracted and adapted from Weiss's book, "*Data Structures and Algorithm Analysis in Java*". They are provided for COP3530 students only. Not to be published or publicly distributed without permission by the publisher.



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Module #5: Hashing (part II)

Outline:

- The collision resolution:
 - (II) Open addressing
 - General ideas
 - Advantages vs Disadvantages
 - (a) Linear probing
 - Primary clustering
 - Expected number of probing
 - Complexity analysis and Java code

Remember...

Separate chaining

■ Advantages

- Used when memory is of concern, easily implemented.

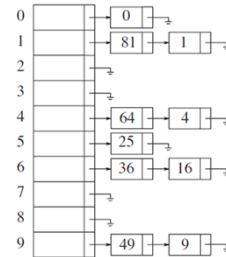
■ Disadvantages

- **Parts of the table/array might never be used.**
- As chains get longer, search time increases to $O(n)$ in the worst case.



Next Question:

- Is there a way to use the “**unused**” space in the table/array instead of using chains to make more space?



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Open Addressing

Main idea: use empty space in the table

Important points:

- All items are stored in the hash table itself.
- In addition to the cell data (if any), each cell keeps one of the three states: **EMPTY, OCCUPIED, DELETED**.
- While inserting, if a collision occurs, alternative cells are tried until an empty cell is found.
- **Deletion (lazy deletion):** When a key is deleted the slot is marked as **DELETED**.
- **Probe sequence:** A probe sequence is the sequence of array indexes that is followed in searching for an empty cell during an insertion, or in searching for a key during find or delete operations.

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Open Addressing



- The most common probe sequences are of the form:

$$h_i(\text{key}) = (h(\text{key}) + c(i)) \bmod \text{TableSize},$$

where $i = 0, 1, \dots, \text{TableSize}-1$ and $c(0) = 0$.

- All items are stored in the hash table itself.
- The function $c(i)$ is used to resolve collisions.
- Similarly, to find item with the key k , we examine the same sequence of locations in the same order.
- For a given hash function $h(\text{key})$, the only difference in the **open addressing collision resolution** techniques is in the definition of the function $c(i)$.

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Open Addressing



- **Advantages of Open Addressing:**

- ☐ All items are stored in the hash table itself. There is no need for another data structure.

- **Disadvantages of Open Addressing:**

- ☐ The keys of the objects to be hashed must be distinct.
- ☐ Dependent on choosing a proper table size.
- ☐ Requires the use of a three-state (**EMPTY**, **OCCUPIED**, **DELETED**) flag in each cell.

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Open Addressing Linear probing

Linear function: $c(i) = i$

- If $h(\text{key}) = \text{key} \% \text{TableSize}$ is already occupied then
 - try $(h(\text{key}) + 1) \% \text{TableSize}$. If occupied...
 - try $(h(\text{key}) + 2) \% \text{TableSize}$. If occupied...
 - try $(h(\text{key}) + 3) \% \text{TableSize}$. If occupied...
- **Example:** insert keys {89, 18, 49, 58, 69}

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49	49	49
1					58	58
2						69
3						
4						
5						
6						
7						
8			18	18	18	18
9		89	89	89	89	89

- Note that this table is relatively empty but blocks of occupied cells start forming. This effect, known as **primary clustering**, means that **keys tend to cluster around table locations that they originally hash to**.

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Open Addressing. Linear probing

If position $h(\text{key}) = \text{key} \bmod \text{TableSize}$ is **occupied** then

Apply the **linear probing**

i^{th} probe was $(h(\text{key}) + i) \% \text{TableSize}$, $i = 1, 2, 3, 4, \dots$

- **Example:** insert {5, 15, 6, 3, 27, 8}

					5				
0	1	2	3	4	5	6	7	8	9
					5	15			
0	1	2	3	4	5	6	7	8	9
					5	15	6		
0	1	2	3	4	5	6	7	8	9
			3		5	15	6		
0	1	2	3	4	5	6	7	8	9
			3		5	15	6	27	
0	1	2	3	4	5	6	7	8	9
			3		5	15	6	27	8
0	1	2	3	4	5	6	7	8	9

Primary clustering

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Open Addressing. Linear probing

- **insert** finds a free table position using a **linear probe function**
- What about **find**?
 - Must use same **probe function** to follow the path for the data
 - **Unsuccessful search** when reach **empty position**
- What about **delete**?
 - Must use "lazy" deletion.
 - Marker indicates "no data here, but don't stop probing"
 - "Real" deletion (clean table) – off-line process (rehash)

□ Example:

find(109) = 1

find(58) = null (T[8], T[9], T[0], T[1], and T[2] ≠ 58, T[3]=null)

delete(38) → T[8] = "no data, don't stop" ← Lazy Deletion!

find(8), T[8] ? 8, no data, move to next

T[9] ? 8, 19 ≠ 8, move to next

T[0] ? 0, 0 = 0, YES!, find(8) = 0

0	8
1	109
2	10
3	/
4	/
5	/
6	/
7	/
8	38
9	19

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Open Addressing. Linear probing

$$(h(\text{key}) + i) \% \text{TableSize}$$

- **Trivial fact:** For any $\lambda < 1$ ($N < \text{TableSize}$), linear probing will find an empty cell. **So, no infinite loop unless table is full.**
- **Non-trivial fact:**
 - For **insertions and searches** the expected number of probes using linear probing is:
 - For **unsuccessful searches:**

$$\frac{1}{2} \left(1 + \frac{1}{(1-\lambda)^2} \right)$$

Example: For $\lambda=1/2$ the # of probes < 2.5
 For $\lambda=1/4$ the # of probes < 1.38
 - For **successful searches:**

$$\frac{1}{2} \left(1 + \frac{1}{(1-\lambda)} \right)$$

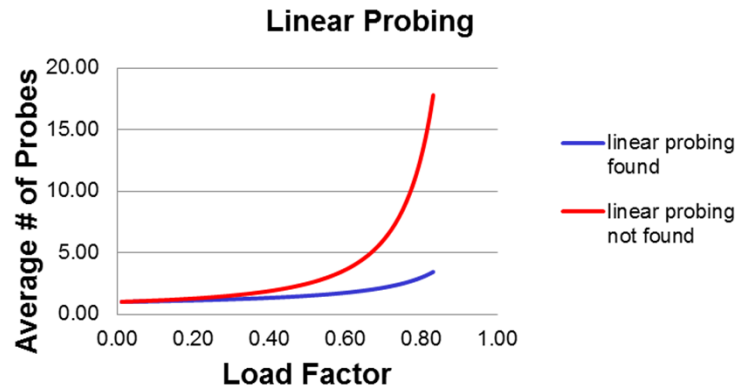
Example: For $\lambda=1/2$ the # of probes < 1.5
 For $\lambda=1/4$ the # of probes < 1.16

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Open Addressing. Linear probing

■ Facts!

- Need to leave sufficient empty space in the table to get good performance
- Linear-probing performance degrades rapidly as table gets full (i.e. when $\lambda \rightarrow 1$ then the number of probes is increased)



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Linear probing. Java Code

- See Java Code for Linear probing in:

<http://algs4.cs.princeton.edu/34hash/LinearProbingHashST.java.html>

Authors: Robert Sedgewick and Kevin Wayne

- Hashing visualization in:

<http://iswsa.acm.org/mphf/openDSAPerfectHashAnimation/perfectHashAV.html>

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