University of California San Diego

ECE 107 Project

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ECE 107: Electromagnetism

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14 March 2023

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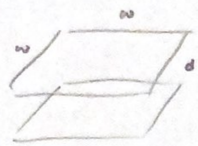


$\frac{V_0}{2}$ $S^{(1)}$ surface

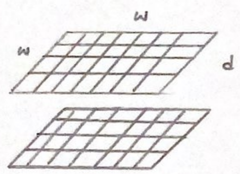
$-\frac{V_0}{2}$ $S^{(2)}$ surface

Formulate an integral equation for unknown surface charge density distribution ρ_s on the plates

$$V(r) = \int_S \frac{1}{4\pi\varepsilon_0 |r - r'|} \rho_s(r')\, dS'$$

$$\Rightarrow \iint_{S^{(1)}+S^{(2)}} \frac{1}{4\pi\varepsilon_0 |r - r'|} \rho_s(r')\, dS' = \begin{cases} \frac{V_0}{2} & r \in S_1 \\ -\frac{V_0}{2} & r \in S_2 \end{cases}$$

Formulate a discretized problem



$\frac{V_0}{2}$ $S^{(1)}$ surface

$-\frac{V_0}{2}$ $S^{(2)}$ surface

Divide surface into $N = N_{S^{(1)}} + N_{S^{(2)}}$ small patches

$N_{S^{(1)}}$, $N_{S^{(2)}}$ are number of patches on $S^{(1)}$, $S^{(2)}$

$$\Rightarrow \sum_{n=1}^{N} \iint_{S_n} \frac{\rho_s(r')\, dS'}{4\pi\varepsilon_0 |r_m - r'|} = \begin{cases} \frac{V_0}{2} & r_m \in S_1 \\ -\frac{V_0}{2} & r_m \in S_2 \end{cases}$$

$$\Rightarrow \sum_{n=1}^{N} Z_{mn} Q_n = V_m \qquad \Rightarrow \underline{\underline{Z}}\,\underline{Q} = \underline{V}$$

$\leftarrow$ matrix equations (linear system of algebric equations)

where Z : $N \times N$ matrix $\qquad Z_{mn} = \frac{1}{\Delta S_n} \iint_{S_n} \frac{ds}{4\pi\varepsilon_0 |r_m - r'|}$

self patch

two close patches

$$\Rightarrow \begin{cases} Z_{mm} \simeq \frac{1}{2\varepsilon_0 \sqrt{\pi \Delta S_n}} & ; \text{ self patch} \\ Z_{mn} \simeq \frac{1}{2\varepsilon_0 \Delta S_n} \left(-d + \sqrt{\frac{\Delta S_n}{\pi} + d^2} \right) & x_m = x_n \ \& \ y_m = y_n \end{cases}$$

two far patches

$$Z_{mn} \simeq \frac{1}{4\pi\varepsilon_0 |r_m - r_n|} \quad ; \text{ otherwise}$$

Q : $N \times 1$ vector $\qquad Q_n = \rho_s(r_n)\, \Delta S_n$

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$\underline{V}$: $N \times 1$ vector $\qquad V_m = \begin{cases} \frac{V_0}{2} & r_m \in S_1 \\ -\frac{V_0}{2} & r_m \in S_2 \end{cases}$

Solution : $\underline{Q} = \underline{\underline{Z}}^{-1} \underline{V}$

total charge on surface 1

$$C = \frac{Q_{S1}}{V_0} \quad \Rightarrow \quad C = \frac{\sum_{n=1}^{N_{S1}} Q_n}{V_0}$$

```
% ECE 107 Project

% handling the geometry
a = 1e-2, b = 1e-2, d = 3e-3
```

```
a = 0.0100
b = 0.0100
d = 0.0030
```

```
nx = 50, ny = 50, eps = 1e-8
```

```
nx = 50
ny = 50
eps = 1.0000e-08
```

```
e0 = 8.85e-12
```

```
e0 = 8.8500e-12
```

```
N = nx*ny*2
```

```
N = 5000
```

```


% r matrix for patch locations storage
r = zeros(N, 3);

delx = a/nx;
dely = b/ny;
dels = delx*dely;

cnt = 0;

% looping to store the patch locations
for k = 1:2
    for i = 1:nx
        for j = 1:ny
            cnt = cnt + 1;

            r(cnt, 1) = delx*(i-1);
            r(cnt, 2) = dely*(j-1);
            r(cnt, 3) = d*(k-1);
        end
    end
end
```

```
% handling the main solver
Z = zeros(N, N);

for i = 1:N
    for j = 1:N
        Rmn = norm(r(i, :) - r(j, :));

        % two far patches
        if Rmn >= (delx - eps)
```

```
            Z(i, j) = 1 / ( 4*pi*e0*Rmn );

        % two close patches
        elseif Rmn < (delx - eps) && Rmn > eps
            Z(i, j) = (1/(2*e0*dels))*(-d + sqrt( (dels/pi) + d^2 ));

        % self patches
        else
            Z(i, j) = 1 / ( 2*e0*sqrt(pi*dels) );
        end
    end
end
```

```
% post processing
% Problem 2

V0 = 2
```

```
V0 = 2
```

```
V = zeros(N, 1);
V(1:N/2, :) = ones(N/2, 1)*(V0/2);
V((N/2+1):N, :) = ones(N/2, 1)*(-V0/2);

Q = inv(Z)*V
```

Q = 5000×1

10^{-14} ×

```
    0.2282
    0.1506
    0.1368
    0.1289
    0.1240
    0.1208
    0.1184
    0.1166
    0.1153
    0.1142
      ⋮
```

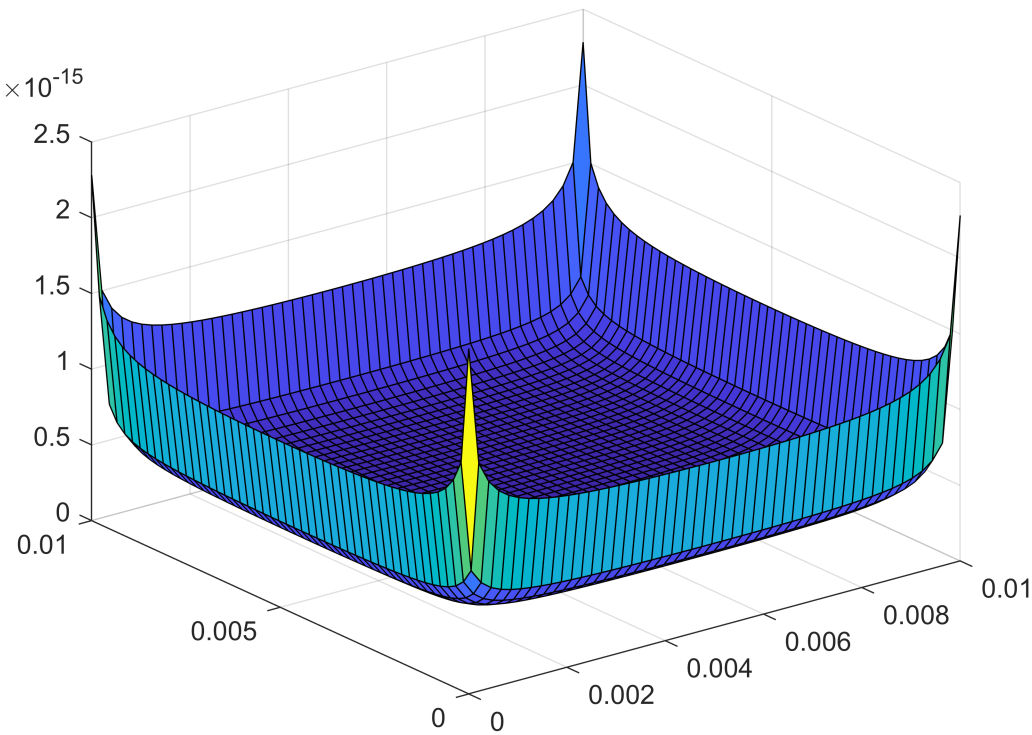
```
Qtotal = sum(Q(1:N/2));
C = Qtotal/V0
```

```
C = 5.2436e-13
```

```
% Problem 3
Qtop = Q(1:N/2, 1);
Qdis = reshape(Qtop, ny, nx);

x_tick = linspace(0, a, nx);
y_tick = linspace(0, b, ny);
[X, Y] = meshgrid(x_tick, y_tick);

surf(X, Y, Qdis)
```




```
% Problem 4
dmin = 1e-4
```

```
dmin = 1.0000e-04
```

```
dmax = 4e-2
```

```
dmax = 0.0400
```

```

d_list = linspace(dmin, dmax, 77)
```

```
d_list = 1×77
    0.0001    0.0006    0.0011    0.0017    0.0022    0.0027    0.0032    0.0038 ···
```

```
C_cal = [];

for i = 1:length(d_list)
    d = d_list(i);

    r = get_loc(N, nx, ny, delx, dely, d);
    Z = get_Z(N, r, delx, eps, e0, dels, d);

    Q = inv(Z)*V;
    Qtotal = sum(Q(1:N/2));
    C = Qtotal/V0;

    C_cal = [C_cal C];
```

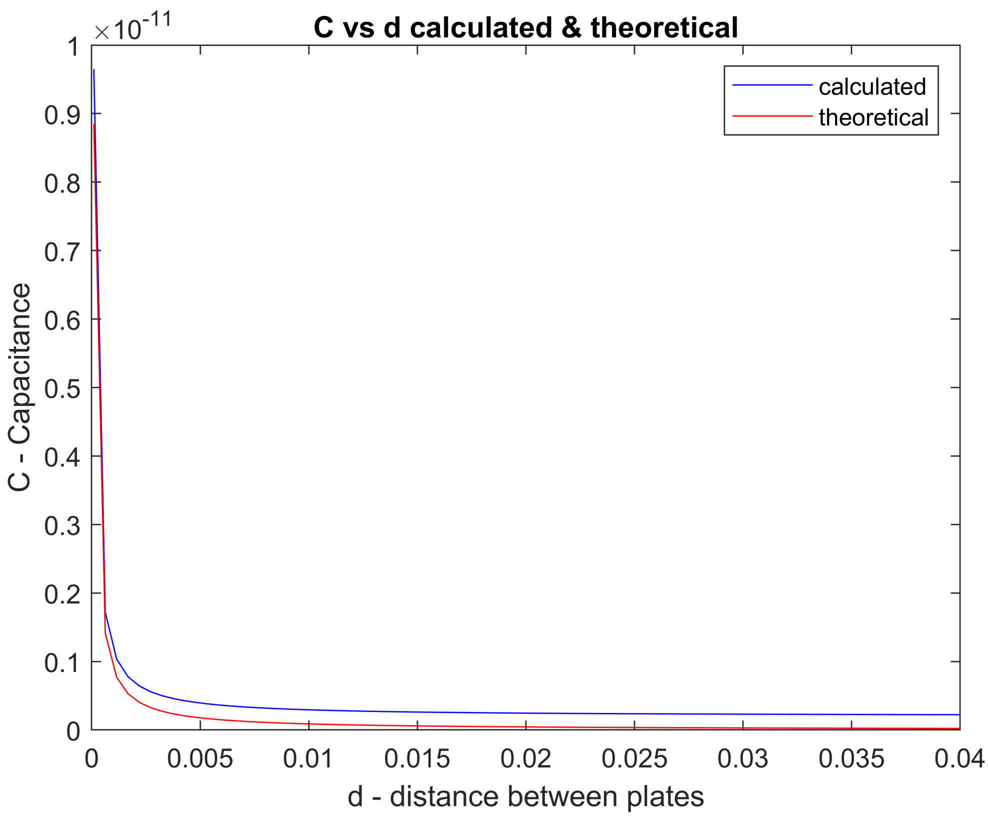
```
end

C_theo = (e0*a*b)./d_list
```

```
C_theo = 1×77
10^-11 ×
    0.8850    0.1416    0.0770    0.0528    0.0402    0.0325    0.0272    0.0234 ···
```

```
figure
plot(d_list, C_cal, 'blue')

hold on
plot(d_list, C_theo, 'red')
title('C vs d calculated & theoretical')
xlabel('d - distance between plates')
ylabel('C - Capacitance')
legend('calculated', 'theoretical')
hold off
```



Comment on when this formula is closer to the results obtained via the numerical solution:

The formula $C = \epsilon_0 w^2/d$ is closer to the results obtained via the numerical solution when the distance d between the two plates of the capacitor are smaller than 1mm.

```matlab
function r = get_loc(N, nx, ny, delx, dely, d)
    r = zeros(N, 3);
    cnt = 0;

    % looping to store the patch locations
    for k = 1:2
        for i = 1:nx
            for j = 1:ny
                cnt = cnt + 1;

                r(cnt, 1) = delx*(i-1);
                r(cnt, 2) = dely*(j-1);
                r(cnt, 3) = d*(k-1);
            end
        end
    end

end


% handling the main solver
function Z = get_Z(N, r, delx, eps, e0, dels, d)
    Z = zeros(N, N);

    for i = 1:N
        for j = 1:N
            Rmn = norm(r(i, :) - r(j, :));

            % two far patches
            if Rmn >= (delx - eps)
                Z(i, j) = 1 / ( 4*pi*e0*Rmn );

            % two close patches
            elseif Rmn < (delx - eps) && Rmn > eps
                Z(i, j) = (1/(2*e0*dels))*(-d + sqrt( (dels/pi) + d^2 ));

            % self patches
            else
                Z(i, j) = 1 / ( 2*e0*sqrt(pi*dels) );
            end
        end
    end

end
```