MGSC 662: Island Warriors Optimization

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1 Introduction

The video game industry encompasses diverse genres, from action and adventure to role-playing games (RPGs) and turn-based strategy games. Game development and evolution centers on critical design decisions regarding gameplay mechanics, difficulty scaling, and player engagement. Development teams regularly confront questions about the nature and scope of changes needed to enhance the gaming experience.

This report examines *Island Warrior*, a prototype RPG where players traverse a series of islands, accumulating weapons and resources to strengthen their characters before facing a final boss. The development team aims to create a game that resonates with players by aligning gameplay design with modern lifestyles, ensuring an engaging experience that accommodates varying gaming habits.

Game completion time serves as a critical metric for designing targeted marketing strategies. Players with time constraints may gravitate toward shorter, more efficient completion paths, while others might prefer extensive exploration and progression. Research suggests an inverse correlation between completion time and players' willingness to play—a metric encompassing player engagement and satisfaction.

The project employs advanced optimization techniques to develop a model that minimizes completion time while maintaining high player engagement. The methodology incorporates non-convex mixed-integer programming (MIP) and scenario analysis to optimize in-game routing and gameplay mechanics. Drawing from the Traveling Salesman Problem (TSP), the development team explores optimization strategies for the prototype RPG, combining game design expertise with data analysis to refine the player experience.

2 Problem Description and Formulation

2.1 Problem Context and Business Relevance

The gaming industry faces a critical challenge in optimizing player engagement while maintaining game completion efficiency. Our project addresses this challenge through the lens of *Island Warriors*, a prototype role-playing game that serves as a test bed for optimization techniques in game design. The game represents a complex business problem where developers must balance multiple competing objectives: ensuring games are completable within reasonable time frames, maintaining player engagement throughout the experience, and creating meaningful progression systems that encourage continued play.

The project is constructed with a nonlinear and non-convex MIP with the minimization objective function that denotes the accumulated amount of time a gamer spends through a series of islands. The objective function is subject to linear and nonlinear constraints to regulate the in-game behaviors and ensure harmonious game settings.

The decision variables denote decisions a gamer has to make while they are traveling through the islands involving both binary and continuous numbers, i.e., whether to visit an island, whether to pick up an item there, or how much money they would spend to increase their strength, etc.

2.2 Dataset Overview

Since this project is based on a fictitious game, the dataset was created from scratch. The game includes the following key components: Islands, Weapons, Characters, Money, Ticket Prices, and Difficulty Levels.

2.2.1 Islands

Island names and ticket prices are organized as follows:

- Island names are stored in a list for easy referencing.
- Ticket prices are represented as a nested list, where each element corresponds to the cost of travel between two islands:
  - The first element of the first nested list represents the price for traveling from Island 1 to itself, with the value 0.
  - The second element of the first nested list indicates the price to travel from Island 1 to Island 2, and so on.
- Travel costs increase with distance; moving between adjacent islands is cheaper than traveling between distant ones. This discourages players from skipping to later islands immediately.

Island difficulty:

- Difficulty levels for each island are stored in a list, allowing segmentation based on their challenge levels.
- Higher difficulty levels correspond to islands with higher numeric values.
- This system encourages progression through easier islands first.

2.2.2 Weapons

Weapons in the game are categorized as follows:

1. Weapon categories: A list specifies the type of weapons available in the game.
2. Base weapon power: A nested list contains power values for each weapon.
3. Weapon types: A nested list categorizes weapons by their type.

Both the base weapon power and weapon type lists align with specific islands, enabling seamless association of a weapon's type, power, and location.

2.2.3 Characters and Modifiers

The game features five unique characters with attributes and modifiers that enhance gameplay:

1. Attributes:
   - Starting power: The initial strength of each character.

- Starting money: The resources available to each character at the game's start.

These attributes are stored in lists indexed to match the character list.

2. Modifiers:

   - Define weapon proficiencies for each character.
   - Nested list structure: Each character's proficiencies are stored in a nested list. For example, the first list corresponds to the thief's proficiencies. Example (Thief): 0.3 for two-handed swords; 1 for bows; 2 for daggers; -1 for staff; 1 for mace

These modifiers act as soft constraints, steering players toward weapon choices aligned with a character's strengths while preserving their freedom to make unconventional decisions. This approach enhances strategic depth without compromising player agency.

2.3 Mathematical Framework and Variable Definition

The optimization model employs a sophisticated combination of binary and continuous variables to capture the complexity of gameplay decisions. At its core, the model tracks player movement through a network of islands using binary variables $x_{ij} \in \{0, 1\}$, where x_{ij} represents travel from island i to island j. This movement system builds upon traditional traveling salesman problem frameworks but incorporates additional complexity through resource management and power progression systems.

The power progression system, central to gameplay optimization, operates through multiple interrelated variables. Player power (p_i) at each island represents a continuous value derived from collected items and monetary investments. The relationship between power and game difficulty introduces non-linearity to our model through the inverse relationship $d_i \cdot p_i = D_i$, where d_i represents the adjusted difficulty and D_i represents the base difficulty of island i.

This non-linear relationship captures the essential RPG mechanic where increased player power reduces the time required to overcome challenges.

2.3.1 Objective Function and Core Mechanics

The primary optimization objective minimizes the total adjusted completion time while maintaining meaningful gameplay progression. This objective is mathematically expressed as:

$$\text{Minimize} \quad \sum_{i \in N} x_{ij} \cdot d_i$$

This formulation represents more than simple time minimization—it captures the essence of efficient gameplay while accounting for the necessary power acquisition and progression that makes the game engaging. The objective function works in concert with our willingness-to-play metric, creating a balance between speed and engagement.

The core gameplay mechanics are implemented through a series of interconnected constraints. Network flow constraints ensure logical progression through the game world while allowing for strategic path selection. The constraint set

$$\sum_{j \in N} x_{ij} = \sum_{i \in N} x_{ij}$$

ensures flow conservation at each island, while specialized constraints for the starting and ending islands ($x_{0j} = 1$ and $x_{i14} = 1$) create a proper gameplay arc.

2.3.2 Modeling Assumptions and Design Decisions

Regarding game mechanics, we made the deliberate choice to implement a non-linear power scaling system through the relationship between player power and island difficulties . This inverse relationship reflects common industry practice in RPG design, where increased player power reduces the time required to overcome challenges. While real games often implement even more complex scaling systems involving randomness or dynamic difficulty adjustment, our non-linear approach provides a reasonable approximation of power-to-difficulty scaling while maintaining model solvability.

The model assumes that item power values and character modifiers remain constant and deterministic, simplifying the problem while ensuring computational feasibility.

Resource management forms another critical set of assumptions in our model. We implemented a flexible spending system where players can utilize their monetary resources at any visited island, with instant power gains from spending. This design choice reflects modern gaming trends that favor player agency over rigid resource management systems. Travel costs between islands are assumed to be fixed and known in advance, though real games might implement dynamic travel costs based on player progression or game events. The decision to remove inventory space limitations was both a practical consideration for model simplicity and a reflection of contemporary game design philosophies that prioritize engagement over inventory management constraints.

Our final set of assumptions concerns the game world structure. We assumed that island difficulties remain constant regardless of visit order, and that power gains from items and spending are immediately effective. Each island can be visited at most once (except the starting island), and the final boss must be defeated to complete the game. These assumptions create a structured optimization problem while maintaining sufficient complexity to model realistic game progression systems.

These assumptions, while simplifying some aspects of real game design, create a tractable optimization problem that captures the essential elements of RPG progression systems. The balance between theoretical necessity and practical applicability allows our model to provide meaningful insights for game design decisions while remaining computationally feasible.

3 Numerical implementation and results

3.1 First model simple TSP model

The implementation framework was developed iteratively through a network of 15 distinct islands, each featuring unique characteristics and challenges. We structured this approach to progressively incorporate complexity, evolving from a basic routing optimization to a comprehensive game progression model.

Our initial implementation utilized a classical Traveling Salesman Problem (TSP) framework, incorporating standard constraints for subtour elimination and complete vertex visitation. However, this foundational approach, while mathematically rigorous, proved inadequate for capturing the strategic elements inherent in adventure game design.

This limitation prompted subsequent refinements to better align with actual gameplay mechanics.

3.2 Network System

In the standard Traveling Salesman Problem (TSP), only inter-node travel costs are considered, providing no mechanism to capture conditions or complexities within each node. To address this shortcoming, we developed a network comprising 15 islands, each assigned a difficulty value between 100 and 10,000 units. These islands are connected through a travel cost matrix designed to ensure logical progression across the game world. Although the classical TSP constraints—particularly the requirements to prevent subtours and to visit all nodes—limit player freedom, we retained certain foundational TSP elements. Specifically, the model preserves the condition that each island is visited exactly once, as well as predefined start and end points.

a. General Flow Balance:

$$\sum_{i \neq j} x_{ij} = \sum_{i \neq j} x_{ji} \quad \forall i \in N$$

b. Start and End Points:

$$\sum_{j=1..n-1} x_{0j} = 1$$

$$\sum_{i=0..n-2} x_{in-1} = 1$$

$$\sum_{i \neq 14} x_{i0} = 0$$

$$x_{14,0} = 1$$

c. Visit Correspondence:

$$\sum_{i} x_{ij} \geq y_i \quad \forall i \in N$$

$$\sum_{j} x_{ij} \geq y_i \quad \forall i \in N$$

d. Prevention of Backward Travel:

$$x_{ij} + x_{ji} \leq 1 \quad \forall i, j \in N, i \neq j$$

e. Single Visit Requirements:

$$\sum_{i \neq j} x_{ij} = y_i \quad \forall i \in N$$

$$\sum_{i \neq j} x_{ij} = y_i \quad \forall i \in N$$

e. No Subtour:

$$u_i - u_j + n \cdot x_{ij} \leq n - 1 \quad \forall i, j \in \{1, \ldots, 14\}, i \neq j$$

3.3 Features (characters and items)

To enhance gameplay, we introduced four distinct playable characters. When character selection was modeled as a binary decision variable, the thief emerged as the strongest initial choice, making it particularly suitable for novice players:

$$\sum_k c_k = 1, c_k \in \{0, 1\}$$

To evaluate the relative performance and viability of each character, we subsequently treated the character choice as a supervised factor, examining completion times across all four characters:

$$c_i = 1, i \in \{0, 1, 2, 3, 4\}$$

In addition, our model incorporates collectible items as rewards on each island. Players may select at most one item per island. Item values increase player power, and to preserve character uniqueness, specific modifiers are applied so that certain characters benefit more from particular weapon types. We imposed no constraints on inventory capacity, allowing players complete freedom in item collection. The relevant constraints are as follows:
a. Visit-Dependent Collection:

$$z_{ik} \leq y_i \quad \forall i \in N, \forall k \in K$$

b. Backpack Consistency:

$$b_{ik} = z_{ik} \quad \forall i \in N, \forall k \in K$$

c. Single Item per Island:

$$\sum_{k \in K} z_{ik} \leq 1 \quad \forall i \in N$$

3.4 Money System

To introduce an additional layer of strategy, we incorporated a monetary system. Each character begins with a predetermined amount of currency, which can be used both to cover travel costs between islands and to purchase temporary power enhancements. The travel pricing model is represented by a 15×15 ferry fare matrix, with individual travel costs ranging from 30 to 1,200 currency units. While players are granted substantial freedom in how they manage these funds, they must develop effective budgetary strategies to ensure that travel expenses are always covered.

The corresponding constraints are defined as follows:
a. Money Balance:

$$money = \sum_{k \in K}(c_k \times character\_money_k) - \sum i \in N \sum_{j \neq i}(x_{ij} \times Ferry\_ticket\_priceij) - \sum_{i \in N} spend\_amount_i$$

b. Spending Conditions:

$$s_i \leq y_i \quad \forall i \in N$$

$$spend\_amount_i \leq money \times s_i \quad \forall i \in N$$

$$\sum_{i \in N} spend\_amount_i \leq money$$

3.5 Power System

The power component of the model integrates both the equipment acquired during gameplay and the temporary power enhancements obtained through monetary expenditure.
a. Power Level per Island:

$$p_i = \sum_{k\in K}(z_{ik}\times Weapon\_Powerik)+\sum_{k\in C}(c_k\times Modifier_k)+\alpha\times spend\_amount_i \quad \forall i \in N, \text{ where } w = weapo$$

b. Final Boss Power Requirement:

$$p_{n-1} \geq 8000$$

3.6 Difficulty

We built a realistic difficulty update system using time to describe difficulty, creating an inverse proportion function to adjust completion time based on player's current power level.

$$adjusted\_difficulties_i \times p_i = Island\_difficulty[i]$$

3.7 Willingness to Play

We recognized that minimizing completion time alone does not fully capture the nuances of player engagement and the underlying business implications. To address this, we introduced a variable representing the player's "willingness to play" (WTP). We use an inverse proportion function to reflect the diminishing marginal impact of additional playtime on player willingness. For example, while one extra hour might have a significant effect on a new player, it is less meaningful for a player who has already invested 300 hours.

$$WTP = \frac{k\cdot\sum_{i=1}^{n} y_i + l\cdot\sum_{i=1}^{n} b_i}{\sum_{i=1}^{n} y_i\cdot \text{adjusted\_difficulties}i}$$

3.8 Result Analysis

The optimized solution identified the Thief(Character 0) as the most advantageous character choice. Rather than visiting all 15 islands, the model selected a route incorporating only 10 islands. The model's optimized strategy achieved a completion time of 14.77 units and a willingness-to-play value of 37.23, suggesting a balanced approach that fosters player engagement without unduly prolonging gameplay.

Metric	Value
Optimal Completion Time	14.77
Willingness Value	37.23
Total Islands Visited	10
Selected Character	Character 0
Total Money Spent	336.48(3)→498.89(9)→753.62(13)= 1588.99(in total)

More specifically: The optimization revealed a sophisticated resource allocation strategy throughout the journey. The model strategically distributed resources across three critical points: an investment of 336.48 units at Island 7 for early-game development, 498.89 units at Island 9 for mid-game power enhancement. And 753.62 units at Island 13 culminating in a significant final investment at Island 13 to prepare for the final boss encounter. This cumulative expenditure of 1,588.99 units demonstrates the model's capability to balance resource conservation with essential power acquisition.

Beginning with a substantial initial power acquisition of 9,000 points at the starting island, the strategy progresses through carefully selected power enhancement milestones. Notable acquisitions include a 7,000-point power increase at Island 8, a critical 22,200-point power spike at Island 11, and a final 15,000-point enhancement at Island 14. This progression ensures sufficient capabilities for the final challenge while maintaining optimal resource efficiency by avoiding excessive power accumulation that could result in unnecessary time investment during the journey.

4 Problem Extensions

4.1 Scenario Analysis

To assess the feasibility of adding talents to specific characters, we conducted a series of tests combining characters with various talents. These tests utilized the same model as before, with an added constraint to select a specific character for each scenario. Talent effects were directly incorporated into the dataset, and each combination was analyzed independently to derive meaningful insights.

The scenarios are:

- No Talent
- Talent 1: Starting money increased by $1000
- Talent 2: Item value increased by 20%
- Talent 3: Travel cost decreased by 40%
- Talent 4: Difficulty of islands decreased by 8%

Each scenario yielded the following results:

4.1.1 Key Interpretations

- **Thief:** Talent 1 yields the highest willingness to play, though the differences between talents are relatively minor. Any talent addition improves gameplay experience.
- **Warrior:** Talent 1 significantly enhances willingness to play. While completion times differ across talents, the improvement is consistent, suggesting any talent addition is beneficial.
- **Mage:** Talent 1 is the most effective. However, Talents 2 and 4 result in significantly higher completion times, making them less suitable.
- **Archer:** Talent 1 provides the highest willingness to play, while Talents 2 and 4 show minimal improvements and may require adjustments.
- **Maid:** Talent 4 provides the highest willingness to play, but the fastest completion time occurs with Talent 2, suggesting discrepancies that need further balancing.

Character	Talent	Time	Willingness to Play	Number of Weapons	Travel Route
Thief	No	19.86	34.74	9	0-3-1-7-4-8-6-2-5-9-13-14
	1	12.27	69.30	10	0-3-1-7-4-8-6-2-5-9-13-12-14
	2	14.78	52.78	9	0-3-1-7-4-8-6-2-5-9-13-14
	3	14.29	59.50	10	0-3-1-7-4-8-6-2-5-9-13-12-14
	4	14.20	54.92	9	0-3-1-7-4-8-6-2-5-9-13-14
Warrior	No	42.58	17.85	10	0-3-1-7-4-8-6-2-5-9-10-13-14
	1	25.80	38.38	12	0-3-1-7-4-5-10-11-8-6-2-9-13-12-14
	2	38.56	20.23	9	0-3-1-7-4-8-6-2-5-9-13-14
	3	34.09	26.99	11	0-3-1-7-4-5-10-11-8-6-2-9-13-14
	4	35.66	21.87	9	0-3-1-7-4-8-6-2-5-9-13-14
Mage	No	186.13	4.03	9	0-3-1-7-4-5-10-11-8-6-2-9-13-12-14
	1	57.38	11.33	5	0-3-1-7-4-8-6-2-9-13-14
	2	185.60	5.01	9	0-3-1-7-4-5-10-11-8-6-2-9-13-12-14
	3	86.84	7.49	5	0-3-1-7-4-8-6-2-9-13-14
	4	171.24	5.43	9	0-3-1-7-4-5-10-11-8-6-2-9-13-12-14
Archer	No	134.86	6.67	12	0-3-1-7-4-5-10-11-8-6-2-9-13-12-14
	1	28.24	30.09	10	0-3-1-7-4-8-6-2-5-9-13-12-14
	2	60.20	11.79	8	0-3-1-7-4-8-6-2-9-13-14
	3	46.07	18.45	10	0-3-1-7-4-8-6-2-5-9-13-12-14
	4	65.69	10.81	8	0-3-1-6-2-9-13-14
Maid	No	397.52	1.08	3	0-1-7-4-3-11-8-6-2-9-10-13-12-14
	1	179.47	3.84	2	0-3-11-8-6-2-9-10-13-14
	2	178.44	3.87	2	0-3-11-8-6-2-9-10-13-14
	3	253.68	2.72	3	0-3-11-8-6-2-9-10-13-14
	4	357.50	4.56	3	0-3-11-8-6-2-9-10-13-12-14
height					

Table 1: Summary of Talent Performance by Character

4.1.2 Recommendations for Game Balance

- **Adjustments to Talents:** Talents 2 and 4 should be refined to align more closely with Talents 1 and 3, ensuring balanced gameplay across all characters.
- **Tailored Talent Assignments:** Characters with lower baseline willingness to play should be prioritized for talent enhancements to maintain feasibility and enjoyment.

4.1.3 Identified Issues

1. **Warrior:**
   (a) Insufficient starting money for travel (resolved by increasing starting money to $2,000).
   (b) Uniform modifiers (2x) limit power scaling, which cannot be offset through additional spending.
2. **Maid:**
   (a) Limited weapon synergies and lower overall modifiers.
   (b) Requires near-perfect item selection to remain competitive.
   (c) Route adjustments were required for optimal gameplay in the final scenario.

4.1.4 Purpose

The primary goal of this analysis was to identify the most suitable talent for each character based on the "willingness to play" metric. Talents with significantly lower scores compared to the highest-performing talent indicate poor suitability for the associated character. This ensures players are provided with engaging and enjoyable gameplay experiences, aligning character and talent combinations effectively.

By leveraging these insights, developers can refine game mechanics and optimize talent distribution, enhancing both character feasibility and overall player satisfaction.

5 Recommendations and Conclusions

Through this project, the shortest path for game completion was identified, and strategies for enhancing characters with talents were evaluated through scenario analysis. These findings provided valuable insights into customer segmentation based on allowable gaming time and iterative plans to enhance the gaming experience. Referring to the data published on Statista shown in Figure 1, which illustrates the varying average gaming times across different age groups, we can combine these insights with the minimum game completion time derived from the analysis.

The numerical results highlight critical insights for game design. The optimal progression curve, featuring power scaling from 2,500 to 22,200 units with strategically placed power spikes, supports sustained player engagement and establishes meaningful progression milestones. A content utilization rate of 66.7% balances player interest with manageable complexity, while the completion time of 14.77 units aligns with casual gaming sessions, ensuring accessibility and depth for a diverse audience.

Despite these successes, the model revealed certain limitations. Infeasible solutions during scenario analysis were likely caused by inconsistencies in dataset scaling, particularly for parameters such as ferry ticket prices, starting powers, and weapon attributes. However, such issues had been mitigated through scaling the dataset, in which the weapons' power modifiers for some characters had been properly adjusted.

The non-convexity and non-linearity of the objective function further increased computational complexity, limiting scalability to larger game worlds. Additionally, the static difficulty values in the model may not fully reflect the dynamic scaling used in modern games, and fixed resource constraints may overlook the evolving economies of live service games.

To address these challenges, scaling the dataset through normalization and tightening variable bounds are recommended to improve computational efficiency. For imparting broader insights to game design and development, incorporating development costs into the optimization model could help identify cost-effective strategies. Additionally, introducing multiple datasets, each representing a unique world, could offer players diverse gaming experiences within a single game. However, converging these datasets into a coherent experience would require considerable effort in scaling and boundary adjustments to avoid excessively inflating the feasible area.

This project demonstrates the potential of optimization techniques to inform game design while highlighting areas for refinement to ensure scalability, computational efficiency, and practical applicability.

A Appendix: Supplementary Figures

Characteristic	2019	2020	2021	2022	2023
15 to 19 years	73.8	112.8	86.4	98.4	98.4
20 to 24 years	54.6	84.6	63	87	70.8
25 to 34 years	34.2	43.2	42.6	37.8	37.2
35 to 44 years	17.4	19.8	19.2	19.2	21
45 to 54 years	13.2	20.4	16.8	12.6	19.8
55 to 64 years	12.6	16.8	19.8	15	17.4
65 to 74 years	18.6	24	26.4	29.4	21
75 years and over	25.2	30.6	30	28.8	28.8

Figure 1: Gaming Time Across Different Age Groups (2019–2023).

Reference

Clement, J. (2023, August 30). *U.S. daily time spent playing games/computer use by age 2019.* Statista.
https://www.statista.com/statistics/502149/average-daily-time-playing-games-and-using-computer-us-by-age/