ENM 360: Introduction to Data-driven Modeling

Lecture #20: Sampling methods

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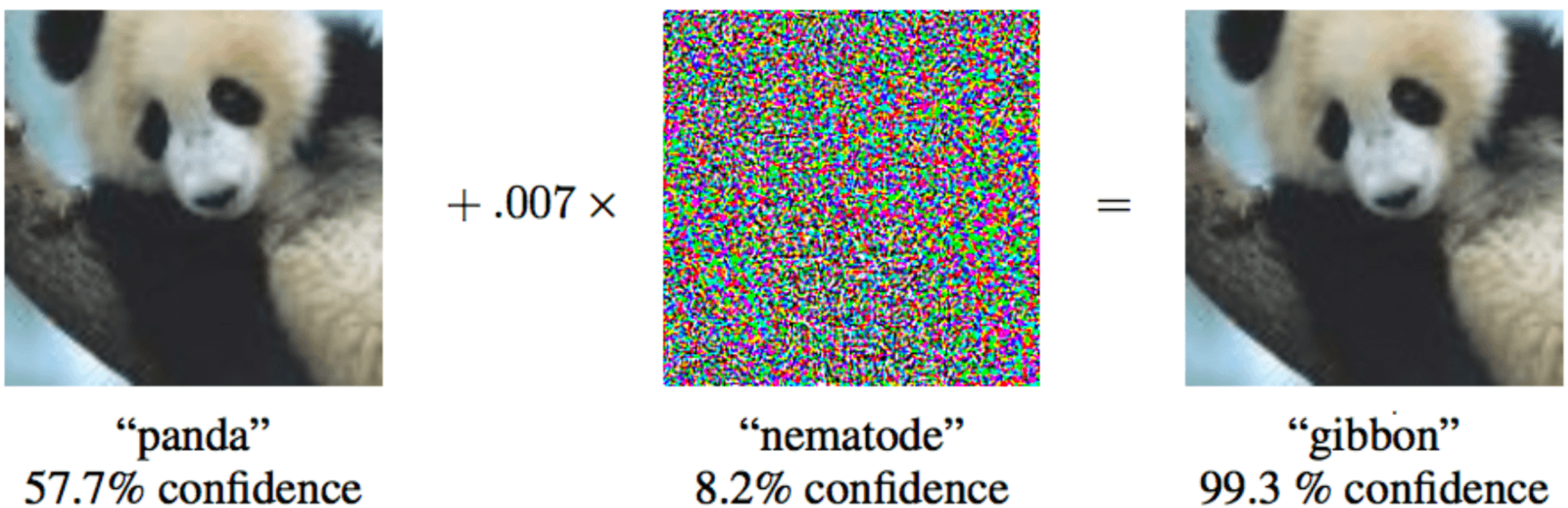
AI bloopers


24:01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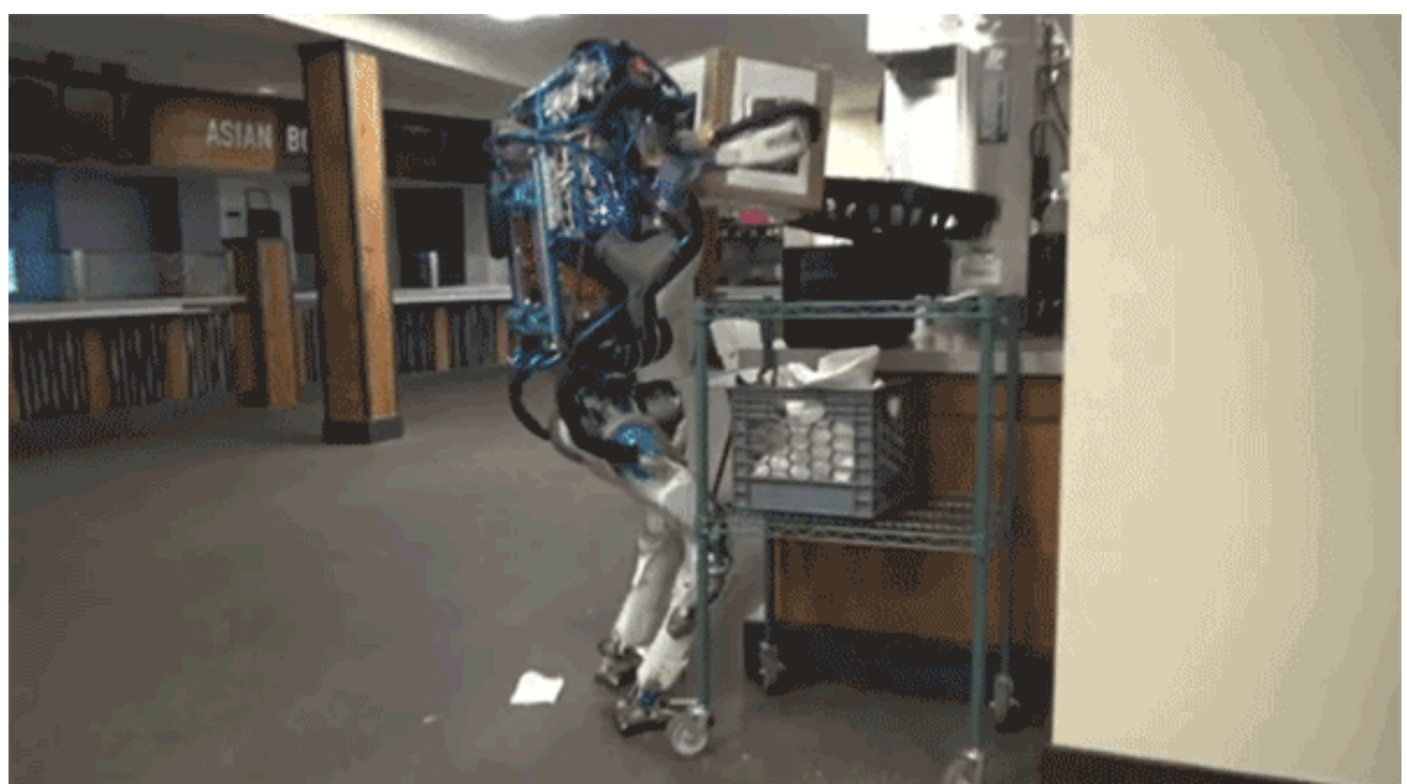
AI bloopers



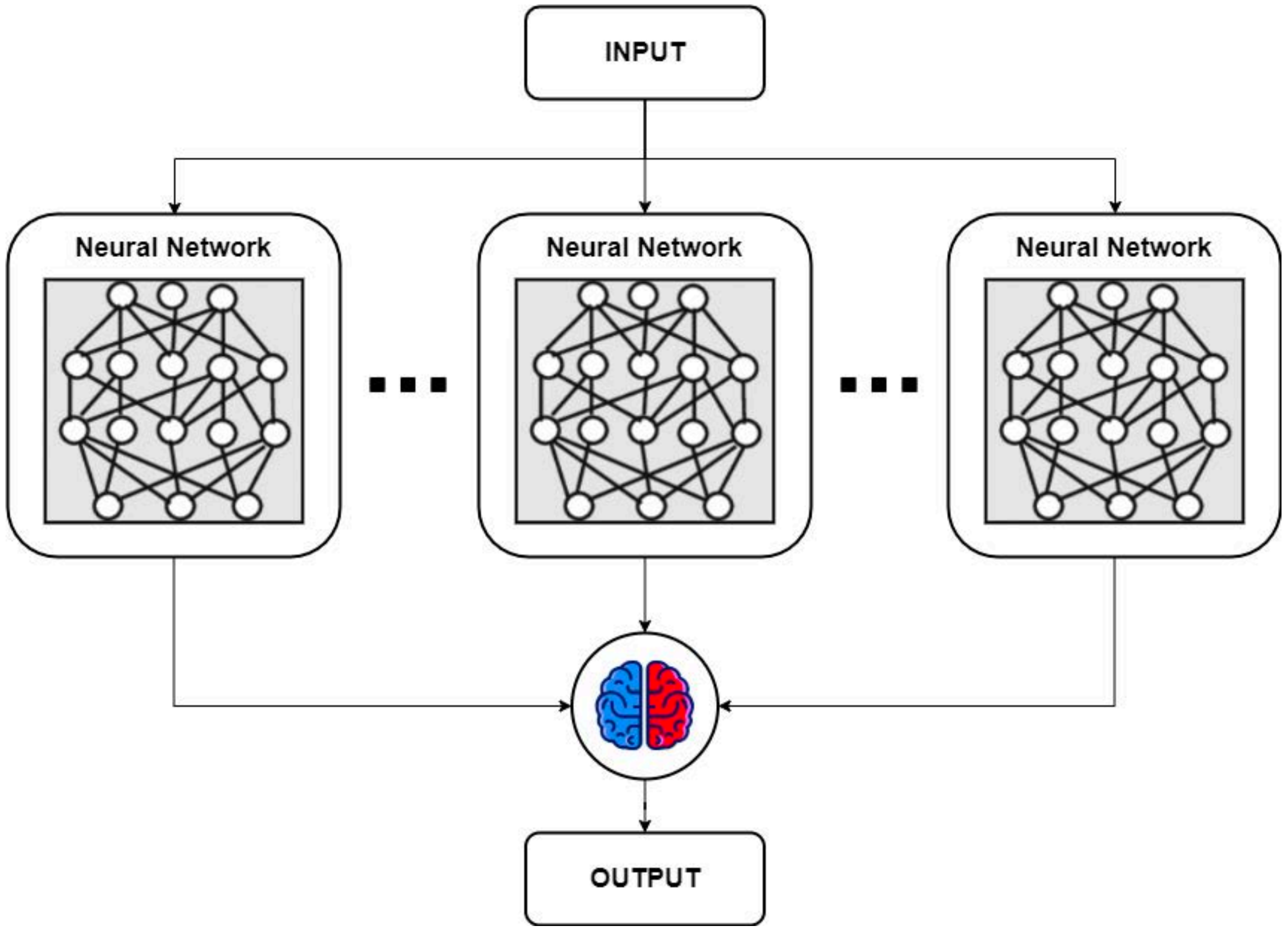
A need for robustness and uncertainty quantification

https://losslandscape.com/

A need for robustness and uncertainty quantification

Becomes particularly important when:

- We are working with small data-sets (over-fitting regime).
- We need to make high-consequence decisions.
- We require performance/accuracy guarantees.
- We work under a limited budget.



$$p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta|)p(\theta)}{p(\mathcal{D})} = \frac{p(\mathcal{D}|\theta|)p(\theta)}{\int p(\mathcal{D}|\theta|)p(\theta)d\theta}$$

The frequentist approach:
Ensemble averaging

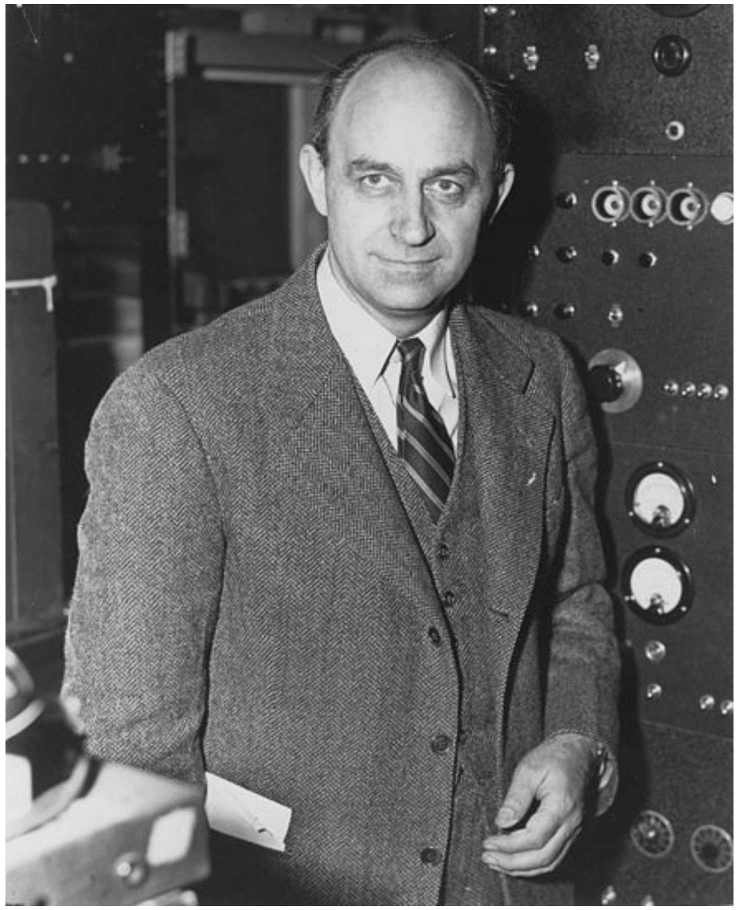
The Bayesian approach:
Probabilistic programming

Monte Carlo approximation

$$\mathbb{E}_{x\sim p(x)}[f(x)] = \int f(x)p(x)dx \approx \frac{1}{n}\sum_{i=1}^{n} f(x_i),$$

$$\text{where } x_i \text{ are drawn iid from } p(x)$$

Monte Carlo approximation



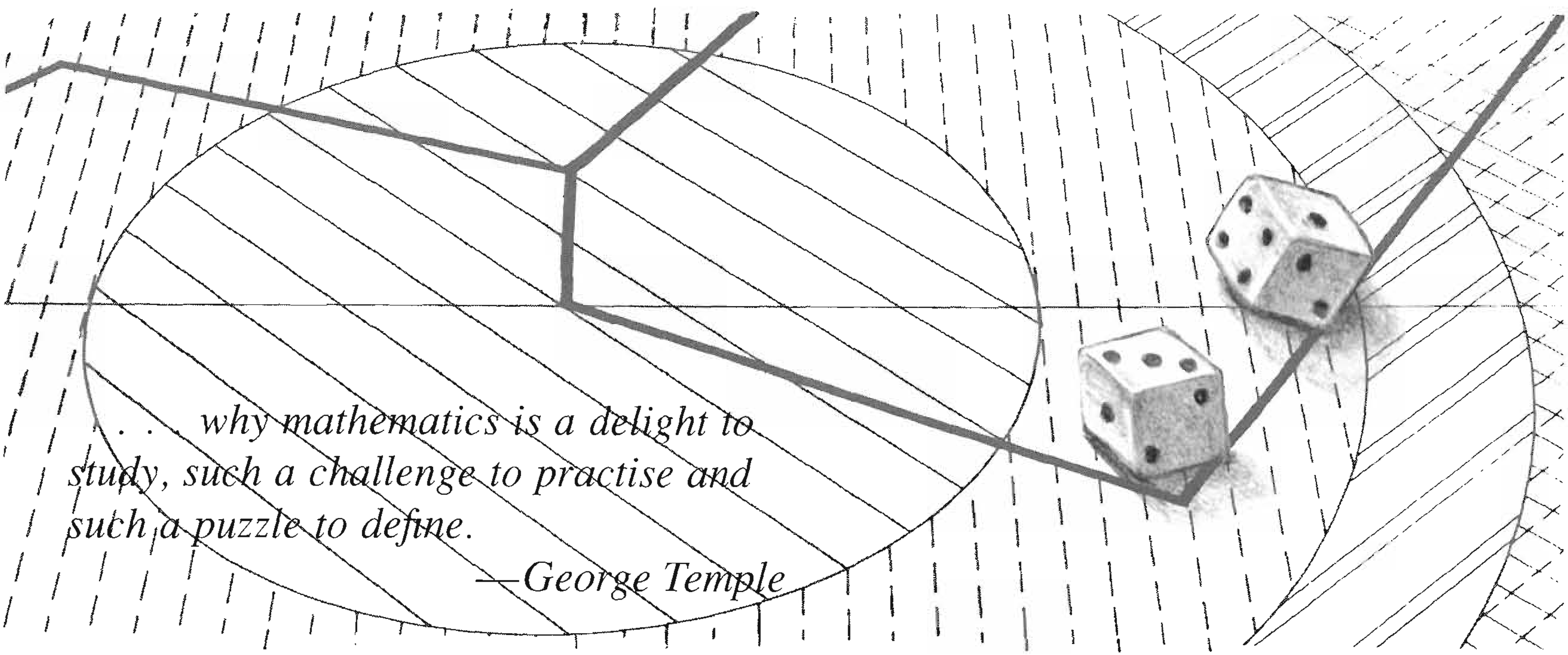
Enrico Fermi

John von Neumann

Stan Ulam

THE BEGINNING *of the* MONTE CARLO METHOD

by N. Metropolis



. . . why mathematics is a delight to study, such a challenge to practise and such a puzzle to define.

—George Temple

Monte Carlo approximation

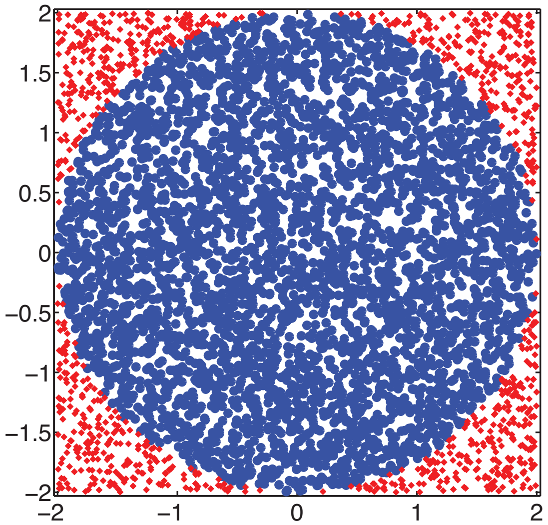
Monte Carlo approximation

Example: estimating π by Monte Carlo integration

MC approximation can be used for many applications, not just statistical ones. Suppose we want to estimate π. We know that the area of a circle with radius r is πr^2, but it is also equal to the following definite integral:

$$I = \int_{-r}^{r} \int_{-r}^{r} \mathbb{I}(x^2 + y^2 \leq r^2) dx dy \tag{2.99}$$

Hence $\pi = I/(r^2)$. Let us approximate this by Monte Carlo integration. Let $f(x, y) = \mathbb{I}(x^2 + y^2 \leq r^2)$ be an indicator function that is 1 for points inside the circle, and 0 outside, and let $p(x)$ and $p(y)$ be uniform distributions on $[-r, r]$, so $p(x) = p(y) = 1/(2r)$. Then



$$\begin{aligned} I &= (2r)(2r) \int \int f(x, y) p(x) p(y) dx dy & (2.100) \\ &= 4r^2 \int \int f(x, y) p(x) p(y) dx dy & (2.101) \\ &\approx 4r^2 \frac{1}{S} \sum_{s=1}^{S} f(x_s, y_s) & (2.102) \end{aligned}$$

We find $\hat{\pi} = 3.1416$ with standard error 0.09 (see Section 2.7.3 for a discussion of standard errors). We can plot the points that are accepted/ rejected as in Figure 2.19.