ENM 360: Introduction to Data-driven Modeling

Lecture #2: Primer on Linear Algebra and Scientific Computing

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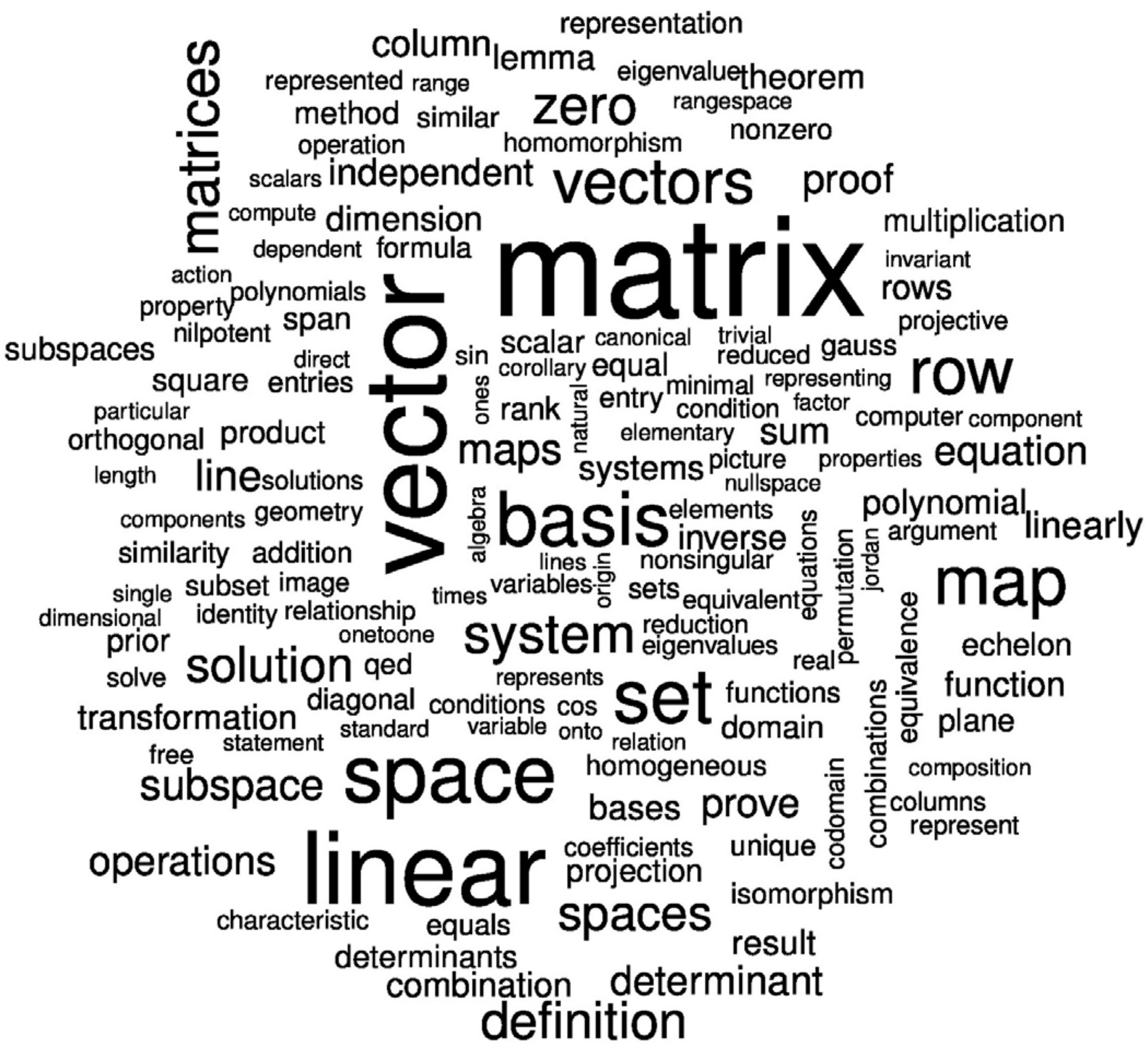
Lecture outline



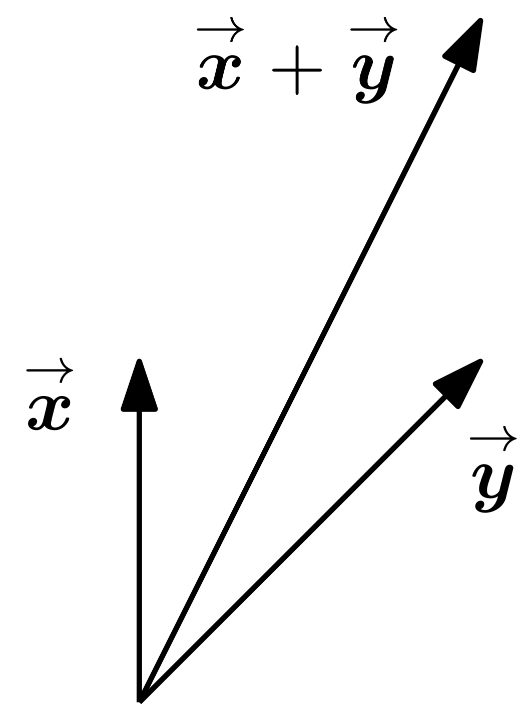
Scientific computing

Linear algebra

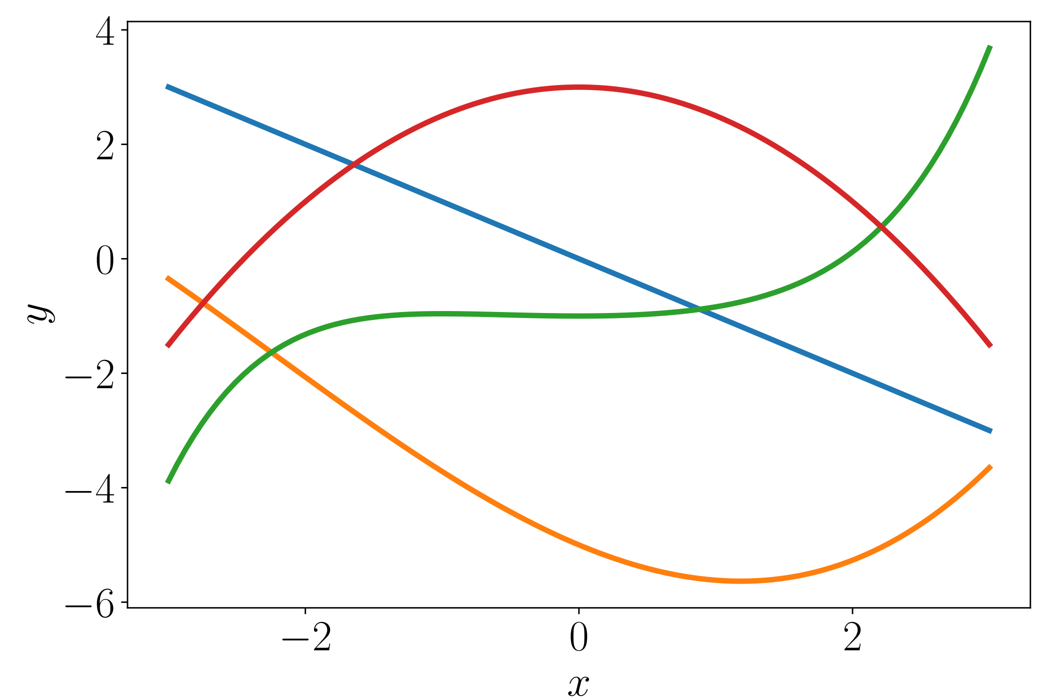
Matrix/vector
calculus/operations

Vectors

- Basic definitions
- Vector operations (e.g. addition, subtraction, multiplication, etc.)
- Linear combinations
- Dot products
- Norms
- Vector/Linear spaces
- Linear (in)dependance
- Bases



(a) Geometric vectors.



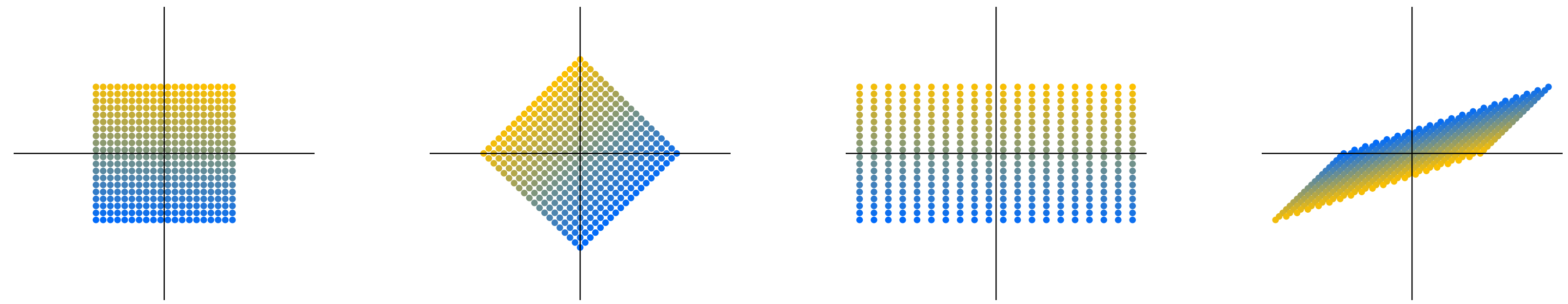
(b) Polynomials.

Useful resources:

- *Gilbert Strang's lectures at MIT OCW: https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/video-lectures/*
- *Pavel Grinfeld's series on linear algebra: https://www.youtube.com/playlist?list=PLlXfTHzgMRUKXD88IdzS14F4NxAZudSmv*

Matrices

- Basic definitions
- Matrix operations (e.g. addition, subtraction, multiplication, etc.)
- Unit matrices, transposes, inverses
- Basic properties
- Norms
- Linear transformation of vectors
- Eigenvalues and eigenvectors
- Linear systems



(a) Original data. (b) Rotation by $45°$. (c) Stretch along the horizontal axis. (d) General linear mapping.

$$\underbrace{\boldsymbol{A}}_{n\times k}\underbrace{\boldsymbol{B}}_{k\times m} = \underbrace{\boldsymbol{C}}_{n\times m}$$

For $\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \in \mathbb{R}^{2\times 3}$, $\boldsymbol{B} = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{3\times 2}$, we obtain

$$\boldsymbol{AB} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 5 \end{bmatrix} \in \mathbb{R}^{2\times 2},$$

$$\begin{aligned} 2x_1 + 3x_2 + 5x_3 &= 1 \\ 4x_1 - 2x_2 - 7x_3 &= 8 \\ 9x_1 + 5x_2 - 3x_3 &= 2 \end{aligned}$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & -2 & -7 \\ 9 & 5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}$$

Useful resources:

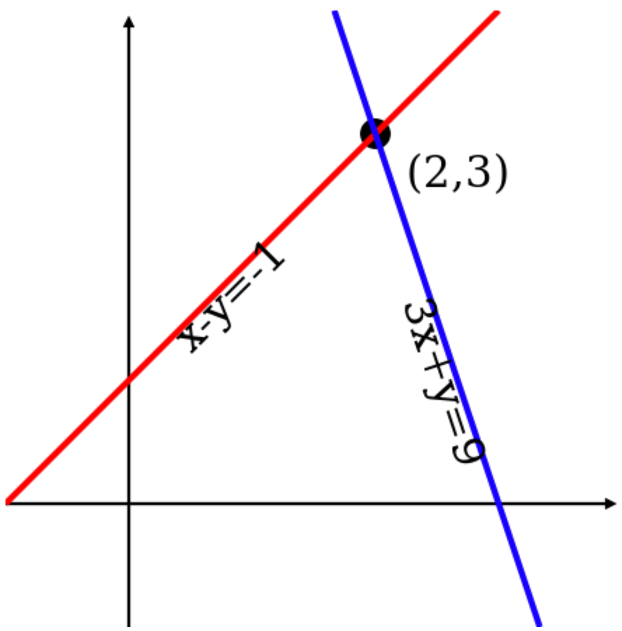
- *https://see.stanford.edu/materials/lsoeldsee263/Additional1-notes-matrix-primer.pdf*
- *https://see.stanford.edu/materials/lsoeldsee263/Additional2-matrix_crimes.pdf*

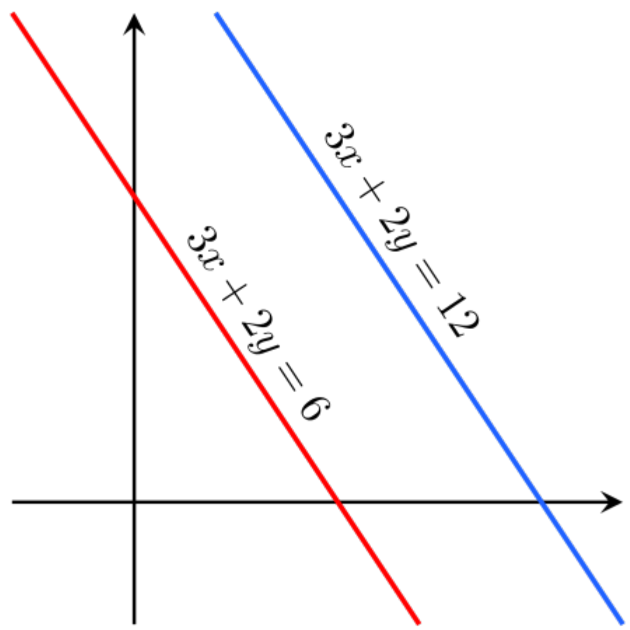
Linear systems

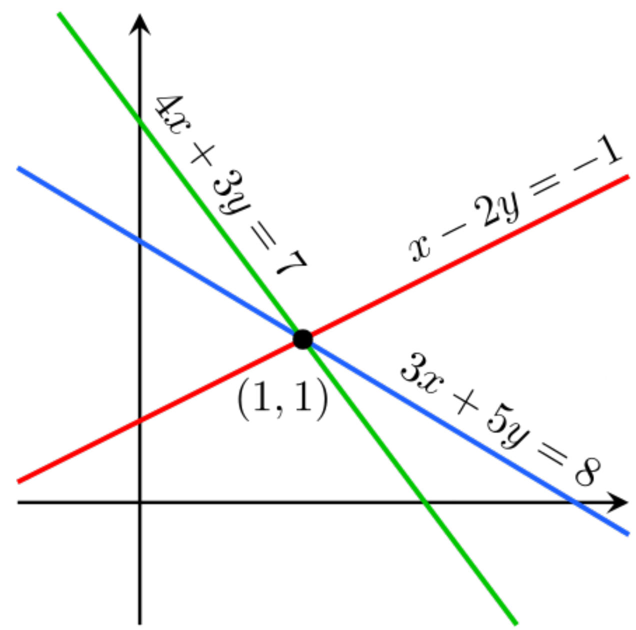
- Direct solvers:
  - Gauss elimination/LU decomposition
  - Cholesky decomposition (SPD matrices)
  - QR decomposition
  - SVD
- Iterative solvers:
  - Jacobi iterations
  - Gauss-Seidel
  - Successive over-relaxation (SOR)
  - Krylov subspace methods (conjugate gradients, etc.)

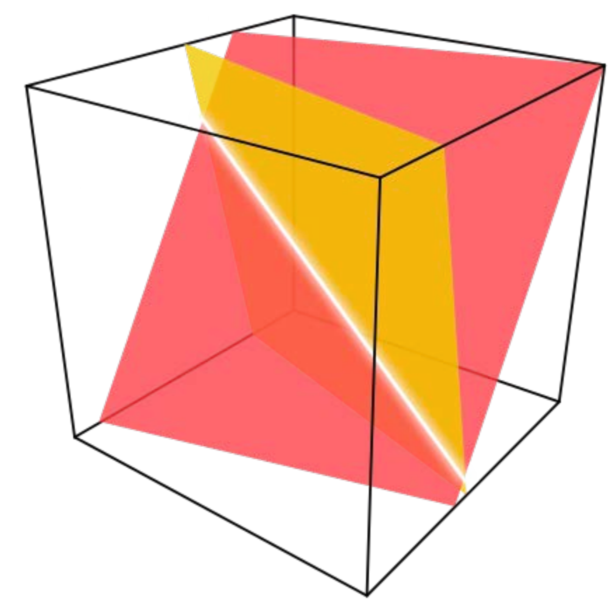
$$
\begin{aligned}
2x_1 + 3x_2 + 5x_3 &= 1 \\
4x_1 - 2x_2 - 7x_3 &= 8 \\
9x_1 + 5x_2 - 3x_3 &= 2
\end{aligned}
$$

$$
\begin{bmatrix} 2 & 3 & 5 \\ 4 & -2 & -7 \\ 9 & 5 & -3 \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
=
\begin{bmatrix} 1 \\ 8 \\ 2 \end{bmatrix}
$$









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