Physics 360/Math 360

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Assignment 8, Due Monday March 29th

1 Vector calculus

1.1 Line Integrals

Boas §6.8 (Line integrals) # 1, 6,
Extra credit: # 8, 15

1.2 Green's Theorem

Problems from §6.9 (Green's Theorem) # 2, 3,
Extra credit: # 6

1.3 Divergence and Divergence Theorem

Problems from §6.10 # 1
Extra credit: # 12

1.4 Stokes' Theorem

Problems from §6.11 # 1, 2
Extra credit: # 16

1.5 Div, grad, curl

You're covering a lot of this in your math class, so we won't do a ton of problems here.

1.5.1 Fields with zero and non-zero divergence and curl

Write down a vector field with the following characteristics (a different field for each item in the list). In each case, you are not allowed to re-use one of the fields from class. In each case, it's acceptable if the listed conditions are not met for all point, but you must then tell me at least one point where the conditions are met (e.g. "the divergence is zero at the origin, but positive at $(2, 2)$")

- non-zero divergence, zero curl
- zero divergence, non-zero curl
- zero divergence, zero curl
- non-zero divergence, non-zero curl

1.5.2 Murder Mystery

One often runs into the following problem: given a field $\mathbf{V}$, find $\mathbf{A}$ such that $\mathbf{V} = \boldsymbol{\nabla}\mathbf{F}$ or $\mathbf{V} = \boldsymbol{\nabla} \times \mathbf{A}$. As explained in the section on conservative fields, if you can write $\mathbf{V}$ as the gradient of another field $\mathbf{F}$, we know that $\mathbf{V}$ is *conservative*, and we call $\mathbf{F}$ the *potential field* for $\mathbf{V}$. Additionally, is easy to show that, if vV can be written as $\boldsymbol{\nabla} \times \mathbf{A}$, then $\boldsymbol{\nabla} \cdot \mathbf{V} = 0$. To be clear: you can't always write $\mathbf{V}$ as the curl of some other field; we're trying to figure something out about the cases where you *can*.

In this problem, you will use the "murder mystery" method (used in several texts, but I think this name comes from the Oregon Bridge Project folks).

First example: say you're given

$$\mathbf{F} = y\hat{\mathbf{i}} + (x + 2y)\hat{\mathbf{j}} \tag{1}$$

and you want to know if it's conservative. You decide to check by determining if it can be written as the gradient of some other field U. Well, what do we know?

We know that, if $\mathbf{U} = \boldsymbol{\nabla}F$, $U_x = \frac{\partial F}{\partial x}$, so we can immediately guess that $F = xy$ (note: it could easily be something else, like $xy + \sin(4000e^{\pi y^2})$, but our strategy will be to start with the easy guess and make it more complicated if it needs to be later on).

We also know that $\frac{\partial U}{\partial y} = F_y = (x + 2y)$ so we can integrate to guess that $U = xy + xy^2$. Are these two guesses compatible? Yes. Taking the partial of $\frac{\partial U}{\partial x}$ gives us the y we were looking for.

Please read http://www.math.oregonstate.edu/BridgeBook/book/math/mmm for a bit more in the way of details, complete with some very good pictures. The actual homework problem here is the one at the bottom of that page: find a potential function for the vector field

$$\mathbf{G} = yz\hat{\mathbf{i}} + (xz + z)\hat{\mathbf{j}} + (xy + y + 2z)\hat{\mathbf{k}} \tag{2}$$