Physics 360/Math 360 MWF 10:00 - 10:50 PM, Room: Dennis 209

Instructor: Michael Lerner, CST 213, Phone: 765-983-1784, Email: lernemi@earlham.edu

Office Hours: T 10-11, W 4:00-5:00 and by appointment. I also have an open-door policy, and you're encouraged to stop in to ask questions whenever my door is open. That's most of the time.

Course goals

- Students will develop a level of mathematical sophistication that will allow them to confidently and competently explore further material outside of a classroom setting.
- Students will be able to apply the standard tools of mathematical physics (Fourier series and transforms, series solutions to partial differential equations, Bessel functions, spherical harmonics, etc.) to provide quantitative insight into physical systems.
- Students will learn basic scientific Python programming.
- Students will be able to apply numerical integration to provide quantitative insight into physical systems.

Required Textbooks *

Boas, Mathematical Methods in the Physical Sciences, 3E The third edition of this text is both an excellent resource text as well as a good text to learn from.

Prerequisite Math 320, differential equations.

Grading Policy

- Class preparation/participation, Moodle/Piazza participation: 10%
- One mini-midterm, open book, 5%
- Two midterms, each 10%, for a total of 20%
- One final, 15%
- Independent project, 15%
- Homework, 35%

Attendance Policy: Students are expected to attend classes regularly. A student who incurs an excessive number of absences may have some or all of the class preparation/participation grade (10%) deducted at the discretion of the instructor.

Academic Integrity: http://www.earlham.edu/policies-and-handbooks/community/student-code-of-conduct/

Important Dates: they're all in the syllabus.

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*You can likely substitute recommended texts for the required texts, but please discuss it with me first so that I can OK your choices and make sure that you'll have access to the correct homework problems

Major Topics:

- Power Series
- Computational Methods
- Complex Numbers
- Series Solution of Second Order Linear Equations
- Vector Analysis
- Fourier Series and Transforms
- Laplace Transforms
- Dirac Delta Functions
- Partial Differential Equations
- Bessel Functions
- Numerical integration

Recommended Textbooks

Boyce and DiPrima, Elementary Differential Equations and BVPs This was the differential equations book last semester. It's an excellent reference for PDEs later in the term.

The Feynman Lectures on Physics, Volume II This has some very nice explanations of the vector calculus concepts that we'll be covering, taken from a physicist's perspective. This part of Feynman is also extremely conversational and readable. While every self-respecting scientist should, of course, own a hard copy, you can read the whole thing online with beautiful typesetting at http://www.feynmanlectures.info/

Gelfand and Fomin, Calculus of Variations The best introduction I know of to calculus of variations. It's an $11 Dover paperback.

Emmy Noether's Wonderful Theorem by Dwight E. Neuenschwander will be the basis for our discussion of Noether's Theorem, if we cover Noether's theorem. This book is really well written, and goes much farther than we'll have time for.

Kusse and Westwig, Mathematical Physics This text is extremely well written. It doesn't have quite the right focus for this particular class.

Arfken and Weber, Mathematical Methods for Physicists This encyclopedic volume makes for a good reference, but is a bit too dry to learn from in this course.

Schey, div grad curl and all that This is an extremely conversational introduction to/refresher on vector calculus

Prerequisites, co-requisites, etc.†

Prerequisite Math 320, differential equations. **If you have not taken this, talk to me ASAP**

Co-requisite Math 350, multivariate calculus.

Strongly Suggested Physics 125, analytical physics I.

Suggested Math 310, linear algebra. Really, this makes everything easier.

† Please contact the instructor if you're considering taking 350 as a co-requisite. Doing so requires that I carefully coordinate my syllabus with whoever happens to be teaching 350 at the time.

This class has, in some iterations, spent a week covering **infinite series** (Boas sections 1.1-1.8). This year, we will assume that students are sufficiently familiar with those topics as they were covered in Math 280. **If this is not the case, let me know as soon as possible**.

Course Goals: One of the great strengths of a modern physics background is that it allows you to address problems in an enormous range of fields, from physics itself to economics, biology, ecology, computer science, complex systems, physical chemistry, geology, and engineering. The primary objective of this course is to provide a systematic introduction to mathematical techniques that will serve you both in future physics courses and in modeling interesting problems in your domain of choice. A significant secondary objective is to develop a level of mathematical sophistication that will allow you to confidently and competently explore further material on your own.

As might be gathered from the preceding paragraph, a significant fraction of the results of modern scientific research can only be accurately expressed in the language of advanced mathematics. A glance at any of the most prestigious scientific journals in your field of interest will quickly confirm this. Thus, even experimental scientists or those interested primarily in field work need to achieve a certain degree of mathematical sophistication if they are to understand the conceptual foundation and interpretation of research results in their own fields. This course is intended to provide you with a working knowledge of those techniques most commonly encountered in science and engineering. In the process of learning these techniques, you will also acquire experience in the physical interpretation of mathematical models of physical problems.

Structure There will be three class meetings each week. The class meetings will generally consist of lectures, discussion of assignments, and student presentation of assigned problems.

It is very important that students study the appropriate text assignments before coming to class and that they be prepared either to ask questions or to discuss questions related to the assigned material. The subject matter of this course is sufficiently complex that it is neither desirable nor possible to cover all relevant details in class. The primary value of lectures and class discussions in this course is to explain and illustrate points which students find difficult or obscure. It is essential, therefore, that students put in the advance work required to identify such points; the instructor cannot anticipate all of them. It is hoped that all of us will work together to produce a class atmosphere which is conducive to lively and interesting discussions of the material.

Tests: There will be three midterms and a final. The mini-midterm will be open-book and untimed. The other tests will be timed and closed-book. All will be self-scheduled exams, to be picked up and turned in at the front desk of the science library.

Piazza: Preclass: We often run into students who say things like "I totally understand the physics ... I just cant do the problems." This highlights the extremely important point that we need time in class both to discuss theoretical concepts and to work directly on problem solving. Readings are assigned below for every class period. By midnight before each class period, you must comment on Piazza (piazza.com/earlham/summer2018/physmath360). Your comment can be in one of the three following categories: (1) something interesting from the reading (2) something confusing from the reading (3) an answer to someone elses question. In order to make sure we have a good, constructive discussion atmosphere, please note that disrespectful comments will receive no credit. You can post anonymously; instructors can see who made each post, but students cannot.

Homework: This is basically a course in problem-solving techniques. The most common and the most damaging mistake a student can make in such a course is to yield to the temptation to try and master the material by reading the text and listening to the lectures while working a minimum of problems. While reading and listening can give you valuable new ideas, problem-solving skills are only efficiently developed by solving as many problems as possible. For most students the understanding and knowledge gained from this course will be in direct proportion to the number of problems which they successfully complete. The best approach is to maximize the percentage of your study time devoted to a disciplined effort to solve problems.

Homework will be assigned each approximately once per week. As with all Physics classes, you will find that your understanding is greatly improved if you start the assignments early in the week, rather than late in the week.

You get five free late days. After that, each additional weekday that an assignment is late will result in a deduction of a full letter grade for that assignment.

Grading policy: several of the problems assigned in this class are quite challenging. Others are just rote computation. For the more challenging problems, my goal is to have you make the strongest possible effort towards **understanding** the solution. Thus, if you cannot fully solve the problem, say whatever you can about the way in which a solution would proceed from where you stop; say whatever you can about the qualitative behavior of a solution; say whatever you can about the physical meaning of the solution.

Homework problems will be graded on roughly the same scale as used in Physics 125 and 235:

5 Solution is complete and well-written

4 Solution is missing minor parts or some important explanations

3 Solution is missing major parts and/or has few if any explanations

2 At least one major portion of the problem correct

1 Very little coherent initial effort was expended

0 No initial solution was submitted

Independent project: The primary goals of this course are involve developing a level of mathematical sophistication that will allow you to confidently and competently explore further material outside of a lecture setting. So, in order to make sure we're walking the walk, you'll each pick either an interesting problem to model or an interesting technique to learn. You'll write a short paper and present the results to the class. You'll be expected to start on this halfway through the semester, and we'll discuss it in more detail at that point. Examples might include

- The Fast Fourier Transform (FFT)
- Physical representations of Fourier Transforms, including circuits and optics
- The discrete Fourier Transform, including application to time series data, consideration of how many useful samples you have in a dataset in Fourier space, etc.
- Struve functions
- Complex analysis including itegrating around poles, Cauchy Integral, Residue theorem (**this is a hard topic to nail down in terms of applications; you must carefully verify your proposal with me first!**)
- Tensors, covariant vs. contravariant, pseudo-vectors, index notation
- Black-Scholes Theorem (economics)
- Applications of Noether's Theorem to economic systems
- Modern techniques such as the immersed-boundary method
- Coding and examination of more complex numerical integrators
- Green-Kubo relations and the Fluctuation Theorem
- Wavelet Transforms
- Real world applications of any of the techniques studied in this course
- Numerical integration (e.g. write your own RK4 and Velocity Verlet, compare the two for some real application)
- Interesting topic of your choice!

Note: If there is sufficient demand, it is likely possible to include at least one of the above topics in the main body of the course itself.

Monday	Wednesday	Friday
Jan 8th	10th **1** **Power Series** Boas 1.10-1.13 Interval of Convergence, Theorems, **Taylor Series**	12th **2** Boas 1.14-1.15 $tan(x)$, $e^{tan(x)}$, Accuracy of Approximations, Some applications 2.1-2.4 **Complex Numbers** Definitions and Terminology
15th **3** **Complex Numbers** Boas 2.4-2.6 Definitions and Terminology, Complex Algebra, Complex Infinite Series	17th **4** Boas 2.7-2.10 Power Series, Disk of Convergence, Functions of complex numbers, Euler's Formula, Powers and Roots of $\mathbb{C}$	19th **5** Boas 2.11-2.15 Powers and Roots of $\mathbb{C}$, Exp and Trig Functions, Hyperbolic Functions, Logarithms, Complex Roots and powers, Inverse Trig/Hyperbolic Funcs
22nd **6** **Series Solution of Second Order Linear Equations** Boas 12.1-12.2 The basic idea, making tables, spherical harmonics (Legendre)	24th **7** Boas 12.5 Generating function, expansion of a potential **Mini-midterm**	26th **8** Boas 12.7-12.8 Orthogonality and Normalization
29th **9** Boas 12.11 The Method of Frobenius	31st **10** Boas 12.12-12.13 Bessel's equation	Feb 2nd **11** Boas 12.14 Graphs, zeros and numerical computation of Bessel
5th **12** **Fourier Series, Fourier and Laplace Transforms** Boas 7.1-7.2 SHM, Wave Motion, Periodic Functions	7th **13** Boas 7.3-7.5 Applications of Fourier Series, Average Value of a Function, Fourier Coefficients, Kronecker delta	9th **14** Boas 7.6-7.7 Dirichlet Conditions, Complex Fourier Series
12th **15** Boas 7.8-7.9 Other Intervals, Even and Odd Functions	14th **16** Boas 7.12 Fourier Transforms	16th **Early Semester Break**

Monday	Wednesday	Friday
19th **17** Orthogonal Functions (Michael at BPS, John Howell takes charge)	21st **18** Boas 8.8-8.9 Laplace Transforms (Michael at BPS, John Howell continues to run the show)	23rd **19** Boas 8.10-8.11 Convolutions and Dirac Delta Functions
26th **20** Boas 8.11 More Dirac Delta Functions	28th **21** Boas 8.12 Green's functions Boas 2.16 Application: Optics **Midterm 1**	Mar 2nd **22** **Vector Analysis** Boas 6.1-6.5 Review of basic concepts including Triple Products, Fields, mention tensors
5th **23** Boas 6.6 Directional Derivative, Gradient	7th **24** Boas 6.7-6.8 Expressions involving ∇, Line integrals	9th **25** Boas 6.9 Green's Theorem in the Plane
12th **Spring Break**	14th **Spring Break**	16th **Spring Break**
19th **26** Boas 6.10 Divergence and Divergence Theorem	21st **27** Boas 6.11 Curl and Stokes Theorem **Project topics**	23rd **28** **Partial Differential Equations** Boas 13.1 - 13.2 Intro, Laplace's Equation, Steady-state Temperature in a Rectangular Plate **Paragraph about project**
26th **29** Boas 13.3 Diffusion/Heat Flow Equation, Schrödinger Equation	28th **30** Boas 13.4 Wave Equation, Vibrating String	30th **31** Last day to drop Boas 13.5 Steady-state temperature in a cylinder
Apr 2nd **32** Boas 13.6 Vibration of a Circular Membrane (in-class discussion)	4th **33** **Project workday** **Project draft 1** Draft due 8:00 AM Tomorrow	6th **34** Boas 13.6 Vibration of a Circular Membrane (group worksheet and movies) briefly Boas 13.7 Steady-state temperature in a sphere Stochastic differential equations and diffusion

Monday	Wednesday	Friday
9th **35** **Project workday** $\boxed{\textbf{Midterm 2}}$	11th **36** **More PDEs** Boas 13.8 Poisson's Equation and a return to Green's functions	13th **37** Boas 13.9 Integral Transform Solutions of PDEs
16th **38** **Project workday, led by guest** (Michael at AACR)	18th **Epic Expo** (no class)	20th **39** **Numerical Integration** MGLNotes Numerical Integration with Euler, Verlet and Runge-Kutta $\boxed{\textbf{Project draft 2}}$
23rd **40** MGLNotes code for integrators, special cases and series approximations	25th **41** MGLNotes Symplectic Integration and code examples	27th **42** $\boxed{\text{Last class}}$
30th Finals	$\boxed{\text{May 2nd}}$ Finals	4th **43**

Much of this syllabus has been taken from syllabi provided by Ray Hivley and John Howell.