

B. Herman and J. Roberts

# Nuclear Reactor Core Methods

April 9, 2012

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Lists of abbreviations, symbols and the like are easily formatted with the help of the Springer-enhanced `description` environment.

PWR	Pressurized Water Reactor
BWR	Boiling Water Reactor
ANM	Analytic Nodal Method



# **Part I**

## **Fundamentals**

Lorem ipsum...



# Chapter 1

## Neutron Transport Equation

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### 1.1 Terminology

Definition of all terms (flux, current etc.) Just a copy paste of 106 notes I am sure

### 1.2 Derivation of Neutron Transport Equation

Jeremy I am sure you have this done from 106.



## Chapter 2

# Multigroup Neutron Diffusion Equation

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### 2.1 Continuous Energy Diffusion Equation

This section will contain the derivation of the continuous form of the diffusion equation from the neutron transport equation.

### 2.2 Derivation of Multigroup Diffusion Equation

This section will contain the derivation of the multigroup diffusion equation from the continuous energy diffusion equation



## Chapter 3

# Finite Difference Methods

### 3.1 Taylor Series

The finite difference method relies heavily on the mathematical concept of Taylor Series. If we take a function,  $f(x)$ , the independent variable  $x$  can be discretized into many points as shown in Figure . If the value of the function is known at  $x_i$ , the value at  $x_{i+1}$  can be determined by a Taylor series expansion at  $x_i$ ,

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f^{(3)}(x_i)}{3!}h^3 + \dots + \frac{f^{(n)}(x_i)}{n!}h^n + \dots \quad (3.1)$$

In Eq. (3.1),  $f^{(3)}$  represents the  $n$ -th derivative of the function and  $h$  is the spacing between points,  $h = x_{i+1} - x_i$ .

The expansion shown above is exact if the number of terms in the Taylor series expansion is taken to infinity. Of course, this is not practical for computational methods and therefore we truncate the series at a finite number of terms. The error present caused by the truncation is known as truncation error. Instead of representing the full Taylor expansion of a function, we will truncate the expression after a few number of terms and represent the truncation error with  $\mathcal{O}(h^n)$ . In this representation of the truncation error,  $n$  represents the order of convergence. Order of convergence means that as the grid is refined by a factor of two for example, the truncation error will reduce on the order of  $2^n$ . This does not imply that one method is better than the other, just merely a concept of convergence rate due to truncation effects. For example, if we expand a function to second order, we would rewrite Eq. (3.1) this as

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \mathcal{O}(h^3). \quad (3.2)$$

As we approximate differentials, we can keep track of this truncation error to determine order of convergence of our methods. This is one way to ensure that our discretization method and implementation of solution algorithms are correct.

### 3.2 Approximation of Differentials

There are many different approximations of differentials that can be constructed based on Taylor series. We will first consider the approximation of first order derivatives. The first approximation is a *first order forward difference* where we use information about a point just to the right,  $x_{i+1}$ , to infer the derivative at  $x_i$ . If we perform a Taylor expansion about point  $x_{i+1}$  to first order we get

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \mathcal{O}(h^2). \quad (3.3)$$

This equation can be solved for the derivative of the function at  $x_i$

$$f'_{for}(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + \mathcal{O}(h), \quad (3.4)$$

where  $f'_{for}(x_i)$  represents the first order forward difference approximation to the derivative at  $x_i$ .

The opposite approximation is to consider a point to the left,  $x_{i-1}$ , to infer the derivative at  $x_i$ , known as the *first order backward difference*. Here, we take a Taylor expansion to the left,

$$f(x_{i-1}) = f(x_i) - f'(x_i)h + \mathcal{O}(h^2). \quad (3.5)$$

Solving for the derivative we can arrive at

$$f'_{bac}(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} + \mathcal{O}(h). \quad (3.6)$$

Comparing Eqs. (3.4) and (3.6) we see that the formulation looks the same in that it is always the right point minus the left point in the numerator of the fraction. The only difference is the sign in the truncation error is reversed. Therefore, we can expect that one of these approximations will under-predict the true answer and the other one will over-predict. Again both of these methods are first order methods.

The last simple approximation of a first derivative is a *second-order central difference*. In this method we look at both left and right points. We can Taylor expand each of these to second order to get

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \mathcal{O}(h^3) \quad (3.7)$$

$$f(x_{i-1}) = f(x_i) - f'(x_i)h + \frac{f''(x_i)}{2!}h^2 - \mathcal{O}(h^3). \quad (3.8)$$

Subtracting the  $x_{i-1}$  equation from the  $x_{i+1}$ , we are left with

$$f(x_{i+1}) - f(x_{i-1}) = 2f'(x_i)h + \mathcal{O}(h^3). \quad (3.9)$$

Solving for the derivative at  $x_i$  we arrive at the second order central difference approximation

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1}))}{h} - \mathcal{O}(h^2). \quad (3.10)$$

From the resulting expression, this approximation method does not depend on the value of the function at  $x_i$  and that the scheme is second order convergent.

### ***3.2.1 Nonuniform Spacing***

## **3.3 Finite Difference Multigroup Diffusion Equation**





## Chapter 4

### Finite Volume Methods

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### Finite Element Methods

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## Chapter 6

# Stationary Iterative Methods

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This chapter will contain the idea of iterative methods, and talk about Jacobi and Gauss - Siedel, example should be provided either for fission source iterations or energy group sweep. Also should include SOR method.



## Chapter 7

# Nonstationary Iterative Methods - Krylov Subspace Methods

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REF: <http://www.netlib.org/utk/papers/templates/node9.html>

Intro to Krylov Methods Arnoldi Iterations - Gram-Schmidt etc?





## Chapter 8

# Conjugate Gradient

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Specific example - Conjugate Gradient



## Chapter 9

### GMRES

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Specifically derive out GMRES with givens rotations. Preconditioning JFNK?



## Chapter 10

### Power Iteration

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Derive out the power iteration method and give example.



## Chapter 11

### Nonlinear Iteration

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Newton Iteration - with GMRES JFNK





## Chapter 12

### Chebyshev Acceleration Method

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Chebyshev Acceleration of Power iteration



## Chapter 13

### Time Stepping Methods

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Forward Euler (Explicit) Backward Euler (Implicit) Runge-Kutta (4th order mostly used in spatial kinetics) Adams-Moulton Adams-Bashforth



## **Part II**

# **Reactor Statics**

Lorem ipsum...

## Chapter 14

### Classical Nodal Methods - Flare Model

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Summer course on nodal methods (Herman office)





## Chapter 15

# Analytic Nodal Method

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Derivation of Analytic Nodal Method with example code Smith Master Thesis



## Chapter 16

### Nodal Expansion Method

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- Bandini Thesis



## **Part III**

# **Reactor Dynamics**

Lorem ipsum...

## Appendix A

### Chapter Heading

*All's well that ends well*

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#### A.1 Section Heading

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##### A.1.1 Subsection Heading

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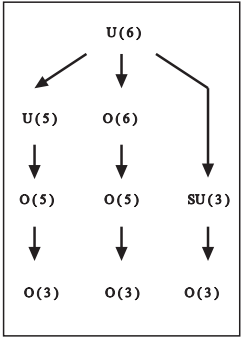
For multiline equations we recommend to use the `eqnarray` environment.

$$\begin{array}{l} \mathbf{a} \times \mathbf{b} = \mathbf{c} \\ \mathbf{a} \times \mathbf{b} = \mathbf{c} \end{array} \quad (\text{A.1})$$

##### A.1.1.1 Subsubsection Heading

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Furtheron please use the

**Fig. A.1** Please write your figure caption here



L<sup>A</sup>T<sub>E</sub>X automatism for all your cross-references and citations as has already been described in Sect. A.1.1.

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

**Table A.1** Please write your table caption here

Classes	Subclass	Length	Action Mechanism
Translation	mRNA <sup>a</sup>	22 (19–25)	Translation repression, mRNA cleavage
Translation	mRNA cleavage	21	mRNA cleavage
Translation	mRNA	21–22	mRNA cleavage
Translation	mRNA	24–26	Histone and DNA Modification

<sup>a</sup> Table foot note (with superscript)



# Glossary

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