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A unified algorithm framework for the facility location and service area problems

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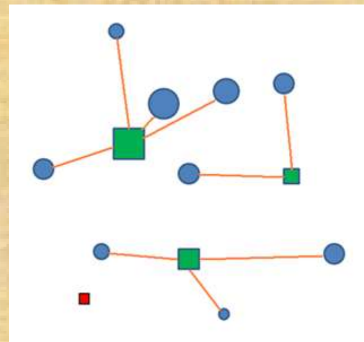
Outline

- Research background
- Problems and formulations
- Algorithm framework
- Experiments
- Conclusion

1 Research background

Location problems

- Continuous location problems
- Discrete location problems
  - PMP: P-median problem
  - PCP: P-center problem
  - MCLP: maximum covering location problem
  - LSCP: location set covering problem
  - FLP: facility location problem
    - UFLP: uncapacitated facility location problem
    - CFLP: capacitated facility location problem
      - SSCFLP: single-source CFLP



Minimize $\sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$

Subject to: $\sum_{i \in I} x_{ij} = 1, \forall j \in J$

$\sum_{j \in J} d_j x_{ij} \leq q_i y_i, \forall i \in I$

$x_{ij} = \{0,1\}, \forall i \in I, j \in J$

$y_i = \{0,1\}, i \in I$

single-source capacitated facility location problem (SSCFLP)

Competitive methods for SSCFLP

- MILP modeling
- Cut and Solve (Yang et al. 2012)
- Kernel Search (Guastaroba & Speranza 2012, 2014)
- Lagrangian heuristic
- Local-search based Heuristics: Tabu, GRASP, VLNS…
  - Multi-exchange (Ahuja et al. 2004, Tran et al. 2017)
- Population-based Heuristics:
  - Evolutionary algorithm: EA, GA…

Challenges on solving SSCFLP

- There are a large number of algorithms, however, only several of them have been tested on the large and difficult benchmark instances.

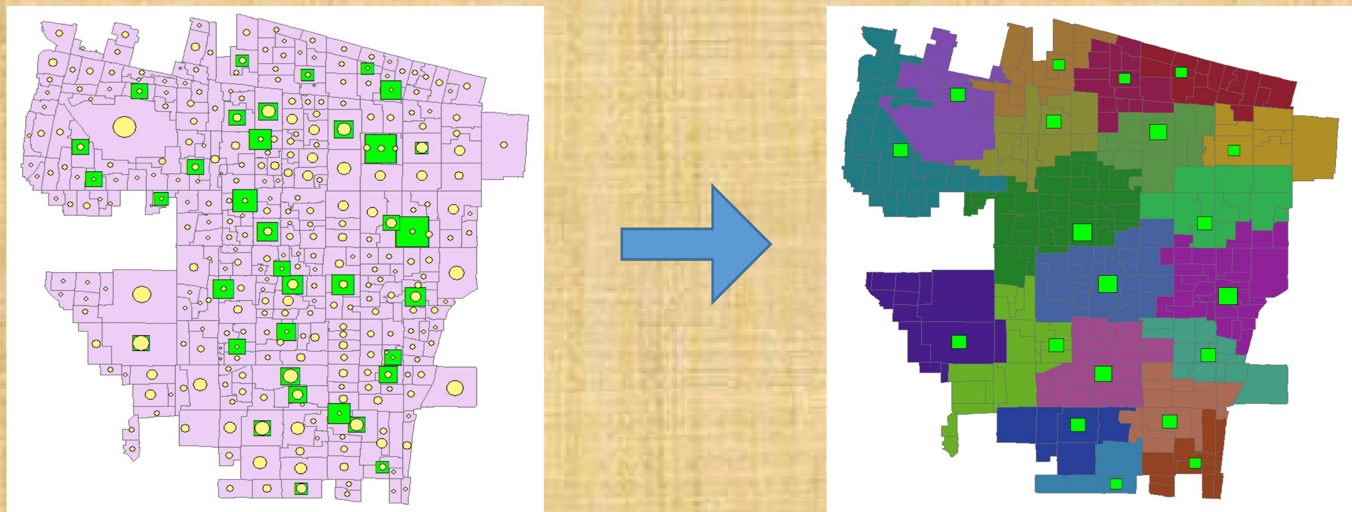
Dataset	Num. of instances	Num. facility	Num. demand	Difficulty
Holmberg (1999)	71	10-30	50-200	"Easy"
Beasley (1988)	36	16-100	50-1000	"Easy" - hard
Yang (2012)	20	30-80	200-400	Hard
Avella (2009)	100	300-1000	300-1500	Very hard

- Effectiveness, efficiency and easiness of the algorithms

Algorithm	Effectiveness (Solution quality, optimality)	Efficiency (Computation time, memory usage)	Easiness (Design difficulty, parameter setting…)
MILP	***	*	***
Kernel Search, Cut-and-Solve	***	*	**
Multi-Exchange Heuristic	***	**	*
Lagrangian Heuristic	*	***	***

New problems

- CKFLP: CFLP with K-facilities (Aardal et al. 2014),
- FLSAP: SSCFLP with connective service areas
- CKFLSAP: SSCFLP with K-facilities and connective service areas



- It is harder to solve the new problems than the classical problems!

Research motivation

- Is it possible to design an effective and efficient algorithm to solve location problems?

Algorithm	Effectiveness	Efficiency)	Easiness
MILP	***	*	***
Kernel Search, Cut-and-Solve	***	*	**
Multi-Exchange Heuristic	***	**	*
Lagrangian Heuristic	*	***	***
New algorithm?	***	***	***

- Is it possible to design a unified algorithm to solve both the classical and the new problems?
  - SSCFLP, CFLSAP, CKFLP, CKFLSAP?
- Yes…!?

2 Problems and formulations

- A summary of the location and/or allocation problems

Problem	Location choice	Facility capacity	Facility cost	Number of facilities	Demand assignment	Demand split	Transportation cost	Contiguity	NP-Hard
Single-source capacitated facility location problem (SSCFLP)	✓	✓	✓		✓		✓		✓
Facility location and service area problem (FLSAP)	✓	✓	✓		✓		✓	✓	✓
Capacitated K-facility location problem (CKFLP)	✓	✓	✓	✓	✓		✓		✓
Capacitated K-facility location and service area problem (CKFLSAP)	✓	✓	✓	✓	✓		✓	✓	✓
Capacitated facility location problem (CFLP)	✓	✓	✓		✓	✓	✓		✓
Uncapacitated facility location problem (UFLP)	✓		✓		✓	✓	✓		✓
Capacitated P-median problem (CPMP)	✓	✓		✓	✓		✓	✓	✓
P-median problem (PMP)	✓			✓	✓		✓	✓	✓
Facility service area problem (SAP)		✓			✓		✓	✓	✓
Generalized assignment problem (GAP)		✓			✓		✓		✓
Transportation problem (TP)		✓			✓	✓	✓		✗
Political districting problem(PDP) *defined as a location problem	✓	✓		✓	✓		✓	✓	✓

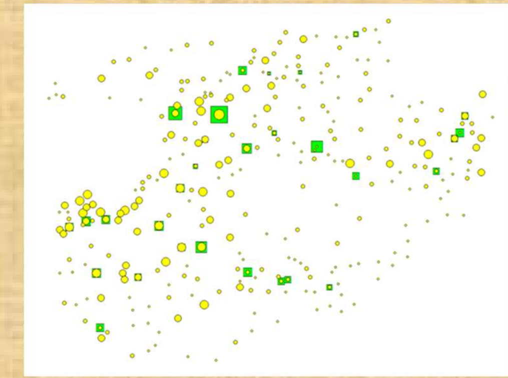
Problem description

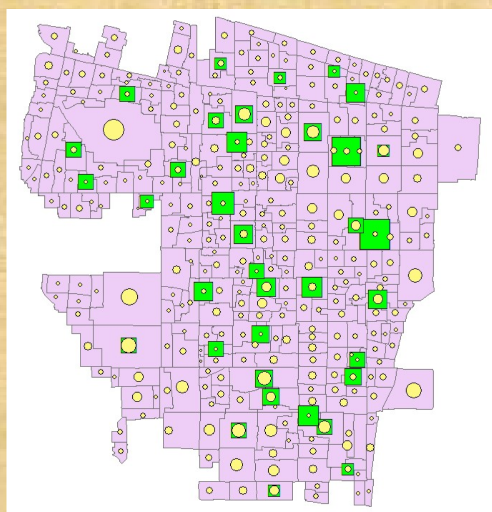
Parameters:

- Demand set $D = \{1, 2 \ldots n\}$, each unit i has demand d_i;
- Location set $S = \{s_1, s_2 \ldots s_m\}$, each location s_k has service capacity q_k and setup cost f_k;
- d_{ik}: distance from unit i to unit k;
- c_{ik}: unit transportation cost from unit i *to* unit k;
- N_i: set of neighborhood units of unit i;

Decision variables:

- Binary y_k: location s_k is selected or not;
- Binary x_{ik}: unit i is assigned to location s_k or not;
- Non-negative integer f_{ijk}: flow from unit i to unit j in service area of s_k.





Facility location and service area problem (CKFLSAP)

Minimize $\sum_{k\in S} f_k y_k + \sum_{i\in D}\sum_{k\in S} d_i c_{ik} x_{ik}$ (1)

Subject to: $\sum_{k\in S} x_{ik} = 1, \forall i \in D$ (2)

$\sum_{i\in D} d_i x_{ik} \le q_k y_k, \forall k \in S$ (3)

$\sum_{k\in S} y_k \le K$ (4)

$f_{ijk} \le (n-K)x_{ik}, \forall i \in D, j \in N_i, \forall k \in S$ (5)

$f_{ijk} \le (n-K)x_{jk}, \forall i \in D, j \in N_i, \forall k \in S$ (6)

$\sum_{j\in N_i} f_{ijk} - \sum_{j\in N_i} f_{jik} \ge x_{ik}, \forall i \in D, \forall k \in S, i \ne k$ (7)

$f_{kjk} = 0, \forall k \in S, j \in N_k$ (8)

$x_{ik} = \{0,1\}, \forall i \in D, k \in S$ (9)

$y_k = \{0,1\}, k \in S$ (10)

$f_{ijk} \ge 0, \forall i \in D, j \in N_i, k \in S$ (11)

Facility cost and transportation cost

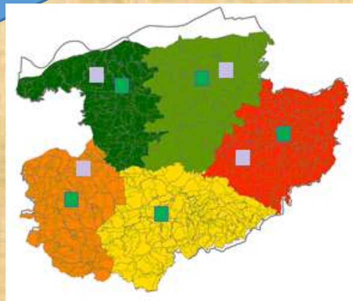
Assignment constraints

Capacity constraints

Constraint on number of facilities

Constrains on contiguity

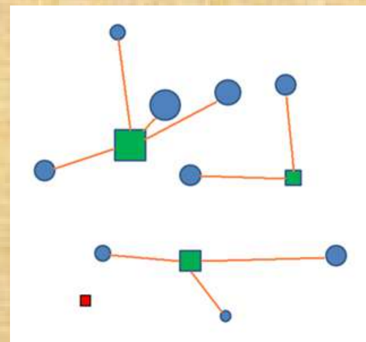
Decision variables



Capacitated Facility location problem (CFLP)

Minimize	$\sum_{k\in S} f_k y_k + \sum_{i\in D}\sum_{k\in S} d_i c_{ik} x_{ik}$	(1)
Subject to:	$\sum_{k\in S} x_{ik} = 1, \forall i \in D$	(2)
	$\sum_{i\in D} d_i x_{ik} \le q_k y_k, \forall k \in S$	(3)
	$\cancel{\sum_{k\in S} y_k \le K}$	(4)
	$\cancel{f_{ijk} \le (n-K)x_{ik}, \forall i \in D, j \in N_i, \forall k \in S}$	(5)
	$\cancel{f_{ijk} \le (n-K)x_{jk}, \forall i \in D, j \in N_i, \forall k \in S}$	(6)
	$\cancel{\sum_{j\in N_i} f_{ijk} - \sum_{j\in N_i} f_{jik} \ge x_{ik}, \forall i \in D, \forall k \in S, i \ne k}$	(7)
	$\cancel{f_{kjk} = 0, \forall k \in S, j \in N_k}$	(8)
	$x_{ik} = \{0,1\}, \forall i \in D, k \in S$	(9)
	$y_k = \{0,1\}, k \in S$	(10)
	$\cancel{f_{ijk} \ge 0, \forall i \in D, j \in N_i, k \in S}$	(11)

- Single-source CFLP(SSCFLP): *Xik*={0,1}
- Classical CFLP (or (demand-split CFLP) : *Xik*=[0,1]



Uncapacitated facility location problem (UFLP)

Minimize $$\sum_{k\in S} f_k y_k + \sum_{i\in D} \sum_{k\in S} d_i c_{ik} x_{ik} \tag{1}$$

Subject to: $$\sum_{k\in S} x_{ik} = 1, \forall i \in D \tag{2}$$

$$\cancel{\sum_{i\in D} d_i x_{ik} \le q_k y_k, \forall k \in S} \tag{3}$$

$$\cancel{\sum_{k\in S} y_k \le K} \tag{4}$$

$$\cancel{f_{ijk} \le (n-K)x_{ik}, \forall i \in D, j \in N_i, \forall k \in S} \tag{5}$$

$$\cancel{f_{ijk} \le (n-K)x_{jk}, \forall i \in D, j \in N_i, \forall k \in S} \tag{6}$$

$$\cancel{\sum_{j\in N_i} f_{ijk} - \sum_{j\in N_i} f_{jik} \ge x_{ik}, \forall i \in D, \forall k \in S, i \ne k} \tag{7}$$

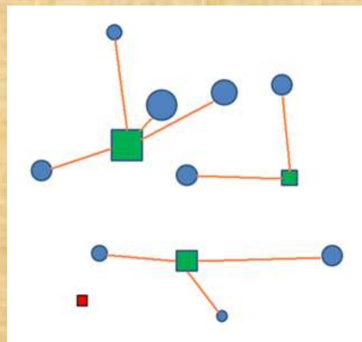
$$\cancel{f_{kjk} = 0, \forall k \in S, j \in N_k} \tag{8}$$

$$x_{ik} = \{0,1\}, \forall i \in D, k \in S \tag{9}$$

$$y_k = \{0,1\}, k \in S \tag{10}$$

$$\cancel{f_{ijk} \ge 0, \forall i \in D, j \in N_i, k \in S} \tag{11}$$

- Single-source: x_{ik}={0,1}
- or demand-split: x_{ik}=[0,1]



Capacitated P-Median Problem(CPMP)

Minimize $\cancel{\sum_{k\in S} f_k y_k} + \sum_{i\in D} \sum_{k\in S} d_i c_{ik} x_{ik}$ (1)

Subject to: $\sum_{k\in S} x_{ik} = 1, \forall i \in D$ (2)

$\sum_{i\in D} d_i x_{ik} \le q_k y_k, \forall k \in S$ (3)

$\cancel{\sum_{k\in S} y_k \le K}$ $\boxed{\sum_{k\in S} y_k = K}$ (4)

$\cancel{f_{ijk} \le (n-K) x_{ik}, \forall i \in D, j \in N_i, \forall k \in S}$ (5)

$\cancel{f_{ijk} \le (n-K) x_{jk}, \forall i \in D, j \in N_i, \forall k \in S}$ (6)

$\cancel{\sum_{j\in N_i} f_{ijk} - \sum_{j\in N_i} f_{jik} \ge x_{ik}, \forall i \in D, \forall k \in S, i \ne k}$ (7)

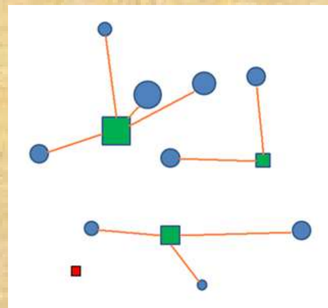
$\cancel{f_{kjk} = 0, \forall k \in S, j \in N_k}$ (8)

$x_{ik} = \{0,1\}, \forall i \in D, k \in S$ (9)

$y_k = \{0,1\}, k \in S$ (10)

$\cancel{f_{ijk} \ge 0, \forall i \in D, j \in N_i, k \in S}$ (11)

- Classical PMP: remove (3)
- demand-split: x_{ik}=[0,1]



Generalized assignment problem(GAP)

Minimize $\cancel{\sum_{k\in S} f_k y_k} + \sum_{i\in D}\sum_{k\in S} d_i c_{ik} x_{ik}$ (1)

Subject to: $\sum_{k\in S} x_{ik} = 1, \forall i \in D$ (2)

$\sum_{i\in D} d_i x_{ik} \le q_k \cancel{y_k}, \forall k \in S$ (3)

$\cancel{\sum_{k\in S} y_k \le K}$ (4)

$\cancel{f_{ijk} \le (n-K)x_{ik}, \forall i \in D, j \in N_i, \forall k \in S}$ (5)

$\cancel{f_{ijk} \le (n-K)x_{jk}, \forall i \in D, j \in N_i, \forall k \in S}$ (6)

$\cancel{\sum_{j\in N_i} f_{ijk} - \sum_{j\in N_i} f_{jik} \ge x_{ik}, \forall i \in D, \forall k \in S, i \ne} k$ (7)

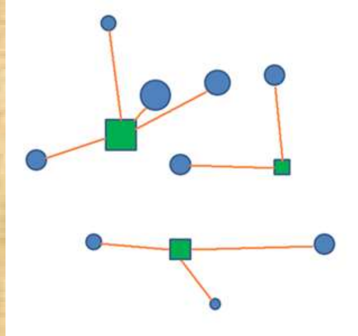
$\cancel{f_{kjk} = 0, \forall k \in S, j \in N_k}$ (8)

$x_{ik} = \{0,1\}, \forall i \in D, k \in S$ (9)

$\cancel{y_k = \{0,1\}, k \in S}$ (10)

$\cancel{f_{ijk} \ge 0, \forall i \in D, j \in N_i, k \in S}$ (11)

- Transportation problem (TP): Xik=[0,1]



Service area problem(SAP)

Minimize $$\cancel{\sum_{k\in S} f_k y_k} + \sum_{i\in D}\sum_{k\in S} d_i c_{ik} x_{ik} \qquad (1)$$

Subject to: $$\sum_{k\in S} x_{ik} = 1, \forall i \in D \qquad (2)$$

$$\sum_{i\in D} d_i x_{ik} \le q_k \cancel{y_k}, \forall k \in S \qquad (3)$$

$$\cancel{\sum_{k\in S} y_k \le K} \qquad (4)$$

$$f_{ijk} \le (n-K)x_{ik}, \forall i \in D, j \in N_i, \forall k \in S \qquad (5)$$

$$f_{ijk} \le (n-K)x_{jk}, \forall i \in D, j \in N_i, \forall k \in S \qquad (6)$$

$$\sum_{j\in N_i} f_{ijk} - \sum_{j\in N_i} f_{jik} \ge x_{ik}, \forall i \in D, \forall k \in S, i \ne k \qquad (7)$$

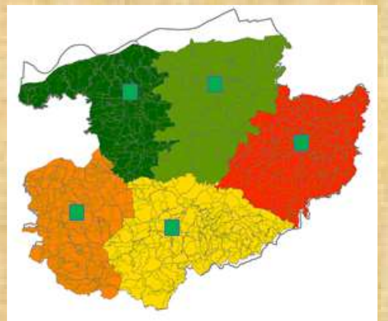
$$f_{kjk} = 0, \forall k \in S, j \in N_k \qquad (8)$$

$$x_{ik} = \{0,1\}, \forall i \in D, k \in S \qquad (9)$$

$$\cancel{y_k = \{0,1\}, k \in S} \qquad (10)$$

$$f_{ijk} \ge 0, \forall i \in D, j \in N_i, k \in S \qquad (11)$$

- Demand assignment
- Contiguity of service areas



Political districting problem(PDP)

Compactness

Balance

Contiguity

$$\text{Minimize} \quad \cancel{\sum_{k\in S} f_k y_k} + \sum_{i\in D}\sum_{k\in S} d_i c_{ik} x_{ik} \qquad (1)$$

$$\text{Subject to:} \quad \sum_{k\in S} x_{ik} = 1, \forall i \in D \qquad (2)$$

$$(1-\varepsilon)\overline{P}y_k \le \sum_{i\in D} d_i x_{ik} \le (1+\varepsilon)\overline{P}y_k, \forall k \in S, \overline{P} = \sum_{i\in D} d_i/K \qquad (3)$$

$$\sum_{k\in S} y_k = K \qquad (4)$$

$$f_{ijk} \le (n-K)x_{ik}, \forall i \in D, j \in N_i, \forall k \in S \qquad (5)$$

$$f_{ijk} \le (n-K)x_{jk}, \forall i \in D, j \in N_i, \forall k \in S \qquad (6)$$

$$\sum_{j\in N_i} f_{ijk} - \sum_{j\in N_i} f_{jik} \ge x_{ik}, \forall i \in D, \forall k \in S, i \ne k \qquad (7)$$

$$f_{kjk} = 0, \forall k \in S, j \in N_k \qquad (8)$$

$$x_{ik} = \{0,1\}, \forall i \in D, k \in S \qquad (9)$$

$$y_k = \{0,1\}, k \in S \qquad (10)$$

$$f_{ijk} \ge 0, \forall i \in D, j \in N_i, k \in S \qquad (11)$$

- PDP is defined as a special case of CKFLSAP

3 Algorithm framework

- A unified algorithm for FLPs
  - Phase I: Initial solution
    - Lagrangian-relaxation based heuristic
    - Linear-programming based heuristic
    - Sub-problem
  - Phase II: Iterative improvement
    - Location search: ADD, DROP, SWAP, Multi-exchange
    - Assignment search: k-unit move (k=1,2,3)
  - Phase III: Set-partitioning

A unified algorithm framework

- Building blocks for FLP algorithms

Solver **MIP modeling:** FLSAP, CFLP, UFLP, CPMP, GAP, SAP, GAP/TP+SAP, K-medoids+SAP…
Heuristic: LR, Greedy, VND, ILS, SA, Tabu, GRASP, GA, EA …
Hybrid: LR+ILS, ILS+VND, GA+SPP, ILS+SPP …

Problem parameter setting; Solver parameter setting…

Search strategy
Acceptance rules: first, best, annealing, tabu …
Diversity search: multi-starts, ruin-repair, selection…
Exploring search: path-relink, restart best solution…
Leaning: evaluation of operators, parameter adjusting

Initial Solution
Region growth, Drop heuristic, Relaxation, K-medoids ….

Search operators
Assign search: single/multi-unit move
Location search: add, drop, swap, multi-exchange
Evolution: evaluation, selection, crossover, mutation

Relaxation
Lagrangian relax.
Linear relax.
Rounding…

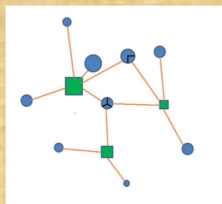
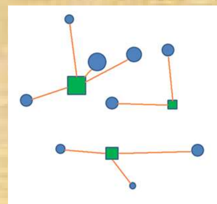
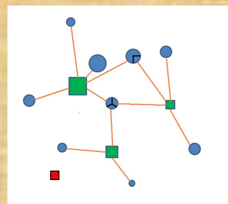
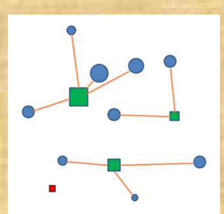
Basic functions Data input and output; Solution feasibility check, contiguity repair, objective evaluation; Boundary and neighbor search; Historic info. Record …

Models Variables, Objective function, Constraints on assignment, capacity, contiguity and number of facilities

Problem definition Spatial units, attributes and neighbors; Distance and cost matrix; Objective and constraints

MIP model building and solving

- Formulating and solving MIP models such as TP, GAP, FLP, SSCFLP/CKFLP, PMP/CPMP, SAP, FLSAP/CKFLSAP;

- Applications:
  - Solve a problem directly;
  - Solve a problem by solving its sub-problems, for examples
    - FLSAP=FLP-split + SAP, or FLP + SAP
    - PDP= TP+SAP, or K-medoids + SAP
  - Solve a sub-problem to construct the initial solution for heuristics.

Lagrangian relaxation for GAP

- GAP model

Minimize $\sum_{i \in D} \sum_{k \in S} d_i c_{ik} x_{ik}$

Subject to: $\sum_{k \in S} x_{ik} = 1, \forall i \in D$

$\sum_{i \in D} d_i x_{ik} \leq q_k, \forall k \in S$

$x_{ik} = \{0,1\}, \forall i \in D, k \in S$

- LR model

Minimize $\sum_{i \in D} \sum_{k \in S} (d_i c_{ik} x_{ik} - u_i) + \sum_{i \in D} u_i$

Subject to: $\sum_{i \in D} d_i x_{ik} \leq q_k, \forall k \in S$

$x_{ik} = \{0,1\}, \forall i \in D, k \in S$

- Linear decomposition(LD)

Minimize $\sum_{i \in D} (d_i c_{ik} x_{ik} - u_i), k \in S$

Subject to: $\sum_{i \in D} d_i x_{ik} \leq q_k$

$x_{ik} = \{0,1\}, \forall i \in D$

- Relationships

$$f(LR) = \sum_{k \in S} f(LD_k) \leq F(AP)$$

Lagrangian relaxation for SSCFLP

- FLP model

$$
\begin{aligned}
\text{Minimize} \quad & \textstyle\sum_{k\in S} f_k y_k + \sum_{i\in D} \sum_{k\in S} d_i c_{ik} x_{ik} \\
\text{Subject to:} \quad & \textstyle\sum_{k\in S} x_{ik} = 1, \forall i \in D \\
& \textstyle\sum_{i\in D} d_i x_{ik} \le q_k y_k, \forall k \in S \\
& x_{ik} = \{0,1\}, \forall i \in D, k \in S \\
& y_k = \{0,1\}, k \in S
\end{aligned}
$$

- LR model

$$
\begin{aligned}
\text{Minimize} \quad & \textstyle\sum_{k\in S} f_k y_k + \sum_{i\in D} \sum_{k\in S} (d_i c_{ik} x_{ik} - u_i) + \sum_{i\in D} u_i \\
\text{Subject to:} \quad & \textstyle\sum_{i\in D} d_i x_{ik} \le q_k y_k, \forall k \in S \\
& x_{ik} = [0,1], \forall i \in D, k \in S \\
& y_k = \{0,1\}, k \in S
\end{aligned}
$$

- Linear decomposition(LD)

$$
\begin{aligned}
\text{Minimize} \quad & f_k + \textstyle\sum_{i\in D} (d_i c_{ik} x_{ik} - u_i) \\
\text{Subject to:} \quad & \textstyle\sum_{i\in D} d_i x_{ik} \le q_k \\
& x_{ik} = [0,1], \forall i \in D
\end{aligned}
$$

$$F(LR) = \textstyle\sum_{k\in S} min(0, f_k + F(LD_k)) \le F(FLP) \qquad \textbf{?!}$$

Lagrangian-relaxation heuristic

(1) Set Lagrange multipliers u, upper bound *ub*=∞ and lower bound *lb*=-∞;

(2) While the stop criteria is not met:

(3) Solve F(LR), get y_k, x_{ik};

(4) If F(LR)>*lb*:

(5) *lb*= F(LR);

(6) Construct a feasible solution of F(FLP) based on y_k and x_{ik};

(7) If F(FLP)<*ub*: *ub*=F(FLP);

(8) Adjust multipliers u based on y_k, x_{ik},*ub* and *lb;*

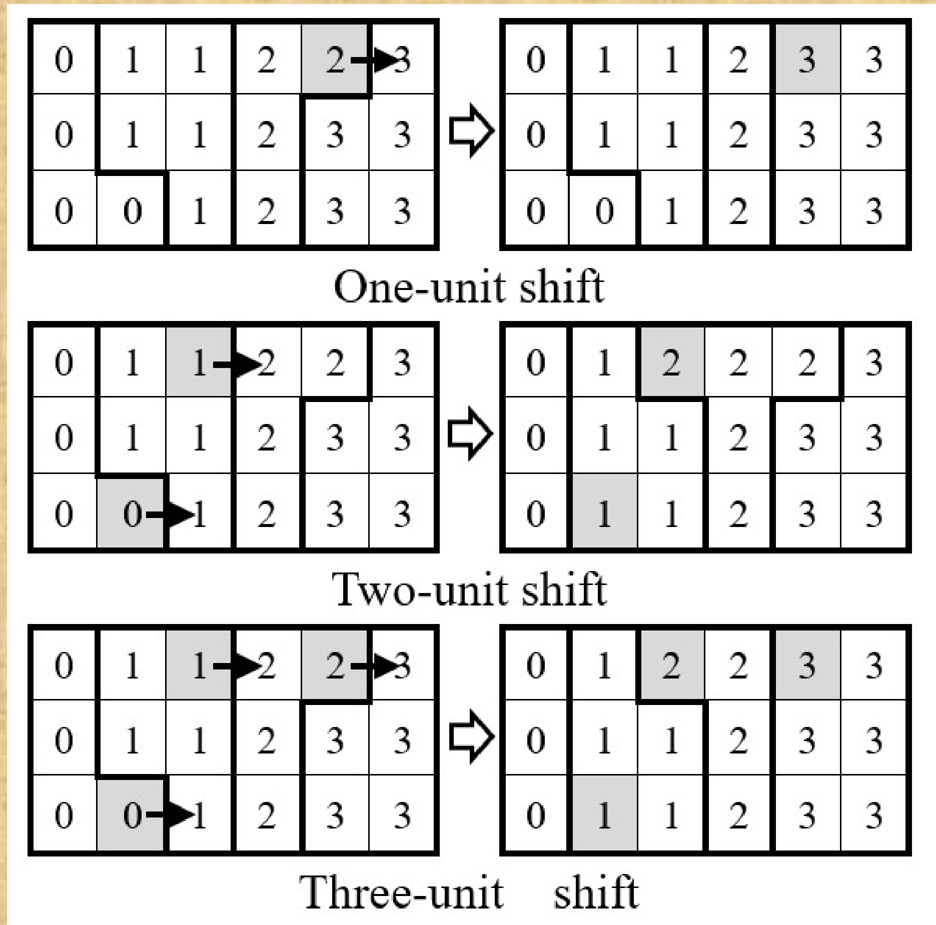
(9) Output *ub*, *lb* and the best feasible solution.

Key techniques: feasible solution of F(FLP)

Assignment operators

- One-unit move (O(n))
- Two-unit move (O(n^2))
- Three-unit move (O(n^3))

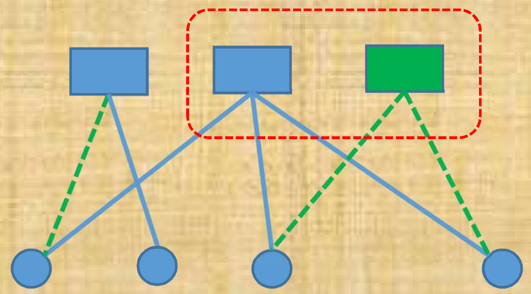
- Implemented based on boundary units and their neighborhood service areas.



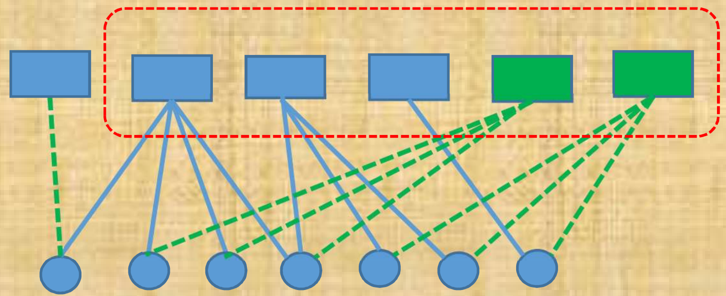
Location search operators

- ADD: greedily select a candidate facility to open;
- DROP: greedily select a facility to close;
- SWAP: greedily select a candidate facility to open, and a facility to close;
- Multi-Exchange: based on a hyper graph, greedily select *r* candidate facilities to open, and *s* facilities to close;

- The operators are designed based on a cost-reduction hypergraph.



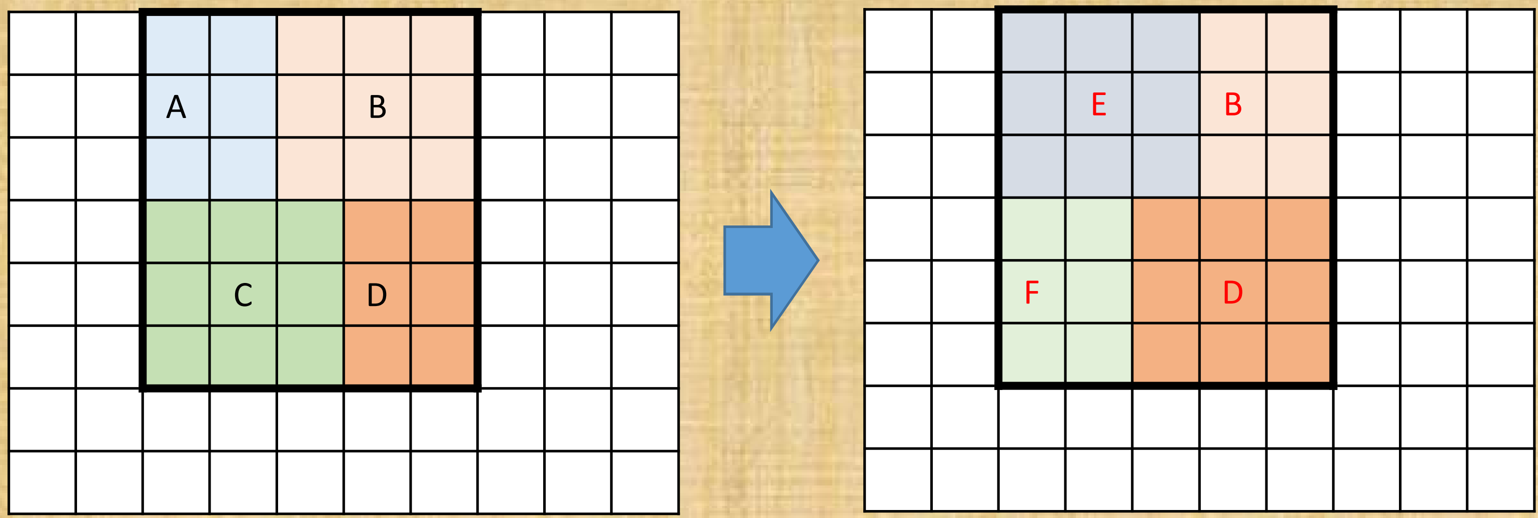
SWAP



Multi-Exchange

Spatially constrained location search

- Randomly select a connective geographic region;
- For all candidate locations, greedily select new locations, and assign all the demand units to new locations;
- The computational complexity of ADD, DROP, SWAP and multi-exchange can be reduced evidently;
- Transportation cost is usually increased or decreased slightly;
- Contiguity areas can be easily maintained.



SPP model for selecting a better solution

- Let set Ω={1,2,3…} be the service areas identified by heuristics;
- For each area *i*, these is a set of assigned units U_i and cost c_i (facility cost and transportation cost);
- SPP(set partitioning problem) model:

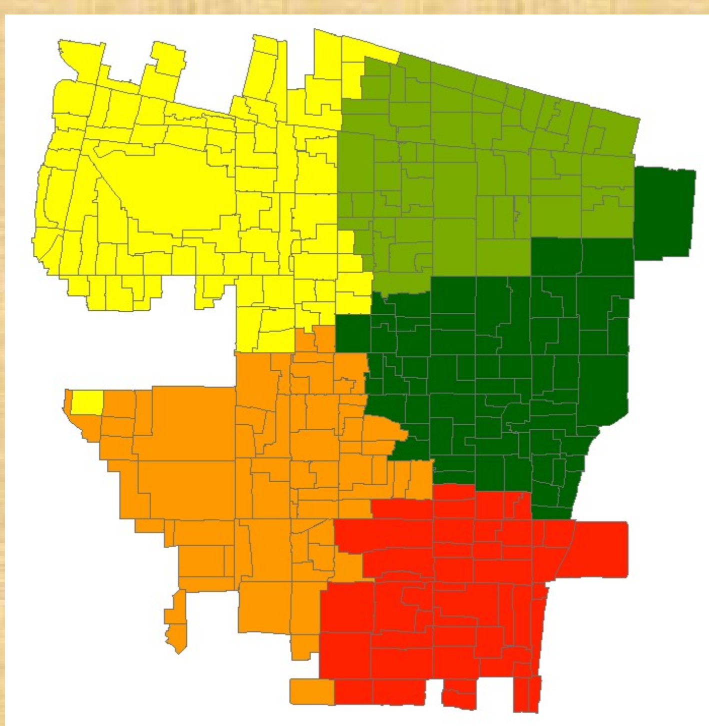
$$\text{Minimize} \quad \sum_{i \in \Omega} c_i x_i$$

$$\text{Subject to:} \quad \sum_{i \in \Omega, j \in U_i} x_i = 1, \forall j \in V$$

$$x_i \in \{0,1\}, \forall i \in \Omega$$

Operations for areal contiguity

- Check of areal contiguity: spanning tree method;
- Repair for areal contiguity: delete fragmented units and reassign them to their; neighborhood service areas.
- Find boundary units;
- Find neighborhood service areas.

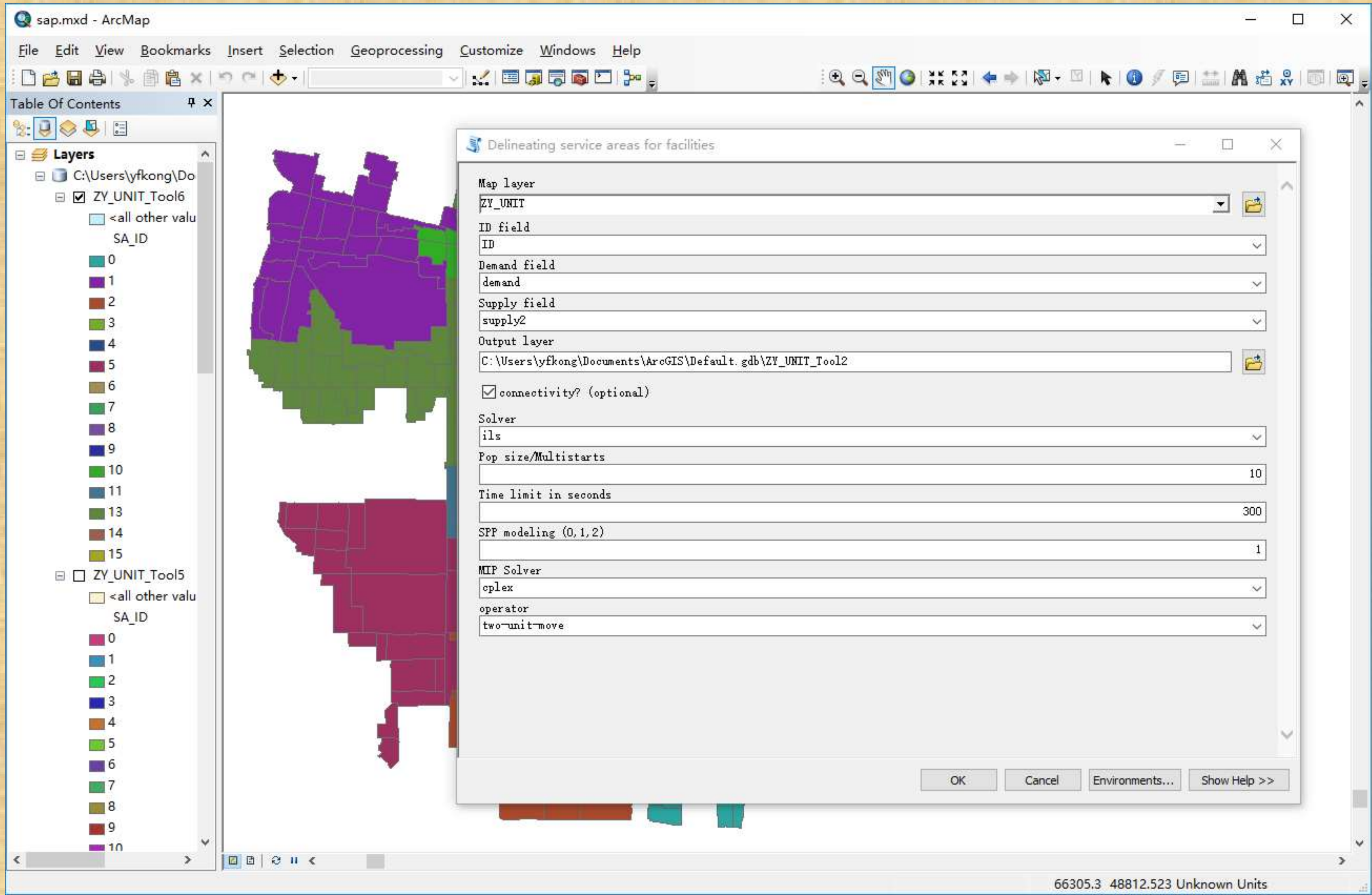


Hybrid heuristic: LR+ILS+SPP

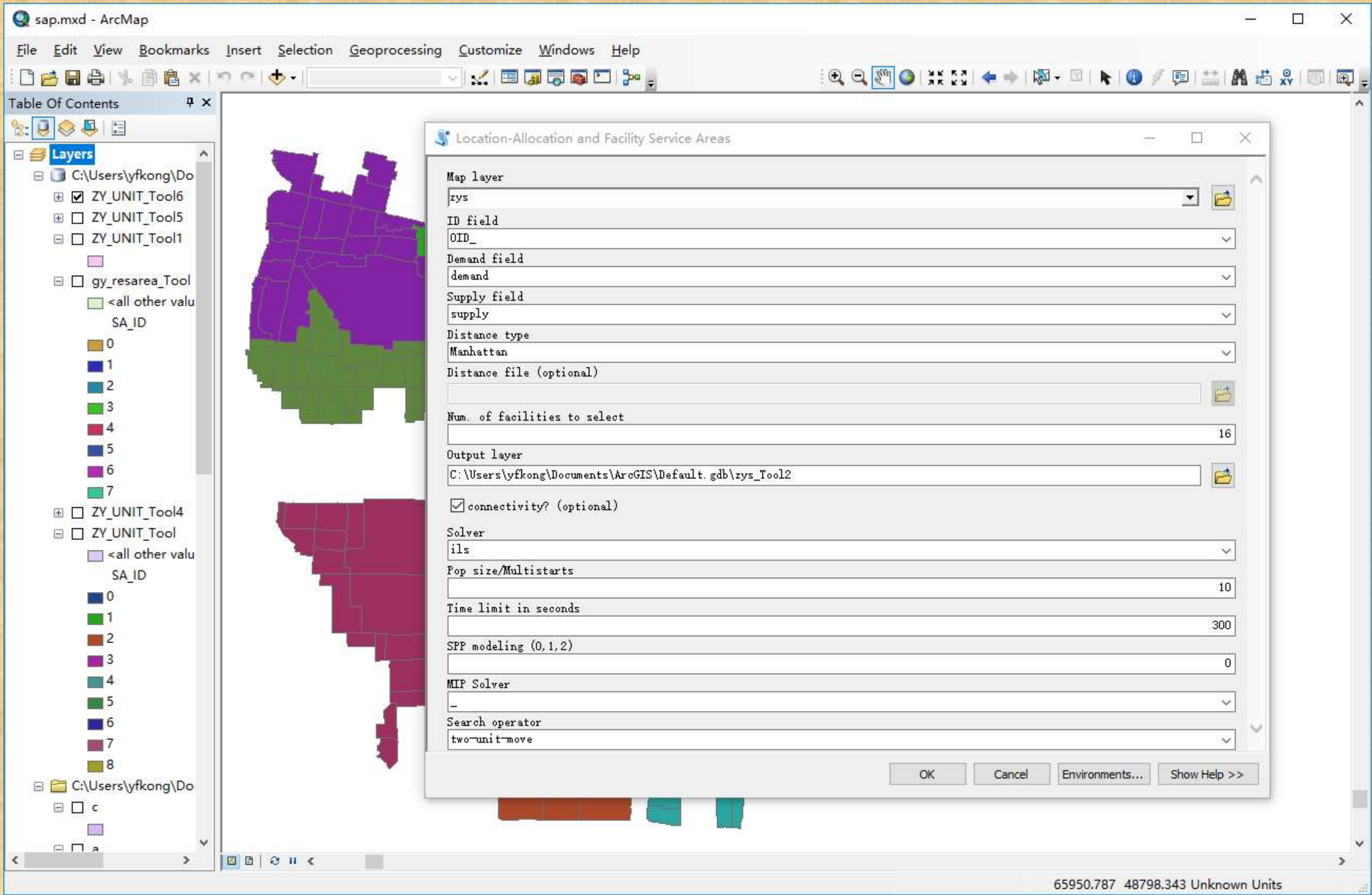
Parameters: m (number of best solutions), t (time limit)

- Phase I: Initial solution
  - Lagrangian-relaxation heuristic
  - Select m-best solutions
- Phase II: Iterative improvement
  - Randomly select a solution from the m-best solutions
  - Solution search using the following operators randomly
    - Spatially constrained DROP, SWAP and Multi-Exchange
    - Ruin-and-Repair
  - Assignment search: one-unit move, two-unit move
  - Update the m-best solutions
- Phase III: Set-partitioning

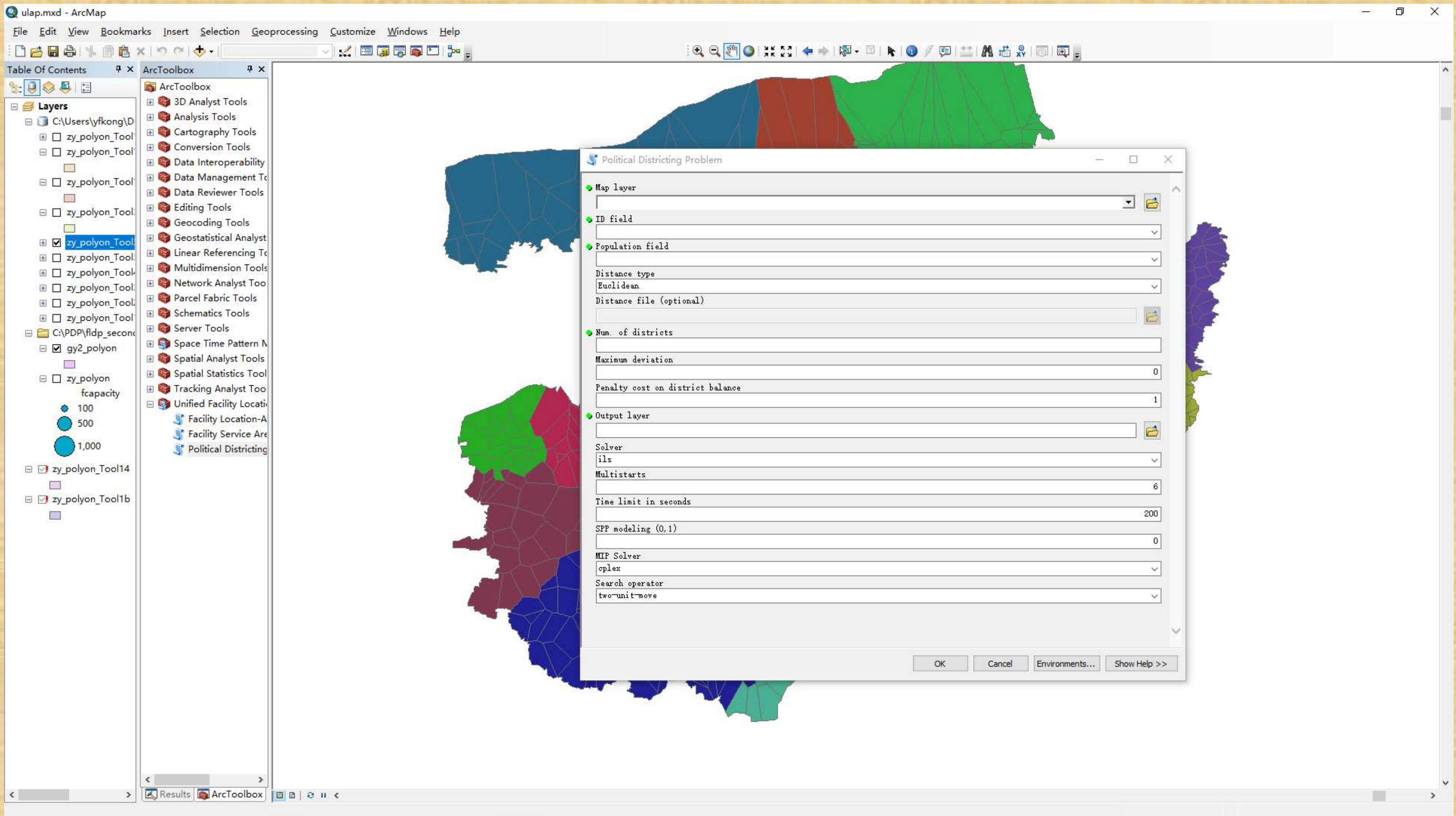
ArcGIS tool: Facility Service Area Problem



ArcGIS tool: Facility Location and Service Area Problem



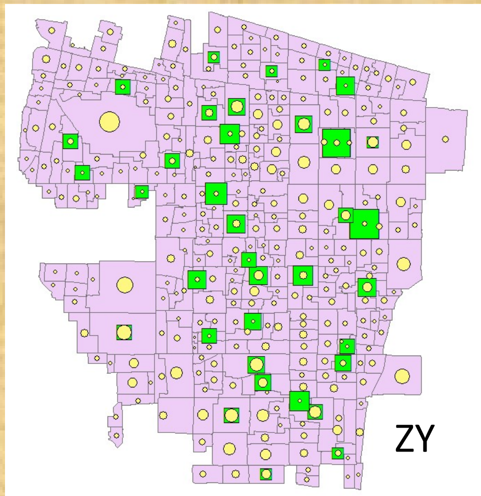
ArcGIS tool: Political Districting Problem

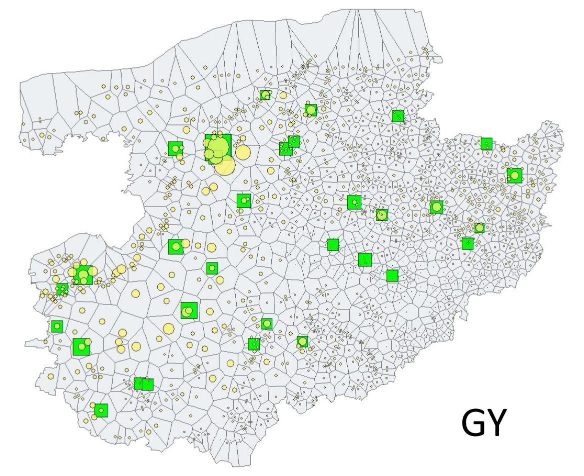


4 Experiments

- Algorithms: MILP, LR+ILS+SPP
- Computing environment:
  - HP desktop with Inter Core i7-6700 CPU @3.4GHz, 8G Ram
  - Win10, PyPy, CPLEX 12.6, ArcGIS 10.4
- Benchmark instances:

Dataset	Num.inst.	Instance size	Problem
ZY, GY	2	36*324, 34*1287	SSCFLP, FLSAP, CKFLSAP
Holmberg (1999)	71	16*50, 25*50, 50*50	SSCFLP
Beasley (1988)	36	10~50 * 30~200, 100*1000	SSCFLP
Yang (2012)	20	30*200, 60*200, 60*300, 80*400	SSCFLP
Avella (2009)	100	300*300, 300*1500, 500*500, 700*700, 1000*1000	SSCFLP





Solution results for instances of ZY and GY

- Time limit: 100s
- Run 10 times on each instance
- Solution gaps (compared with optimal or near-optimal solutions from MILP modeling)

	Problem	Num. of facilities	Gap/%
ZY	SSCFLP	-	0.31%
GY	SSCFLP	-	0.48%
ZY	FLSAP	-	0.43%
GY	FLSAP	-	0.10%
ZY	CKFLP	13-22	0.06%~1.56%
GY	CKFLP	18-30	0.42%~1.20%
ZY	CKFLSAP	13-22	0.04%~1.42%
GY	CKFLSAP	18-30	0.30%~0.80%

Results for Beasley and Holmberg instances

- Solution gaps compared with optimal solutions.

Dataset	Num. of inst.	Time limit/s	Num. of optimality	Gaps/%
cap61-cap124	24	3	21	0.00-0.07
capa1-capc4	12	100	0	0.00~0.19
p1-p24	24	3	24	0
p25-p40	16	10	11	0.01-0.46
p41-p55	15	5	12	0.01-0.07
p56-p71	16	10	13	0.07-0.30

instance	Gap/%
Capa1	0.03
Capa2	0.00
Capa3	0.00
Capa4	0.00
Capb1	0.00
Capb2	0.08
Capb3	0.19
Capb4	0.00
Capc1	0.02
Capc2	0.00
Capc3	0.19
capc4	0.00

SSCFLP heuristic results for Yang instances

Solution gaps compared with optimal solutions.

		LR+ILS+SPP		Kernel Search (2014)		Cut-Solve (2012)	
Instance	Optimal	Time/s	Gap/%	Time/s	Gap%	Time/s	Gap/%
30_200_1	30181	50	0.25%	964.378	0	396.5	0
30_200_2	28923	50	0.71%	1021.459	0	571.9	0
30_200_3	28131	50	0.69%	11.232	0	48.33	0
30_200_4	28152	50	0.05%	55.52	0	66.1	0
30_200_5	27646	50	0.00%	3.822	0	20.2	0
60_200_1	27977	50	0.40%	1637.051	0	798.2	0
60_200_2	29704	50	0.32%	2702.611	0.01%	44636.8	0
60_200_3	27993	50	0.03%	28.564	0	1524	0
60_200_4	27691	50	0.08%	1132.063	0	1047.1	0
60_200_5	29195	50	0.23%	2701.831	0	32978.2	0
60_300_1	35648	50	0.36%	2238.729	0.01%	2051.4	0
60_300_2	35474	50	0.11%	72.368	0	225.9	0
60_300_3	33872	50	0.27%	85.987	0	1910.9	0
60_300_4	33096	50	0.28%	577.528	0	873.2	0
60_300_5	30918	50	0.20%	10.67	0	1611.8	0
80_400_1	39318	50	0.83%	616.419	0	3998.3	0
80_400_2	37076	50	0.46%	987.061	0	1583.9	0
80_400_3	43859	50	0.31%	2702.83	0	15358.4	0
80_400_4	37344	50	0.02%	35.366	0	5208	0
80_400_5	43508	50	0.57%	2703.875	0	8899	0

Solution results for TBED1 instances

Solution gaps compared with optimal or best known solutions.

Instance	Size	Time/s	Gap/%
i300 (1-5)	300*300	200	0.28-1.08
i300 (6-10)	300*300	200	0.23-0.86
i300(11-15)	300*300	200	0.08-0.66
i300(16-20)	300*300	200	0.02-0.40
i3001500 (1-5)	300*1500	200	0.01-0.07
i3001500 (6-10)	300*1500	200	0.00-0.04
i3001500 (11-15)	300*1500	200	0.00-0.03
i3001500 (16-20)	300*1500	200	0.00-0.00
i500 (1-5)	500*500	500	0.96-1.30
i500 (6-10)	500*500	500	0.32-0.67
i500(11-15)	500*500	500	0.11-0.90
i500(16-20)	500*500	500	0.06-0.36

Instance	Size	Time/s	Gap/%
i700 (1-5)	300*300	1000	0.65-1.98
i700 (6-10)	300*300	1000	0.10-1.17
i700(11-15)	300*300	1000	0.16-0.72
i700(16-20)	300*300	1000	0.13-0.34
i1000 (1-5)	1000*1000	1000	0.50-1.81
i100 0(6-10)	1000*1000	1000	0.30-0.88
i1000 (11-15)	1000*1000	1000	0.24-0.57
i1000 (16-20)	1000*1000	1000	0.19-0.40
* Gap: the gap to the optimal or best known solution.			

TBED1: the hardest benchmark instances for SSCFLP.
Kernel Search: average solution time=3628s (Guastaroba et al. 2012)
Multi-Exchange: average solution time=3701s (Tran et al. 2017)

5 Conclusion

- A unified algorithm framework was introduced to solve facility location and service area problems. The hybrid algorithm LR+ILS+SPP can be used to solve problems such as PMP/CPMP, UFLP/CFLP/SSCFLP/CKFLP, SAP/FLSAP/CKLPSAP, and PDP.
- Experiments show that multiple location problems can be solved effectively and efficiently.

Dataset	Num.inst.	Instance size	Gap/%	Time/3
ZY, GY	2	36*324, 34*1287	0.04-1.56	100
Holmberg	71	16*50, 25*50, 50*50	0.00-0.46	3-10
Beasley	36	10~50 * 30~200, 100*1000	0.00-0.19	3,100
Yang	20	30*200, 60*200, 60*300, 80*400	0.02-0.83	50
Avella	100	300*300, 300*1500, 500*500, 700*700, 1000*1000	0.00-1.98	200-1000

Further research

- More effective spatially-constrained location search for solving FLPs;
- Further testing on the new location problems: CKFLP, FLSAP, and CKFLSAP;
- Hyper-heuristic method for adaptively selecting parameters, operators, and algorithms based on instance analysis and online learning;
- Development of application tools in GIS software.

Questions or suggestions?

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https://github.com/yfkong/unified

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Yunfeng Kong, A unified metaheuristic framework and key algorithmic mechanisms for the regionalization problem (41871307), 2019.01-2022.12