Causal Inference Crash Course Part 2: Defining Some Causal Models

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Causal Inference Series

1) Foundations
2) **Defining Some ATE/ATET Causal Models**
3) ATE/ATET Inference, Asymptotic Theory, and Bootstrapping
4) Best Practices: Outliers, Class Imbalance, Feature Selection, and Bad Control
5) Heterogeneous Treatment Effect Models and Inference
6) Difference-in-Difference Models for Panel Data
7) Regression Discontinuity Models
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Overview

- This presentation will define some propensity-matching based models:
  A. Ordinary Least Squares (OLS)
  B. Propensity Binning with Regression adjustment
  C. Inverse propensity weighting
  D. Double machine learning - Partial Linear Model (PLM)
  E. Double machine learning - Interactive Regression Model (IRM)
- For each model, we will define the estimator and its properties.
- So yes, there will be a lot of math.

Ordinary Least Squares (OLS)

- We have an outcome Y_i, pre-treatment features X_i, and a treatment indicator T_i. We want to know the causal relationship between T_i and Y_i.
- We can estimate this relationship by estimating an OLS model:

$$Y_i = \hat{\beta} X_i + \hat{\tau} T_i + \epsilon_i$$

- Where $(\hat{\beta}, \hat{\tau})$ are estimated to minimize the mean squared error:

$$argmin_{\hat{\beta},\hat{\tau}} \{ \frac{1}{N} \sum_{i=1}^{N} (Y_i - \hat{\beta} X_i - \hat{\tau} T_i)^2 \}$$

- We know OLS is simple but why is it causal?

Why is OLS Causal?

- The OLS model estimates $(\hat{\beta}, \hat{\tau})$ based on the assumption that the mean squared error is zero, conditional on (X_i, T_i)
- This is defined as the moment conditions: $E\left[(Y_i - \hat{\beta}X_i + \hat{\tau}T_i)\times X_i)\right] = 0$ and $E\left[(Y_i - \hat{\beta}X_i + \hat{\tau}T_i)\times T_i)\right] = 0$
  - After conditioning on X_i , the unexplained variation in Y_i is mean independent of treatment.
  - Therefore, they correspond to the unconfoundedness assumption
- Under mean independence, $\hat{\tau}$ is an unbias ATE / ATET estimate.

OLS as a propensity-based matching model

- OLS implicitly estimates a propensity score
- Recall that the OLS estimator is:

$$\hat{\beta} = (X_i{}' X_i)^{-1} X' Y_i = \frac{cov(X_i, Y_i)}{var(X_i, X_i)}$$

- From the Frisch-Waugh-Lovell theorem, we can be more specific about $\hat{\tau}$. (Appendix has more details)

$$\hat{\tau} = \frac{cov(\tilde{T}_i, Y_i)}{var(\tilde{T}_i, \tilde{T}_i)}$$

- Where $\tilde{T}_i = E[T_i|X_i] - T_i$. Well, $E[T_i|X_i]$ is the propensity score!

Propensity Binning with Regression Adjustment

- What is Regression Adjustment?
- Recall the potential outcome (Neyman-Rubin) framework:
  - Average Treatment Effect on the Treated (ATET) = $Y_i(1,1) - Y_i(1,0)$
  - Average Treatment Effect (ATE) = $\mathrm{E}_1(Y_i(1,1), Y_i(0,1)) - \mathrm{E}_0[Y_i(0,0), Y_i(1,0)]$
    - where E_x is the weighted average of observed and counterfactuals for $T = x$.
- The problem is that we do not observe $Y_i(0,1)$ and $Y_i(1,0)$
- Regression Adjustment model asks: "What if we treat estimating $Y_i(0,1)$ and $Y_i(1,0)$ as a pure prediction problem and predict out of sample?"

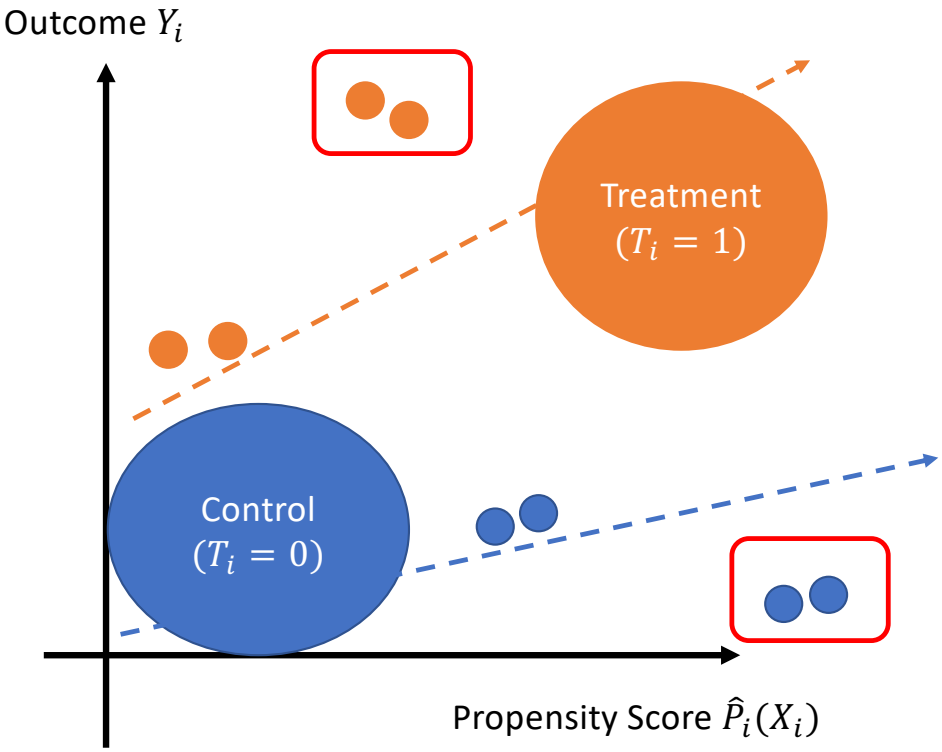
Regression Adjustment Algorithm

1. Start with the control ($T_i = 0$) sample. Train your favorite ML model to predict $Y_{i,T_i=0}$ using $X_{i,T_i=0}$. Call this trained model $g_0(X_i)$. Do the same with the treatment ($T_i = 1$) sample, call the trained model $\mathrm{g}_1(X_i)$.
2. Estimate the outcomes under treatment and control with $\mathrm{g}_0(X_i)$ and $\mathrm{g}_1(X_i)$. This gives you $\widehat{Y_i(1,1)}$, $\widehat{Y_i(0,1)}$, $\widehat{Y_i(1,0)}$, and $\widehat{Y_i(0,0)}$.
3. Estimate ATE or ATET:
    1. $\mathrm{ATET} = \widehat{Y_i(1,1)} - \widehat{Y_i(1,0)}$
    2. $ATE = \{\, \widehat{Y_i(1,1)}, \widehat{Y_i(0,1)} \,\} - \{\, \widehat{Y_i(1,0)}, \widehat{Y_i(0,0)} \,\}$

Regression Adjustment is OLS

- Regression Adjustment is OLS in disguise.
- Intuitively, this is because the OLS is also extrapolating the potential outcomes with a single model, instead of multiple. (Appendix has the technical explanation.)
- Therefore, Regression Adjustment implicitly estimates a propensity score because OLS does too.

Propensity Binning as Insurance Against Outliers



- We show in the dotted lines the predicted outcomes for control and treatment.
- But we have outliers, based on propensity score, which are influencing our predicted outcomes.
- How can we flexibly accommodate the outliers?

Propensity Binning with Regression Adjustment

- We estimate a propensity score, $\hat{P}_i(X_i)$, and then divide the data into segments of $\hat{P}_i(X_i)$.
- For each segment, implement the Regression Adjustment model.

Inverse Propensity Weighting

- This approach is inspired by sampling methods.
- Suppose you have :
  - (A) treatment observation with a propensity score of 0.99
  - (B) treatment observation with a propensity score of 0.01
- You most likely have a lot of (A), but not a lot of (B). So you want to give (B) more weight in your analysis because it happens very rarely.

Inverse Propensity Weighting Model Definition

- The Inverse Propensity Weighting (IPW) estimator for the ATE is:

$$\frac{1}{N}\sum_{i=1}^{N}\left[\frac{T_i\,Y_i}{\hat{P}(X_i)}-\frac{(1-T_i)Y_i}{1-\hat{P}(X_i)}\right]$$

- For ATET:

$$\frac{1}{N}\sum_{i=1}^{N}\hat{P}(X_i)\left[\frac{T_i\,Y_i}{\hat{P}(X_i)}-\frac{(1-T_i)Y_i}{1-\hat{P}(X_i)}\right]$$

- We will unpack the ATE to intuitively understand it.

Inverse Propensity Weighting Intuition

$$\frac{1}{N}\sum_{i=1}^{N}\left[\frac{T_i\, Y_i}{\hat{P}(X_i)} - \frac{(1-T_i)Y_i}{1-\hat{P}(X_i)}\right]$$

- When we divide a treatment observation by the propensity score, we are increasing its importance when it is less likely to be treated.
- Note that the denominator of $\frac{1}{N}\frac{1}{\hat{P}(X_i)}$ approximates taking the average of just the treated observations.

Advantages and Disadvantages

- **Advantages:** Weighting provides flexible form and only requires diagnosing with propensity score
- **Disadvantage**: since we divide by the propensity score, you risk imprecise estimates if you have a lot of propensity scores near zero or one.
  - This means the variance can explode and be very large.
- Solutions are:
  - Drop these observations;
  - Replace these observations' propensity scores with a pre-determined value (like 0.001 or 0.999) or unconditional probability of treatment.

Double Machine Learning (DML)

- Yes, we are finally here.
- At a high-level, DML is a more flexible version of the previous models
- Note that DML relies on the same assumptions as the other models presented here
- We will cover the propensity-matching based models from the Chernozhukov et al. (2016) paper.
  - Partial Linear Model
  - Interactive Regression Model

What can ML do for causal inference?

- Off-the-shelf ML models also cannot be used for inference.
  - Exceptions: generalized random forests (Athey et al. 2018); neural nets (Farrell et al. 2020)
- We can use ML in two ways:
- (1) Estimate a better propensity score. Recall that for OLS: $\hat{\tau} = \frac{cov(\tilde{T}_i, Y_i)}{var(\tilde{T}_i, \tilde{T}_i)}$
- (2) Estimate better counterfactuals as we do for Regression Adjustment models

High-Level Strategy for DML

- We use two ML strategies so we can do inference:
- (1) Regularization based on residualization
  - Based on Frisch-Waugh-Lovell theorem (recall this from the OLS slides?)
  - Compare residuals that are constructed to be independent except due to the variation of interest (Neyman orthogonality)
- (2) Sample-splitting to prevent overfitting
  - Cross-validation is important to make sure prediction is not biased

DML – Partial Linear Model Motivation

- The partial linear model is a form of OLS. We allow a more flexible prediction of Y_i and T_i
- OLS:

$$Y_i = \hat{\beta} X_i + \hat{\tau} T_i + \epsilon_i$$
$$Y_i = g_0(X_i) + \hat{\tau} T_i + \epsilon_i$$
$$Y_i - g_0(X_i) = g_0(X_i) - g_0(X_i) + \hat{\tau} T_i + \epsilon_i$$
$$Y_i - g_0(X_i) = \hat{\tau} T_i + \epsilon_i$$

- We replace $\hat{\beta} X_i$ with $g_0(X_i)$. From this we can show that estimating a regression of the residualized Y_i based on $g_0(X_i)$ on T_i estimates the treatment effect.
- OLS also requires a residualized T_i, from the Frisch-Waugh-Lovell theorem.

DML – Partial Linear Model Setup

- The partial linear model is a form of OLS.
- OLS:

$$Y_i = \hat{\beta} X_i + \hat{\tau} T_i + \epsilon_i$$

- DML – Partial Linear Model:

$$Y_i = g_0(X_i) + \hat{\tau} T_i + \epsilon_i$$
$$T_i = m_0(X_i) + \nu_i$$

- Assumptions are still the same as OLS (especially unconfoundedness)
- We still assume the treatment effect is linearly additive

Partial Linear Model Procedure

$$Y_i = g_0(X_i) + \hat{\tau} T_i + \epsilon_i$$
$$T_i = m_0(X_i) + \nu_i$$

- First Stage:
  1. Predict Y_i using X_i with sample-splitting, get $\hat{Y}_i$
  2. Predict T_i using X_i with sample-splitting , get $\hat{T}_i$

  Sample-splitting

  3. Calculate the residuals for Y_i and T_i.
     Specifically, $\tilde{Y}_i = Y_i - \hat{Y}_i$ and $\tilde{T}_i = T_i - \hat{T}_i$

  Residualization

- Second Stage:
  - Estimate this OLS model:

$$\tilde{Y}_i = \hat{\tau} \tilde{T}_i + \zeta_i$$

What ML models can be used?

- You can use essentially any ML model for the first stage to generate $\hat{Y}_i$ and $\hat{T}_i$
- These are called the “nuisance parameters” because we care about the quality of the prediction, not the theoretical properties of the ML model.

DML – Interactive Regression Model

- The partial linear model is simple and intuitive because it is a more flexible version of OLS.
- Despite this, we are restricted by the assumption that treatment linearly interacts with the outcome.
- It also assumes that the treatment effect is the same for all observations. Specifically, that the average treatment effect (ATE) is the same as the average treatment effect on the treated (ATET).

Interactive Regression Model (IRM) Specification

$$\hat{\tau}_{ATE} = E[\left(\hat{Y}_{1,i} - \hat{Y}_{0,i}\right) + \frac{T_i\left(Y_i - \hat{Y}_{1,i}\right)}{\hat{p}_i} - \frac{(1-T_i)\left(Y_i - \hat{Y}_{0,i}\right)}{1-\hat{p}_i}]$$

$$\hat{\tau}_{ATET} = \frac{T_i(Y_i - \hat{Y}_{0,i})}{p \times \hat{p}_i} - \frac{\hat{p_i}(1-T_i)(Y_i - \hat{Y}_{0,i})}{p \times (1-\hat{p}_i)}$$

- When we estimate ATET, we no longer need $\hat{Y}_{1,i}$
- Estimating the IRM model has the same first stage as PLM. The only difference is the second stage.
- Now we will explain the components of ATE, which generalize to ATET.

Explaining the IRM Model, Part 1

$$\hat{\tau}_{ATE} = \left(\hat{Y}_{1,i} - \hat{Y}_{0,i}\right) + \frac{T_i(Y_i - \hat{Y}_{1,i})}{\hat{p}_i} - \frac{(1 - T_i)\left(Y_i - \hat{Y}_{0,i}\right)}{1 - \hat{p}_i}$$

- This part is just regression adjustment, which is bias if there is large estimation error
- These parts are the estimation error of the outcome for treatment and control units, which are weighted by propensity scores
- We combine regression adjustment and propensity weighting for a "doubly robust" approach (more details in the appendix) where we can correct for our regression adjustment estimates.

Explaining the IRM Model, Part 2

- Rewriting the previous equation:

$$\hat{\tau}_{ATE} = \left(\hat{Y}_{1,i} - \hat{Y}_{0,i}\right) + \frac{T_i(Y_i - \hat{Y}_{1,i})}{\hat{p}_i} - \frac{(1 - T_i)\left(Y_i - \hat{Y}_{0,i}\right)}{1 - \hat{p}_i}$$

$$= \left(\hat{Y}_{1,i} + \frac{T_i(Y_i - \hat{Y}_{1,i})}{\hat{p}_i}\right) - \left(\hat{Y}_{0,i} - \frac{(1 - T_i)\left(Y_i - \hat{Y}_{0,i}\right)}{1 - \hat{p}_i}\right)$$

- Therefore, we are correcting our estimates of $\hat{Y}_{1,i}$ and $\hat{Y}_{0,i}$.
- Applying this observation level correction means that there is variation at treatment estimates at the observation level.

Conclusion

- This presentation defines some propensity-matching based models:
  A. Ordinary Least Squares (OLS)
  B. Propensity Binning with Regression adjustment
  C. Inverse propensity weighting
  D. Double machine learning - Partial Linear Model (PLM)
  E. Double machine learning - Interactive Regression Model (IRM)
- Remember they all rely on the same assumptions!

Appendix Slides

Frisch-Waugh-Lovell Theorem

- The OLS estimator is based on the Frisch-Waugh-Lovell theorem, or "partialing out".
- Pay attention. This is the same trick behind double machine learning.
- Start with:

$$cov\left(\tilde{T}_i, Y_i\right) = cov\left(\tilde{T}_i, \beta X_i + \tau T_i + \epsilon_i\right)$$

  - Know that $cov\left(\tilde{T}_i, \beta X_i\right) = 0$ because $\tilde{T}_i = E[T_i|X_i] - T_i$, so it already conditions on X_i.
  - Also know that $cov\left(\tilde{T}_i, \epsilon\right) = 0$ for the same reason.
- Then: $cov\left(\tilde{T}_i, Y_i\right) = cov\left(\tilde{T}_i, \tau T_i\right) = \tau\ var(\tilde{T}_i, \tilde{T}_i)$

Is LASSO an improvement on OLS?

Model	Ordinary Least Squares (OLS)	Least Absolute Shrinkage and Selection Operator (LASSO)
Objective Function	$argmin_{\hat{\beta},\hat{\tau}}\{\frac{1}{N}\sum_{i=1}^{N}(Y_i - \hat{\beta}X_i - \hat{\tau}T_i)^2\}$	$argmin_{\hat{\beta},\hat{\tau}}\{\frac{1}{N}\sum_{i=1}^{N}(Y_i - \hat{\beta}X_i - \hat{\tau}T_i)^2\}$ $subject\ to\ \sum_{j=1}^{J}\lvert\hat{\beta}_j\rvert + \lvert\hat{\tau}\rvert \leq C$

- LASSO regression coefficients are chosen to maximize prediction, subject to a constraint in the parameters.
- Intuitively, it assumes that coefficients are zero and there are penalties non-zero coefficients.
- Certainly, LASSO has better out-of-sample prediction. But can we use it for causal inference?

Can we use LASSO for causal inference?

- No, we can't. Here is a technical and intuitive explanation.
- Technically, OLS identifies the causal estimate because of this moment condition you can get from solving the optimization problem:

$$E\left[(Y_i - \hat{\beta}X_i + \hat{\tau}T_i) \times T_i)\right] = 0$$

  But you can't get this from a LASSO.
- Intuitively, a LASSO coefficient has two interpretations: the causal estimate of $\hat{\tau}$, and a feature selection of whether T_i is important to the prediction problem.
  - Then the unconfoundedness assumption may no longer hold.

Regression Adjustment is OLS

- Regression Adjustment is OLS in disguise.
- Suppose our train models are linear functions:

$$g_0(X_i) \equiv Y_{0,i} = \hat{\beta}_0 X_i + \epsilon_{0,i} \text{ ; and}$$

$$g_1(X_i) \equiv Y_{1,i} = \hat{\beta}_1 X_i + \epsilon_{1,i}$$

- We can show this maps to an OLS model.
- Note that $Y_i = Y_{1,1}(T_i) + Y_{0,0}(1 - T_i)$ which we can replace with the trained models.

$$Y_i = \hat{Y}_{1,1}(T_i) + \hat{Y}_{0,0}(1 - T_i) = \left(\hat{\beta}_1 X_i + \epsilon_{1,i}\right)(T_i) + (\hat{\beta}_0 X_i + \epsilon_{0,i})\ 1 - T_i)$$

Rearranging: $Y_i = \hat{\beta}_0 X_i + (\hat{\beta}_1 X_i - \hat{\beta}_0 X_i)\ T_i + (\epsilon_{1,i} - \epsilon_{0,i})\ T_i$

Then this is our treatment estimate from OLS, $\hat{\tau}$

The relationship between ATET/ATE and the linearity assumption

- Notation for this slide: $\mathrm{Y_{j,k}}$ is the outcome of the treatment status = j group for the observed group whose treatment status = k
- ATET = $\mathrm{E}[\ \mathrm{Y}_{1,1}] \ - E[Y_{0,1}]$
- ATE = $\ p\ E[Y_{1,1}] + (1-p)E[Y_{1,0}] \ - p\ E[Y_{0,1}] + (1-p)E[Y_{0,0}]$
- ATET = ATE when $E[Y_{1,0}]$ = $E[Y_{1,1}]$ and $E\left[Y_{0,1}\right] = E[Y_{0,0}]$
- In other words, we think that the treatment group's baseline outcome is the same as the control group's baseline outcome, and vice versa.
- The linearity assumption in OLS assumes ATET = ATE

Appendix Slides – Doubly Robust Models

Doubly Robust

- What if the propensity score is wrong?
  - It can be wrong because we do not have enough features, or we have the incorrect model specification.
- This causes a problem because recall that we can estimate the treatment effect using the difference between treatment status and the propensity score. Recall from the OLS slides:

$$\hat{\tau} = \frac{cov(\tilde{T}_i, Y_i)}{var(\tilde{T}_i, \tilde{T}_i)}$$

  where $\tilde{T}_i = E[T_i|X_i] - T_i$.

- Ideally $\tilde{T}_i$ represents the conditionally random variation in treatment. But if the propensity model is wrong, then it also incorporates model error.
- Then, we have the wrong value for $\hat{\tau}$!

Model mis-specification

- The same logic applies to predicting the counterfactual outcome.
- Model mis-specification isn’t completely solved with a good ML model.
- For example, a counterfactual outcome prediction is an extrapolation exercise.

Doubly Robust Mental Model

- A “doubly robust” model attempts to address model mis-specification.
- It incorporates:
  - (1) propensity score; and
  - (2) counterfactual models

such that it will give you the correct value for $\hat{\tau}$, as long as one of the models is correct.

- There are a lot of functional forms, but we’ll show a popular one next.

Augmented Inverse Propensity Weight (AIPW) Model Definition

- from Robins, Rotnitzky, and Zhao (1994)
- ATE:

$$\frac{1}{N}\sum_{i=1}^{N}\left\{\left[\frac{T_i\,Y_i}{\hat{P}}-\frac{(1-T_i)Y_i}{1-\hat{P}}\right]-\frac{T_i-\hat{P}}{\hat{P}\left(1-\hat{P}\right)}\left[\left(1-\hat{P}\right)\left(\hat{Y}_{i,1}\right)+\hat{P}\left(\hat{Y}_{i,0}\right)\right]\right\}$$

- We will build intuition why this works under misspecification of either $\hat{P}$ or $\hat{Y}_{i,1}$/ $\hat{Y}_{i,0}$.
- This is similar to a double machine learning implementation

AIPW - $\hat{Y}_{i,1}$ / $\hat{Y}_{i,0}$ are wrong

$$\frac{1}{N}\sum_{i=1}^{N}\{\left[\frac{T_i\,Y_i}{\hat{P}}-\frac{(1-T_i)Y_i}{1-\hat{P}}\right]-\frac{T_i-\hat{P}}{\hat{P}(1-\hat{P})}\left[(1-\hat{P})(\hat{Y}_{i,1})+\hat{P}(\hat{Y}_{i,0})\right]\}$$

- If $\hat{Y}_{i,1}$ / $\hat{Y}_{i,0}$ are wrong, but $\hat{P}$ is right, then then in $\hat{P}=E[T_i]$ so term is zero in expectation.
- Therefore, in expectation, we are left with this:

$$\frac{1}{N}\sum_{i=1}^{N}\left[\frac{T_i\,Y_i}{\hat{P}}-\frac{(1-T_i)Y_i}{1-\hat{P}}\right]$$

- Which is the inverse propensity weighting model

AIPW - $\hat{P}$ is wrong

- Note we can re-arrange the AIPW estimator to look like this:

$$\frac{1}{N}\sum_{i=1}^{N}\frac{T_i\left(Y_i-\hat{Y}_{i,1}\right)}{\hat{P}}+\hat{Y}_{i,1}\ -\frac{(1-T_i)\left(Y_i-\hat{Y}_{i,0}\right)}{1-\hat{P}}-\hat{Y}_{i,0}$$

- If $\hat{P}$ is wrong, but $\hat{Y}_{i,1}$ / $\hat{Y}_{i,0}$ are right, then these terms are zero in expectation.
- Therefore, in expectation, we are left with this:

$$\frac{1}{N}\sum_{i=1}^{N}\hat{Y}_{i,1}\ -\hat{Y}_{i,0}$$

- Which is the regression adjustment model

More Misspecification Problems

- Doubly robust methods allow us to have misspecification in $\hat{P}$ or $\hat{Y}_{i,1}$/ $\hat{Y}_{i,0}$.
- We can get even more flexible with ML models and relax functional forms.
- Flexibility becomes important if X_i becomes high dimensional.
- Note that X_i include generated features.
  - For example:
  - Interact Prime status with previous spending
  - Basis function transformations of previous spending (polynomials, segments, etc.)

Appendix Slides – Instrumental Variables

Instrumental Variables (IV) - Motivation

- The unconfoundedness assumption may not be satisfied. There is selection bias into T_i unexplained by X_i
- Are we blocked? Not necessarily.
- Suppose we have a feature Z_i that directly determines T_i but does not directly determine Y_i
- In other words, Z_i only affects Y_i through T_i

IV Two-Stage Model

$$Y_i = \beta X_i + \tau T_i + \epsilon_i$$
$$T_i = \alpha X_i + \gamma Z_i + \psi_i$$

- We can use this model to estimate τ in a two stage process.
- 1. Estimate T_i, get $\hat{T}_i$
- 2. Estimate $Y_i = \hat{\beta} X_i + \hat{\tau} \hat{T}_i + \epsilon_i$

- Why does this work? We study the necessary assumptions and intuition

IV Assumptions

$$Y_i = \beta X_i + \tau T_i + \epsilon_i$$
$$T_i = \alpha X_i + \gamma Z_i + \psi_i$$

- (1) Exclusion Restriction: $cov(\epsilon_i, Z_i) = 0$, in other words Z_i is uncorrelated $\mathrm{Y_i}$ conditional on X_i and T_i.
  - Example of a violation: suppose that we want to know the impact of a missed promise ($\mathrm{T_i}$) on future spending ($\mathrm{Y_i}$). We propose using an extreme weather event ($\mathrm{Z_i}$), like an earthquake, as an instrument for getting a missed promise. This would not work because an earthquake would affect future spending for reasons unrelated to a missed promise occurring.
- (2) Strong instrument: $cov(T_i, Z_i) \neq 0$, in other words Z_i is a strong predictor of T_i

Intuitively, why does this work?

- In the second stage, we estimate $Y_i = \hat{\beta} X_i + \hat{\tau}\, \hat{T}(X_i, Z_i) + \epsilon_i$
- Then, variation in $\hat{T}$ depends on X_i and Z_i. Let's focus on the variation due to Z_i.
- If we think back to the Frisch-Waugh-Lovell theorem, the OLS coefficient is based on the difference between T_i and $\hat{T}(X_i)$. Here in the IV setting, we are looking at the difference between $\hat{T}(X_i, Z_i)$ and $\hat{T}(X_i)$.
- Therefore, we are relying in variation in T_i based on Z_i which we assume is exogeneous to Y_i conditional on X_i.