



Physics

Honor General Physics I

Module 1 : Mechanics basics

Module 2 : Energy and Oscillations

Potential Energy and Conservation of Energy

Potential Energy

- Conservative Force
 - Conservative Force are forces for which $W_1 = -W_2$ is always true
 - Examples: gravitational force, spring force
 - Otherwise we could not speak of their potential energies
- Gravitational Potential Energy

$$U(y) = mgy$$

- Elastic Potential Energy

$$U(x) = \frac{1}{2}kx^2$$

Conservation of Mechanical Energy

- Reading a Potential Energy Curve

Work Done on a System by an External Force

- Power

$$P_{avg} = \frac{\Delta E}{\Delta t}$$

$$P = \frac{dE}{dt}$$

Conservation of Energy

Equilibrium

- Static equilibrium
 - $F_{net} = 0$
 - $\tau_{net} = 0$

Elasticity

- $Stress = modulus \times strain$
- Young's modulus, **E**, used for tension/compression (拉力/压力) :

$$\frac{F}{A} = E \frac{\Delta L}{L}$$

- Shear modulus, **G**, used for shearing (剪切力) :
 - Δx is along a different axis than L

$$\frac{F}{A} = G \frac{\Delta x}{L}$$

- Bulk modulus, **B**, used for hydraulic compression (液压压力) :
 - Relates pressure to volume change

$$p = B \frac{\Delta V}{V}$$

Gravitation

Newton's Law of Gravitation

- Gravitation and the principle of Superposition

$$F = G \frac{m_1 m_2}{r^2}$$

- hell's theorem

- Gravitation Near Earth's surface

- Combine $F = \frac{GMm}{r^2}$ and $F = ma_g$

$$a_g = \frac{GM}{r^2}$$

$$F_N - ma_g = m(-\omega^2 R)$$

$$g = a_g - \omega^2 R$$

$$a_g \approx 9.8 \text{ m/s}^2$$

$$\omega \approx 7.3 \times 10^{-5} \text{ rad/s}$$

$$R \approx 6357 \text{ km}$$

- Gravitation Inside Earth

$$F = \frac{GmM_{ins}}{r^2}$$

$$\rho = \frac{M_{ins}}{\frac{4}{3}\pi r^3} = \frac{M}{\frac{4}{3}\pi R^3}$$

$$F = \frac{GMm}{R^3} r$$

Gravitational Potential Energy

$$U = -\frac{GMm}{r}$$

$$K + U = \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = 0$$

$$v = \sqrt{\frac{2GM}{R}}$$

Satellites: Orbits and Energy

- Escape Speed

$$m \frac{v^2}{r} = \frac{GMm}{r^2}$$

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$K = -\frac{U}{2} \text{ (circular orbit)}$$

$$E = K + U = -\frac{GMm}{2r} \text{ (circular orbit)}$$

$$T^2 = \frac{4\pi^2}{GM}r^3$$

Oscillations

Simple Harmonic Motion

- Frequency
- Period

$$T = \frac{1}{f}$$

$$x(t) = x_m \cos(\omega t + \phi)$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$v(t) = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \phi)$$

$$a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -\omega^2 x_m \cos(\omega t + \phi) = -\omega^2 x(t)$$

$$F = ma = -m\omega^2 x$$

$$\omega = \sqrt{\frac{k}{m}} \text{ (Linear simple harmonic oscillation)}$$

Energy in Simple Harmonic Motion

$$U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2 \cos^2(\omega t + \phi)$$

$$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

$$E = U + K = \frac{1}{2}kx_m^2$$

An Angular Simple Harmonic Oscillator

$$\tau = -\kappa\theta$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

Pendulums, Circular motion

$$\tau = -L(F_g \sin\theta)$$

$$\alpha = -\frac{mgL}{I}\theta$$

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$T = 2\pi\sqrt{\frac{F}{g}} = 2\pi\sqrt{\frac{I}{mgh}}$$

Damped Simple Harmonic Motion

$$F_d = -bv$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$x(t) = x_m e^{\frac{-bt}{2m}} \cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$E(t) \approx \frac{1}{2}Kx_m^2 e^{\frac{-bt}{m}}$$

Waves

Sinusoidal Waves

- Transverse Waves

$$y(x, t) = y_m \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} \text{ (angular wave number)}$$

$$\omega = \frac{2\pi}{T} \text{ (angular frequency)}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \text{ (frequency)}$$

- Longitudinal Waves
Sound Waves

$$B = -\frac{\Delta p}{\Delta V/V} = \rho v^2$$

$$v = \sqrt{\frac{B}{\rho}}$$

$$I = \frac{P_s}{4\pi r^2} \text{ (Intensity)}$$

Wave Speed

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f \text{ (wave speed)}$$

- Wave Speed on a Stretched String

$$\mu = \frac{m}{l} \text{ (linear density)}$$

$$v = \sqrt{\frac{\tau}{\mu}} \text{ (speed) } (\tau \text{ is tension in N})$$

Energy and Power of a Wave Travelling along a String

$$dK = \frac{1}{2}dmu^2$$

$$\frac{dK}{dt} = \frac{1}{2} \frac{dm}{dt} u^2 = \frac{1}{2} \mu v \omega^2 y_m^2 \cos^2(kx - \omega t)$$

$$\left(\frac{dK}{dt}\right)_{avg} = \frac{1}{4} \mu v \omega^2 y_m^2 = \left(\frac{dU}{dt}\right)_{avg}$$

$$P_{avg} = \frac{d(K + U)}{dt} = 2\left(\frac{dK}{dt}\right)_{avg} = \frac{1}{2} \mu v \omega^2 y_m^2$$

$$a_y = \frac{d^2 y}{dt^2}$$

$$\frac{d^2 y}{dx^2} = \frac{\mu}{\tau} \frac{d^2 y}{dt^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \text{ (wave equation)}$$

Interference of Waves

$$y'(x, t) = y_1(x, t) + y_2(x, t)$$

$$If :$$

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx - \omega t + \phi)$$

$$s.t.$$

$$y'(x, t) = [2y_m \cos \frac{1}{2}\phi] \sin(kx - \omega t + \frac{1}{2}\phi)$$

- Sound Interference

$$\phi = \frac{\Delta L}{\lambda} 2\pi$$

Phasors

Standing Waves

If :

$$y_1(x, t) = y_m \sin(kx - \omega t)$$

$$y_2(x, t) = y_m \sin(kx + \omega t)$$

s.t.

$$y'(x, t) = [2y_m \sin kx] \cos \omega t$$

- Resonance
- A pipe open at both ends

$$\sin(kL) = 0 \rightarrow kL = \frac{2\pi}{\lambda} L = n\pi$$

$$f = \frac{v}{\lambda} = \frac{nv}{2L}, n = 1, 2, 3, \dots$$

- A pipe closed at one end and open at the other

$$f = \frac{v}{\lambda} = \frac{nv}{4L}, n = 1, 3, 5, \dots$$

Doppler's Effect

$$f' = f \frac{v \pm v_D}{v \pm v_S}$$

$$\sin \theta = \frac{v}{v_S}$$

Fluids

Fluids basics

Pascal's Principle

Archimedes' Principle

Ideal Fluids

Bernoulli's Equation

Module 3 : Thermodynamics

The First Law of Thermodynamics

Temperature

- Kelvins is an SI base quantity

$$T = (273.16 \text{ K}) \left(\lim_{p \rightarrow 0} \frac{p}{p_3} \right)$$

$$T_C = T - 273.15$$

$$T_F = \frac{9}{5}T_C + 32^\circ$$

Zeroth law of thermodynamics

- if A and B are each in thermal equilibrium with T , then A and B are in thermal equilibrium with each other.

Absorption of Heat

Heat capacity - C

specific heat - c

heat of transformation - L

$$Q = C \Delta T = C(T_f - T_i)$$

$$Q = cm \Delta T = cm(T_f - T_i)$$

$$Q = Lm$$

Substance	State	Specific Heat (Cal/g·K)	Specific Heat (J/kg·K)	Molar Specific Heat (J/mol·K)
Lead	Solid	0.0305	128	26.5
Tungsten	Solid	0.0321	134	24.8
Silver	Solid	0.0564	236	25.5
Copper	Solid	0.0923	386	24.5
Aluminum	Solid	0.215	900	24.4
Brass	Solid	0.092	380	-
Granite	Solid	0.19	790	-
Glass	Solid	0.20	840	-
Ice (−10°C)	Solid	0.530	2220	-
Mercury	Liquid	0.033	140	-
Ethyl alcohol	Liquid	0.58	2430	-
Seawater	Liquid	0.93	3900	-
Water	Liquid	1.00	4187	-

First law of thermodynamics

$$W = \int dW = \int_{v_i}^{v_f} p dV$$

$$\Delta E_{int} = E_{int,f} - E_{int,i} = Q - W$$

$$dE_{int} = dQ - dW$$

Process	Restriction	Consequence
Adiabatic	$Q = 0$	$\Delta E_{int} = -W$
Constant volume	$W = 0$	$\Delta E_{int} = Q$
Closed cycle	$\Delta E_{int} = 0$	$Q = W$
Free expansion	$Q = W = 0$	$\Delta E_{int} = 0$

Heat Transfer Mechanisms

- Thermal Conduction
rate - P_{cond}

$$P_{cond} = \frac{Q}{t} = kA \frac{T_H - T_C}{L}$$

- Convection
- Thermal Radiation (Stefan-Boltzmann law)
 $\sigma = 5.6704 \times 10^{-8} W/m^2 \cdot K^4$

$$P_{rad} = \sigma \epsilon A T^4$$

$$P_{abs} = \sigma \epsilon A T_{env}^4$$

Kinetic Theory of Gases

Avogadro's Number

$$N_A = 6.02 \times 10^{23} mol^{-1}$$

$$n = \frac{M_{sam}}{M} = \frac{M_{sam}}{m N_A} = \frac{N}{N_A}$$

Ideal Gas

R is a constant = $8.31 J/mol \cdot K$

k is the Boltzmann constant = $1.380649 \times 10^{-23} J/K$

$$pV = nRT$$

$$pV = NkT$$

$$W = nRT \ln \frac{V_f}{V_i}$$

Pressure, Temperature, and rms Speed

$$p = \frac{nMv_{rms}^2}{3V}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Gas	Molar Mass (g/mol)	v_{rms} (m/s)
Hydrogen (H ₂)	2.02	1920
Helium (He)	4.00	1370
Water vapor (H ₂ O)	18.0	645
Nitrogen (N ₂)	28.0	517
Oxygen (O ₂)	32.0	483
Carbon dioxide (CO ₂)	44.0	412
Sulfur dioxide (SO ₂)	64.1	342

Translational Kinetic Energy

$$K_{avg} = \frac{1}{2}m(v^2)_{avg} = \frac{1}{2}mv_{rms}^2 = \frac{1}{2}m\frac{3RT}{2N_A} = \frac{3}{2}kT$$

Mean Free Path

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 \frac{N}{V}}$$

- Proof:

$$\lambda = \frac{\text{length of path}}{\text{number of collisions}} = \frac{v_{ave} \Delta t}{\pi d^2 v_{rel} \Delta t \frac{N}{V}}$$

The Distribution of Molecular Speed

$$P(v) = 4\pi \left(\frac{M}{2\pi RT} \right)^{\frac{3}{2}} v^2 e^{-\frac{Mv^2}{2RT}}$$

$$\int_0^\infty P(v) dv = 1$$

$$frac = \int_{v1}^{v2} P(v) dv$$

$$v_{avg} = \int_0^\infty v P(v) dv = \sqrt{\frac{8RT}{\pi M}} \text{ (average speed)}$$

$$v_{rms} = \sqrt{\int_0^\infty v^2 P(v) dv} = \sqrt{\frac{3RT}{M}} \text{ (rms speed)}$$

$$v_p = \sqrt{\frac{2RT}{M}} \text{ (most probable speed)}$$

The Molar Specific Heats of An Ideal Gas

- Molar Specific Heat at Constant Volume

$$E_{int} = \frac{3}{2} nRT$$

$$Q = nC_V \Delta T \text{ (constant volume)}$$

$$\Delta E_{int} = Q - W$$

$$W = p\Delta V = 0$$

$$C_V = \frac{3}{2} R = 12.5 J/mol \cdot K \text{ (monatomic gas)}$$

- Molar Specific Heat at Constant Pressure

$$C_p = \frac{Q}{n\Delta T}$$

$$\Delta E_{int} = Q - W$$

$$W = p\Delta V = nR\Delta T$$

$$C_p = C_V + R = \frac{5}{2}R(\text{monatomic})$$

Degree of Freedom and Molar Specific Heats

$$C_V = \left(\frac{f}{2}\right)R = 4.16 f \text{ J/mol} \cdot K$$

Molecule Type	Example	Translational	Rotational	Total f	C_V (Equation 19-51)	$C_P = C_V + R$
Monatomic	He	3	0	3	$\frac{3}{2}R$	$\frac{5}{2}R$
Diatomic	O ₂	3	2	5	$\frac{5}{2}R$	$\frac{7}{2}R$
Polyatomic	CH ₄	3	3	6	$3R$	$4R$

The Adiabatic Expansion of an Ideal Gas

$$nC_V dT = dE_{int} = dQ - dW = -p dV$$

$$pV = nRT \rightarrow dp V + p dV = nR dT$$

$$C_P = C_V + R \rightarrow V dp + p dV \cdot \frac{C_P}{C_V} = 0$$

$$\ln p + \ln V \cdot \frac{C_P}{C_V} = \text{constant}$$

$$\rightarrow pV^\gamma = \text{constant}$$

Entropy and the Second Law of Thermodynamics

Entropy

let $\Delta E_{int} = 0$ as temperature keeps the same

$$W = \int P dV = Q$$

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ_{rev}}{T} \text{ (define the entropy change)}$$

For ideal gas, we have:

$$dE_{int} = dQ - dW$$

$$dW = p dV$$

$$dE_{int} = nC_V dT$$

$$dQ = p dV + nC_V dT$$

$$\frac{dQ}{T} = nR \frac{dV}{V} + nC_V \frac{dT}{T}$$

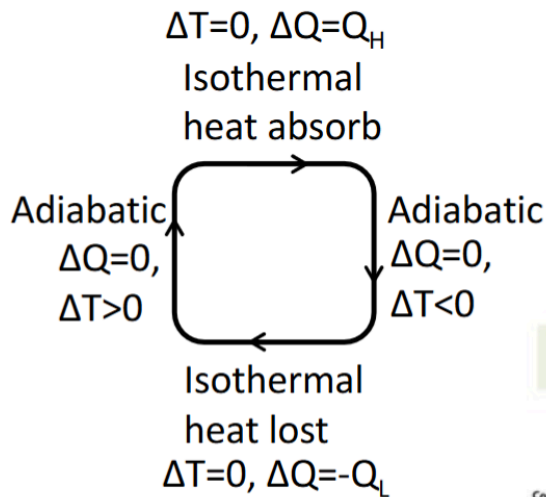
$$\Delta S = \int_i^f \frac{dQ}{T} = nR \int_i^f \frac{dV}{V} + nC_V \int_i^f \frac{dT}{T} = nR \ln \frac{V_f}{V_i} + nC_V \ln \frac{T_f}{T_i}$$

$$\Delta S \geq 0 \text{ (second law of thermodynamics)}$$

Entropy in the Real World: Engines

- Carnot Engine

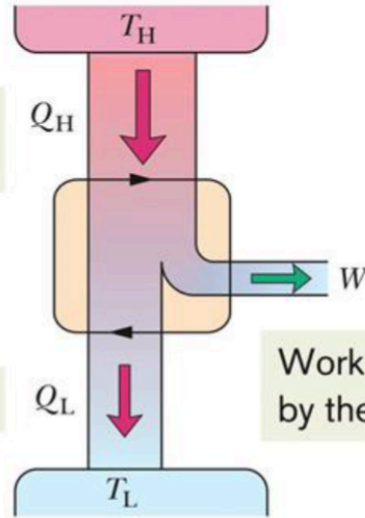
Schematic of a Carnot engine



Heat is absorbed.

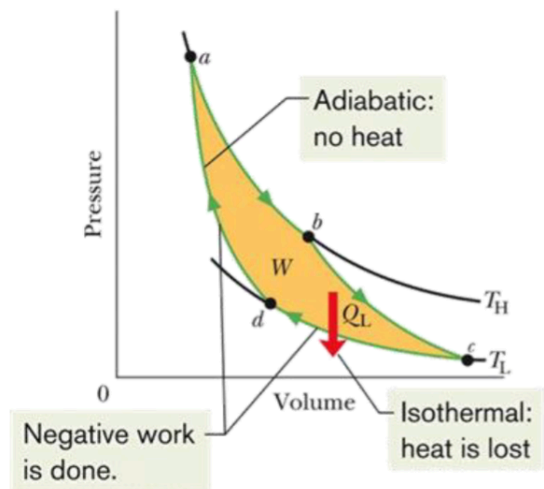
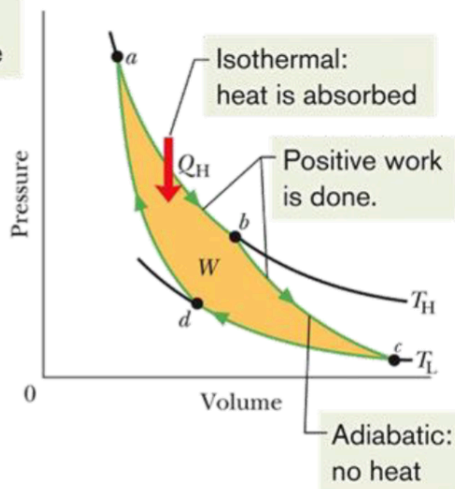
Heat is lost.

Work is done by the engine.



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Stages of a Carnot engine



$$\eta = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|}$$

$$\eta_C = 1 - \frac{T_L}{T_H} \text{ (Efficiency of Carnot Engine)}$$

$$W = nR(T_H - T_L) \ln\left(\frac{V_b}{V_a}\right)$$

$$Q_H = nRT_H \ln\left(\frac{V_b}{V_a}\right)$$

$$\eta = \frac{W}{Q} < \eta_C \text{ (For Stirling Efficiency)}$$

$$\varepsilon_A \leq 1 - \frac{T_L}{T_H} = \varepsilon_C \text{ (For Arbitrary Engine Efficiency)}$$

Refrigerators and Real Engines

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_H|}{|W|} \text{ (any refrigerator)}$$

$$W = nR(T_L - T_H) \ln\left(\frac{V_b}{V_a}\right)$$

$$Q_H = -nRT_H \ln\left(\frac{V_b}{V_a}\right)$$

$$K_C = \frac{T_H}{T_L - T_H} \text{ (Carnot refrigerator)}$$

A Statistical View of Entropy

$$W = C_N^{n_1} = \frac{N!}{n_1! \cdot n_2!} \text{ (multiplicity of configuration)}$$

Label	n_1	n_2	Multiplicity W	Calculation of W	Entropy (10^{-23} J/K)
I	6	0	1	$\frac{6!}{6!0!} = 1$	0.00
II	5	1	6	$\frac{6!}{5!1!} = 6$	2.47
III	4	2	15	$\frac{6!}{4!2!} = 15$	3.74
IV	3	3	20	$\frac{6!}{3!3!} = 20$	4.13
V	2	4	15	$\frac{6!}{2!4!} = 15$	3.74
VI	1	5	6	$\frac{6!}{1!5!} = 6$	2.47
VII	0	6	1	$\frac{6!}{0!6!} = 1$	0.00

$$\sum W = 64$$

$$S = k \ln W \text{ (Boltzmann's entropy equation)}$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

When N is very large:

$$\ln N! \approx N(\ln N) - N \text{ (Stirling's approximation)}$$

Free expansion problem

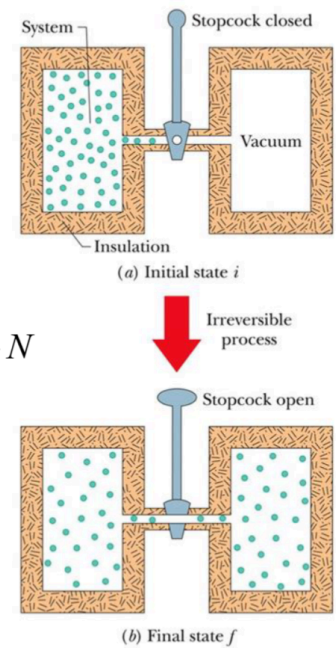
- Initially, all molecules are in left chamber:

$$W_i = \frac{N!}{N! 0!} = 1 \quad S_i = k \ln W_i = k \ln 1 = 0$$

- Finally, half of molecules are in either side:

$$W_f = \frac{N!}{(N/2)! (N/2)!} \quad \ln N! \approx N(\ln N) - N$$

$$\begin{aligned} S_f &= k \ln W_f = k \ln(N!) - 2k \ln[(N/2)!] \\ &= k[N(\ln N) - N] - 2k[(N/2) \ln(N/2) - (N/2)] \\ &= Nk \ln 2 = nR \ln 2. \end{aligned}$$



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Words

- Monatomic: 单原子
- diatomic: 双原子
- adiabatic: 绝热
- isothermal: 等温
- isentropic: 等熵