Topology Preliminary Exam Study Guide

NDSU

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Chapter 1

Point-Set Topology

Since Point-Set Topology is a prerequisite, this section will strictly be problems that provide utility to solve problems in Algebraic and Differential topology.

Exercise 1. If $f : X \to Y$ is continuous and Y is both compact and Hausdorff, then the graph

$$G = \{(x, y) \in X \times Y | y = f(x)\}$$

is closed in $X \times Y$.

Exercise 2. Let $f : X \to Y$ be a continuous bijection. If X is compact and Y is Hausdorff, then f is a homeomorphism.

Exercise 3 (Spring 19)**.** Give an example of a space that is connected, but not locally path connected. Give an example of a topological space that is locally path connected but not locally simply connected

Exercise 4 (Spring 18)**.** Let X be a Hausdorff topological space. Show that there exists a compact topological space Y such that X is a subspace of Y and $Y \backslash X$ consists of a single point.

Exercise 5 (Spring 18)**.** For distinct positive integers m and n, prove that R^m is not homeomorphic to R^n.

Exercise 6 (Fall 17)**.** A topological space X is called locally Euclidean if for some fixed $n \leq 1$ and for all $p \in X$ there exists an open set U containing p such that U is homeomorphic to R^n. Is it true that a locally Euclidean second countable space is Hausdorff?

Exercise 7 (Fall 17)**.** Let X be a Hausdorff topological space and U an open subset of X. Show that if $\{K_n | n \in \mathbb{N}\}$ is a nested sequence of compact subsets of X with $\bigcap_{n=1}^{\infty} K_n \subset U$, then there exists $n \in \mathbb{N}$ such that $K_n \subset U$.

Exercise 8 (Spring 17)**.** Let X be a Hausdorff topological space and let $F, K \subset X$ be disjoint compact subsets. Show that there exist open sets $U, V \subset X$ such that $F \subset U, K \subset V$ and $U \cap V = \emptyset$.

Exercise 9 (Spring 16)**.** Prove the following.(a) A closed subset of a compact topological space is compact.(b) A compact subset of a Hausdorff topological space is closed.

Exercise 10 (June 14)**.** Find an example of a topological space which is of class a)T0 but not T1. b)T 1but not T2. c)T2 but not T3. d)T3 but not T4.

Chapter 2

Algebraic Topology