

Algebra Preliminary Exam Study Guide

NDSU

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Chapter 1

Rings

Definition 1. A Ring R is a set together with two binary operations $+$ and \times such that

1. $(R, +)$ is an abelian group,
2. \times is associative
3. $(a + b) \times c = (a \times c) + (b \times c)$ for all $a, b, c \in R$.

Definition 2. A ring is commutative ring if \times is commutative and is said to have identity if there is a $1 \in R$ such that

$$1 \times a = a \times 1 = a \quad \text{for all } a \in R$$

Exercise 1. An element is called idempotent if $x^2 = x$. Show that if each element a in the ring R is idempotent, then R is a commutative ring. (Note: R is called a boolean ring)

We can add more structure to the rings with the following definition.

Definition 3. Let R be a ring with identity 1, where $1 \neq 0$. We say R is a division ring if every nonzero element $a \in R$ has a multiplicative inverse. If R is commutative, then we say R is a field.