數值分析

作業三

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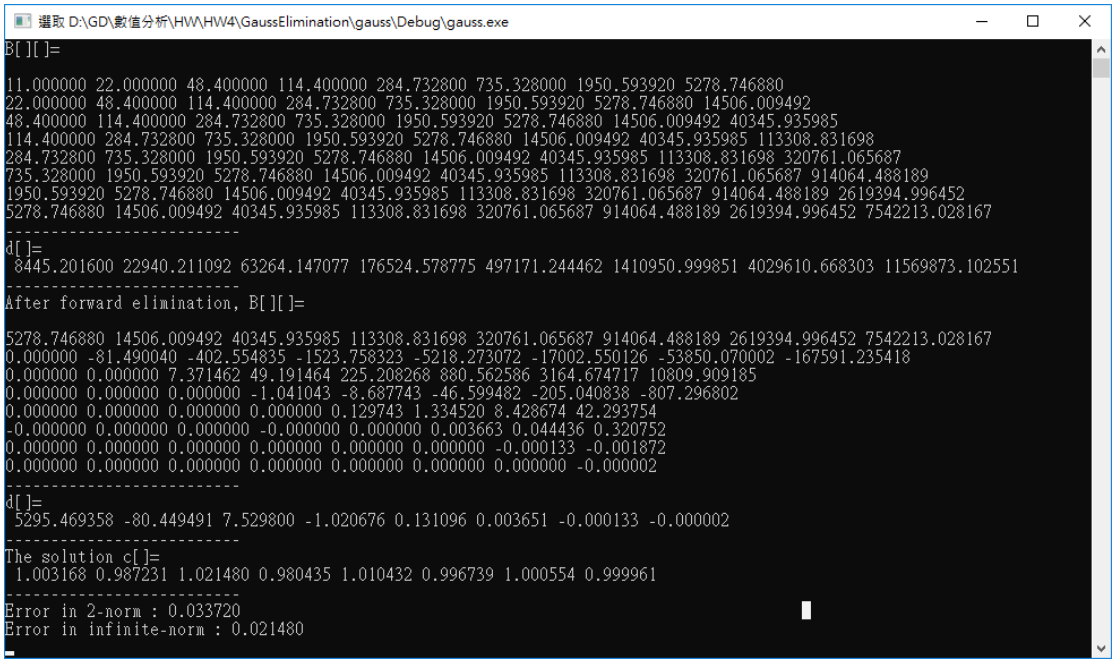
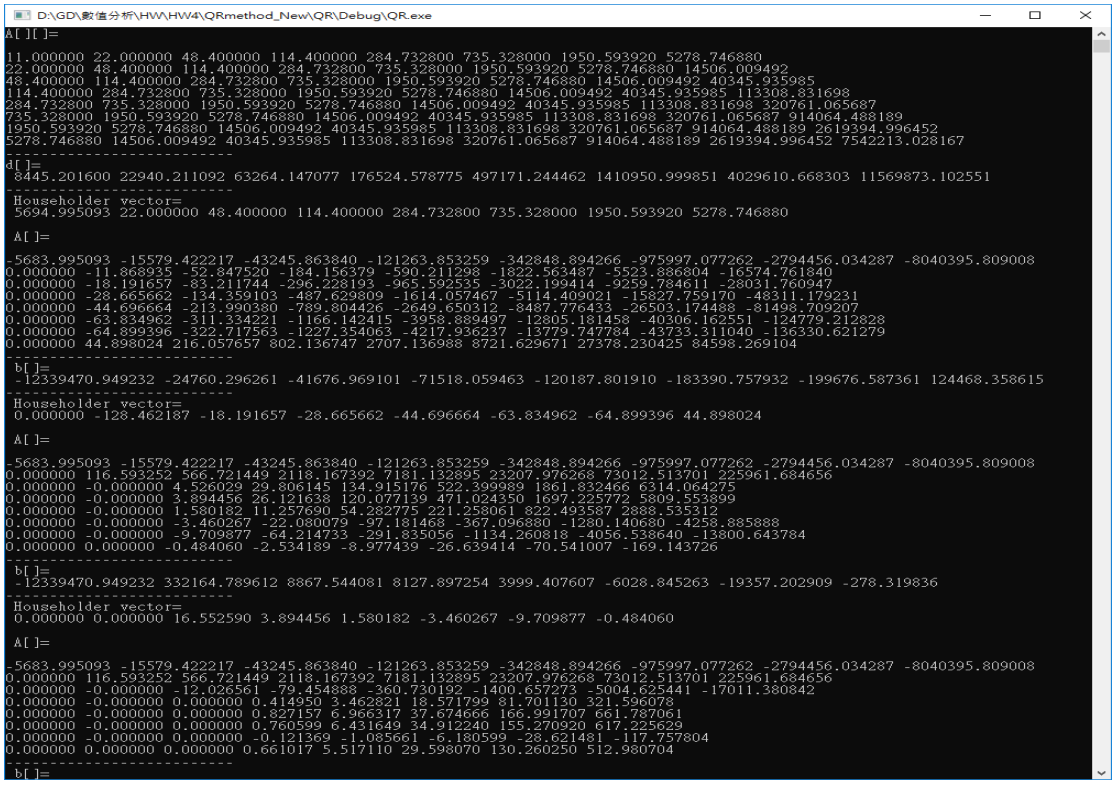
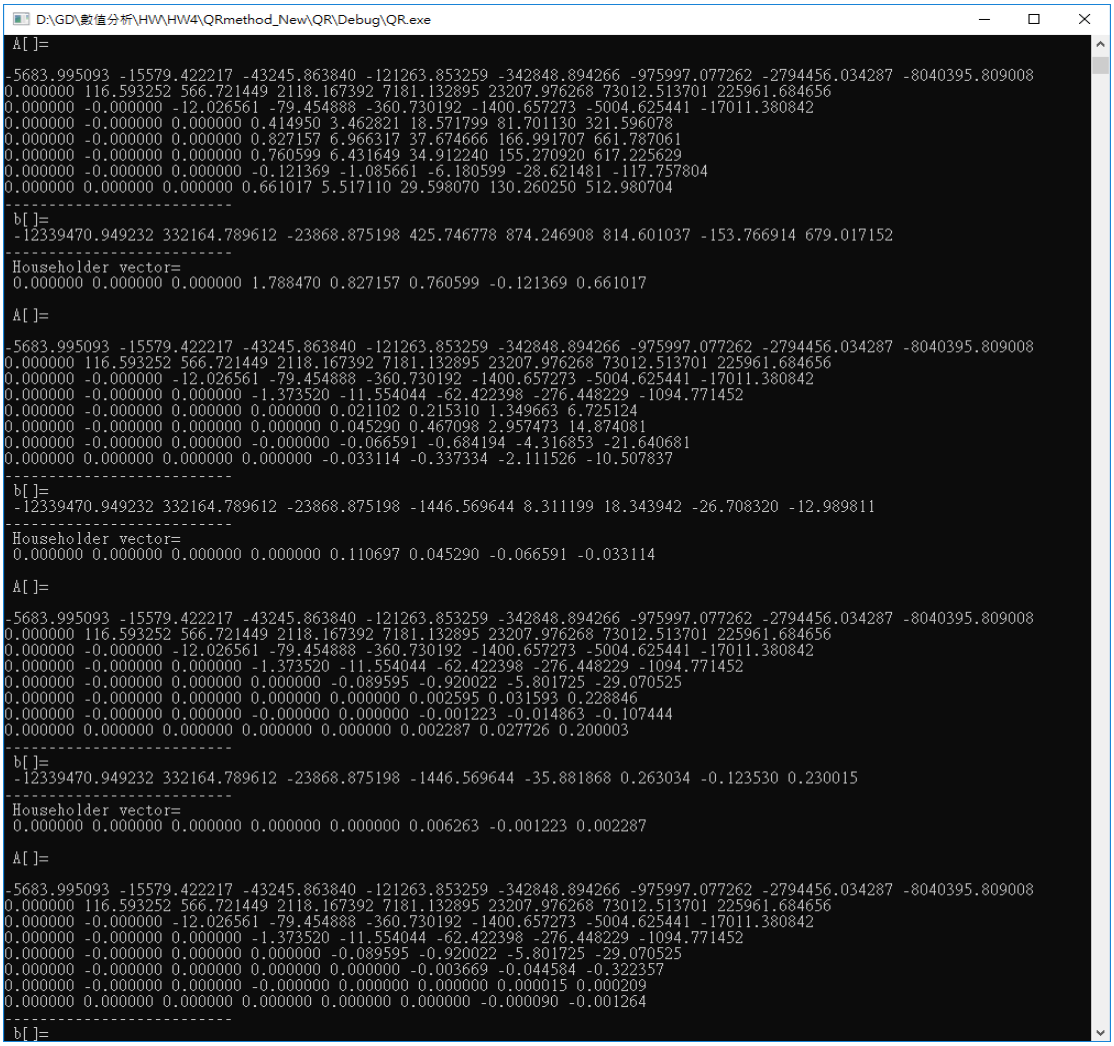
A.

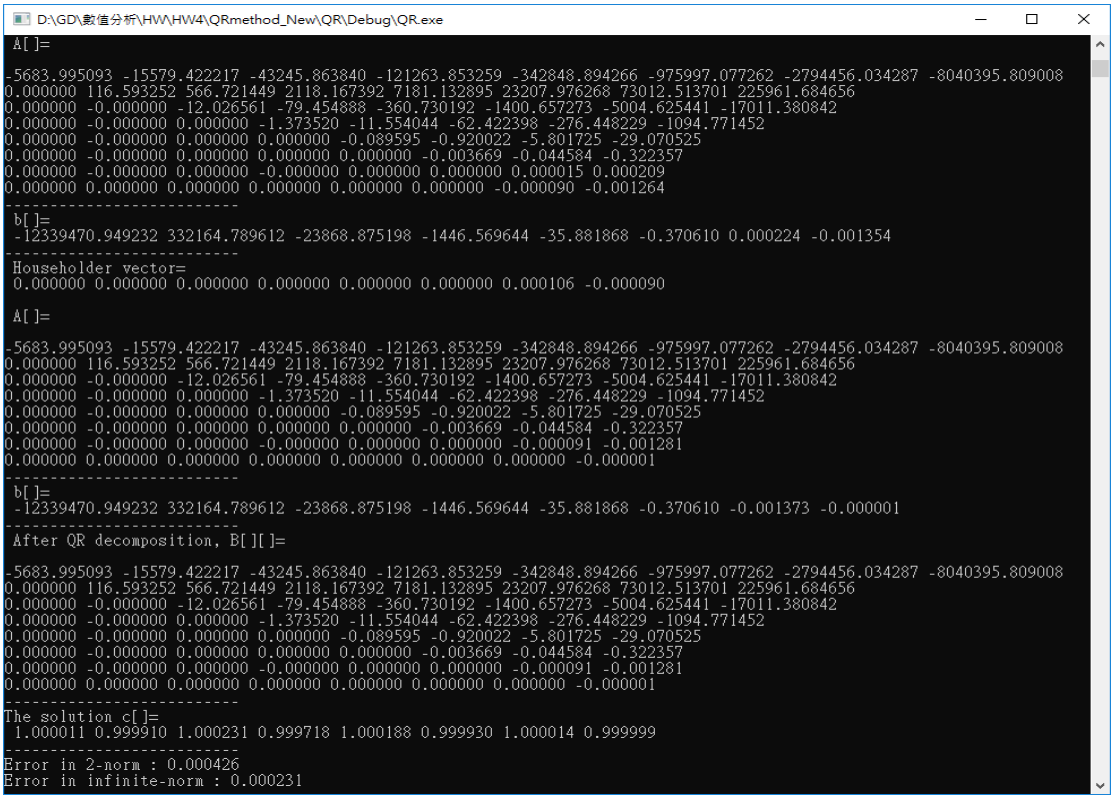
1. gauss_solver:



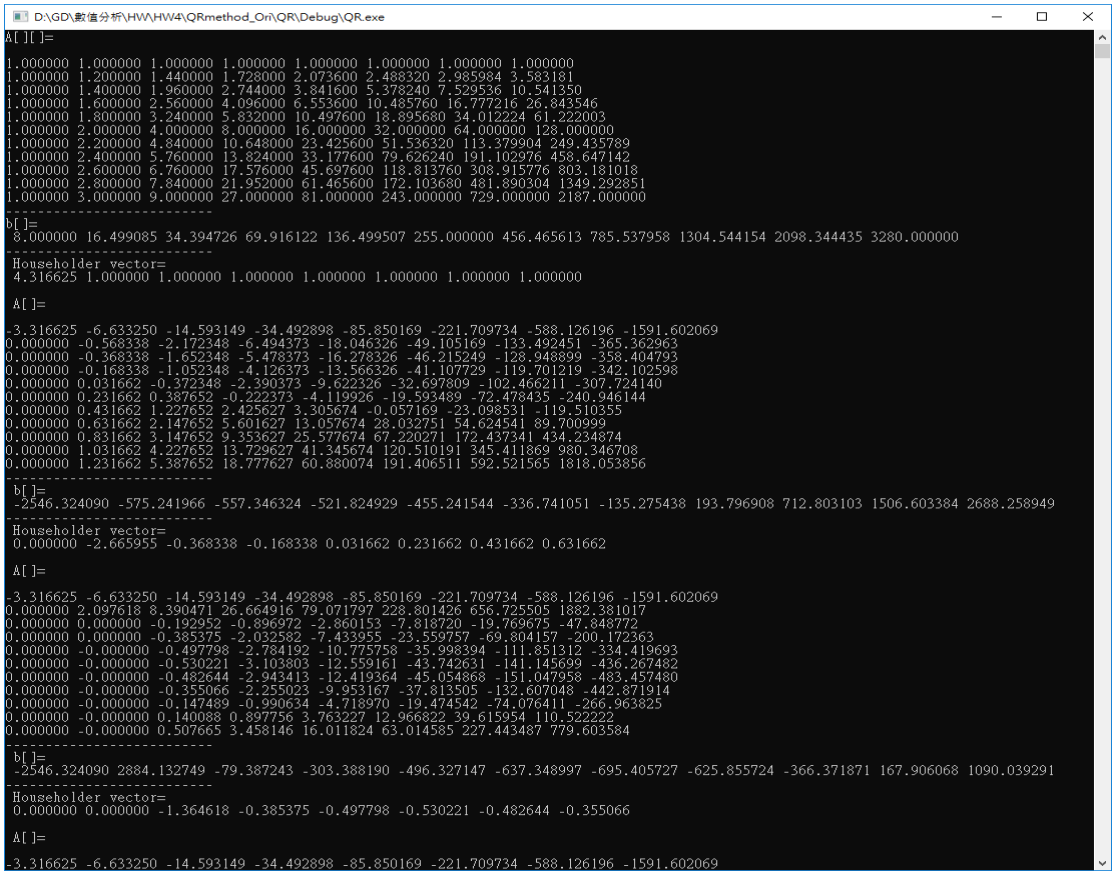
2. QR_Solver for New System:

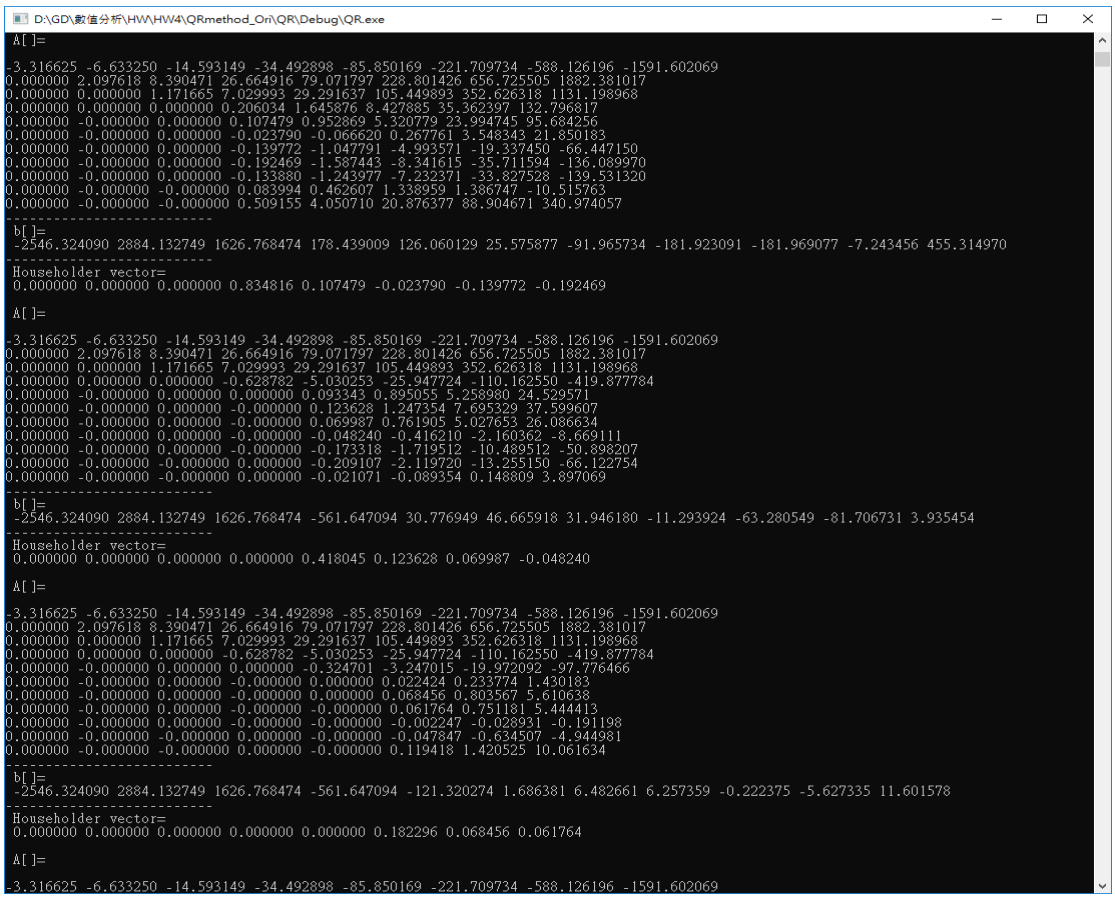



D:\GD\数值分析\HW\HW4\QRmethod_New\QR\Debug\QR.exe
A[]=
-5683.995093 -15579.422217 -43245.863840 -121263.853259 -342848.894266 -975997.077262 -2794456.034287 -8040395.809008
0.000000 116.593252 566.721449 2118.167392 7181.132895 23207.976268 73012.513701 225961.684656
0.000000 -0.000000 -12.026561 -79.454888 -360.730192 -1400.657273 -5004.625441 -17011.380842
0.000000 -0.000000 0.000000 0.414950 3.462821 18.571799 81.701130 321.596078
0.000000 -0.000000 0.000000 0.827157 6.966317 37.674666 166.991707 661.787061
0.000000 -0.000000 0.000000 0.760599 6.431649 34.912240 155.270920 617.225629
0.000000 -0.000000 0.000000 -0.121369 -1.085661 -6.180599 -28.621481 -117.757804
0.000000 0.000000 0.000000 0.661017 5.517110 29.598070 130.260250 512.980704
b[]=
-12339470.949232 332164.789612 -23868.875198 425.746778 874.246908 814.601037 -153.766914 679.017152
Householder vector=
0.000000 0.000000 0.000000 1.788470 0.827157 0.760599 -0.121369 0.661017
A[]=
-5683.995093 -15579.422217 -43245.863840 -121263.853259 -342848.894266 -975997.077262 -2794456.034287 -8040395.809008
0.000000 116.593252 566.721449 2118.167392 7181.132895 23207.976268 73012.513701 225961.684656
0.000000 -0.000000 -12.026561 -79.454888 -360.730192 -1400.657273 -5004.625441 -17011.380842
0.000000 -0.000000 0.000000 -1.373520 -11.554044 -62.422398 -276.448229 -1094.771452
0.000000 -0.000000 0.000000 0.000000 0.021102 0.215310 1.349663 6.725124
0.000000 -0.000000 0.000000 0.000000 0.045290 0.467098 2.957473 14.874081
0.000000 -0.000000 0.000000 -0.000000 -0.066591 -0.684194 -4.316853 -21.640681
0.000000 0.000000 0.000000 0.000000 -0.033114 -0.337334 -2.111526 -10.507837
b[]=
-12339470.949232 332164.789612 -23868.875198 -1446.569644 8.311199 18.343942 -26.708320 -12.989811
Householder vector=
0.000000 0.000000 0.000000 0.000000 0.110697 0.045290 -0.066591 -0.033114
A[]=
-5683.995093 -15579.422217 -43245.863840 -121263.853259 -342848.894266 -975997.077262 -2794456.034287 -8040395.809008
0.000000 116.593252 566.721449 2118.167392 7181.132895 23207.976268 73012.513701 225961.684656
0.000000 -0.000000 -12.026561 -79.454888 -360.730192 -1400.657273 -5004.625441 -17011.380842
0.000000 -0.000000 0.000000 -1.373520 -11.554044 -62.422398 -276.448229 -1094.771452
0.000000 -0.000000 0.000000 0.000000 -0.089595 -0.920022 -5.801725 -29.070525
0.000000 -0.000000 0.000000 0.000000 0.002595 0.031593 0.228846
0.000000 -0.000000 0.000000 -0.000000 0.000000 -0.001223 -0.014863 -0.107444
0.000000 0.000000 0.000000 0.000000 0.000000 0.002287 0.027726 0.200003
b[]=
-12339470.949232 332164.789612 -23868.875198 -1446.569644 -35.881868 0.263034 -0.123530 0.230015
Householder vector=
0.000000 0.000000 0.000000 0.000000 0.000000 0.006263 -0.001223 0.002287
A[]=
-5683.995093 -15579.422217 -43245.863840 -121263.853259 -342848.894266 -975997.077262 -2794456.034287 -8040395.809008
0.000000 116.593252 566.721449 2118.167392 7181.132895 23207.976268 73012.513701 225961.684656
0.000000 -0.000000 -12.026561 -79.454888 -360.730192 -1400.657273 -5004.625441 -17011.380842
0.000000 -0.000000 0.000000 -1.373520 -11.554044 -62.422398 -276.448229 -1094.771452
0.000000 -0.000000 0.000000 0.000000 -0.089595 -0.920022 -5.801725 -29.070525
0.000000 -0.000000 0.000000 0.000000 0.000000 -0.003669 -0.044584 -0.322357
0.000000 -0.000000 0.000000 -0.000000 0.000000 0.000000 0.000015 0.000209
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 -0.000090 -0.001264
b[]=


D:\GD\数值分析\HW\HW4\QRmethod_New\QR\Debug\QR.exe
A[]=
-5683.995093 -15579.422217 -43245.863840 -121263.853259 -342848.894266 -975997.077262 -2794456.034287 -8040395.809008
0.000000 116.593252 566.721449 2118.167392 7181.132895 23207.976268 73012.513701 225961.684656
0.000000 -0.000000 -12.026561 -79.454888 -360.730192 -1400.657273 -5004.625441 -17011.380842
0.000000 -0.000000 0.000000 -1.373520 -11.554044 -62.422398 -276.448229 -1094.771452
0.000000 -0.000000 0.000000 0.000000 -0.089595 -0.920022 -5.801725 -29.070525
0.000000 -0.000000 0.000000 0.000000 0.000000 -0.003669 -0.044584 -0.322357
0.000000 -0.000000 0.000000 -0.000000 0.000000 0.000000 0.000015 0.000209
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 -0.000090 -0.001264
b[]=
-12339470.949232 332164.789612 -23868.875198 -1446.569644 -35.881868 -0.370610 0.000224 -0.001354
Householder vector=
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000106 -0.000090
A[]=
-5683.995093 -15579.422217 -43245.863840 -121263.853259 -342848.894266 -975997.077262 -2794456.034287 -8040395.809008
0.000000 116.593252 566.721449 2118.167392 7181.132895 23207.976268 73012.513701 225961.684656
0.000000 -0.000000 -12.026561 -79.454888 -360.730192 -1400.657273 -5004.625441 -17011.380842
0.000000 -0.000000 0.000000 -1.373520 -11.554044 -62.422398 -276.448229 -1094.771452
0.000000 -0.000000 0.000000 0.000000 -0.089595 -0.920022 -5.801725 -29.070525
0.000000 -0.000000 0.000000 0.000000 0.000000 -0.003669 -0.044584 -0.322357
0.000000 -0.000000 0.000000 -0.000000 0.000000 0.000000 -0.000091 -0.001281
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 -0.000001
b[]=
-12339470.949232 332164.789612 -23868.875198 -1446.569644 -35.881868 -0.370610 -0.001373 -0.000001
After QR decomposition, B[][]=
-5683.995093 -15579.422217 -43245.863840 -121263.853259 -342848.894266 -975997.077262 -2794456.034287 -8040395.809008
0.000000 116.593252 566.721449 2118.167392 7181.132895 23207.976268 73012.513701 225961.684656
0.000000 -0.000000 -12.026561 -79.454888 -360.730192 -1400.657273 -5004.625441 -17011.380842
0.000000 -0.000000 0.000000 -1.373520 -11.554044 -62.422398 -276.448229 -1094.771452
0.000000 -0.000000 0.000000 0.000000 -0.089595 -0.920022 -5.801725 -29.070525
0.000000 -0.000000 0.000000 0.000000 0.000000 -0.003669 -0.044584 -0.322357
0.000000 -0.000000 0.000000 -0.000000 0.000000 0.000000 -0.000091 -0.001281
0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 -0.000001
The solution c[]=
1.000011 0.999910 1.000231 0.999718 1.000188 0.999930 1.000014 0.999999
Error in 2-norm : 0.000426
Error in infinite-norm : 0.000231

3. QR_Solver For Original System:





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D:\GD\数值分析\HW\HW4\QRmethod_Ori\QR\Debug\QR.exe
 A[]=

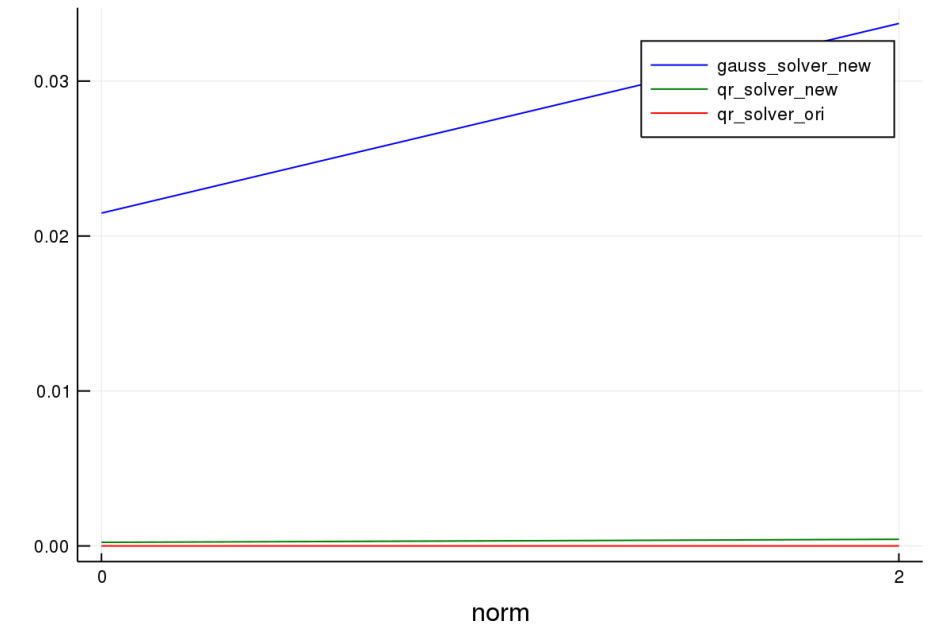
-3.316625 -6.633250 -14.593149 -34.492898 -85.850169 -221.709734 -588.126196 -1591.602069
0.000000 2.097618 8.390471 26.664916 79.071797 228.801426 656.725505 1882.381017
0.000000 0.000000 1.171665 7.029993 29.291637 105.449893 352.626318 1131.198968
0.000000 0.000000 0.000000 -0.628782 -5.030253 -25.947724 -110.162550 -419.877784
0.000000 -0.000000 0.000000 0.000000 -0.324701 -3.247015 -19.972092 -97.776466
0.000000 -0.000000 0.000000 -0.000000 0.000000 -0.159872 -1.918463 -13.704715
0.000000 -0.000000 0.000000 -0.000000 0.000000 0.000000 -0.004645 -0.072842
0.000000 -0.000000 0.000000 -0.000000 -0.000000 0.000000 0.021974 0.316503
0.000000 -0.000000 0.000000 -0.000000 -0.000000 0.000000 -0.002407 -0.004676
0.000000 -0.000000 -0.000000 0.000000 -0.000000 0.000000 -0.069612 -0.972539
0.000000 -0.000000 -0.000000 0.000000 -0.000000 0.000000 0.010640 0.147083
-------------------------
 b[]=
 -2546.324090 2884.132749 1626.768474 -561.647094 -121.320274 -15.783051 -0.077487 0.338477 -0.007083 -1.042151 0.157723
-------------------------
 Householder vector=
 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 -0.078599 0.021974

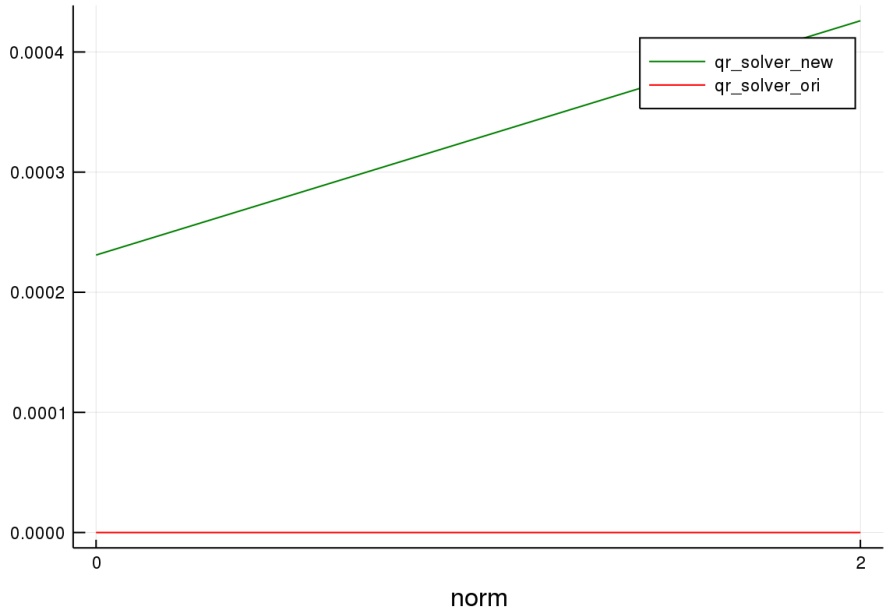
 A[]=

-3.316625 -6.633250 -14.593149 -34.492898 -85.850169 -221.709734 -588.126196 -1591.602069
0.000000 2.097618 8.390471 26.664916 79.071797 228.801426 656.725505 1882.381017
0.000000 0.000000 1.171665 7.029993 29.291637 105.449893 352.626318 1131.198968
0.000000 0.000000 0.000000 -0.628782 -5.030253 -25.947724 -110.162550 -419.877784
0.000000 -0.000000 0.000000 0.000000 -0.324701 -3.247015 -19.972092 -97.776466
0.000000 -0.000000 0.000000 -0.000000 0.000000 -0.159872 -1.918463 -13.704715
0.000000 -0.000000 0.000000 -0.000000 0.000000 0.000000 0.073955 1.035364
0.000000 -0.000000 0.000000 -0.000000 -0.000000 0.000000 0.000000 0.006679
0.000000 -0.000000 0.000000 -0.000000 -0.000000 0.000000 -0.000000 0.029259
0.000000 -0.000000 -0.000000 0.000000 -0.000000 0.000000 -0.000000 0.008953
0.000000 -0.000000 -0.000000 0.000000 -0.000000 0.000000 0.000000 -0.002939
-------------------------
 b[]=
 -2546.324090 2884.132749 1626.768474 -561.647094 -121.320274 -15.783051 1.109318 0.006679 0.029259 0.008953 -0.002939
-------------------------
 After QR decomposition, A[][]=

-3.316625 -6.633250 -14.593149 -34.492898 -85.850169 -221.709734 -588.126196 -1591.602069
0.000000 2.097618 8.390471 26.664916 79.071797 228.801426 656.725505 1882.381017
0.000000 0.000000 1.171665 7.029993 29.291637 105.449893 352.626318 1131.198968
0.000000 0.000000 0.000000 -0.628782 -5.030253 -25.947724 -110.162550 -419.877784
0.000000 -0.000000 0.000000 0.000000 -0.324701 -3.247015 -19.972092 -97.776466
0.000000 -0.000000 0.000000 -0.000000 0.000000 -0.159872 -1.918463 -13.704715
0.000000 -0.000000 0.000000 -0.000000 0.000000 0.000000 0.073955 1.035364
0.000000 -0.000000 0.000000 -0.000000 -0.000000 0.000000 0.000000 0.006679
0.000000 -0.000000 0.000000 -0.000000 -0.000000 0.000000 -0.000000 0.029259
0.000000 -0.000000 -0.000000 0.000000 -0.000000 0.000000 -0.000000 0.008953
0.000000 -0.000000 -0.000000 0.000000 -0.000000 0.000000 0.000000 -0.002939
-------------------------
The solution c[]=
 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000
-------------------------
Error in 2-norm : 0.000000
Error in infinite-norm : 0.000000
```

B.





C.

QR 在解原系統時的表現良好，誤差為零。

由第一張圖可知誤差 gauss > qr_new > qr_ori

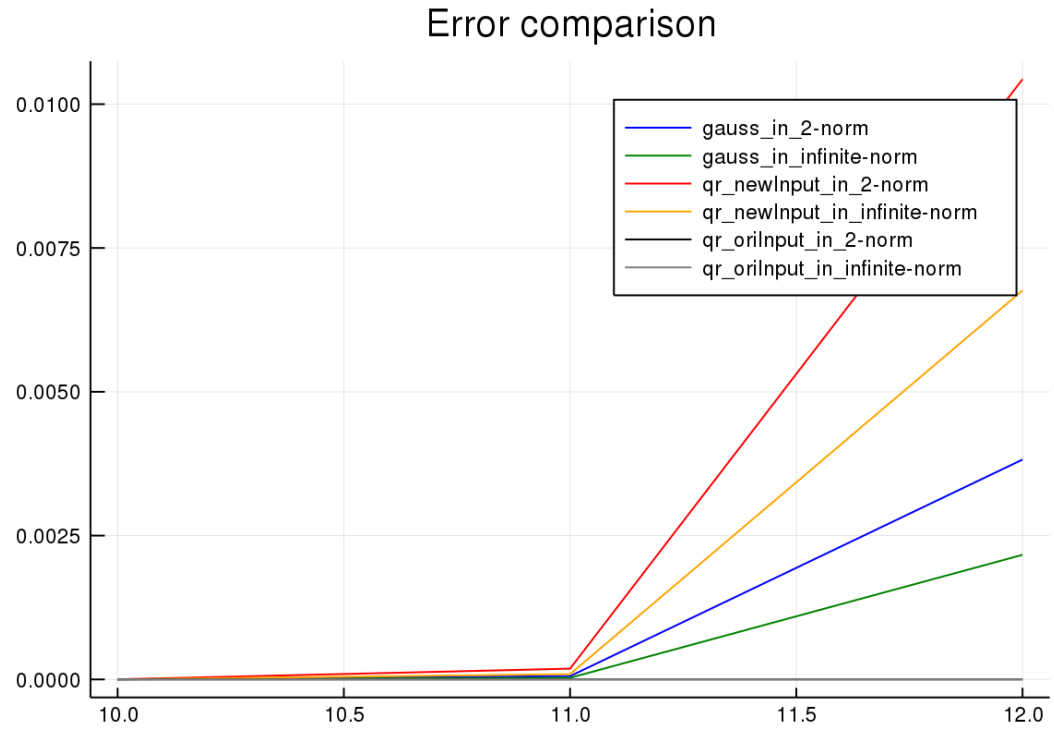
D.

1. gauss_solver 在解新系統時誤差為 m = 13 ,n = 9，2-norm error = 0.8425672

qr_solver 在解新系統的時候誤差在 m = 13 ,n = 9 時 2-norm error = 7.28984

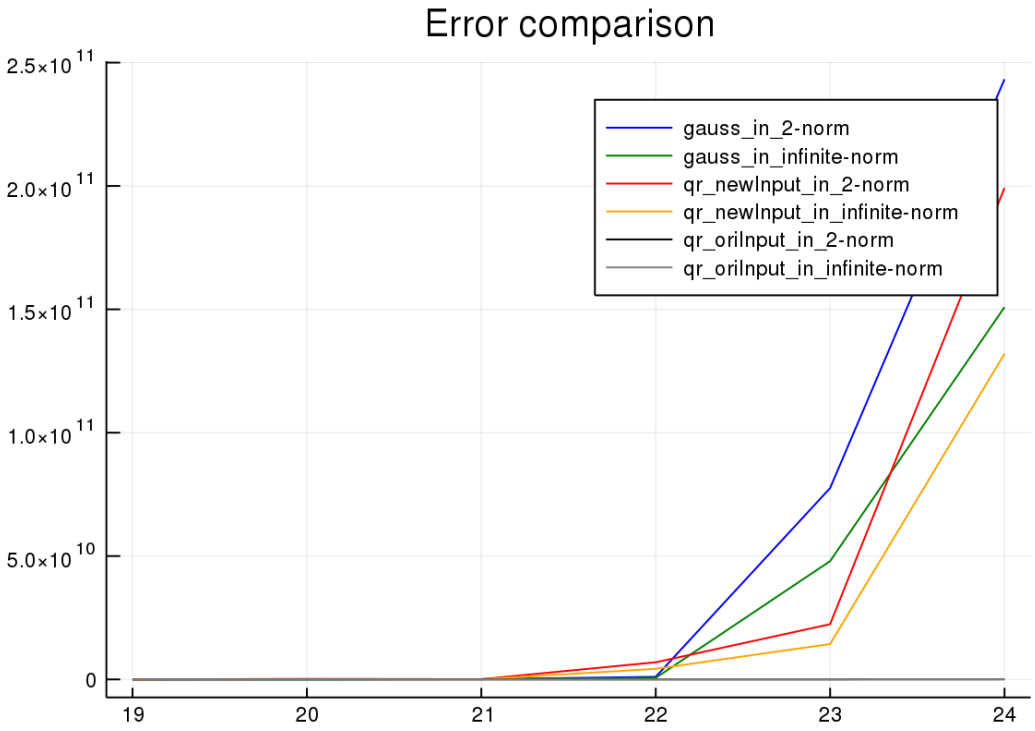
3. qr_solver 在解原系統的時候誤差在 m = 13,n = 9 時 2-norm error = 0

4. qr_solver 在解原系統的時候誤差在 m = 19,n = 15 時 2-norm error = 4.87069



E.更多比較

1. 19 <= m <= 24 , n = m - 4



由此圖可以觀察出 qr_solver 在一開始失去精準度時的速度是比 gauss_solver 慢的，可是多了三個 degree 和三個 sample point 後可以看出 qr_solver 的發散速度會大於 gauss_solver。

2. 在原系統比較兩種 solver 的誤差，方陣大小 19<=m<=21, n = m