# Homework 2

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## 2.34

### a.

Recall that our MLEs are $\widehat{\mu} = 4395.147$ and $\widehat{\sigma} = 1882.499$. Then we can find our asymptotic covariance.

$$\begin{aligned}
\text{AsyCov}(\widehat{\mu}, \widehat{\sigma}^2) &= \frac{1}{n} I^{-1}(Y, \mu, \sigma^2) \\
&\approx \frac{1}{35}\widehat{\sigma}^2 \begin{bmatrix} 1 & -0.423 \\ -0.423 & 1.824 \end{bmatrix}^{-1} \\
&= \frac{1}{35} 1882.499^2 \begin{bmatrix} 112264 & 26035 \\ 26035 & 61548.4 \end{bmatrix} \\
&= 1882.499^2 \begin{bmatrix} 0.031679 & 0.00734662 \\ 0.00734662 & 0.0173679 \end{bmatrix} \\
&= \begin{bmatrix} 112264 & 26035 \\ 26035 & 61548.4 \end{bmatrix}
\end{aligned}$$

### b.

$$\widehat{Q} = \widehat{\sigma}(-\log(-\log(0.993))) + \widehat{\mu}$$

Notice that

$$\begin{bmatrix} \widehat{\mu} \\ \widehat{\sigma} \end{bmatrix} \sim N_2\Big( \begin{bmatrix} \mu \\ \sigma \end{bmatrix}, \sigma^2 \begin{bmatrix} 0.031679 & 0.00734662 \\ 0.00734662 & 0.0173679 \end{bmatrix} \Big)$$

$$\begin{aligned}
\widehat{Q} &= \begin{bmatrix} 1 & -\log(-\log(0.993)) \end{bmatrix} \begin{bmatrix} \widehat{\mu} \\ \widehat{\sigma} \end{bmatrix} \\
&\sim N\Big(\widehat{\mu} - \widehat{\sigma}(-\log(-\log(0.993))), \begin{bmatrix} 1 & -\log(-\log(0.993)) \end{bmatrix} \widehat{\sigma}^2 \begin{bmatrix} 0.031679 & 0.00734662 \\ 0.00734662 & 0.0173679 \end{bmatrix} \begin{bmatrix} 1 & -\log(-\log(0.993)) \end{bmatrix}^T \Big) \\
&= N\Big(4395.147 - 1882.499(-\log(-\log(0.993))), \\
&\widehat{\sigma}^2 \begin{pmatrix} 1 & -\log(-\log(0.993)) \end{pmatrix} . \begin{pmatrix} 0.031679 & 0.00734662 \\ 0.00734662 & 0.0173679 \end{pmatrix} . \begin{pmatrix} 1 & -\log(-\log(0.993)) \end{pmatrix}^T \Big) \\
&= N(13729.2, 1883617)
\end{aligned}$$

## 2.42