# Random Latex Notes

# Jimmy Hickey

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Studying for ST703 Exam 1

$$\overline{X - \overline{x}} = \sum (x_i - \overline{x}) = 0$$

$$Cov(X, Y) = E\left((X - E(X)) \cdot (Y - E(Y))\right)$$
for  $\theta = \beta_1 - \beta_2$ 

$$Var(\hat{\beta}_1 - \hat{\beta}_2) = Var(\hat{\beta}_1) + Var(\hat{\beta}_2) - 2Cov(Var(\hat{\beta}_1), Var(\hat{\beta}_2))$$

$$stderr = \frac{\sigma}{\sqrt{n}}$$

$$stderr(\hat{\beta}_0) = \sqrt{s^2 \left(\frac{1}{n} + \frac{(x_0 - \overline{x})^2}{\sum (x_i - \overline{x})^2}\right)}$$

$$stderr(\hat{\beta}_1) = \sqrt{\frac{s^2}{\sum (x_i - \overline{x})^2}}$$

$$s^2 = MSE$$

Covariance Matrix = 
$$\sigma^2 \cdot (X^T X)^{-1}$$

$$\sigma = \sqrt{MSE} \quad \forall x = x_i$$

$$T = \frac{\hat{\theta} - \theta_0}{SE(\hat{\theta})}$$

$$SSRegn = \sum_i (\hat{y}_i - \bar{y})^2$$

$$SSE = \sum_i (y_i - \hat{y}_i)^2$$

$$SST = \sum (y_i - \bar{y})^2$$

 $R(\beta_1, \beta_2 | \beta_0) = \text{full model SSE of ANOVA}$ 

$$R(\beta_2|\beta_0) = \text{full model SSE}$$
 - Type II SS  $\beta_1$ 

We are 90

# 2019-10-11

Studying for ST520 midterm

$$E[\hat{\theta}] = \theta$$

$$P[D|E] \approx 0 \land P[D|\bar{E}] \approx 0 \Rightarrow \theta \approx \psi$$

In a prospective study,  $n_{1+}$  and  $n_{2+}$  are fixed.

$$[(X_1 > r_1) \land (X_1 + X_2 > r)] \lor (X_1 > r)$$

We will estimate  $\Delta$  using  $\hat{\Delta} = \bar{Y}_1 - \bar{Y}_2$ , where  $\bar{Y}_1$  is the sample average response of the  $n_1$  patients receiving treatment A, and  $\bar{Y}_2$  is the sample average response of the  $n_2$  patients receiving treatment B. Notice  $\hat{\Delta}$  is most efficient when  $\pi = 0.5$ .

Start with m balls labeled A and m balls labeled B. Randomly pick a ball for the first patient and assign the treatment indicated by the ball. If the patient recieves A, then replace that A ball with a B.

#### 2019-10-08

Studying for ST703 Exam 2

Source
 DF
 SS
 MS
 F

 Model
 
$$t-1$$
 $\sum_{i=1}^{t} \sum_{j=1}^{n_1} \left( \bar{y}_{i+} - \bar{y}_{++} \right)^2$ 
 $\frac{SSM}{df_M}$ 
 $\frac{MSM}{MSE}$ 

 Error
  $\sum_{i=1}^{t} (n_i) - t$ 
 $\sum_{i=1}^{t} \sum_{j=1}^{n_i} \left( y_{ij} - \bar{y}_{i+} \right)^2$ 
 $\frac{SSE}{df_E}$ 

 Total
  $\sum_{i=1}^{t} (n_i) - 1$ 
 $\sum_{i=1}^{t} \sum_{j=1}^{n_1} \left( y_{ij} - \bar{y}_{++} \right)^2$ 
 $\frac{SSE}{df_E}$ 

 $\hat{\beta}$ 

 $X\hat{\beta}$ 

	Partial SS (III)	Sequential SS (I)
	used in t-test	incremental
$\overline{X_1}$	$R(\beta_1 \beta_0,\beta_2,\beta_3)$	$R(\beta_1 \beta_0)$
$X_2$	$R(\beta_2 \beta_0,\beta_1,\beta_3)$	$R(\beta_2 \beta_0,\beta_1)$
$X_3$	$R(\beta_3 \beta_0,\beta_1,\beta_2)$	$R(\beta_3 \beta_0,\beta_1,\beta_2)$

$$\begin{split} Var(a_1\hat{\beta}_1 + a_2\hat{\beta}_2) &= a_1^2 Var(\hat{\beta}_1) + a_2^2 Var(\hat{\beta}_2) + 2a_1 a_2 Cov(\hat{\beta}_1, \hat{\beta}_2) \\ |\hat{\theta}| &\geq t_{df_{error}, \alpha/2} SE(\hat{\theta}); \text{ Fisher} \\ &\geq t_{df_{error}, \frac{1}{k} \frac{\alpha}{2}} SE(\hat{\theta}); \text{ Bonferroni} \\ &\geq q_{t, df_{error}, \alpha} \cdot \sqrt{\frac{1}{2}} SE(\hat{\theta}); \text{ Tukey-Kramer} \\ &\geq \sqrt{(t-1) F_{df_{error}, \alpha}^{t-1}} SE(\hat{\theta}); \text{ Scheffe} \end{split}$$

 $\nu$  is the df used to estimate  $\sigma^2$ .  $W_i$  is the sample mean for group i.

$$\mu_P - \mu = 3$$

$$\mu_T - \mu = 3$$

$$\mu_S - \mu = -6$$

$$\mu_E - \mu = 0$$

$$\mu_C - \mu = 0$$

if null is true this subtraction is 0 so  $MSModel = \sigma^2 = MSE$ Pooled variance = MSESample Variance =  $\frac{1}{\sqrt{2\pi}} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{SSTotal}{2\pi}$ 

Sample Variance =  $\frac{1}{N-1} \sum (y_i - \bar{y})^2 = \frac{SSTotal}{N-1}$ Remember

$$t = \frac{\hat{\theta} - \theta_0}{SE(\hat{\theta_0})} \Rightarrow \frac{\hat{\mu}_1 - \hat{\mu}_2 - \theta_0}{\sqrt{MSE(1/n_1 + 1/n_2)}}$$

2-sided F p-value =  $2\Big[1-F_{t_{n-1}}(|t_{obs}|)\Big]$ 

Interpretation Example:  $\beta_0$  is the effect from location 5 for initial weight = 0,  $\beta_{1-4}$  are the differences in effects between locations 1-4 and 5 for fixed initial weight.

 $E_{ij} \sim N(0, \sigma^2)$  where  $\sigma^2$  is the population variance of the response

## 2019-11-18

Studying for 701 midterm 2

$$1 - \frac{u}{u+v} = \frac{v}{u+v}$$

$$\begin{split} M_X(t) &= \int_0^\infty e^{tx} \frac{1}{\Gamma(p/2)2^{p/2}} \cdot x^{p/2-1} e^{-x/2} dx \\ &= \int_0^\infty \frac{1}{\Gamma(p/2)2^{p/2}} x^{p/2-1} \cdot e^{-x/2+tx} dx \\ &= \int_0^\infty \frac{1}{\Gamma(p/2)2^{p/2}} x^{p/2-1} \cdot e^{-x/2(1-2t)} dx \\ &= (1-2t)^{-(p/2-1)} \int_0^\infty \frac{1}{\Gamma(p/2)2^{p/2}} \cdot x^{p/2-1} e^{-x/2(1-2t)} (1-2t)^{p/2-1} dx \\ &= (1-2t)^{-(p/2-1)} \int_0^\infty \frac{1}{\Gamma(p/2)2^{p/2}} \cdot (x(1-2t))^{p/2-1} e^{-x/2(1-2t)} dx \end{split}$$

$$u = x(1 - 2t)$$

$$du = (1 - 2t)dx$$

$$\begin{split} &= (1-2t)^{-(p/2-1)} \int_0^\infty \frac{1}{\Gamma(p/2) 2^{p/2}} \cdot (u)^{p/2-1} e^{-u/2} \frac{1}{1-2t} du \\ &= \frac{(1-2t)^{-p/2+1}}{1-2t} \cdot 1 \\ &= (1-2t)^{-p/2} \end{split}$$

## 2019-11-19

Clinical trials homework 5

$$1 - m(x)\hat{S}(t)$$

#### 2019-12-06

dots

#### 2019-12-07

ST703 final cheat sheet

Source df SS

A 
$$a-1$$
B  $b-1$ 
AB  $(a-1)(b-1)$ 
Error  $N-ab$ 
Total  $N-1$ 

$$\frac{1}{2}[1,1,-1,-1]$$

$$\frac{1}{2}[1,-1,0]$$

$$[0,1,0,-1]$$

$$[0,0,1,-1]$$
SS
$$\frac{\sum_{i}\sum_{j}\sum_{k}(\overline{y}_{i++}-\overline{y}_{+++})^{2}}{\sum_{i}\sum_{j}\sum_{k}(y_{ijk}-\overline{y}_{i++}-\overline{y}_{+++})^{2}}$$

$$\frac{1}{2}[1,-1,0]$$

 $\sigma_T$  accounts for variability among treatment effects; this component contributes to the variance of the observed response because the observed response depends on the effect of a randomly selected treatment

[1, -1, -1, 1]

 $\sigma$  accounts for the variability of the replicate measurements for the same treatment

$$E(Y_{ij}) = \mu$$

$$Var(Y_{ij}) = \sigma_T^2 + \sigma^2$$

$$Cov(Y_{ij}, Cov(Y_{il})) = Cov(T_i + E_{ij}, T_i + E_{il}) = \sigma_T^2 \text{ for } j \neq l$$

$$Cov(Y_{ij}, Y_{kl}) = 0 \text{ for } i \neq k$$

$$\sigma_T^2 = 0$$
 if  $SE = \sqrt{c_1 M S_1 + \dots + c_k M S_k}$  then 
$$E(Y_{ijk}) = 0$$
 
$$Var(Y_{ijk}) = \sigma_A^2 + \sigma_B^2 + \sigma_{AB}^2 + \sigma^2$$
 
$$Cov(Y_{ijk}, Y_{ijl}) = \sigma_A^2 + \sigma_B^2 + \sigma_{AB}^2$$
 
$$Cov(Y_{ijk}, Y_{iml}) = \sigma_A^2$$
 
$$Cov(Y_{ijk}, Y_{qjl}) = \sigma_B^2$$

$$E(Y_{ijk}) = \mu + \alpha_i$$

$$Var(Y_{ijk}) = \sigma_B^2 + \sigma_{\alpha B}^2 + \sigma^2$$

$$Cov(Y_{ijk}, Y_{ijl}) = \sigma_B^2 + \sigma_{\alpha B}^2 \text{ for } k \neq l$$

$$Cov(Y_{ijk}, Y_{iml}) = 0 \text{ for } j \neq m$$

$$Cov(Y_{ijk}, Y_{qjl}) = \sigma_B^2 \text{ for } i \neq q$$

if assuming  $b_i = b$  and  $n_{ij} = n$ , DFB(A) = a(b-1) and DFE = nab - ab

$$E(Y_{ijk}) = \mu$$

$$Var(Y_{ijk}) = \sigma_A^2 + \sigma_{B(A)}^2 + \sigma^2$$

$$Cov(Y_{ijk}, Y_{ijl}) = \sigma_A^2 + \sigma_{B(A)}^2 \text{ for } k \neq l$$

$$Cov(Y_{ijk}, Y_{iml}) = \sigma_A^2 \text{ for } j \neq m$$

$$Cov(Y_{ijk}, Y_{qjl}) = 0 \text{ for } i \neq q$$

$$E(Y_{ijk}) = \mu + \alpha_i$$

$$Var(Y_{ijk}) = \sigma_{B(A)}^2 + \sigma^2$$

$$Cov(Y_{ijk}, Y_{ijl}) = \sigma_{B(A)}^2 \text{ for } k \neq l$$

$$Cov(Y_{ijk}, Y_{iml}) = 0 \text{ for } j \neq m$$

$$Cov(Y_{ijk}, Y_{qjl}) = 0 \text{ for } i \neq q$$

#### 2019-12-08

ST520 final cheat sheet

$$T_n = \frac{\overline{Y}_1 - \overline{Y}_2}{\sqrt{\frac{\sigma_1}{n_1} + \frac{\sigma_2}{n_2}}}$$

$$\phi(n, \Delta_A, \theta) = \frac{\Delta_A}{\sqrt{\frac{\sigma_1}{n_1} + \frac{\sigma_2}{n_2}}}$$

$$SE[KM(t)] = KM(t) \left[ \sum_{deathtimes \ u \le t} \frac{d(u)}{n(u)(n(u) - d(u))} \right]^{1/2}$$

Notice that  $\lambda_1(t) \leq \lambda_0(t) \Rightarrow S_1(t) > S_0(t)$ Compare  $T_n$  to  $\mathcal{Z}_{\alpha}$ 

$$b_j = c(\alpha, K, \Phi) \times j^{(\Phi - 0.5)}$$

 $\Phi = 0$ : O'Brien Fleming

 $\Phi = 0.5$ : Pocock

$$E_{\Delta^*}(V_k) = \sum_{j=1}^k j \cdot P_{\Delta^*}(|T(t_1)| < b_1 \wedge \dots \wedge |T(T_{j-1})| < b_{j-1} \wedge |T(T_j)| > b_j)$$

$$AI = \frac{IF(\alpha, \beta, K, \Phi)}{K} \cdot I^{FS} \cdot E_{\Delta^*}(V)$$

 $\delta_i$  unknown b/c inclusion and exclusion principles and whether or not the patient wants to be in the study So sampling is unknown. Thus, this is a biased sample of the target population

#### 2019-12-10

ST701 Final Exam cheat sheet

Convergence in distribution

 $X_n \stackrel{d}{\to} X$  if  $F_{X_n}(x) \to F_X(x)$  for all x such that  $F_X$  is continuous If c is a constant  $X_n \stackrel{d}{\to} c \Leftrightarrow X_n \stackrel{p}{\to} c$ 

$$M_{X_n}(t) \to M_X(t) \Rightarrow X_n \stackrel{d}{\to} X$$

If  $X_n \stackrel{d}{\to} X$  and  $Y_n \stackrel{p}{\to} c$ 

$$aX_n + bY_n \xrightarrow{d} aX + bc$$

$$X_nY_n \xrightarrow{d} cX$$

$$X_n/Y_n \xrightarrow{d} X/c \ (c \neq 0)$$

$$X_N \stackrel{as}{\to} X \Rightarrow X_n \stackrel{p}{\to} X$$

$$X_N \stackrel{L_2}{\to} X \Rightarrow X_n \stackrel{p}{\to} X$$

$$X_N \stackrel{d}{\to} X \Rightarrow X_n \stackrel{d}{\to} X$$

$$X_N \stackrel{d}{\to} c \Leftrightarrow X_n \stackrel{p}{\to} c$$

CLT

Let  $X_i$  iid RV with  $E(X_i) = \mu$  and  $V(X_i) = \sigma^2$  and  $\overline{X} = 1/n \sum X_i$ .

$$\frac{\sqrt{n}(\overline{X} - \mu)}{\sigma} \stackrel{d}{\to} N(0, 1)$$

#### 2019-12-11

520 Final

$$\overline{\pi} = \frac{\pi_1 \cdot n_1}{n_1 + n_2} + \frac{\pi_2 \cdot n_2}{n_1 + n_2}$$

# 2012-12-12

703 Final

$$\begin{aligned} \mathcal{Z}_{\alpha/2} \cdot SE(\theta) &\leq \frac{length}{2} \\ \\ \mathcal{Z}_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} &\leq \frac{length}{2} \\ \\ \mathcal{Z}_{\alpha/2} \cdot \sqrt{\frac{p(1-p)}{n}} &\leq \frac{2 \cdot 0.02}{2} \end{aligned}$$