

a. Compute  $B^+$ .

$$\begin{aligned} B^+ &= (B) \\ B \rightarrow D \quad B^+ &= (B, D) \\ D \rightarrow A \quad B^+ &= (A, B, D) \\ A \rightarrow BCD \quad B^+ &= (A, B, C, D) \\ BC \rightarrow DE \quad B^+ &= (A, B, C, D, E) \end{aligned}$$

b. Prove (using Armstrongs axioms) that  $AF$  is a superkey.

$$\begin{aligned} (AF)^+ \rightarrow (A) \quad (AF)^+ &= (A) && (Reflexivity) \\ (AF)^+ \rightarrow (F) \quad (AF)^+ &= (A, F) && (Reflexivity) \\ A \rightarrow BCD \quad (AF)^+ &= (A, B, C, D, F) && (FD1) \\ A \rightarrow BC \rightarrow DE \quad (AF)^+ &= (A, B, C, D, E, F) && (Transitivity) \\ R \subseteq (AF)^+, AF &\text{ is a superkey.} \end{aligned}$$

c. Compute a canonical cover for the above set of functional dependencies  $F$ ; give each step of your derivation with an explanation.

Using the union rule  $B \rightarrow D$  is extraneous.

$$\begin{aligned} F' &= \{A \rightarrow BCD, BC \rightarrow DE, D \rightarrow A\} \\ C &\text{ is extraneous in } BC \rightarrow DE. \\ (B)^+ &= (A, B, C, D, E) \\ F'' &= \{A \rightarrow BCD, B \rightarrow DE, D \rightarrow A\} \\ D &\text{ is extraneous in } A \rightarrow BCD. \\ (A)^+ &= (A, B, C, D, E) \\ F''' &= F_c = \{A \rightarrow BC, B \rightarrow DE, D \rightarrow A\} \end{aligned}$$

d. Give a 3NF decomposition of  $r$  based on the canonical cover.

$$\{(A, B, C), (B, D, E), (D, A), (A, F)\}$$

e. Give a BCNF decomposition of  $r$  using the original set of functional dependencies.

$$\{(A, B, C, D), (A, E, F)\} \Rightarrow \{(BD), (ABC), (AEF)\}$$

f. Can you get the same BCNF decomposition of  $r$  as above, using the canonical cover?

No you cannot.