The Pragmatic Theory solution to the Netflix Grand Prize

Martin Piotte
Martin Chabbert
August 2009
Pragmatic Theory Inc., Canada
nfpragmatictheory@gmail.com

Table of Contents

1 Introduction

Recommender systems give companies a way to effectively target their products and services, thus improving their potential for revenue. These systems are also interesting from the customer standpoint because they are presented with alternatives focused on their likes and dislikes, thus lowering the number of unwanted propositions. One of the keys to building a good recommender system is to find meaningful and systematic patterns in the data, in order to identify the concrete tastes of customers.

When Netflix sought out to improve their *Cinematch* recommender system, this is the part of their solution that they targeted. They put together an open, online, competition where participants are given a set of their customers' movie ratings and are asked to predict the ratings that these customers would give on a different set of movies. While this is not the only component that controls the quality of their recommender system, improving the accuracy of these predicted ratings will certainly indicate that the customers' tastes are better captured.

Team *BellKor's Pragmatic Chaos* was the first to achieve a prediction accuracy improvement of more than 10% over *Cinematch*. This team is an international coalition comprised of teams *BellKor*, *Big Chaos* and *Pragmatic Theory*. The joined team includes three participants from *BellKor*: Yehuda Koren, Robert Bell and Chris Volinsky; two participants from *Big Chaos*: Andreas Töscher and Michael Jahrer; and two participants from *Pragmatic Theory*, the authors of this document: Martin Piotte and Martin Chabbert.

The complete solution, as included in the winning July 26th 2009 submission which yielded a 0.8554 quiz set RMSE, is described over three papers, one for each of the joined teams. This document presents the solution from team *Pragmatic Theory*. It includes all methods and techniques, from the most innovative and complex aspects to some of the more naive early efforts. As the name of our team implies, our strategy in this competition was to try anything and everything; to leave no stone unturned. Although we have always tried to choose our methods logically, some of the resulting predictors may not have a proper theoretical or psychological grounding; they were instead selected for their contribution to the blended prediction accuracy. Also, because of this pragmatic approach, not all of the concepts presented here will be usable in a real world recommender system. Still, we believe that our approach has allowed us to find original and creative ideas and to bring known algorithms to a new level, which should, in turn, allow for significant improvement to recommendation engines.

The first section presents some general concepts that are used across multiple models. The second section gives detailed explanations on all of the models that were included as part of the final blend, focusing on the more crucial and innovative models. When relevant, the parameter values that were used for each predictor instance are given. The third section presents the different blending techniques that were used. The final section discusses how the models and techniques presented here can be used to build a more accurate and efficient recommender system. Note that the list of all predictors, with their respective RMSE, is provided in an annex section.

It is assumed the reader is familiar with and understands the content of the Progress Prize papers for 2007 and 2008.

2 Common Concepts

This section describes concepts, methods and results that are reused across multiple models and algorithms we have developed. To avoid repetition, these elements are described once here, while the following sections simply refer to the appropriate element when needed.

2.1 Time variables

It has become obvious over the duration of the Netflix Prize competition that the date data contained useful information for the generation of accurate models. However, values derived from the date are often more useful than the raw date itself.

One derived value that we found most useful is the number of ratings a user has made on a given day. We call this measure frequency, as it measures a number of events occurring in fixed amount of time. We speculate that the usefulness of this measure comes from the fact that Netflix subscribers use the rating interface in two different ways. Users are initially invited to rate movies they have seen in the past to enable the recommender to learn their preference. After this initial period, users will often only rate movies they have just seen. We believe that ratings provided immediately after having seen the movie and ratings provided months or years afterwards have different characteristics. When a user selects a rating, we cannot know how long ago he has seen this movie. However, the number of ratings provided on a given day provides a hint: a large number of ratings clearly indicates that at least some of the movies have not been seen in the immediate past[1]. Similarly, users initializing the system are unlikely to rate only one movie at a time, so single ratings are a good hint that the movie was seen in the recent past.

In addition to the frequency measure, we also use the following time related variables in different models:

- Elapsed time between a given rating and the first rating of the user;
- Elapsed time between a given rating and the mean rating date of the user;
- Elapsed time between a given rating and the median rating date of the user;
- Elapsed time between a given rating and the first rating of the movie;
- Elapsed time between a given rating and the mean rating date of the movie;
- Elapsed time between a given rating and the median rating date of the movie;
- Absolute date value.

Time is always measured in number of days.

2.2 Baselines

Some models require a simple approximation of the ratings as an element of the model. We developed two such simple approximations, which we called baseline_1 and baseline_2.

[1] There is an instance in the dataset of a user rating 5446 movies in a single day. It is clearly not possible that all these movies have been seen by a single person in the recent past.

2.2.1 Baseline1

In Baseline1, a rating is approximated as:

$$baseline_1(u,m) = \mu + b_{baseline1,m}(m) + b_{baseline1,u}(u) \quad (1)$$

Where:

- u is the user;
- m is the movie
- $baseline_1$ is the value of the baseline
- μ is the global mean
- $b_{baseline1,m}$ is a movie bias
- $b_{baseline1,u}$ is a user bias

$b_{baseline1,m}$ and $b_{baseline1,u}$ are chosen to minimize the following expression over the training set:

$$\sum_u \sum_{m\ rated\ by\ u} (r - baseline_1(u,m))^2 + \left(\lambda_m + \frac{\alpha_m}{N_m(m)}\right) b^2_{baseline1,m}(m) + \left(\lambda_u + \frac{\alpha_u}{N_u(u)}\right) b^2_{baseline1,u}(u) \quad (2)$$

Where:

- r is a rating from the training set;
- λ_m, α_m, λ_u and α_u are regularization parameters;
- N_m is the number of ratings of movie m;
- N_u is the number of ratings of user u.

The model is trained using alternating least squares regression for three iterations, starting with the user parameters. This method is described in [12] Section 4. The regularization parameters were chosen using the assisted manual selection (see Section 2.4.2) to minimize the error on the probe set:

λ_u	0.0987708
α_u	4.65075
λ_m	0
α_m	0

Note that the zero values for λ_m and α_m actually means that no regularization is applied to the movie coefficients.

2.2.2 Baseline2

Baseline2 is similar to baseline1; however the movie bias term is multiplied by a user scale factor to model the user rating variance:

$$baseline_2(u,m) = \mu + b_{baseline2,m}(m)\left(1 + s_{baseline2}(u)\right) + b_{baseline2,u}(u) \quad (3)$$

Where:

- u is the user;
- m is the movie
- $baseline_2$ is the value of the baseline
- μ is the global mean
- $b_{baseline2,m}$ is a movie bias
- $b_{baseline2,u}$ is a user bias
- $1+s_{baseline2}$ is the scale applied to the movie bias

The +1 offset in the scale factor ensures the scale remains close to 1 for users with very few ratings. $s_{baseline2}$, $b_{baseline1,m}$ and $b_{baseline1,u}$ are chosen to minimize the following expression over the training set:

$$(r - baseline_2(u,m))^2 + \left(\lambda_m + \frac{\alpha_m}{N_m(m)}\right) b^2_{baseline2,m}(m) + \left(\lambda_u + \frac{\alpha_u}{N_u(u)}\right) b^2_{baseline2,u}(u) + \left(\lambda_s + \frac{\alpha_s}{N_u(u)}\right) s^2_{baseline2}(u) \quad (4)$$

Where:

- r is a rating from the training set;
- λ_s, α_s, λ_m, α_m, λ_u and α_u are regularization parameters;
- N_m is the number of ratings of movie m;
- N_u is the number of ratings of user u.

The model is trained using alternating least squares regression for three iterations, starting with the movie parameters. The regularization parameters were optimized using Nelder-Mead simplex method to minimize the error on the probe set:

λ_m	0.00234879
α_m	0.0610418
λ_u	0.0903385
α_u	4.91299
λ_s	0.0753211
α_s	3.11288

2.3 Non-linear envelopes

Model accuracy can often be improved by transforming the output through a non-linear transformation that limits output between 1 and 5. Such a transformation has been suggested in [3], where the transformation is a sigmoid type function limited between 1 and 5.

The principal problem with this function is that its inflection point occurs at the middle of the range, i.e. at 3. However, this is significantly below the training data average. Better accuracy can be achieved by shifting the inflection point, as in the following function:

$$\sigma_x(x) = \begin{cases} 1 + \dfrac{2(\sigma_0 - 1)}{1 + e^{\frac{-2(x-\sigma_0)}{\sigma_0 - 1}}} & if\ x < \sigma_0 \\ 2\sigma_0 - 5 + \dfrac{2(5 - \sigma_0)}{1 + e^{\frac{-2(x-\sigma_0)}{5-\sigma_0}}} & if\ x \geq \sigma_0 \end{cases} \quad (5)$$

Where:

- σ_0 is a parameter defining the level of the inflection point.

σ_x has the following properties:

- $\sigma_x(\sigma_0) = \sigma_0$
- $\sigma'_x(\sigma_0) = 1$
- $\sigma"_x(\sigma_0) = 0$

We define variants by selecting different expressions for σ_0:

$\sigma_0(x)$	$\sigma_0 = 3$	Symmetrical sigmoid type function
$\sigma_1(x)$	$\sigma_0 = \mu$	Inflection point shifted to the global average μ.
$\sigma_2(x)$	$\sigma_0 = \mu + b_{baseline1,u}(u)$	Inflection point shifted to a user-specific bias based on Baseline1.
$\sigma_3(x)$	$\sigma_0 = \mu + b_{baseline2,m}(m)\ s_{baseline2}(u) + b_{baseline2,u}(u)$	Inflection point shifted to a sample specific bias based on Baseline$_2$.[2]
$\sigma_4(x)$	$\sigma_0 = baseline_2(u, m)$	Inflection point shifted to Baseline2.

It is interesting to note that any "linear" model can be transformed by wrapping it through one of the proposed envelope functions. Often, regularization that was optimized for the linear model is still adequate for the non-linear version. For models learned through various type of gradient descent, the increase in program complexity required to compute the derivative of the envelope function is very small.

2.4 Parameter selection

All statistical models used in our solution require some form of meta-parameter selection, often to reach proper regularization. We used the following methodology to select these meta-parameters.

We train all our models twice. The first time, we train our model on the training data set where the probe set has been removed. When a model uses feedback from movies with unknown ratings, we use the probe set and a random half of the qualifying set for the feedback. We select meta-parameters to optimize the accuracy on the probe set.

The second pass involves retraining the model with the selected meta-parameters, after including the probe set in the training data. The complete qualifying set is used for feedback from rated movies with unknown ratings.

[2] The intent was to use Baseline$_2$, but the expression provided was used by mistake. $\sigma_4(x)$ corrects this error.

Every time parameter selection or validation on the probe set is mentioned in this document, it always refers to optimizing the probe set accuracy of a model or combination of models trained on data excluding the probe set.

We experimented with several approaches to determine optimal meta-parameters.

2.4.1 Nelder-Mead Simplex Method

The simplest and most successful way of selecting meta-parameters has been to use the Nelder-Mead Simplex Method (see [13] and [14]). It is a well known multivariable unconstrained optimization method that does not require derivatives. The algorithm suggests meta-parameter vectors to be tested, the model is then evaluated and the accuracy measured on the probe set. The Nelder-Mead algorithm requires only the measured accuracy on the probe set as feedback. This is repeated until no more significant improvement in accuracy is observed.

2.4.2 Assisted manual selection

While the Nelder-Mead Simplex Method is very simple to use and very powerful, it requires many model evaluations, typically ten times the number of meta-parameters being selected which leads to long execution time for large and complex models. We used different ways to manually shorten the meta-parameter selection time:

- We reused meta-parameters values between similar models (which could be slightly sub-optimal);
- We evaluated meta-parameters on models with smaller dimension, then reused the value found for the larger models;
- We used some of the meta-parameters from another model, while running the Nelder-Mead on the remaining meta-parameters.
- We used the Nelder-Mead algorithm to select a few multiplication constants we applied to the meta-parameters selected for a different variant.

This was used only as a time saving measure. Better results can always be obtained by running the Nelder-Mead algorithm on all meta-parameters simultaneously.

2.4.3 APT2

We also used the APT2 automatic parameter tuning described in [4] in one instance.

2.4.4 Manual selection with hint

Another approach that we experimented with applies only to models defined as a sum of multiple terms:

$$\hat{r} = \sum_{i=1}^{n} f_i(u, m, d) \tag{6}$$

For one such model, it is possible to extend it by multiplying each term by a factor:

$$\widehat{r'} = \sum_{i=1}^{n} \left(1 + a_i + \frac{b_i}{N_u} + \frac{c_i}{N_m}\right) f_i(u, m, d) \tag{7}$$

Where:

- $\hat{r}$ is the initial estimate
- $\widehat{r'}$is the extended estimate
- $N_u$is the number of movies rated by user *u*
- $N_m$is the number of users rating movie *m*
- *u*, *m*, *d* are the user, movie and date
- *a*, *b*, *c* are coefficients assigned to each term *f*

The values of *a*, *b*, *c* are estimated by computing a linear regression to minimize the error of $\widehat{r'}$ over the probe set. The values of a, b and c are then used as a hint on how to manually adjust the regularization parameters used to estimate the terms $f_i(u, m, d)$.

The idea is that for the correct regularization, the values of a, b, c should be close to zero. If different from zero, the sign of a, b, c suggests the direction the regularization should change, and gives some idea of the change magnitude.

We can use the following example to illustrate the methodology, using the following simple model:

$$\hat{r}(u, m) = \sum_{i} p(u, i) q(m, i) \tag{8}$$

Where:

- $\hat{r}(u, m)$ is the predicted rating;
- p(u) is a vector representing latent features for each user;
- q(m) is a vector representing latent features for each movie.

The model is trained by minimizing the following error function over the rated user-movie pairs in the dataset.

$$error = \sum_{u} \sum_{m\ rated\ by\ u} \left[\left(r(u, m) - \sum_{i} p(u, i) q(m, i) \right)^2 + \lambda_p \sum_{i} p(u, i)^2 + \lambda_q \sum_{i} q(m, i)^2 \right] \tag{9}$$

When trained using alternating least squares (see [1]), we found that the best accuracy is achieved by making $\lambda_p$and $\lambda_q$vary according to the number of observations of a given parameter:

$$\lambda_p = \frac{\alpha_p}{N_u} + \beta_p \tag{10}$$

$$\lambda_q = \frac{\alpha_q}{N_m} + \beta_q \tag{11}$$

Where:

- α is the part that varies inversely with the number of observations;
- β is a fixed value

N is the number of observations using the parameter. In this example N_u is the number of movies rated by u, while N_m is the number of users having rated m.

For the example above, the regularization hints work in the following way:

- $a_i > 0$ suggests to decrease the value of β_p and β_q
- $a_i < 0$ suggests to increase the value of β_p and β_q
- $b_i > 0$ suggests to decrease the value of α_p
- $b_i < 0$ suggests to increase the value of α_p
- $c_i > 0$ suggests to decrease the value of α_q
- $c_i < 0$ suggests to increase the value of α_q

Overall, we found that the Nelder-Mead algorithm is simpler to use and converges faster, but we included a description of the hint method since it was used in part of the final submission.

2.4.5 Manual

For most of the early models, the meta-parameters were simply chosen through trial and error to optimize (roughly) the probe set accuracy.

2.4.6 Blend optimization

The ultimate goal of the Netflix Prize competition is to achieve highly accurate rating predictions. The strategy we have followed, which has been followed by most participants, is to blend multiple predictions into a single, more accurate prediction. Because of this, the quality of a model or prediction set is not determined by its individual accuracy, but by how well it improves the blended results.

Accordingly, during automatic parameter selection with the Nelder-Mead simplex method, we sometimes used as objective function the result of a linear regression of the model being optimized with the set of our best predictors. This induced the parameter selection algorithm to find parameters that might not provide the best model accuracy, but would lead a better blending result. This is most helpful with the neighbourhood models.

When this technique was used, we indicate it in the appropriate section. When not indicated, optimization was made on the individual model accuracy.

2.5 Regularized metrics

Several of our algorithms use an estimate of a user's (or movie's) metrics, like the rating average or standard deviation. Because of sparseness, using the user statistic without regularization is often undesirable. We estimate a regularized statistic by adding, to the user rating set, the equivalent to *n*

ratings taken from the training set population. If we take the rating average and rating standard deviation as example, we obtain:

$$\hat{\mu} = \frac{n_u \mu_u + n\mu}{n_u + n} \tag{12}$$

$$\hat{\sigma} = \sqrt{\frac{n_u \overline{r_u^2} + n\overline{r^2}}{n_u + n} - \hat{\mu}^2} \tag{13}$$

Where:

- $\hat{\mu}$ is the regularized average
- $\hat{\sigma}$ is the regularized standard deviation;
- n_u is the number of movies rated by user *u*;
- *n* is the regularization coefficient, i.e. the equivalent of drawing *n* typical samples from the global population;
- μ_u is the average rating of the user *u*;
- μ is the average rating of the training set;
- $\overline{r_u^2}$ is the average of the user ratings squared;
- $\overline{r^2}$ is the average of the training set ratings squared.

The same methodology can be used to compute regularized higher moments, like the skew, or the fraction of ratings given as 1, 2, 3, 4 or 5, etc.

2.6 Iterative training

Model parameters are typically learned through an iterative process: gradient descent or alternating least squares.

Typically, model parameters are seeded with zero values. If this causes a singularity (zero gradient), then small random values are used instead. When some special seeding procedure was used, it is described in the appropriate section.

Typically, training is performed until maximum accuracy on the probe set is obtained. When a different stopping rule is used, for example a fixed number of iterations, it is indicated in the appropriate section.

Unless specified otherwise, L2 regularization is used for all models. It is often described as weight decay as this is how it appears when the model is trained using gradient descent. When the method described in Section 2.5 is used, it is referred to explicitly. Special regularization rules are specified with each model when appropriate.

3 Models

This section of the document describes each model included in the solution one by one. Many of these models are very similar. Also, some of them are early versions that were naive, poorly motivated or even simply incorrect. However, it was in the nature of the competition that the best winning strategy was to include many of these deficient models, even if to obtain only a very small marginal improvement. We would expect a commercial recommender system to use only the best fraction of the model presented here, and to achieve almost the same accuracy. Note that we have not described our failed attempts. This explains some missing algorithms in numbered model families.

3.1 BK1/BK2 integrated models

This model is an integrated model based on the model described in Section 2.3 of [2]. [3]

In [2], the user latent features have a time component that varies smoothly around the average rating date for a user. We augment this smooth variation by a parameter *h(u, t)* that allows for per day variation of the time component, which can give a user-specific correction to the time-dependent latent feature. Interestingly, it also helps to model a user that is in reality a group of two persons sharing an account. If we assume that only one person enters ratings on a given day, then *h(u, t)* can act as a selector between which of the persons is actually providing the ratings on that day.

In some instances we also added a non-linear envelope which improves accuracy, as described in the Common Concepts section.

The model is described by the following equations:

$$\widehat{dev(u,t)} = k_1(t - \bar{t_u})^{k_4} - \overline{dev_u} \tag{14}$$

$$\begin{aligned} z(u,m,t) = \mu + b_m(m,t_{30}) + b_u(u) + b_{u1}(u)\widehat{dev(u,t)} + k_2 b_{u2}(t) \\ + \sum_i^n q_i(m)\left[p_i(u) + p_{1i}(u)(\widehat{dev(u,t)} + h(u,t)) + k_5 p_{2i}(u,t) + \frac{1}{\sqrt{|N(u)|}}\sum_{j\in N(u)} y_i(j)\right] \\ + \frac{1}{\sqrt{|R^k(m;u)|}}\sum_{j\in R^k(m;u)}(r(u,j) - baseline_1(u,j))\,w_{m,j} + \frac{1}{\sqrt{|N^k(m;u)|}}\sum_{j\in N^k(m;u)} c_{m,j} \end{aligned} \tag{15}$$

$$\hat{r}(u,m,t) = \begin{cases} z(u,m,t) & linear\ model \\ \sigma_2(z(u,m,t)) & non\text{-}linear\ model \end{cases} \tag{16}$$

Where:

- $\widehat{dev(u,t)}$ is the time varying function described in [2] to model time-dependent ratings;
- k_1 is a scaling factor used to stabilize the regularization, and thus allows the same regularization parameters for similar terms of the model;
- *t* is the date of the rating;

[3] The acronym BK was chosen as the name of a number of integrated model variants because they were inspired from the model described by team BellKor in [2].

- $\bar{t_u}$ is the mean rating date for user *u*;
- k_4 is an exponent applied to the date deviation;
- $\overline{dev_u}$ is an offset added to make the mean value of $\widehat{dev(u,t)}$ equals to zero (but in some experiments we left $\overline{\mathrm{dev_u}} = 0$)
- μ is the global rating average in the training data;
- t_{30} is the date value mapped to an integer between 1 and 30 using equal time intervals;
- b_m is the movie bias (same as b_i in [2]);
- b_u is the user time independent bias (same as in [2]);
- b_{u1} is the time-dependent part of the user bias (same as $b_u^{(1)}$ in [2]);
- b_{u2} is the per-day user bias (same as $b_u^{(2)}$ in [2]);
- *n* is the length of the latent feature vectors;
- *q* is the movie latent feature vector of length *n*;
- *p* is the time independent user latent feature vector of length *n;*
- p_1 is the time-dependent user latent feature vector of length *n;*
- p_2 is the per-day user latent feature vector of length *n;*
- *h* is a per user and per day correction to $\widehat{dev(u,t)}$ for user latent features;
- *y(j)* is the implicit feedback vector of length *n* for movie *j* rated by user *u* as in [2];
- *N(u)* is the set of movies rated by *u*, including movies for which the rating is unknown;
- $R^k(m; u)$ is the subset of movies with known ratings by *u* that are among the *k* neighbours of movie *m;*
- $N^k(m; u)$ is the subset of movies rated by *u* (rating known or not) that are among the *k* neighbours of movie *m;*
- *r(u, j)* is the rating given by user *u* to movie *j*;
- $Baseline_1$ is defined in the Common Concepts section;
- $w_{m,j}$ is the weight given to the rating deviation from the baseline for the movie pair (*m, j*);
- $c_{m,j}$ is an offset given to the movie pair (*m, j*).
- σ_2 is a non-linear function described in the Common Concepts section.

The neighbouring movies of movie *m* are selected as the set of movies that maximize the following expression:

$$\frac{\rho_{m,j} n_{m,j}}{n_{m,j} + \alpha_\rho} \tag{17}$$

Where:

- $\rho_{m,j}$ is the Pearson's correlation between the ratings of movies *m* and *j* computed over users having rated both;
- $n_{m,j}$ is the number of occurrences of movies *m* and *j* rated by the same user;
- α_ρ is a shrinkage coefficient set to 100.

The parameters are learned through stochastic gradient descent by minimizing the squared error over the training set. The training samples for each user are sorted by date, so that earlier dates are evaluated first during training. b_m, b_w, b_{u1} and b_{u2} are trained using a learning rate γ_1 and a weight decay of λ_6. *q, p,* p_1, p_2 and *y* are trained using a learning rate γ_2 and a weight decay of λ_7. *w* and *c* are trained using a learning rate γ_3 and a weight decay of λ_8. *h is* trained using a learning rate γ_9 and a weight decay of λ_9. γ values are decreased by a factor $\Delta\gamma$ at each iteration, i.e. γ values are multiplied by (1 - $\Delta\gamma$). All the meta-parameters are selected by validation on the probe set. The values were selected using a combination of manual selection, APT2 (described in [4]) and Nelder-Mead Simplex Method. During the selection of the meta-parameters, the maximum number of iterations is limited to 30.

The following table shows the combinations used in the solution. Many of these models differ only in their regularization parameters, because they were intermediate steps in the search for optimal meta-parameters. We found that keeping some of these intermediate attempts provided a small improvement to the blend. Note that, in all these variants, p_2 is forced to zero.

Variant	n	k	σ_2	$\overline{dev_u}$	k_1	k_2	k_4	k_5	$\Delta\gamma$
bk1-a50[4]	50	n/a	no	0	0.0363636	0.909091	0.4	1	0.0785714
bk1-a50-2	50	n/a	no	0	0.0363636	0.909091	0.4	1	0.0785714
bk1-a200	200	n/a	no	0	0.0363636	0.909091	0.4	1	0.0785714
bk1-a1000	1000	n/a	no	mean	0.0363636	0.909091	0.4	1	0.0785714
bk1-b200-1	200	300	no	0	0.04	1	0.4	1	0.1
bk1-b200-2	200	300	no	0	0.0363636	0.909091	0.4	1	0.0785714
bk1-b200-5	200	300	no	mean	0.0341776	1.07444	0.4	1	0.085444
bk1-b200-6	200	300	no	mean	0.0362749	0.951543	0.4	1	0.0845112
bk1-b1000	1000	300	no	mean	0.0362749	0.951543	0.4	1	0.0845112
bk1-c200	200	1000	no	mean	0.0362749	0.951543	0.4	1	0.0845112
bk2-b200h	200	300	no	mean	0.0425157	0.994899	0.4	1	0.0918146
bk2-b200hz	200	300	yes	mean	0.0425157	0.994899	0.4	1	0.0918146

Variant	λ_6	λ_7	λ_8	λ_9	γ_1	γ_2	γ_3	γ_9
bk1-a50	0	0.015	0	0	0.007	0.007	0	0
bk1-a50-2	0	0.015	0	0	0.007	0.007	0	0
bk1-a200	0	0.015	0	0	0.007	0.007	0	0
bk1-a1000	0	0.015	0	0	0.007	0.007	0	0
bk1-b200-1	0.005	0.015	0.015	0	0.007	0.007	0.001	0
bk1-b200-2	0	0.015	0.015	0	0.007	0.007	0.001	0
bk1-b200-5	0.0042722	0.0138561	0.0218166	0	0.00598108	0.0074641	0.00085444	0
bk1-b200-6	0.000330738	0.0159766	0.031863	0	0.00753674	0.00774446	0.000152311	0
bk1-b1000	0.000330738	0.0159766	0.031863	0	0.00753674	0.00774446	0.000152311	0
bk1-c200	0.000330738	0.0159766	0.031863	0	0.00753674	0.00774446	0.000152311	0
bk2-b200h	0.00342766	0.0141807	0.0256555	0.000373603	0.00765716	0.00770752	0.000134346	0.0406664
bk2-b200hz	0.00342766	0.0141807	0.0256555	0.000373603	0.00765716	0.00770752	0.000134346	0.0406664

Learning p_2 simultaneously with all other parameters requires a huge amount of memory storage. Typically, this cannot be done in a home computer main memory, and disk storage must be used. Instead of doing this, we used the following alternative. We first train the model without the p_2 term. After convergence, the model is retrained using the movie parameters learned during pass #1 and learning only the user parameters including p_2 this time in pass #2. The idea is that during pass #2, the

[4] Due to a bug, sorting of the training sample per date was incorrect for this variant.

users become independent from each other (since the movie parameters are now fixed), and thus is necessary to store the p_2 values only for one user at a time.

The following table shows the retraining parameters for the sets included in the solution:

Variant	Pass #1	k_5	$\Delta\gamma$	λ_9	γ_1	γ_2	γ_9
bk1-a50-2x	bk1-a50-2	0.901726	0.0743765	0.00219152	0.00861301	0.00636511	0.0235341
bk1-a200x	bk1-a200	0.901726	0.0743765	0.00219152	0.00861301	0.00636511	0.0235341
bk1-a1000x	bk1-a1000	0.901726	0.0743765	0.00219152	0.00861301	0.00636511	0.0235341
bk1-b200-5x	bk1-b200-5	0.85359	0.0837099	0.00265673	0.00805462	0.00595674	0.0209891
bk1-b200-6x	bk1-b200-6	0.89084	0.0699494	0.00239747	0.00798123	0.00615092	0.021551
bk1-c200x	bk1-c200	0.850717	0.0728293	0.00240786	0.00831418	0.00639189	0.0205951

3.2 BK3 integrated model

This model extends BK1/BK2 by including movie latent features that are time-dependent. The movie bias is also modified to include a time invariant bias and a frequency dependent correction. The frequency measure is defined in the Common Concepts section.

The model is described by the following equations:

$$\widehat{dev(u,t)} = k_1(t-\bar{t_u})^{k_4} - \overline{dev_u} \tag{18}$$

$$\begin{aligned} z(u,m,t) = \mu + b_m(m) + b_{mt}(m,t_{30}) + b_{mf}(m,f_{30}) + b_u(u) + b_{u1}(u)\widehat{dev(u,t)} + k_2 b_{u2}(t) \\ + \sum_i^n [q_i(m) + q_{t,i}(m,t_8) + q_{f,i}(m,f_8)] \Bigg[p_i(u) + p_{1i}(u)(\widehat{dev(u,t)} + h(u,t)) \\ + \frac{1}{\sqrt{|N(u)|}} \sum_{j\in N(u)} y_i(j) \Bigg] + \frac{1}{\sqrt{|R^k(m;u)|}} \sum_{j\in R^k(m;u)} (r(u,j) - baseline_1(u,j))\, w_{m,j} \\ + \frac{1}{\sqrt{|N^k(m;u)|}} \sum_{j\in N^k(m;u)} c_{m,j} \end{aligned} \tag{19}$$

$$\hat{r}(u,m,t) = \begin{cases} z(u,m,t) & linear\ model \\ \sigma_2(z(u,m,t)) & non-linear\ model \end{cases} \tag{20}$$

Where:

- $\widehat{dev(u,t)}$ is the time varying function described in [2] to model time-dependent ratings;
- k_1 is a scaling factor used to stabilize the regularization, and thus allow the same regularization parameters for similar terms of the model;
- t is the date of the rating;
- $\bar{t_u}$ is the mean rating date for user u;
- k_4 is an exponent applied to the date deviation;
- $\overline{dev_u}$ is an offset added to make the mean value of $\widehat{dev(u,t)}$ equals to zero;
- μ is the global rating average in the training data;
- t_{30} (t_8) is the date value mapped to an integer between 1 and 30 (8): mapping boundaries are selected so that each one contains roughly the same number of samples from the data set;

- f_{30} (f_8) is the frequency value mapped to an integer between 1 and 30 (8): mapping boundaries are selected so that each one contains roughly the same number of samples from the data set;
- b_m is the movie bias;
- b_{mt} is the date dependent movie bias correction;
- b_{mf} is the frequency dependent movie bias correction;
- b_u is the user time independent bias;
- b_{u1} is the time-dependent part of the user bias;
- b_{u2} is the per-day user bias;
- q is the movie latent feature vector of length n;
- q_t is the date dependent correction vector of length n for the movie latent features;
- q_f is the frequency dependent correction vector of length n for the movie latent features;
- p is the time independent user latent feature vector of length n;
- p_1 is the time-dependent user latent feature vector of length n;
- h is a per-day correction to $\widehat{dev(u,t)}$ for user latent features;
- $y(j)$ is the implicit feedback vector of length n for movie j rated by a user;
- $N(u)$ is the set of movies rated by u, including movies for which the rating is unknown;
- $R^k(m; u)$ is the subset of movies with known ratings by u that are among the k neighbours of movie m;
- $N^k(m; u)$ is the subset of movies rated by u (rating known or not) that are among the k neighbours of movie m;
- $r(u, j)$ is the rating given by user u to movie j;
- $Baseline_1$ is defined in the Common Concepts section;
- $w_{m,j}$ is the weight given to the rating deviation from the baseline for the movie pair (m, j);
- $c_{m,j}$ is an offset given to the movie pair (m, j).
- σ_2 is a non-linear function described in the Common Concepts section.

The neighbouring movies are selected as in BK1/BK2.

The parameters are learned through stochastic gradient descent by minimizing the squared error over the training set. The training samples for each user are sorted by date. b_m, b_{mt}, b_{mf}, b_u, b_{u1} and b_{u2} are trained using a learning rate γ_1 and a weight decay of λ_6. q, p, p_1, p_2 and y are trained using a learning rate γ_2 and a weight decay of λ_7. q_t is trained using a learning rate γ_{11} and a weight decay of λ_{11}. q_f is trained using a learning rate γ_{12} and a weight decay of λ_{12}. w and c are trained using a learning rate γ_3 and a weight decay of λ_8. h is trained using a learning rate γ_9 and a weight decay of λ_9. γ values are decreased by a factor $\Delta\gamma$ at each iterations. All the meta-parameters are selected by validation on the probe set. The values were selected using a combination of manual selection and the Nelder-Mead Simplex Method. During the selection of the meta-parameters, the maximum number of iterations is limited to 30.

The following table shows the combinations used in the solution.

Variant	n	k	σ_2	k_1	k_2	k_4	$\Delta\gamma$
bk3-a0z	0	n/a	yes	0.0362749	0.951543	0.4	0.0845112
bk3-a50	50	n/a	no	0.042979	1.0	0.4	0.11587
bk3-b200	200	300	no	0.042979	1.0	0.4	0.11587
bk3-c50	50	n/a	no	0.036	1.0	0.4	0.085121
bk3-c50x[5]	50	n/a	no	0.0362749	0.951543	0.4	0.0845112
bk3-c100[6]	*100*	*n/a*	*no*	0.0362749	0.951543	0.4	0.0845112
bk3-d200	200	300	no	0.0362749	0.951543	0.4	0.0845112
bk3-d200z	200	300	yes	0.0362749	0.951543	0.4	0.0845112

Variant	λ_6	λ_7	λ_8	γ_1	γ_2	γ_3
bk3-a0z	0.000330738	0	0	0.00753674	0	0
bk3-a50	0.00138669	0.0092125	0	0.00790541	0.00525646	0
bk3-b200	0.00138669	0.0092125	0.030711	0.00790541	0.00525646	0.000199005
bk3-c50	0.0010603	0.0192911	0	0.00747126	0.00733103	0
bk3-c50x	0.000330738	0.0192911	0	0.00753674	0.00733103	0
bk3-c100	*0.000330738*	*0.0192911*	*0*	*0.00753674*	*0.00733103*	*0*
bk3-d200	0.000330738	0.0192911	0.031863	0.00753674	0.00733103	0.000152311
bk3-d200z	0.000330738	0.0192911	0.031863	0.00753674	0.00733103	0.000152311

Variant	λ_9	λ_{11}	λ_{12}	γ_9	γ_{11}	γ_{12}
bk3-a0z	0	0	0	0	0	0
bk3-a50	0.000373603	0.0092125	0	0.0406664	0.00525646	0
bk3-b200	0.000373603	0.0092125	0	0.0406664	0.00525646	0
bk3-c50	0.000373603	0.040012	0.0216398	0.0406664	1.48091×10^{-5}	0.000106117
bk3-c50x	0.000373603	0.040012	0.0216398	0.0406664	1.48091×10^{-5}	0.000106117
bk3-c100	0.000373603	0.040012	0.0216398	0.0406664	1.48091×10^{-5}	0.000106117
bk3-d200	0.000373603	0.040012	0.0216398	0.0406664	1.48091×10^{-5}	0.000106117
bk3-d200z	0.000373603	0.040012	0.0216398	0.0406664	1.48091×10^{-5}	0.000106117

3.3 BK4 integrated model

This model is our most sophisticated time-dependent integrated model. It uses a very rich time modeling and precise regularization.

One of the significant differences of this model compared to the BK1/BK2/BK3 is that several time effects are modeled as low rank matrix factorizations. For example, a time-dependent user bias $b_u(u, t)$ can be approximated as the inner-product of a user vector and a time vector.

Another significant difference is the introduction of a time-dependent scale factor s(u, t) inspired from [9]. Scaling is important to capture the difference in rating behaviour from different users. Some users will rate almost all movies with the same rating, while other will use the full scale more uniformly. Terms in the model that include only movie and time cannot reflect this spectrum of behaviour unless they are scaled by a user-specific parameter. These include the movie bias, the implicit feedback contribution to

[5] This variant uses the 2006 ratings from the KDD cup (http://www.netflixprize.com/community/viewtopic.php?pid=6555)

[6] This variant is not used in the solution, but it is included here because it is referred to in the Logistic Transformation section.

the latent features and the implicit feedback contribution of the neighbourhood model. It is important to note that the same scale value is used for all three. The other terms do not require explicit scaling as they already contain an implicit per-user scaling factor.

The time-dependent latent features are weighted by a sigmoid function in this model, which offers the advantage of being restricted to [0, 1]. This sigmoid function is offset for each user so that the expression has a mean of zero for the user or movie it applies to. The sigmoid captures the main time-dependent correction, but it is used in conjunction with a finer grain correction term. It is interesting to note that even the implicit feedback vector *y* also becomes date dependent in the model (see also Section 3.12.4 Milestone).

The neighbourhood part is modified to underweight ratings far apart in time.

The model is defined by the following equations:

$$\begin{aligned} z(u,m,t) &= \mu + b_u(u,t) + b_m(m,t)\, s(u,t) \\ &+ \sum_i^n q_i(m,t) \left[p_i(u,t) + \frac{s(u,t)}{\sqrt{|N(u)|}} \sum_{j \in N(u)} y_i(j,t) \right] \\ &+ \sum_{j \in R^k(m;u)} w_w(m,j)(r(u,j) - baseline_2(u,j))\, w_{m,j} \\ &+ \sum_{j \in N^k(m;u)} w_c(m,j) s(u,t) c_{m,j} \end{aligned} \tag{21}$$

$$b_u(u,t) = b_{u0}(u) + \sum_i^{n_{bu}} b_{u1,i}(u) \left[b_{uf,i}(f) + b_{ut,i}(t_u) \right] + b_{u2}(u,t) \tag{22}$$

$$b_m(m,t) = b_{m0}(m) + \sum_i^{n_{bm}} b_{m1,i}(m) \left[b_{mf,i}(f) + b_{mt,i}(t_m) \right] \tag{23}$$

$$s(u,t) = 1 + s_0(u) + \sum_i^{n_{su}} s_{1,i}(u) \left[s_{f,i}(f) + s_{t,i}(t_u) \right] + s_2(u,t) \tag{24}$$

$$q(m,t) = q_0(m) + q_1(m) \left[\sigma\left(f\alpha_q + \beta_q \right) + q_t(m, t_{36}) + q_f(m, f_{36}) \right] \tag{25}$$

$$p(u,t) = p_0(u) + p_1(u) \left[\sigma\left(t_u \alpha_p + \beta_p \right) + p_2(u,t) \right] \tag{26}$$

$$y(j,t) = y_0(j) + y_1(j) \left[\sigma\left(f\alpha_y + \beta_y \right) + y_t(j, t_{36}) + y_f(j, f_{36}) \right] \tag{27}$$

$$\sigma(x) = \frac{1}{1 + e^{-x}} \tag{28}$$

$$w_w(m,j) = \frac{\frac{1}{1 + \beta_w t_\Delta(m,j)}}{\sqrt{\sum_{l \in R^k(m;u)} \left(\frac{1}{1 + \beta_w t_\Delta(m,l)} \right)^2}} \tag{29}$$

$$w_c(m,j) = \frac{\frac{1}{1 + \beta_w t_\Delta(m,j)}}{\sqrt{\sum_{l \in N^k(m;u)} \left(\frac{1}{1 + \beta_w t_\Delta(m,l)} \right)^2}} \tag{30}$$

$$\hat{r}(u,m,t) = \begin{cases} z(u,m,t) & linear\ model \\ \sigma_2(z(u,m,t)) & non-linear\ model \\ \sigma_3(z(u,m,t)) & non-linear\ model \\ \sigma_4(z(u,m,t)) & non-linear\ model \end{cases} \quad (31)$$

Where:

- $z(u,m,t)$ is the linear approximation of the user *u* rating for movie *m* at time *t*;
- μ is the global rating average in the training data;
- $b_u(u, t)$ is the time-dependent user bias;
- $b_m(m, t)$ is the time-dependent movie bias;
- *s(u, t)* is the time-dependent scale factor for user *u*;
- *q(m, t)* is the time-dependent movie latent feature vector of length *n*;
- *p(u, t)* is the time-dependent user latent feature vector of length *n*;
- *y(j, t)* is the time-dependent implicit feedback vector of length *n* for movie *j*;
- *N(u)* is the set of all movies rated by *u*, including movies for which the rating is unknown;
- $R^k(m; u)$ is the subset of movies with known ratings by *u* that are among the *k* neighbours of movie *m;*
- $N^k(m; u)$ is the subset of movies rated by *u* (rating known or not) that are among the *k* neighbours of movie *m;*
- $w_w(m, j)$ is the weight associated with the neighbourhood relation between movies *m* and *j* rated by user *u;*
- *r(u, j)* is the rating provided by *u* for movie *j*;
- *baseline2* is described in the Common Concepts section;
- $w_{m,j}$ is the weight given to the movie pair (*m, j*) to the rating deviation from the baseline;
- $c_{m,j}$ is an offset given to the movie pair (*m, j*).
- $b_{u0}(u)$ is the time independent user bias;
- $b_{u1}(u)$ is a vector of length n_{bu} which form the user part of the low rank matrix factorization of the time-dependent part of the user bias;
- b_{uf} and b_{ut} are vectors of length n_{bu} which form the time part of the low rank matrix factorization of the time-dependent part of the user bias: b_{uf} is selected based on frequency, b_{ut} based on the deviation from the median date;
- *f* is the frequency (see the Common Concepts section for a definition);
- t_u is the difference in days between the current day and the median day for the user;
- b_{u2} is a per user and per day user bias correction, as explained in [2];
- $b_{m0}(u)$ is the time independent movie bias;
- $b_{m1}(u)$ is a vector of length n_{bm} which form the movie part of the low rank matrix factorization of the time-dependent part of the movie bias;
- b_{mf} and b_{mt} are vectors of length n_{bm} which form the time part of the low rank matrix factorization of the time-dependent part of the movie bias: b_{mf} is selected based on frequency, b_{mt} based on the deviation from the median date;
- t_m is the difference in days between the current day and the median rating day for the movie;

- $s_0(u)$ is the time independent user scale;
- $s_1(u)$ is a vector of length n_{su} which form the user part of the low rank matrix factorization of the time-dependent part of the user scale;
- s_f and s_t are vectors of length n_{su} which form the time part of the low rank matrix factorization of the time-dependent part of the user scale: s_f is selected based on frequency, s_t based on the deviation from the median date;
- s_2 is a per user and per day user scale correction, similar in concept to b_2;
- q_0 is a vector of length n representing the time independent movie latent features;
- q_1 is a vector of length n representing the time-dependent movie latent features;
- $\sigma(f\alpha_q + \beta_q)$ is the weight given to q_1 based on the frequency (α_q and β_q are constants found by validation on the probe set), the sigmoid value is offset per movie to have a zero mean over the movie ratings.
- q_t is an adjustment to the q_1 weight for a given movie within a certain date range identified by t_{36};
- q_f is an adjustment to the q_1 weight for a given movie within a certain frequency range identified by f_{36};
- t_{36} identifies one of 36 time intervals chosen uniformly between the first and last date of the training data;
- f_{36} identifies one of 36 frequency intervals according to a logarithmic scale between 1 and 5446 (the maximum of ratings made by one user on a single day in the training data);
- p_0 is a vector of length n representing the time independent user latent features;
- p_1 is a vector of length n representing the time-dependent user latent features;
- $\sigma(t_u\alpha_p + \beta_p)$ is the weight given to p_1 based on the date deviation from the user median date (α_p and β_p are constants found by validation on the probe set)), the sigmoid value is offset per user to have a zero mean over the user ratings;
- p_2 is an adjustment to the p_1 weight for a given user on a given day (note that p_2 is the same as h in BK3: it is not a vector, but a weight scaling a vector, which keeps the number of free parameters smaller);
- $y_0(j)$ is a vector of length n representing the time independent implicit feedback for movie j;
- $y_1(j)$ is a vector of length n representing the time-dependent implicit feedback for movie j;
- $\sigma(f\alpha_y + \beta_y)$ is the weight given to y_1 based on the frequency (α_y and β_y are constants found by validation on the probe set) , the sigmoid value is offset per movie to have a zero mean over the movie ratings;
- y_t is an adjustment to the y_1 weight for a given movie within a certain date range identified by t_{36} (the date considered here is the date of movie j);
- y_f is an adjustment to the y_1 weight for a given movie within a certain frequency range identified by f_{36} (the date considered here is the date of movie j);
- t_Δ is the absolute value of the number of days between the ratings of movies m and j (or l) in the neighbourhood model;

- β_w is a constant that determines how the neighbour weight decays when t_Δ increases ($\beta_w$is determined by validation on the probe set);
- σ_2, σ_3 and σ_4 are non-linear envelope functions described in the Common Concepts section.

The neighbouring movies of movie *m* are selected as the set of *k* movies that maximize the following expression:

$$\frac{\rho_{m,j} n_{m,j}}{n_{m,j} + \alpha_\rho} \tag{32}$$

Where:

- $\rho_{m,j}$ is the similarity measure described in Appendix 1 of [7];
- $n_{m,j}$ is the number of occurrences of movies *m* and *j* rated by the same user;
- α_ρ is a shrinkage coefficient determined through validation on the probe set.

The parameters are learned through stochastic gradient descent by minimizing the squared error over the training set. In some instances, the training data was sorted by date, in others it was in random order. This model uses a very extensive number of meta-parameters to control learning rate and weight decay (regularization). Learning rates are decreased by a factor $\Delta\gamma$ at each iteration. All the meta-parameters are selected by validation on the probe set. The values were selected using a combination of manual selection and the Nelder-Mead Simplex Method. During the selection of the meta-parameters, the maximum number of iterations is limited to 30.

Parameter	Learning rate	Weight decay	**Parameter**	Learning rate	Weight decay
$\boldsymbol{b_{u0}}$	γ_1	λ_1	$\boldsymbol{q_0}$	γ_{15}	λ_{15}
$\boldsymbol{b_{u1}}$	γ_2	λ_2	$\boldsymbol{p_0}$	γ_{16}	λ_{16}
$\boldsymbol{b_{ut}}$	γ_3	λ_3	$\boldsymbol{y_0}$	γ_{17}	λ_{17}
$\boldsymbol{b_{uf}}$	γ_4	λ_4	$\boldsymbol{q_1}$	γ_{18}	λ_{18}
$\boldsymbol{b_{u2}}$	γ_5	λ_5	$\boldsymbol{p_1}$	γ_{19}	λ_{19}
$\boldsymbol{b_{m0}}$	γ_6	λ_6	$\boldsymbol{y_1}$	γ_{20}	λ_{20}
$\boldsymbol{b_{m1}}$	γ_7	λ_7	$\boldsymbol{w_{m,j}}$	γ_{21}	λ_{21}
$\boldsymbol{b_{mt}}$	γ_8	λ_8	$\boldsymbol{c_{m,j}}$	γ_{22}	λ_{22}
$\boldsymbol{b_{mf}}$	γ_9	λ_9	$\boldsymbol{p_2}$	γ_{23}	λ_{23}
$\boldsymbol{s_0}$	γ_{10}	λ_{10}	$\boldsymbol{q_t}$	γ_{24}	λ_{24}
$\boldsymbol{s_1}$	γ_{11}	λ_{11}	$\boldsymbol{q_f}$	γ_{25}	λ_{25}
$\boldsymbol{s_t}$	γ_{12}	λ_{12}	$\boldsymbol{y_t}$	γ_{26}	λ_{26}
$\boldsymbol{s_f}$	γ_{13}	λ_{13}	$\boldsymbol{y_f}$	γ_{27}	λ_{27}
$\boldsymbol{s_2}$	γ_{14}	λ_{14}			

We also used this model to post-process the residual of other models. In these instances, μ takes the value of the baseline model, instead of the global average.

The following tables show the combinations used in the solution.

Variant	Baseline	n	n_{bu}	n_{bm}	n_{su}	k	envelope	sorted	$\Delta\gamma$
bk4-bias	n/a	0	4	20	4	0	no	no	0.085[7]
bk4-biasZ	n/a	0	4	20	4	0	σ_3	no	0.085[7]
bk4-a50	n/a	50	4	20	4	0	no	no	0.085[7]
bk4-b200	n/a	200	4	20	4	0	no	no	0.085
bk4-c50	n/a	50	4	20	4	0	no	yes	0
bk4-c200[8]	*n/a*	*200*	*4*	*20*	*4*	*0*	*no*	*yes*	*0*
bk4-c500	n/a	500	4	20	4	0	no	yes	0
bk4-c200z	n/a	200	4	20	4	0	σ_2	yes	0
bk4-e50a	n/a	50	4	20	4	0	no	yes	0
bk4-e200	n/a	200	4	20	4	0	no	yes	0
bk4-d50	n/a	50	4	20	4	445	no	yes	0
bk4-d50B128[9]	n/a	50	4	20	4	445	no	yes	0
bk4-d500	n/a	500	4	20	4	445	no	yes	0
bk4-f200z4	n/a	200	4	20	4	445	σ_4	yes	0
drbm160-640-bk4	drbm160-640[10]	0	4	20	4	0	no	no	0.085[7]
frbm300-bk4	frbm300[10]	0	4	20	4	0	no	no	0.085[7]

Variant	α_q	α_p	α_y	β_q	β_p	β_y	α_ρ	β_w
bk4-bias	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
bk4-biasZ	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
bk4-a50	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
bk4-b200	0.121418	0.00792848	0.17656	-8.89293	220.309	11.8247	n/a	n/a
bk4-c50	0.0602044	0.00689306	0.118019	-6.42095	253.658	7.21242	n/a	n/a
bk4-c200	*0.0602044*	*0.00689306*	*0.118019*	*-6.42095*	*253.658*	*7.21242*	*n/a*	*n/a*
bk4-c500	0.0602044	0.00689306	0.118019	-6.42095	253.658	7.21242	n/a	n/a
bk4-c200z	0.0602044	0.00689306	0.118019	-6.42095	253.658	7.21242	n/a	n/a
bk4-e50a	0.0602044	0.00689306	0.118019	-6.42095	253.658	7.21242	n/a	n/a
bk4-e200	0.0602044	0.00689306	0.118019	-6.42095	253.658	7.21242	n/a	n/a
bk4-d50	0.0539445	0.00597875	0.131885	-5.65647	228.851	7.91665	556.041	0.00147446
bk4-d50B128	0.0539445	0.00597875	0.131885	-5.65647	228.851	7.91665	556.041	0.00147446
bk4-d500	0.0539445	0.00597875	0.131885	-5.65647	228.851	7.91665	556.041	0.00147446
bk4-f200z4	0.0602044	0.00689306	0.118019	-6.42095	253.658	7.21242	556.041	0.00147446
drbm160-640-bk4	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
frbm300-bk4	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a

[7] $\Delta\gamma$ is applied only to λ_1 ... λ_9 in this variant.

[8] This variant is not used in the solution, but it is included here because it is referred to in section 3.17.1 Logistic transformation models.

[9] Same as bk4-d50, but training passed the optimal score by 4 iterations to optimize blend.

[10] Described later.

Variant	γ_1	γ_2	γ_3	γ_4	γ_5	γ_6	γ_7	γ_8	γ_9
bk4-bias	0.00927732	0.00383994	0.00413388	0.00112172	0.00785357	0.000565172	0.00348556	0.00135257	0.000550582
bk4-biasZ	0.00949966	0.00423486	0.00408419	0.00145721	0.00893363	0.00065001	0.00399963	0.00132472	0.000584056
bk4-a50	0.00928386	0.00390146	0.00455574	0.00105276	0.00790387	0.000560385	0.00333075	0.00180471	0.000663343
bk4-b200	0.00926881	0.00386355	0.00395821	0.00110231	0.00791167	0.00062045	0.00348421	0.00144601	0.00047121
bk4-c50	0.000977427	0.00246524	0.00247907	0.00024348	0.00255845	0.000249488	0.000801197	0.000488952	0.000283411
bk4-c200	*0.000977427*	*0.00246524*	*0.00247907*	*0.00024348*	*0.00255845*	*0.000249488*	*0.000801197*	*0.000488952*	*0.000283411*
bk4-c500	0.000977427	0.00246524	0.00247907	0.00024348	0.00255845	0.000249488	0.000801197	0.000488952	0.000283411
bk4-c200z	0.000977427	0.00246524	0.00247907	0.00024348	0.00255845	0.000249488	0.000801197	0.000488952	0.000283411
bk4-e50a	0.000977427	0.00246524	0.00247907	0.00024348	0.00255845	0.000249488	0.000801197	0.000488952	0.000283411
bk4-e200	0.000977427	0.00246524	0.00247907	0.00024348	0.00255845	0.000249488	0.000801197	0.000488952	0.000283411
bk4-d50	0.000977427	0.00246524	0.00247907	0.00024348	0.00255845	0.000249488	0.000801197	0.000488952	0.000283411
bk4-d50B128	0.000977427	0.00246524	0.00247907	0.00024348	0.00255845	0.000249488	0.000801197	0.000488952	0.000283411
bk4-d500	0.000977427	0.00246524	0.00247907	0.00024348	0.00255845	0.000249488	0.000801197	0.000488952	0.000283411
bk4-f200z4	0.001051047	0.00265092	0.00266579	0.00026182	0.00275115	0.000268279	0.000861543	0.00052578	0.000304758
drbm160-640-bk4	$6.81554\text{x}10^{-5}$	0.00637472	0.00742226	0.00075181	0.00621834	0.000762552	0.00436458	0.00284615	0.000612556
frbm300-bk4	0.00327814	0.00637155	0.0070985	0.000263037	0.00638937	0.000652377	0.00410398	0.00275532	0.000545367

Variant	γ_{10}	γ_{11}	γ_{12}	γ_{13}	γ_{14}	γ_{15}	γ_{16}	γ_{17}	γ_{18}
bk4-bias	0.00069273	0.00195445	0.000242657	0.00258124	0.00125313	0	0	0	0
bk4-biasZ	0.000721524	0.00218922	0.000276723	0.00260608	0.002015	0	0	0	0
bk4-a50	0.000723742	0.00192782	0.000380401	0.00201626	0.00126792	0.000579691	0.0126334	0.00134527	0
bk4-b200	0.00314745	0.00530448	0.00029168	0.00422727	0.00251472	0.00196582	0.0313884	0.00438109	0.00125546
bk4-c50	0.000756619	0.00215928	0.000134043	0.00311735	0.00111064	0.000630419	0.0111556	0.00145574	0.000583831
bk4-c200	*0.000756619*	*0.00215928*	*0.000134043*	*0.00311735*	*0.00111064*	*0.000630419*	*0.0111556*	*0.00145574*	*0.000583831*
bk4-c500	0.000756619	0.00215928	0.000134043	0.00311735	0.00111064	0.000630419	0.0111556	0.00145574	0.000583831
bk4-c200z	0.000756619	0.00215928	0.000134043	0.00311735	0.00111064	0.000630419	0.0111556	0.00145574	0.000583831
bk4-e50a	0.000756619	0.00215928	0.000134043	0.00311735	0.00111064	0.000630419	0.0111556	0.00145574	0.000583831
bk4-e200	0.000756619	0.00215928	0.000134043	0.00311735	0.00111064	0.000630419	0.0111556	0.00145574	0.000583831
bk4-d50	0.000756619	0.00215928	0.000134043	0.00311735	0.00111064	0.00059987	0.0122602	0.0013921	0.00062067
bk4-d50B128	0.000756619	0.00215928	0.000134043	0.00311735	0.00111064	0.00059987	0.0122602	0.0013921	0.00062067
bk4-d500	0.000756619	0.00215928	0.000134043	0.00311735	0.00111064	0.00059987	0.0122602	0.0013921	0.00062067
bk4-f200z4	0.000813608	0.00232192	0.000144139	0.00335215	0.001194293	0.000680015	0.0120332	0.00157026	0.00062976
drbm160-640-bk4	0.00164907	0.0018943	0.000674635	0.00232701	0.00113583	0	0	0	0
frbm300-bk4	0.00104467	0.00215025	0.000637943	0.00187727	0.000460018	0	0	0	0

Variant	γ_{19}	γ_{20}	γ_{21}	γ_{22}	γ_{23}	γ_{24}	γ_{25}	γ_{26}	γ_{27}
bk4-bias	0	0	0	0	0	0	0	0	0
bk4-biasZ	0	0	0	0	0	0	0	0	0
bk4-a50	0	0	0	0	0	0	0	0	0
bk4-b200	0.0132871	0.0050094	0	0	0	0	0	0	0
bk4-c50	0.0109737	0.00186232	0	0	0	0	0	0	0
bk4-c200	*0.0109737*	*0.00186232*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
bk4-c500	0.0109737	0.00186232	0	0	0	0	0	0	0
bk4-c200z	0.0109737	0.00186232	0	0	0	0	0	0	0
bk4-e50a	0.0109737	0.00186232	0	0	0.01319	$1.48766\text{x}10^{-5}$	$3.86391\text{x}10^{-6}$	0.0011758	0.000688992
bk4-e200	0.0109737	0.00186232	0	0	0.01319	$1.48766\text{x}10^{-5}$	$3.86391\text{x}10^{-6}$	0.0011758	0.000688992
bk4-d50	0.00984111	0.00216959	$8.09761\text{x}10^{-5}$	0.000112904	0	0	0	0	0
bk4-d50B128	0.00984111	0.00216959	$8.09761\text{x}10^{-5}$	0.000112904	0	0	0	0	0
bk4-d500	0.00984111	0.00216959	$8.09761\text{x}10^{-5}$	0.000112904	0	0	0	0	0
bk4-f200z4	0.011837	0.00200883	$7.44038\text{x}10^{-5}$	0.00010374	0.01319	$1.48766\text{x}10^{-5}$	$3.86391\text{x}10^{-6}$	0.0011758	0.000688992
drbm160-640-bk4	0	0	0	0	0	0	0	0	0
frbm300-bk4	0	0	0	0	0	0	0	0	0

Variant	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9
bk4-bias	0.00215151	0.000215459	0.00488126	0.0075601	0.000698833	0.00217856	0.00217834	0.00284388	0.00157188
bk4-biasZ	0.00205254	0.000286509	0.00439127	0.00718939	0.000619491	0.00215052	0.00223284	0.0021786	0.00137225
bk4-a50	0.00148082	0.000232721	0.00490838	0.00757012	0.000497164	0.00143908	0.00231426	0.00312576	0.00157386
bk4-b200	0.00334947	0.00009620	0.00487336	0.00741268	0.00095481	0.00281784	0.00229436	0.00274744	0.00151096
bk4-c50	0.00529725	0.000131135	0.00383613	0.00791377	0.000685716	0.00398966	0.00236947	0.0020634	0.0016331
bk4-c200	*0.00529725*	*0.000131135*	*0.00383613*	*0.00791377*	*0.000685716*	*0.00398966*	*0.00236947*	*0.0020634*	*0.0016331*
bk4-c500	0.00529725	0.000131135	0.00383613	0.00791377	0.000685716	0.00398966	0.00236947	0.0020634	0.0016331
bk4-c200z	0.00529725	0.000131135	0.00383613	0.00791377	0.000685716	0.00398966	0.00236947	0.0020634	0.0016331
bk4-e50a	0.00529725	0.000131135	0.00383613	0.00791377	0.000685716	0.00398966	0.00236947	0.0020634	0.0016331
bk4-e200	0.00529725	0.000131135	0.00383613	0.00791377	0.000685716	0.00398966	0.00236947	0.0020634	0.0016331
bk4-d50	0.00529725	0.000131135	0.00383613	0.00791377	0.000685716	0.00398966	0.00236947	0.0020634	0.0016331
bk4-d50B128	0.00529725	0.000131135	0.00383613	0.00791377	0.000685716	0.00398966	0.00236947	0.0020634	0.0016331
bk4-d500	0.00529725	0.000131135	0.00383613	0.00791377	0.000685716	0.00398966	0.00236947	0.0020634	0.0016331
bk4-f200z4	0.00529725	0.000131135	0.00383613	0.00791377	0.000685716	0.00398966	0.00236947	0.0020634	0.0016331
drbm160-640-bk4	0.00174661	0.00108155	0.00179546	0.00644488	0.000802785	0.00174711	0.00177352	0.00220636	0.00170485
frbm300-bk4	0.00129218	0.000856816	0.00199904	0.00795527	0.000635279	0.0012216	0.00212088	0.00254123	0.00134286

Variant	λ_{10}	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}	λ_{18}
bk4-bias	0.000118224	0.000105524	0.000639624	0.0010123	0.00203708	0	0	0	0
bk4-biasZ	0.000160765	0.000161589	0.000691343	0.000921905	0.00116509	0	0	0	0
bk4-a50	0.00106633	0.000270373	0.000682861	0.000995583	0.00146076	0.0331818	0.0234497	0.00550681	0
bk4-b200	0.00067067	0.00027107	0.00065076	0.00104658	0.00186581	0.0376026	0.0211866	0.00549095	0.0019437
bk4-c50	0.000496923	0.000184172	0.000802785	0.00111961	0.000696543	0.0359022	0.0217831	0.00508029	0.00422895
bk4-c200	*0.000496923*	*0.000184172*	*0.000802785*	*0.00111961*	*0.000696543*	*0.0359022*	*0.0217831*	*0.00508029*	*0.00422895*
bk4-c500	0.000496923	0.000184172	0.000802785	0.00111961	0.000696543	0.0359022	0.0217831	0.00508029	0.00422895
bk4-c200z	0.000496923	0.000184172	0.000802785	0.00111961	0.000696543	0.0359022	0.0217831	0.00508029	0.00422895
bk4-e50a	0.000496923	0.000184172	0.000802785	0.00111961	0.000696543	0.0359022	0.0217831	0.00508029	0.00422895
bk4-e200	0.000496923	0.000184172	0.000802785	0.00111961	0.000696543	0.0359022	0.0217831	0.00508029	0.00422895
bk4-d50	0.000496923	0.000184172	0.000802785	0.00111961	0.000696543	0.0345866	0.0254905	0.0053402	0.0034172
bk4-d50B128	0.000496923	0.000184172	0.000802785	0.00111961	0.000696543	0.0345866	0.0254905	0.0053402	0.0034172
bk4-d500	0.000496923	0.000184172	0.000802785	0.00111961	0.000696543	0.0345866	0.0254905	0.0053402	0.0034172
bk4-f200z4	0.000496923	0.000184172	0.000802785	0.00111961	0.000696543	0.0359022	0.0217831	0.00508029	0.00422895
drbm160-640-bk4	0.000973421	0.000461521	0.000469859	0.00299535	0.00129659	0	0	0	0
frbm300-bk4	0.00113314	0.000867036	0.000723798	0.00252515	0.00116078	0	0	0	0

Variant	λ_{19}	λ_{20}	λ_{21}	λ_{22}	λ_{23}	λ_{24}	λ_{25}	λ_{26}	λ_{27}
bk4-bias	0	0	0	0	0	0	0	0	0
bk4-biasZ	0	0	0	0	0	0	0	0	0
bk4-a50	0	0	0	0	0	0	0	0	0
bk4-b200	0.0946061	0.00769279	0	0	0	0	0	0	0
bk4-c50	0.117364	0.00381951	0	0	0	0	0	0	0
bk4-c200	*0.117364*	*0.00381951*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
bk4-c500	0.117364	0.00381951	0	0	0	0	0	0	0
bk4-c200z	0.117364	0.00381951	0	0	0	0	0	0	0
bk4-e50a	0.117364	0.00381951	0	0	0	0	0	0	0
bk4-e200	0.117364	0.00381951	0	0	9.79141×10^{-6}	0.0067143	0.00867617	0.000814275	0.000874146
bk4-d50	0.092418	0.00560977	0.00864227	0.00819148	0	0	0	0	0
bk4-d50B128	0.092418	0.00560977	0.00864227	0.00819148	0	0	0	0	0
bk4-d500	0.092418	0.00560977	0.00864227	0.00819148	0	0	0	0	0
bk4-f200z4	0.117364	0.00381951	0.00864227	0.00819148	9.79141e-006	0.0067143	0.00867617	0.000814275	0.000874146
drbm160-640-bk4	0	0	0	0	0	0	0	0	0
frbm300-bk4	0	0	0	0	0	0	0	0	0

3.4 BK5 SVD++ model

When a user provides a rating for a movie, he must first determine his own appreciation for this movie. Then, he must decide how to represent this appreciation with an integer rating between 1 and 5. It is interesting to model these two steps separately: the appreciation of the user for the movie, and the transfer function that maps appreciation to ratings.

In this model, we estimate the appreciation *z* with the following function:

$$z = b_m(m) + \sum_i^n q_i(m)\left[p_i(u) + \frac{1}{\sqrt{|N(u)|}}\sum_{j\in N(u)} y_i(j)\right] \quad (33)$$

Where:

- $b_m(m)$ is a bias for movie *m* which represents the basic quality of a movie, independently from user preferences;
- *q(m)* is a vector of length *n* representing the movie latent features;
- *p(u)* is a vector of length *n* representing the user latent features;
- *N(u)* is the set of all movies rated by *u*, including movies for which the rating is unknown;
- *y(j)* is a vector of length *n* for movie *j* representing the implicit feedback associated with user *u* rating movie *j*.

We then estimate the user rating using a third degree polynomial. This allows the model to capture non-linearity in the scale that the user chooses to express his appreciations.

$$\hat{r}(u,m) = a_0(u) + (a_1(u)+1)z + a_2(u)z^2 + a_3(u)z^3 \quad (34)$$

It is important to note that the +1 in $(a_1(u)+1)z$ allows this term to approach z when there is insufficient ratings for a given user to learn $a_1(u)$ properly.

The model is trained by minimizing the following error function through stochastic gradient descent, using λ_1 to λ_8 as weight decays:

$$\min_{a_0,a_1,a_2,a_3,b_m,q,p,y} \sum_{u,m} (r(u,m) - \hat{r}(u,m))^2 + \lambda_1 b_m^2(m) + \lambda_2\|q(m)\|^2 + \lambda_3\|p(u)\|^2 + \lambda_4\|y(j)\|^2 + \lambda_5 a_0^2(u) + \lambda_6 a_1^2(u) + \lambda_7 a_2^2(u) + \lambda_8 a_3^2(u) \quad (35)$$

The learning rates are chosen as $\gamma_1$for $b_m(m)$, $\gamma_2$for $q(m)$, $\gamma_3$for $p(u)$, $\gamma_4$for $y(j)$, $\gamma_5$for $a_0(u)$, $\gamma_6$for $a_1(u)$, $\gamma_7$for $a_2(u)$and $\gamma_8$for $a_3(u)$. The learning rate is fixed during training. The dataset is sorted by date for each user. All the values of λ and γ are selected using a combination of manual selection and Nelder-Mead simplex method. During the selection of the meta-parameters, the maximum number of iterations is limited to 30.

We used the following parameters: n=200, $\gamma_1 = 0.00221326$, $\gamma_2 = 0.00460091$, $\gamma_3 = 0.00308307$, $\gamma_4 = 0.000336822$, $\gamma_5 = 0.00670341$, $\gamma_6 = 0.00262918$, $\gamma_7 = 0.00106459$, $\gamma_8 = 0.000239104$, $\lambda_1 = 0.000690139$, λ_2 =0.00321717, $\lambda_3 = 0.000390205$, $\lambda_4 = 0.00153138$, $\lambda_5 = 0.000628539$, $\lambda_6 = 0.00041564$, $\lambda_7 = 0.00128092$ and $\lambda_8 = 0.00337095$.

Two variants are included in the blend, which differ only by the number of iterations used.

Variant	Number of iterations	Comment
bk5-b200	12	Optimal score on probe set
bk5-b200B089	21	Optimal score on blended results

3.5 Other integrated model variants

This model is an earlier type of integrated model that proved inferior to the BKx models. It is documented here because some variants are included in the solution. Similarly to BK4, it uses a matrix factorization approach to model the time variant user and movie biases. It uses four time variables: the absolute date, the elapsed time from the first rating of the user, the elapsed time from the first rating of the movie and the frequency (number of ratings from the user on that day). All four variables are used for both the user and movie bias, although some correlations are weak, like the impact of the elapsed time from the first rating of the movie on the user bias.

Latent user and movie features are kept time independent, which is probably the major weakness of this model. The neighbourhood part of the integrated model uses the factored neighbourhood approach described in [10].

$$z(u,m,t) = \mu + b_U(u,t) + b_M(m,t) + \sum_i^{n_1} q_{1,i}(m)p_{1,i}(u) + \sum_i^{n_2}\left(q_{2,i}(m)\frac{s_2(u)}{\sqrt{|N(u)|}}\sum_{j\in N(u)} y_i(j)\right) + \sum_i^{n_3}\left(q_{3,i}(m)\frac{s_3(u)}{\sqrt{|R(u)|}}\sum_{j\in R(u)} w_i(j)(r(u,j) - baseline_1(u,j)\right) \tag{36}$$

$$b_U(u,t) = b_{U0}(u) + \sum_{i=1}^{R_U} p_U(u,i)x_U(t_U,i) + \sum_{i=1}^{R_M} p_M(u,i)x_M(t_M,i) + \sum_{i=1}^{R_0} p_0(u,i)x_0(t_0,i) + \sum_{i=1}^{R_f} p_f(u,i)x_f(f,i) \tag{37}$$

$$b_M(m,t) = b_{M0}(m) + \sum_{i=1}^{S_U} q_U(m,i)y_U(t_U,i) + \sum_{i=1}^{S_M} q_M(m,i)y_M(t_M,i) + \sum_{i=1}^{S_0} q_0(m,i)y_0(t_0,i) + \sum_{i=1}^{S_f} q_f(m,i)y_f(f,i) \tag{38}$$

$$\hat{r}(u,m,t) = \begin{cases} z & linear\ model \\ \sigma_0(z) & non-linear\ model \\ \sigma_1(z) & non-linear\ model \end{cases}$$

Where:

- $\hat{r}$ is the estimate for user u rating movie m;
- z is the linear estimate for u rating m;
- μ is the global rating mean;
- $b_U(u, t)$ is the time-dependent user bias;
- $b_M(m, t)$ is the time-dependent movie bias;
- t_U is the number of days between the day u rated m and the first rating of u;
- t_M is the number of days between the day u rated m and the first movie of m;
- t_0 is the absolute date of the rating;

- f is the number of ratings (frequency) made by u on day t_0;
- $b_{U0}(u)$ is a bias specific to u;
- $b_{M0}(m)$ is a bias specific to m;
- $p_U(u)$ is a vector of length R_U for user u;
- $x_U(t_U)$ is a vector of length R_U for time interval t_U;
- $p_M(u)$ is a vector of length R_M for user u;
- $x_M(t_M)$ is a vector of length R_M for time interval t_M;
- $p_0(u)$ is a vector of length R_0 for user u;
- $x_0(t_0)$ is a vector of length R_0 for date t_0;
- $p_f(u)$ is a vector of length R_f for user u;
- $x_f(f)$ is a vector of length R_f for frequency f;
- $q_U(m)$ is a vector of length S_U for movie m ;
- $y_U(t_U)$ is a vector of length S_U for time interval t_U;
- $q_M(m)$ is a vector of length S_M for movie m ;
- $y_M(t_M)$ is a vector of length S_M for time interval t_M;
- $q_0(m)$ is a vector of length S_0 for movie m ;
- $y_0(t_0)$ is a vector of length S_0 for date t_0;
- $q_f(m)$ is a vector of length S_f for movie m ;
- $y_f(f)$ is a vector of length S_f for frequency f;
- q_1, q_2 and q_3 are the movie latent feature vectors, respectively of length n_1, n_2 and n_3 (3 sets of movie latent feature are used, one for regular matrix factorization, one for implicit feedback and one for factored neighbourhood);
- p_1 is the user latent feature vector of length n_1;
- *y(j)* is the implicit feedback vector of length *n* for movie *j*;
- *w(i)* is the vector used in conjunction with p_3 to obtain the neighbourhood weights;
- *N(u)* is the set of movies rated by *u*, including movies for which the rating is unknown;
- *R(u)* is the set of movies rated by *u* for which the rating is known;
- *r(u, j)* is the rating given by user *u* to movie *j*;
- *Baseline*$_1$ is defined in the Common Concepts section;
- $s_2(u)$ is a scaling factor for the implicit feedback: depending on the variant, it is either chosen as a) 1.0; b) the standard deviation of user *u* ratings, regularized as described in the Common Concepts section with a population weight of 50 (σ_u); or c) learned through gradient descent;
- $s_3(u)$ is a scaling factor for the neighbourhood term: depending on the variant, it is either chosen as a) 1.0; or b) learned through gradient descent;
- *M(m)* is the set of users having rated the movie *m* (used below).

The model was trained using stochastic gradient descent. The weight decay parameters (regularization) were chosen to vary linearly with the number of movies rated by a user, or the number of users having rated a movie, as shown below. The learning rate was changed dynamically using the following algorithm:

- For each parameter, we compute the inner-product of the parameter variation during the last two iterations. The inner-product is then normalized by dividing it by the product of the vector lengths.
- If the result is greater than 0.5, meaning the variations were mostly collinear, than the learning rate is increased by 20%.
- If the result is smaller than 0.5, meaning the variations were not collinear, than the learning rate is decreased by 20%.

When the envelope is not used, convergence is accelerated by computing a least square regression on the user parameters at each iteration. This makes a hybrid approach where the movie and time factors are learned through gradient descent, but the user factors through linear regression.

In some variants described below, some model parameters are forced to be non-negative.

Training stopped when best accuracy was achieved on the probe set, or after 100 iterations.

Parameter	Initial learning rate	Weight decay	Parameter	Initial learning rate	Weight decay
$b_{U0}(u)$	0.01	$\lambda_1 + \frac{\alpha_1}{\lvert N(u)\rvert}$	$x_0(t_0, i)$	0.0001[11]	λ_{12}
$b_{M0}(m)$	0.001	$\lambda_2 + \frac{\alpha_2}{\lvert M(m)\rvert}$	$p_U(u, i)$	0.0001[11]	$\lambda_{13} + \frac{\alpha_{13}}{\lvert N(u)\rvert}$
$p_{1,i}(u)$	0.01	$\lambda_3 + \frac{\alpha_3}{\lvert N(u)\rvert}$	$x_U(t_U, i)$	0.0001[11]	λ_{13}
$q_{1,i}(m)$	0.001	$\lambda_3 + \frac{\alpha_4}{\lvert M(m)\rvert}$	$p_M(u, i)$	0.0001[11]	$\lambda_{14} + \frac{\alpha_{14}}{\lvert N(u)\rvert}$
$s_2(u)$ when learned	0.0001	$\lambda_5 + \frac{\alpha_5}{\lvert N(u)\rvert}$	$x_M(t_M, i)$	0.0001[11]	λ_{14}
$q_{2,i}(m)$	0.0001	$\lambda_5 + \frac{\alpha_6}{\lvert M(m)\rvert}$	$q_f(m, i)$	0.0001[11]	$\lambda_{15} + \frac{\alpha_{15}}{\lvert M(m)\rvert}$
$y_i(j)$	0.0001	$\lambda_5 + \frac{\alpha_7}{\lvert M(j)\rvert}$	$y_f(f, i)$	0.0001[11]	λ_{15}
$s_3(u)$ when learned	0.0001	$\lambda_8 + \frac{\alpha_8}{\lvert N(u)\rvert}$	$q_0(m, i)$	0.0001[11]	$\lambda_{16} + \frac{\alpha_{16}}{\lvert M(m)\rvert}$
$q_{3,i}(m)$	0.0001	$\lambda_8 + \frac{\alpha_9}{\lvert M(m)\rvert}$	$y_0(t_0, i)$	0.0001[11]	λ_{16}
$w_i(j)$	0.0001	$\lambda_8 + \frac{\alpha_{10}}{\lvert M(j)\rvert}$	$q_U(m, i)$	0.0001[11]	$\lambda_{17} + \frac{\alpha_{17}}{\lvert M(m)\rvert}$
$p_f(u, i)$	0.0001[11]	$\lambda_{11} + \frac{\alpha_{11}}{\lvert N(u)\rvert}$	$y_U(t_U, i)$	0.0001[11]	λ_{17}
$x_f(f, i)$	0.0001[11]	λ_{11}	$q_M(m, i)$	0.0001[11]	$\lambda_{18} + \frac{\alpha_{18}}{\lvert M(m)\rvert}$
$p_0(u, i)$	0.0001[11]	$\lambda_{12} + \frac{\alpha_{12}}{\lvert N(u)\rvert}$	$y_M(t_M, i)$	0.0001[11]	λ_{18}

The meta-parameters were selected using the *manual selection with hint* methodology described in the Common Concepts section.

[11] 0.001 if non-linear envelope is used

Variant	Envelope	Non-negative	n_1	n_2	n_3	R_0	R_U	R_M	R_f	S_0	S_U	S_M	S_f
integ0-0-0TZ	σ_1	n/a	0	0	0	2	2	2	4	20	20	20	40
integ0-100-100TZ	σ_1	n/a	0	100	100	3	3	1	6	60	20	60	120
integ0-200-200NT	n/a	w and q_3	0	200	200	1	1	1	2	2	2	2	5
integ0-200-200TZ	$\sigma 1$	n/a	0	200	200	1	1	1	2	2	2	2	5
integ20-100-100NT	n/a	w and q_3	20	100	100	1	1	1	2	2	2	2	5
integ40-200-0ST	σ_0	n/a	40	200	0	1	1	1	2	2	2	2	5
integ40-200-0T	n/a	n/a	40	200	0	1	1	1	2	2	2	2	5
integ60-0-0TS	σ_0	n/a	60	0	0	1	1	1	2	2	2	2	5
integ80-80-0TZM	σ_1	q_1	80	80	0	1	1	1	2	2	2	2	5

Variant	$s_2(u)$	$s_3(u)$	λ_1	α_1	λ_2	α_2	λ_3	α_3	α_4	λ_5	α_5	α_6	α_7
integ0-0-0TZ	n/a	n/a	0.005	8	0	1	n/a	n/a	n/a	n/a	n/a	n/a	n/a
integ0-100-100TZ	σ_u	1	0	6.3	0	1	n/a	n/a	n/a	0.005	n/a	50	0
integ0-200-200NT	learned	learned	0.015	5.2	0	6.7	n/a	n/a	n/a	0.022	0	150	75
integ0-200-200TZ	σ_u	1	0	6.3	0	1	n/a	n/a	n/a	0.005	n/a	50	0
integ20-100-100NT	learned	learned	0.034	5.4	0	6	0.028	3	40	0.025	0	100	10^{12}
integ40-200-0ST	learned	n/a	0.02	5	0	0	0.027	1.5	20	0.001	0	50	1^{12}
integ40-200-0T	learned	n/a	0.011	5.17	0	1.6	0.03	3	20	0.036	0	100	100^{12}
integ60-0-0TS	n/a	n/a	0	6.4	0	0	0.0235	2	8	n/a	n/a	n/a	n/a
integ80-80-0TZM	1	n/a	0.011	9	0	0	0.03	1	12	0.001	n/a	33	0

Variant	λ_8	α_8	α_9	α_{10}	λ_{11}	α_{11}	λ_{12}	α_{12}	λ_{13}	α_{13}
integ0-0-0TZ	n/a	n/a	n/a	n/a	0.02	2.5	0.042	0	0.023	3
integ0-100-100TZ	0.035	n/a	10	0	0.02	2.5	0.042	0	0.023	3
integ0-200-200NT	0.18	0	115	95	0.027	3.25	0.043	0.85	0.018	3.75
integ0-200-200TZ	0.035	n/a	10	0	0.01	1	0.02	1	0.01	1
integ20-100-100NT	0.165	0	100	100	0.029	3.5	0.047	1	0.022	4
integ40-200-0ST	n/a	n/a	n/a	n/a	0.011	0	0.027	0	0.012	0
integ40-200-0T	n/a	n/a	n/a	n/a	0.028	2.8	0.045	0.7	0.024	2.5
integ60-0-0TS	n/a	n/a	n/a	n/a	0.011	0.5	0.03	0	0.014	0
integ80-80-0TZM	n/a	n/a	n/a	n/a	0.012	0	0.032	0	0.013	0

Variant	λ_{14}	α_{14}	λ_{15}	α_{15}	λ_{16}	α_{16}	λ_{17}	α_{17}	λ_{18}	α_{18}
integ0-0-0TZ	0.029	4	0.001	0	0.01	1	0.01	0	0.002	60
integ0-100-100TZ	0.029	5	0.002	0	0.01	5	0.01	1	0.003	50
integ0-200-200NT	0.025	3.5	0.0003	0	0.0007	0.5	0.009	0	0.0008	20
integ0-200-200TZ	0.02	4	0.001	0	0.001	1	0.01	0	0.001	10
integ20-100-100NT	0.03	4	0.001	0	0.001	1	0.015	0	0.001	50
integ40-200-0ST	0.022	2.8	0	0	0.006	20	0.009	0	0.002	1
integ40-200-0T	0.034	2	0.001	2	0.001	10	0.013	1	0.001	20
integ60-0-0TS	0.013	8	0.001	0	0.005	5	0.008	0	0.001	0
integ80-80-0TZM	0.023	2.8	0	0	0.006	20	0.009	0	0.002	1

With this model, we experimented with a number of concepts in order to facilitate the selection of meta-parameters (especially learning rate and regularization):

[12] A value of 0 is used by mistake for movies for which the rating is unknown.

- By making the learning adaptable, we expected that the initial learning rate would have little impact on the final outcome. This was relatively successful.
- The manual regularization with hints was intended to evolve into a fully automated regularization, but we were never able to achieve this.

With hindsight, we believe that the methodology used in models BK1 to BK5 is superior:

- Select meta-parameters using the Nelder-Mead Simplex Method.
- Use a fixed learning rates, or learning rates with constant decay factors: when combined with early stopping, this makes the usage of the α_x regularization parameters unnecessary, thus reducing the total number of parameters to be selected.

However, in this section, we documented the actual methodology used, as some of the predictions sets were included in the solution.

3.6 Matrix Factorization 1 model

This section describes a matrix factorization method that we used earlier in the competition. It is based on the alternating least squares matrix factorization method described in [12], but contains a number of extensions.

The principal extension is the addition of time-dependent bias terms *c*, *d* and *f*. They form the equivalent of a matrix factorization of the user-time and movie-time interaction, where the matrix rank is limited to 1.

Another extension is that we augment the user latent features with a number of fixed properties *y*. These are multiplied by learned parameters *z* for each movie. This gives a model with a richer movie parameterization than the user, which is desirable because movies have greater support in the data set than users.

In this model, a rating is approximated by the following equation:

$$\hat{r}(u,m,t) = b_u + b_m + \big(c_0(u) + d_0(m)\big)f_0(t) + \big(c_u(u) + d_u(m)\big)f_u(t_u) + \big(c_m(u) + d_m(m)\big)f_m(t_m) + \Big(c_f(u) + d_f(m)\Big)f_f\big(f_{84}\big) + \sum_i^{n_1} p_i(u)q_i(m) + \sum_i^{n_2} y_i(u)z_i(m) \quad (39)$$

Where:

- $\hat{r}$ is the rating estimate;
- u, m and t are respectively the user, the movie and the date of the rating;
- b_u is a user bias;
- b_m is a movie bias;
- f_0 is a function that associates each date with a learned value (the function is learned simultaneously with the other parameters during training);

- c_0 is the weight given to f_0 for user u, which allows the model to capture the fact that ratings may be affected by certain specific dates, like holidays;
- d_0 is the weight given to f_0 for movie m;
- t_u is the number of days elapsed since the first rating of the user;
- f_u is a function that associates each t_u with a learned value (typically initialized as $\sqrt[3]{t_u}$ and updated during training);
- c_u is the weight given to f_u for user u;
- d_u is the weight given to f_u for movie m;
- t_m is the number of days elapsed since the first rating of this movie;
- f_m is a function which associates each t_m with a learned value (typically initialized as $\sqrt[3]{t_m}$ and updated during training);
- c_m is the weight given to f_m for user u;
- d_m is the weight given to f_m for movie m;
- f_{84} is the number of ratings provided by u on day t (frequency f), regrouped in 84 bins according to a logarithmic progression (f_{84} takes value between 1 for a frequency of 1, and 84 for the maximum frequency observed of 5446);
- f_f is a function which associates each f_{84} with a learned value (typically initialized as $\sqrt[3]{f}$ and updated during training);
- c_f is the weight given to f_f for user u;
- d_f is the weight given to f_f for movie m;
- p is a vector of length n_1 representing the user latent features;
- q is a vector of length n_1 representing the movie latent features;
- y is a vector of length n_2 capturing a number of properties of user u. These properties are selected before the training starts and have fixed values. We will explain below how we select the values;
- z is a vector of length n_2 that captures the weight given to each property for movie m.

The intent behind the y properties is to allow the model to have a richer representation of the movies, which have larger support in the training date. To this end, we optionally added n_2 = 59 properties to each user $y_1, ..., y_{59}$ as follows:

- The square root of the user number of ratings;
- The average, variance and skew of the user ratings, regularized as described in the Common Concepts section and using a population weight of 40;
- The fraction of the user ratings that are respectively a 1, 2, 3, 4 or 5, also regularized as described in the Common Concepts section and using a population weight of 40;
- 50 values taken from the hidden units of the RBM model *trbm50* described in Section 3.11.2 Time RBM.

The model is trained using alternating least squares regression in the following order:

1. Update user coefficients;

2. Update movie coefficients;
3. Update frequency function;
4. Update calendar (absolute date) function;
5. Update elapsed time from first user rating function;
6. Update elapsed time from first movie rating function.

In some variants, the time functions are left at their initial values and steps 3 to 6 are omitted. Results are generated when there is no more accuracy improvement measured on the probe set. Training stops after step 2 or step 6.

Regularization is best explained by using the user coefficient update as an example. When updating the user coefficients for a specific user, all the ratings of the user are considered. Let's call X the vector composed of the collection of all the user coefficients (b_u, c_0, c_u, c_m, c_f, and p). X is found by solving the linear problem $A^TAX=A^TB$ where A and B are determined from the non-user coefficients frozen at a constant value during this step.

First we compute an approximation $\bar{A}$ of the average of the diagonal elements of A^TA over the whole training set. All non-diagonal elements of $\bar{A}$ are zero. We also compute the weighted average $\bar{X}$ of X for all users before the update is computed, where the weight corresponds to the number of ratings of each user in the probe or qualifying set. The regularized solution for X is computed as:

$$X = (A^TA + w\bar{A})^{-1}(A^TB + w\bar{A}\bar{X}) \quad (40)$$

$$w = \propto + \lambda_u N \quad (41)$$

Where α and λ_u are regularization constants, and N is the number of training samples included in the evaluation of X (the number of ratings by user u for the user coefficient updates).

The same process is repeated for the movie, frequency, calendar, elapsed from first user rating and elapsed from first movie rating coefficients. The value of w is computed similarly at all steps, the only difference being the value of λ_m, λ_f, λ_0, λ_{tu} and λ_{tm} used instead of λ_u for the movie, frequency, calendar, elapsed time from first user rating and elapsed time from first movie rating respectively.

In some variants, we force all coefficients to be non-negative (see [12] figure 1 for an example of a non-negative solver). In some others, we invert ratings (1 with 5, 2 with 4) before computing a non-negative solution, which leads to different prediction distribution characteristics. In some variants, the bias (b_u and b_m) are not used (forced to zero), as well as the values of f_f, f_0, f_u and f_m. This model is sometimes evaluated on the residual error of another model, which is useful to introduce a time correction to models that are not date aware.

The meta-parameters were selected manually. The following variants are included in the solution:

Variant	baseline	$b_u\, b_m$	f_f	f_0	f_u	f_m
mf01-20	n/a	learned	zero	zero	zero	zero
mf01-40-3-80	n/a	learned	zero	zero	zero	zero
mf27-20	n/a	learned	learned	zero	learned	learned
mf27-20env50	n/a	learned	learned	zero	learned	learned
mf27-40-3-80	n/a	learned	learned	zero	learned	learned
mfc27-60-10-120	n/a	learned	fixed[13]	zero	fixed	fixed
mfw31-00	n/a	learned	learned	learned	learned	learned
mfw31-05	n/a	learned	learned	learned	learned	learned
mfw31-10	n/a	learned	learned	learned	learned	learned
mfw31-40env50	n/a	learned	learned	learned	learned	learned
mfw31-60-10-120	n/a	learned	learned	learned	learned	learned
mfw31-60-x	n/a	learned	learned	learned	learned	learned
mfw31-80-x	n/a	learned	learned	learned	learned	learned
nmf40-60-10	n/a	zero	zero	zero	zero	zero
nmf80-120-20	n/a	zero	zero	zero	zero	zero
pmf80-120-20	n/a	zero	zero	zero	zero	zero
nmf80-120-20-mf27	nmf80-120-20	zero	zero	zero	zero	zero
pmf80-120-20-mf27	pmf80-120-20	zero	zero	zero	zero	zero
frbm200-mf27	frbm200	learned	learned	zero	learned	learned
trbm50-mf27	trbm50	learned	learned	zero	learned	learned
trbm50-asym3v250-mfw27	trbm50-asym3v250	learned	learned	zero	learned	learned
trbm150-mf27	trbm150	learned	learned	zero	learned	learned

[13] 'fixed' means the value is left to its initial value throughout the training.

Variant	n_1	n_2	non-negative	final step	α	λ_u	λ_m	λ_f	λ_0	λ_{tu}	λ_{tm}
mf01-20	20	0	no	2	50	0.01	0.01	0.01	0.01	0.01	0.01
mf01-40-3-80	40	0	no	2	80	0.03	0.03	0.03	0.03	0.03	0.03
mf27-20	20	0	no	6	50	0.01	0.01	0.01	0.01	0.01	0.01
mf27-20env50	20	59	no	6	50	0.01	0.01	0.01	0.01	0.01	0.01
mf27-40-3-80	40	0	no	6	80	0.03	0.03	0.03	0.03	0.03	0.03
mfc27-60-10-120	60	0	no	2	120	0.1	0.1	0.1	0.1	0.1	0.1
mfw31-00	0	0	no	2	50	0.01	0.01	0.01	0.01	0.01	0.01
mfw31-05	5	0	no	2	50	0.01	0.01	0.01	0.01	0.01	0.01
mfw31-10	10	0	no	2	50	0.01	0.01	0.01	0.01	0.01	0.01
mfw31-40env50	40	59	no	2	80	0.03	0.03	0.03	0.03	0.03	0.03
mfw31-60-10-120	60	0	no	2	120	0.1	0.1	0.1	0.1	0.1	0.1
mfw31-60-x	60	0	no	2	80	0.20	0.10	0.20	0.60	0.20	0.30
mfw31-80-x	80	0	no	2	90	0.40	0.17	0.21	1.00	0.14	0.10
nmf40-60-10	60	0	yes	2	60	0.1	0.1	0.1	0.1	0.1	0.1
nmf80-120-20	80	0	yes	2	120	0.2	0.2	0.2	0.2	0.2	0.2
pmf80-120-20	80	0	inverted	2	120	0.2	0.2	0.2	0.2	0.2	0.2
nmf80-120-20-mf27	80	0	yes	6	50	0.01	0.01	0.01	0.01	0.01	0.01
pmf80-120-20-mf27	80	0	inverted	6	50	0.01	0.01	0.01	0.01	0.01	0.01
frbm200-mf27	0	0	no	6	50	0.01	0.01	0.01	0.01	0.01	0.01
trbm50-mf27	0	0	no	6	50	0.01	0.01	0.01	0.01	0.01	0.01
trbm50-asym3v250-mfw27	0	0	no	2	50	0.01	0.01	0.01	0.01	0.01	0.01
trbm150-mf27	0	0	no	6	50	0.01	0.01	0.01	0.01	0.01	0.01

We also implemented a neighbourhood aware model as described in Section 4.4 of [12]. The idea consists of re-computing the user coefficients for each prediction being made, while giving more weight to ratings most similar to the rating being predicted. We defined two different movie proximity measures used when re-computing user coefficients.

We also used the same logic while transposing users and movies, such that for each prediction being made, *movie* coefficients are recomputed according to a user proximity measure.

The proximity measures are described by the following equations:

$$P_1(m_1, m_2) = \frac{1}{\left(1 + 0.0045\left|t_{m_1} - t_{m_2}\right|\right) \sum_i^{n_1} \frac{\left(q_i(m_1) - q_i(m_2)\right)^2}{\overline{q_i^2}}} \tag{42}$$

$$P_2(m_1, m_2) = \frac{1}{\left(1 + 0.0055\left|t_{m_1} - t_{m_2}\right|\right) mse^2(m_1, m_2)} \tag{43}$$

$$P_3(u_1, u_2) = \frac{1}{\sum_i^{n_1} \frac{\left(p_i(u_1) - p_i(u_2)\right)^2}{\overline{p_i^2}}} \tag{44}$$

$$mse(m_1, m_2) = \frac{100\,\overline{mse} + \sum_{u\ rating\ m1\ and\ m2} \left(r(u, m_1) - r(u, m_2)\right)^2}{100 + n_{m_1, m_2}} \tag{45}$$

Where:

- The functions P_1 and P_2 represent the weight to be given to the rating of m_2 when predicting m_1;

- $t_{m_1} - t_{m_2}$ is the time difference between the ratings of m_1 and m_2;
- $\overline{q_i^2}$ is the average of the squared value of the corresponding movie latent feature over all movies;
- $mse(m_1, m_2)$ is the average squared difference between the ratings of m_1 and m_2 rated by a same user, the value is shrunk toward the average mean squared error ($\overline{mse} \cong 1.94$).
- n_{m_1,m_2} is the number of users having rated both m_1 and m_2;
- P_3 represents the weight to be given to the rating of u_2 when predicting u_1.;
- $\overline{p_i^2}$ is the average of the squared value of the corresponding movie latent feature over all users.

The proximity functions P_1 and P_3 are based on the distance between the latent feature vectors. The proximity function P_2 is based on the mean squared error between movie ratings.

The following variants were computed. The same parameters are used as when training the regular model, except for *mfw31-80-x-m* which uses $\alpha = 130$ instead of $\alpha = 90$.

Variant	Baseline	Proximity function
mf27-20-u	mf27-20	P3
mf27-20env50-m	mf27-20env50	P1
mfc27-60-10-120-m	mfc27-60-10-120	P1
mfw31-05-m	mfw31-05	P1
mfw31-40env50-m	mfw31-40env50	P1
mfw31-40env50-m2	mfw31-40env50	P2
mfw31-60-10-120-m	mfw31-60-10-120	P1
mfw31-80-x-m	mfw31-80-x	P1
nmf80-120-20-m	nmf80-120-20	P1
pmf40-60-10-m	pmf40-60-10	P1
drbm100-500-mfw31-m	drbm100-500-mfw31	P1
frbm100-mf27-m	frbm100-mf27	P1
trbm150-mf27-m	trbm150-mf27	P1

3.7 Matrix Factorization 2 model

This section describes a matrix factorization method that is similar and simpler than Matrix Factorization 1. It is based on the alternating least squares matrix factorization method described in [12], but contains a number of extensions. In this model, a rating is approximated by the following equation:

$$\hat{r}(u,m,t) = \sum_{i=1}^{6} c_i(u,m,t) y_i(u) + \sum_{i=1}^{6} d_i(u,m,t) z_i(m) + \sum_{i}^{n} p_i(u) q_i(m) \quad (46)$$

Where:

- $\hat{r}$ is the rating estimate;
- u, m and t are respectively the user, the movie and the date of the rating;
- c_1 is set to 1.0, so that y_1 acts as a user bias;
- c_2 is set to the cubic root of the time interval between t and the first rating of user u;
- c_3 is set to the cubic root of the time interval between t and the first rating of movie m;
- c_4 is set to the cubic root of the frequency (number of ratings made by user u on day t);

- c_5 is set to the cubic root of the number of ratings of movie *m*;
- c_6 is set to the average rating of *m* in the training data, regularized toward the global mean with a population weight of 40 as explained in the Common Concepts section;
- *y* is a vector of length 6 which holds the weight given to constants c_1 to c_6 for user *u*;
- d_1 is set to 1.0, so that z_1 acts as a movie bias;
- d_2 is set to the cubic root of the time interval between *t* and the first rating of user *u*;
- d_3 is set to the cubic root of the time interval between *t* and the first rating of movie *m*;
- d_4 is set to the cubic root of the frequency (number of ratings made by user *u* on day *t*);
- d_5 is set to the cubic root of the number of ratings of user *u*;
- d_6 is set to the average rating of *u* in the training data, regularized toward the global mean with a population weight of 40 as explained in the Common Concepts section.
- *z* is a vector of length 6 which holds the weight given to constants d_1 to d_6 for movie *m*;
- *p* is a vector of length *n* representing the user latent features;
- *q* is a vector of length *n* representing the movie latent features.

The model is trained using alternating least squares regression for 30 iterations in the following order:

1. Update user coefficients;
2. Update movie coefficients.

Regularization is best explained by using the user coefficient update as an example. When updating the user coefficients for a specific user, all the ratings of the user are considered. Let's call *X* the vector composed of the collection of all the user coefficients (*y* and *p*). *X* is found by solving the linear problem $A^TAX=A^TB$ where A and B are determined from the non-user coefficient frozen at a constant value during this step.

First we compute $\bar{A}$ and $\bar{B}$, the average of A^TA and A^TB over the whole training set. All entries of $\bar{A}$ except the diagonal entries associated with *p* and *q* are forced to zero. The regularized solution for *X* is computed as:

$$X = (A^TA + \alpha\bar{A} + \lambda NJ)^{-1}(A^TB + \alpha\bar{B}) \quad (47)$$

Where α and λ are regularization constants, and *N* is the number of training samples included in the evaluation of *X* (the number of ratings by user *u* for the user coefficient updates). *J* is an identity matrix where the entries corresponding to the values of *y* are set to zero.[14]

The same process is repeated for the movies (in which case *N* is the number of ratings of movie *m*).

In some variants, we force all coefficients to be non-negative. In others, some of the *c* or *d* parameters are not used. This model is sometimes evaluated on the residual error of another model, which is useful to introduce a time correction to models that are not date aware.

[14] We are simply reporting here the regularization method used in this model. With hindsight, we find regularization used in other models more appropriate.

The regularization parameters were selected manually to $\alpha = 50$ and $\lambda = 0.05$. The following variants are included in the solution:

Variant	baseline	Use $c_1 d_1$	Use $c_2 d_2 c_3 d_3$	Use $c_4 d_4$	Use $c_5 d_5 c_6 d_6$	*n*	*non-negative*
nnmf40	n/a	no	no	no	no	40	yes
nnmf80	n/a	no	no	no	no	80	yes
ssvd-04-00	n/a	yes	no	no	no	0	no
ssvd-04-10	n/a	yes	no	no	no	10	no
ssvd-04-40	n/a	yes	no	no	no	40	no
ssvd-07-30-2[15]	n/a	yes	yes	no	no	30	no
ssvd-31-00	n/a	yes	yes	no	yes	0	no
ssvd-31-10	n/a	yes	yes	no	yes	10	no
ssvd-31-20	n/a	yes	yes	no	yes	20	no
ssvd-31-60	n/a	yes	yes	no	yes	60	no
ssvd-39-00	n/a	yes	yes	yes	no	0	no
ssvd-39-05	n/a	yes	yes	yes	no	5	no
ssvd-39-10	n/a	yes	yes	yes	no	10	no
ssvd-63-00	n/a	yes	yes	yes	yes	0	no
ssvd-63-30	n/a	yes	yes	yes	yes	30	no
rbm100-ssvd-07-00	rbm100	yes	yes	no	no	0	no
crbm100-ssvd-07-00	crbm100	yes	yes	no	no	0	no
crbm100-ssvd-07-20	crbm100	yes	yes	no	no	20	no
crbm100x-ssvd-03-00	crbm100x	no	yes	no	no	0	no
crbm200-ssvd-07-00	crbm200	yes	yes	no	no	0	no
trbm50-ssvd-39-00	trbm50	yes	yes	yes	no	0	no
trbm100-ssvd-31-00	trbm100	yes	yes	no	yes	0	no
trbm150-ssvd-07-00	trbm150	yes	yes	no	no	0	no
trbm150-ssvd-39-00	trbm150	yes	yes	yes	no	0	no
nnmf40-ssvd-07-00	nnmf40	yes	yes	no	no	0	no
nnmf80-ssvd-39-00	nnmf80	yes	yes	yes	no	0	no

We also implemented a neighbourhood aware model as described in Section 4.4 of [12]. Like in Section 3.6 Matrix Factorization 1, the idea consists of re-computing the user coefficients for each prediction being made, while giving more weight to ratings most similar to the rating being predicted. We defined one movie proximity measure used when re-computing user coefficients. We also used the same logic while transposing users and movies, such that for each prediction being made *movie* coefficients are recomputed according to a user proximity measure.

$$P_1(m_1, m_2) = \frac{1}{0.001 + \sum_i^n \frac{\left(q_i(m_1) - q_i(m_2)\right)^2}{\overline{q_i^2}}} \tag{48}$$

$$P_2(u_1, u_2) = \frac{1}{0.001 + \sum_i^n \frac{\left(p_i(u_1) - p_i(u_2)\right)^2}{\overline{p_i^2}}} \tag{49}$$

Where:

[15] This variant uses $\alpha = 16$ and $\lambda = 0.02$.

- The functions P_1 represents the weight to be given to the rating of m_2 when predicting m_1;
- $\overline{q_i^2}$ is the average of the squared value of the corresponding movie latent feature over all movies;
- P_2 represents the weight to be given to the rating of u_2 when predicting u_1;
- $\overline{p_i^2}$ is the average of the squared value of the corresponding movie latent feature over all users.

The proximity functions P_1 and P_2 are based on the distance between the latent feature vectors.

The following variants were computed. The same parameters are used as when training the regular model.

Variant	Baseline	Proximity function
ssvd-04-10-m	ssvd-04-10	P_1
ssvd-04-40-m	ssvd-04-40	P_1
ssvd-31-10-m	ssvd-31-10	P_1
ssvd-31-20-m	ssvd-31-20	P_1
ssvd-31-20-u	ssvd-31-20	P_2

3.8 Usermovie model

This is a very early and simple model where the rating is modeled as the sum of a user and a movie bias (b_u and b_m). The values are initialized to zero and updated using the following equations for two iterations.

$$\hat{r}(u,m) = b_m + b_u \tag{50}$$

$$b_m = \frac{\sum_{u \in N(m)} (r(u,m) - b_u) + \alpha_m \overline{b_m}}{(|N(m)| + \alpha_m)} \tag{51}$$

$$b_u = \frac{\sum_{m \in N(u)} (r(u,m) - b_m) + \alpha_u \overline{b_u}}{(|N(u)| + \alpha_u)} \tag{52}$$

Where:

- $\hat{r}(u,m)$ is the estimated rating for user u and movie m;
- b_m is the movie bias for movie m;
- b_u is the user bias for user u;
- $N(m)$ is the subset of users that have rated movie m;
- α_m=20 is the movie shrinkage coefficient;
- $N(u)$ is the subset of movies rated by user u;
- α_u=8.3 is the user shrinkage coefficient;
- $\overline{b_m}$ is the average of $(r(u,m) - b_u)$ over the complete training set;
- $\overline{b_u}$ is the average of $(r(u,m) - b_m)$ over the complete training set.

The following table shows the variant selected in the solution:

Variant
usermovie

3.9 SVDNN model

This model is an early and plain version of a Matrix Factorization model (as described in [1]) where NN represents the number of latent features used. Training is done by alternating least squares regression.

Regularization is similar to Section 3.7 Matrix Factorization 2. Let's call *X* the vector composed of the user coefficients. *X* is found by solving the linear problem $A^TAX=A^TB$ where A and B are determined from the movie coefficients frozen at a constant value during this step.

First we compute $\bar{A}$, the average of A^TA over the whole training set. All non-diagonal entries of $\bar{A}$ are forced to zero. The regularized solution for *X* is computed as:

$$X = (A^TA + \alpha\bar{A} + \lambda NI)^{-1}A^TB \tag{53}$$

Where α and λ are regularization constants, and *N* is the number of training samples included in the evaluation of *X* (the number of ratings by user *u* for the user coefficient updates). *I* is the identity matrix.

The same process is repeated for the movies (in which case *N* is the number of ratings of movie *m*).

The variants that were selected in the solution were trained on the residual errors of the *usermovie* model. Note that the *svdNNx* variant also runs an additional pass of the *usermovie* model on the residuals of the *svdNN* model.

The following table shows the predictors selected in the solution.

Variant
svd02x
svd05x

3.10 BRISMF

We implemented the BRISMF model as described in [11]. We reused all given parameters including number of epochs and regularization. The following table indicates the variants that were used in the solution.

Variant	Comment
brismf40	Based on BRISMF#1UM in [11]
brismf250	Based on BRISMF#250UM in [11]
brismf760	Based on BRISMF#1000UM in [11], but with rank limited to K=760.
brismf760n	Based on SemPosMF#800 in [11], but with rank limited to K=760.

3.11 Restricted Boltzmann Machines

3.11.1 Basic RBM models

We implemented all the RBM types described in [5]. The following variants were included in the solution:

Variant	Description
rbm100	Ordinary RBM from Section 2 of [5], with 100 softmax hidden units. The model is trained using mini-batches of 1000 users, a learning rate of $2x10^{-5}$, a weight decay of 0.02 and a momentum of 0.9.
crbm100	Conditional RBM from Section 4 of [5], with 100 softmax hidden units. The model is trained using mini-batches of 1000 users, a learning rate of $2x10^{-5}$, a weight decay of 0.02 and a momentum of 0.9.
crbm100x	Conditional RBM from Section 4 of [5], with 100 softmax hidden units. The model is trained using mini-batches of 1000 users, a learning rate of $2x10^{-5}$, a weight decay of 0.02 and a momentum of 0.9. The learning rate is progressively reduced by a factor of 8 over the last 6 iterations.
crbm200	Conditional RBM from Section 4 of [5], with 200 softmax hidden units. The model is trained using mini-batches of 1000 users, a learning rate of $2x10^{-5}$, a weight decay of 0.02 and a momentum of 0.9.
drbm100-500	Conditional Factored RBM from Section 5 of [5], with a factorization of rank 100 and 500 softmax hidden unit. The initial learning rate is set to 0.0005 for the hidden unit biases and the rank reducing matrices, 0.005 for all others. The learning rate is reduced exponentially by a factor of 1000 over 100 iterations. Update occurs by batches of 1 user, with no weight decay or momentum.
drbm160-640	Conditional Factored RBM from Section 5 of [5], with a factorization of rank 160 and 640 softmax hidden unit. The same training parameters were used as *drbm100-500*.
urbm20-1000	Conditional Factored RBM from Section 5 of [5], with a factorization of rank 20 and 1000 softmax hidden unit. In this variant, the role of the users and the movies is reversed: each movie is modeled by a number of units, and weights connect the hidden units to the users. The initial learning rate is set to $4.51005x10^{-7}$ for the hidden unit biases and the rank reducing matrices, $6.21418x10^{-5}$ for all others. The learning rate is reduced by a factor of 0.0315713 per iteration. Update occurs by batches of 1 user, with a weight decay of 0.0946444 on all parameters except the hidden unit biases, and no momentum. The meta-parameters were selected using the Nelder-Mead Simplex Method. During the selection of the meta-parameters, the maximum number of iterations is limited to 100.
integ0-0-0TZ-grbm200	RBM with 200 Gaussian visible units as described in [7], running on residuals of *integ0-0-0TZ*. The model is trained using mini-batches of 1000 users, an initial learning rate of $8.18119x10^{-5}$ which decreased by a factor 0.000390623 at each iteration. A weight decay of 0.00012058 + 0.0569244/N is used where N is the support of the respective parameter. Momentum is set to zero. The meta-parameters were selected using the Nelder-Mead Simplex Method. During the selection of the meta-parameters, the maximum number of iterations is limited to 100.
mfw31-10-grbm200	RBM with 200 Gaussian visible units as described in [7], running on residuals of *mfw31-10*. The same training parameters were used as *integ0-0-0TZ-grbm200*.

3.11.2 Time RBM models

We developed a new RBM variant that we called Time RBM which takes into account the dates of the ratings. The Time RBM is based on the Conditional RBM, except that the weights are the product of a movie factor by a time factor.

$$W_{ij}^{k} = A_{ij}^{k} B_{jf} \tag{54}$$

The A_{ij}^{k} are akin to typical RBM weights: they capture the relation between item *i* and hidden unit *j* for rating *k*. The B_{jf} capture the relationship between the hidden unit *j* and the number of ratings of this user on that day (frequency *f*). The update rule becomes:

$$\Delta A_{ij}^{k} = \epsilon\left(\langle B_{jf} v_i^k h_j \rangle_{data} - \langle B_{jf} v_i^k h_j \rangle_{model}\right) \tag{55}$$

$$\Delta B_{jf} = \epsilon\left(\langle \left[\sum_{ik} A_{ij}^{k} v_i^k\right] h_j \rangle_{data} - \langle \left[\sum_{ik} A_{ij}^{k} v_i^k\right] h_j \rangle_{model}\right) \tag{56}$$

The biases and the conditional part do not depend on the date and are computed as in [5]. The weight decay for the B_{jf} is made toward 1.0 instead of the usual 0.0. The model is trained using mini-batches of 1000 users, a learning rate of 2x10-5, a weight decay of 0.02 and a momentum of 0.9.

Instead of using the rating frequency, we also tried using the raw date in the same way. The following variants are included in the solution:

Variant	Description
frbm100	Time RBM using frequency, with 100 softmax hidden units.
frbm200	Time RBM using frequency, with 200 softmax hidden units.
frbm200x	Time RBM using frequency, with 200 softmax hidden units. Instead of using one iteration of the mean field equation to make predictions, we use the expression defined in equation (7) of [5].
frbm300	Time RBM using frequency, with 200 softmax hidden units.
frbm300x	Time RBM using frequency, with 200 softmax hidden units. Instead of using one iteration of the mean field equation to make predictions, we use the expression defined in equation (7) of [5].
trbm50	Time RBM using date, with 50 softmax hidden units.
trbm100	Time RBM using date, with 100 softmax hidden units.
trbm150	Time RBM using date, with 150 softmax hidden units.

3.11.3 Decomposed RBM models

To generate a prediction, the classic RBM actually computes the probability that a given rating is a 1, 2, 3, 4 or 5. Typically, the prediction is selected as:

$$\hat{r} = p(r=1) + 2p(r=2) + 3p(r=3) + 4p(r=4) + 5p(r=5) \quad (57)$$

Where:

- $\hat{r}$ is the prediction;
- *p(r=x)* is the probability that the rating takes the value *x*.

However, under certain circumstances, it may be better to let the blending algorithm select optimized weights rather than the expected 1, 2, 3, 4 and 5 values. To that end, we also produce the following expressions and generate them as prediction sets. Their individual accuracy is very poor, but they blend well with each other and with the rest.

$$g_1 = p(r=2) + p(r=3) + p(r=4) + p(r=5) \quad (58)$$
$$g_2 = p(r=3) + p(r=4) + p(r=5) \quad (59)$$
$$g_3 = p(r=4) + p(r=5) \quad (60)$$
$$g_4 = p(r=5) \quad (61)$$

We included the following variants in the blend:

Variant	Baseline	Expression
drbm160-640g1	drbm160-640	*g1*
drbm160-640g2	drbm160-640	*g2*
drbm160-640g3	drbm160-640	*g3*
drbm160-640g4	drbm160-640	*g4*
frbm300g1	frbm300	*g1*
frbm300g2	frbm300	*g2*
frbm300g3	frbm300	*g3*
frbm300g4	frbm300	*g4*

3.11.4 Split RBM model

We described in the previous section (3.11.3 Decomposed RBM) that we can produce separate prediction sets from a RBM, each set representing the probability of the rating being above a certain threshold 1, 2, 3 or 4, using expressions *g1, g2, g3* and *g4*. We also experimented with a variant that tried to directly estimate one of *g1, g2, g3* or *g4*.

We replaced the visible units with a single softmax unit per rating. The visible unit is interpreted as the probability that the correct rating is greater than *g*, where *g* one is 1, 2, 3 or 4. The rest of the model works as the Time RBM model using frequency.

The initial learning rate is set to γ and reduced by a factor Δγ per iteration. Update occurs by batches of 1 user, with no weight decay or momentum. The meta-parameter were selected using the Nelder-Mead Simplex Method. During the selection of the meta-parameters, the maximum number of iterations is limited to 100.

This provided results much inferior to the Decomposed RBMs, and they are described here only because some variants were included in the solution:

Variant	γ	Δγ	Comment
frbm2-100g1	0.00106641	0.0570313	Time RBM with frequency, 100 hidden units and g=1.
frbm2-100g2	0.00246875	0.0351563	Time RBM with frequency, 100 hidden units and g=2.
frbm2-100g3	0.00140625	0.00546875	Time RBM with frequency, 100 hidden units and g=3.
frbm2-100g4	0.0021875	0.0265625	Time RBM with frequency, 100 hidden units and g=4.

3.12 Asymmetric models

Asymmetric models are from a family of models that do not directly model user latent features, but instead infer the user tastes by the movie that he rated. This was first described in [6] as NSVD1. The name Asymmetric was used in [7] which also introduced several variations. We describe here our own variations.

One difference in our implementation compared to Paterek's is that we don't use a bias term. Instead, we always run the model on the residual of another model: time-dependent biases, a low rank matrix factorization or a Restricted Boltzmann Machine.

3.12.1 Asymmetric 1 model

This variant differs from NSVD1 in [6] by an extra factor in the expression that accounts for the value of the rating.

$$\hat{r}(u,m) = \frac{1}{\sqrt{|N(u)|}} \sum_{i=1}^{n} q_i(m) \sum_{j \in N(u)} y_i(j)\, c(j, r_j) \tag{62}$$

Where:

- $\hat{r}$ is the predicted rating for user *u* and movie *m*;
- *N(u)* is the set of ratings provided by user *u*, whether or not the rating is known;
- *q* is a vector of length *n* representing the movie latent features;

- *y(j)* is a vector of length *n* representing the implicit feedback from knowing that movie *j* was rated by *u*;
- r_j is the rating given to movie *j*; if not known, then r_j takes the value zero;
- *c* is a correction factor applied according to the rating of movie *j*, it is a table of dimension 6xnumber of movies.

This model was trained using gradient descent with mini-batch of 1000 users, starting with a learning rate of 0.001 which decreased by a factor of 0.25 at each iteration until the rate reached $5x10^{-6}$. It used a weight decay of 0.01 and a momentum of 0.9.

This model was not very successful, but it is documented here as the following variant was included in the solution:

Variant	Baseline	n
ssvd-31-00-asym1-20	ssvd-31-00	20

3.12.2 Asymmetric 3 model

This variant is much closer to NSVD1 in [6]. The main difference is the introduction of a user-specific scale *s(u)*.

$$\hat{r}(u,m) = \frac{s(u)}{\sqrt{|N(u)|}} \sum_{i=1}^{n} q_i(m) \sum_{j \in N(u)} y_i(j) \tag{63}$$

Where:

- $\hat{r}$ is the predicted rating for user *u* and movie *m*;
- *N(u)* is the set of ratings provided by user *u*, whether or not the rating is known;
- *q* is a vector of length *n* representing the movie latent features;
- *y(j)* is a vector of length *n* representing the implicit feedback from knowing that movie *j* was rated by *u*;
- *s(u)* is a user-specific scale, which is either the constant 1, or the value of the user ratings standard deviation regularized with a population weight of 40 as described in the Common Concepts section.

The model was trained using gradient descent with mini-batches of 1000 users and a learning rate which decreased by a factor of 0.21 at each iteration, until the rate reached 10^{-3} of the initial value. It used a weight decay of 0.01 and a momentum of 0.9.

In one variant, we re-compute *s(u)* after training as follows:

- For each user, we compute, through linear regression, the value of *s(u)* that would minimize the prediction error on the training set;
- We then compute a weighted average between the old value and the new value of *s(u)*, where the old value gets a weight of 40, and the new value a weight equal to the number of known ratings of user *u*.

The following variants were included in the final blend:

Variant	Baseline	n	s(u)	initial learning rate
mfw31-05-asym3v250	mfw31	250	standard deviation	0.0004
ssvd-31-00-asym3-20	ssvd-31-00	20	1.0	0.0005
ssvd-31-00-asym3-100	ssvd-31-00	100	1.0	0.0005
ssvd-31-00-asym3-200	ssvd-31-00	200	1.0	0.0005
ssvd-31-00-asym3v-300	ssvd-31-00	300	standard deviation	0.0004
ssvd-31-00-asym3w-200	ssvd-31-00	200	post-processed	0.0004
trbm50-asym3v250	trbm50	250	standard deviation	0.0004

3.12.3 Asymmetric 4 model

This variant was inspired from the sigmoid variants found in [7].

$$\hat{r}(u,m) = s(u)\sum_{i=1}^{n} q_i(m)\sigma\left(b_i + \sum_{j\in N(u)} y_i(j)\right) \tag{64}$$

$$\sigma(x) = \frac{1}{1+e^{-x}} \tag{65}$$

Where:

- $\hat{r}$ is the predicted rating for user *u* and movie *m*;
- *N(u)* is the set of ratings provided by user *u*, whether or not the rating is known;
- *q* is a vector of length *n* representing the movie latent features;
- *y(j)* is a vector of length *n* representing the implicit feedback from knowing that movie *j* was rated by *u*;
- *b* is a vector of length *n* representing a bias in the sigmoid function *σ*;
- *s(u)* is a user-specific scale, which is either the constant 1, or the value of the user ratings standard deviation regularized with a population weight of 40 as described in the Common Concepts section.

The model was trained using gradient descent with mini-batches of 1000 users and an initial learning rate of 0.0005 which decreased by a factor of 0.21 at each iteration, until the rate reached 10^{-3} of the initial value. It used a weight decay of 0.01 and a momentum of 0.9.

The following variants were included in the solution:

Variant	Baseline	n	s(u)
ssvd-31-00-asym4-200	ssvd-31-00	200	1.0
ssvd-31-00-asym4v-200	ssvd-31-00	200	standard deviation

3.12.4 Milestone model

We named our most successful variant *milestone* although the name is an abuse of language. A prediction is generated using the following model:

$$\hat{r}(u,m) = \frac{s(u)}{\sqrt{N(u)}} \sum_{i}^{n} q(m,i) \sum_{j\ rated\ by\ u} \left[w(u,f_j) y_1(j,i) + (1 - w(u,f_j)) y_2(j,i) \right] \quad (66)$$

Where:

- $\hat{r}$ is the predicted rating for user *u* and movie *m*;
- f_j is the frequency, i.e. the number of ratings made by user *u* the day he rated movie *j*;
- *s(u)* is a user-specific scale, which is the value of the user ratings standard deviation regularized with a population weight of 40 as described in the Common Concepts section;
- *N(u)* is the number of movies rated by *u*, whether or not the score is known;
- *q* is a vector of length *n* representing the movie latent features;
- y_1 and y_2 are also vectors of length *n* representing the implicit feedback of having user *u* rating the associated movie;
- *w* is the date dependent weight applied to y_1 and y_2 to produce the user factors.

While Paterek in [6] created user latent factors by summing a vector for each movie rated by the user, instead we add a linear combination of two vectors y_1 and y_2, weighted by *w*. Paterek's idea was that the user tastes could be inferred by the identity of the movies rated by the user, without looking at the individual score. We extended this approach by assuming that the contribution of each movie rated by the user was a function of the date at which the rating was provided, as captured by *w*. The values of *q*, y_1 and y_2 are learned through gradient descent. The values of *w* are chosen as one among the following alternatives:

#0	$w = \begin{cases} 0 & if\ f = 1 \\ 1 & if\ f > 1 \end{cases}$
#3	*w* varies linearly from 0 to 1 over the range of frequencies observed for *u*.
#4	$w = \begin{cases} 0 & if\ f = 1 \\ 1/4 & if\ f = 2 \\ 1/2 & if\ f = 3 \\ 3/4 & if\ f = 4 \\ 1 & if\ f > 4 \end{cases}$
#5	$w = \begin{cases} \frac{f-1}{9} & if\ f < 10 \\ 1 & if\ f \geq 10 \end{cases}$
#6	$w = \frac{1}{1 + \frac{1}{10}(f-1)}$
#7	$w = \frac{1}{1 + \frac{1}{20}(f-1)}$
#9	$w = \frac{1}{1 + \frac{1}{400}(f-1)^2}$

The model was trained using gradient descent with mini-batches of 1000 users, with an initial learning rate of 0.0004 which decreased by a factor of 0.21 at each iteration, until the rate reached 10^{-3} of the initial value. It used a weight decay of 0.01 and a momentum of 0.9.

The following variants are included in the solution:

Variant	Baseline	n	w
mfw31-00milestone3-100	mfw31-00	100	#3
mfw31-00milestone4-100	mfw31-00	100	#4
mfw31-00milestone5-100	mfw31-00	100	#5
mfw31-00milestone6-100	mfw31-00	100	#6
mfw31-00milestone7-100	mfw31-00	100	#7
mfw31-00milestone9-100	mfw31-00	100	#9
mfw31-10-milestone0-150	mfw31-10	150	#0
mfw31-10-milestone5-150	mfw31-10	150	#5
trbm50-milestone0-150	trbm50	150	#0
trbm50-milestone9-150	trbm50	150	#9

In our opinion, this variant of asymmetrical model supersedes the other variants presented in this document. The other forms were described essentially because some old results were included in the solution. The astute reader will also notice the similitude between the *milestone* model and the implicit feedback component in the *BK4* model.

3.13 Global Effects

We implemented Global Effects (GE) and Global Time Effects (GTE) as described in [4]. The various parameters used were also the ones noted in [4]. The following table shows the base predictors that were included in the solution. Note that all of these use the first 14 effects, in the order described in [4].

Variant	Comment
globalEffect14	Global Effects.
gte14b	Global Time Effects. The basic user and movie effects were run only once (as opposed to twice, which is suggested in [4]).

3.14 Basic statistics

The basic statistics do not attempt to model the prediction, but simply reflect measures of the user and movie statistical properties. These properties have a weak correlation with the ratings and can be useful inputs for blending.

The main categories of statistics that were computed are support (count), mean, standard deviation and skew. The probabilities of the ratings 2, 3, 4 and 5 were also computed. These 8 statistics were computed both for each user and for each movie for a total of 16 predictors. Note that none of these are regularized in any way; they are just straight statistics.

The following table indicates the predictors that were used in the solution:

Variant	Comment
mmean	Movie mean
mp5	Probability of the movie being rated 5
mskew	Movie skew
ucount	User support (number of ratings)
uskew	User skew
up2	Probability of the user rating a 2
up3	Probability of the user rating a 3
up4	Probability of the user rating a 4
up5	Probability of the user rating a 5

3.15 Non-linear post-processing model

User rating distributions are most often asymmetrical. This is easy to verify by computing the skew for the rating distribution of each user. The skew varies from very positive for users with many 1's and a few higher ratings, to the very negative for users with many 5's and a few lower ratings. On the other hand, most models have an essentially symmetrical formulation, which makes the predictions inaccurate in certain rating ranges. We propose to post-process a model using a third degree polynomial to correct for this effect.

$$\begin{aligned}\hat{r}(u,m) = x(u,m) &+ a_0(u) + a_1(u)(x(u,m)-\mu_u) + a_2(u)(x(u,m)-\mu_u)^2 + a_3(u)\,(x(u,m)-\mu_u)^3 \\ &+ b_0(m) + b_1(m)(x(u,m)-\mu_m) + b_2(m)(x(u,m)-\mu_m)^2 + b_3(m)\,(x(u,m)-\mu_m)^3\end{aligned} \quad (67)$$

Where:

- $\hat{r}(u,m)$ is the generated prediction for user *u* and movie *m*;
- *x* is the model prediction being post-processed;
- a_0, a_1, a_2, a_3, b_0, b_1, b_2 and b_3 are the coefficients of the polynomial transformation learned during training;
- μ_u is the average ratings of user *u*;

- μ_m is the average ratings of movie *m*;

The transformation parameters a_0, a_1, a_2, a_3, b_0, b_1, b_2 and b_3 are learned by alternating least squares regression, starting with the movies, for two iterations.

Regularization is computed by adding the expression $\alpha + \lambda N$ to the diagonal elements of the regression matrix. α and λ are constants selected through validation on the probe set automatically, using the Nelder-Mead Simplex Method. *N* is the number of training samples in this particular regression matrix (number of movies rated by user *u* for the *a* factors, numbers of users having rated movie *m* for the *b* factors). The values of α and λ are selected independently for each *a* and *b*, so a total of 16 meta-parameters are trained using the Nelder-Mead algorithm.

The following variants are present in the solution:

Variant	Baseline
bk1-a1000-nlpp1	bk1-a1000
bk3-d200z-nlpp1	bk3-d200z
bk4-f200z4-nlpp1	bk4-f200z4
bk3-200gx-nlpp1B105[16]	bk3-200gx
bk5-b200B089-nlpp1B108[16]	bk5-b200B089

3.16 Clustering model

Another post-processing method we developed was based on user and movie clusters. We randomly divided the users in 256 bins. We also randomly divided the movies in 256 bins, creating 256x256=65536 clusters. We then minimize the intra-cluster rating variance by moving users and movies between bins, while keeping the bin sizes roughly equal. This is repeated greedily, until the intra-cluster variance no longer improves significantly. Note that the intra-cluster variance is computed on the residual error from the *ssvd-04-00* model described previously, which basically centers the data.

The idea is that the average value of the cluster is a predictor for all user/movie pairs in the cluster. However, some of the 65536 clusters have fewer known ratings, making the cluster average value imprecise. To improve the prediction accuracy, we redo the same clustering algorithm using only 128 movie and user bins, thus obtaining larger clusters. Similarly, we also computed 64, 32, 16, 8, 4, 2 and 1 bin clusters. The final prediction for a movie/user pair is a weighted average of the mean rating in each cluster the movie/user pair belongs to:

$$\hat{r}(u,m) = \sum_{i=0}^{8} w_i \mu_i \tag{68}$$

$$w_8 = \frac{N_8}{N_8 + \alpha} \tag{69}$$

$$w_i = \left(1 - \sum_{j=i+1}^{8} w_j\right) \frac{N_i}{N_i + \alpha}, for\ 0 < i < 8 \tag{70}$$

[16] This set used direct optimization of the blended score.

$$w_0 = \left(1 - \sum_{j=1}^{8} w_j\right) \tag{71}$$

Where:

- $\hat{r}(u, m)$ is the prediction for user u and movie m;
- μ_x is the average of the cluster with 2^x bins which contains the pair (u,m);
- w are the weights given to each cluster the pair *(u, m)* belongs to;
- N_x is the number of ratings in the cluster using 2^x bins;
- α is a regularization parameter set to 20000.

The following variants are present in the solution:

Variant	Baseline
ssvd-31-20-cluster	ssvd-31-20

This is the only attempt at clustering that was kept in the solution. In general, we found that clustering could not improve the more accurate models (like BK4), and had virtually no impact on the accuracy of the final blend.

3.17 Classification models

Typically, models used for collaborative filtering attempt to fit a real-valued function to minimize the prediction error. Most models do not exploit the knowledge that ratings take discrete values between 1 and 5 in the Netflix dataset[17].

In order to use this fact, we recast the problem into four distinct classification problems:

- What is the probability that the rating is above 1?
- What is the probability that the rating is above 2?
- What is the probability that the rating is above 3?
- What is the probability that the rating is above 4?

If we call p_1, p_2, p_3 and p_4 the respective probability, then we can estimate the probability that a rating r takes a specific value between 1 and 5 as:

- $p(r = 1) = 1 - p_1$;
- $p(r = 2) = p_1 - p_2$;
- $p(r = 3) = p_2 - p_3$;
- $p(r = 4) = p_3 - p_4$;
- $p(r = 5) = p_4$;

Also, a combined prediction can be generated as:

[17] Restricted Boltzmann Machines are an exception to this, as they estimate distinct probabilities for each possible rating outcome.

- $r = (1 - p_1) + 2(p_1 - p_2) + 3(p_2 - p_3) + 4(p_3 - p_4) + 5\ p_4 = 1 + p_1 + p_2 + p_3 + p_4$.

Recasting the collaborative filtering into a classification problem offers a number of advantages:

- It allows the use of a huge body of knowledge and methods related to classification problems. Although we only experimented with logistic transformation, there are a lot of interesting opportunities in exploring other alternatives.
- It allows the use of specialized models to address hard to predict ratings, like the 1's in the dataset.

3.17.1 Logistic transformation models

A simple classification algorithm that can be used is the logistic transformation. In the logistic model, the probability of the rating being above the value *g* is estimated as:

$$p_g = p(r > g) = \sigma(z(u, m, t)) \tag{72}$$

$$\sigma(x) = \frac{1}{1 + e^{-x}} \tag{73}$$

Where:

- p_g is the probability that the rating is above g = 1, 2, 3 or 4.
- σ is the logistic function (sigmoid);
- z is the *logit* (see [15]).

In many applications, the *logit z* is chosen as a linear regression of a number of explanatory variables. In our application, we use some of our pre-existing models as *logit*. So *z* in equation (72) can be the expression of *z* from equation (19), equation (21) or any other suitable model. Interestingly, when recast as logistic transformation, these same models capture different trends in the data, as evidenced by the contribution to the blended accuracy.

Training is done in the same way as the linear version of the model. The output of the logistic function is the probability of this rating being above the value *g*. The error term is the squared difference between the real outcome (0 if false, 1 if true) and the estimated probability. This allows transforming a linear model into a logistic model with minimum code changes.

When recast as a logistic transformation, regularization must be re-adjusted. In order to speed up the process, we simply multiplied all learning rate parameters by a factor α and all weight decays by a factor β. The values of α and β were selected using a combination of manual selection and the Nelder-Mead Simplex Method.

The following variants are included in the solution:

Variant	logit	g	α	β
bk3-200g1	bk3-d200	1	18.4308	0.0295088
bk3-200g2	bk3-d200	2	9.68684	0.057215
bk3-200g3	bk3-d200	3	7.10457	0.0816571
bk3-200g4	bk3-d200	4	10.4069	0.0830019
bk3-100ga1	bk3-c100	1	18.4308	0.0295088
bk3-100ga2	bk3-c100	2	9.68684	0.057215
bk3-100ga3	bk3-c100	3	7.10457	0.0816571
bk3-100ga4	bk3-c100	4	10.4069	0.0830019
bk3-100g1	bk3-c100	1	3.70845	0.00214961
bk3-100g2	bk3-c100	2	5.25	0.01875
bk3-100g3	bk3-c100	3	7.84473	0.0932007
bk3-100g4	bk3-c100	4	5.14063	0.028125
bk4-c200g1	bk4-c200	1	18.43080	0.0295088
bk4-c200g2	bk4-c200	2	9.68684	0.0572150
bk4-c200g3	bk4-c200	3	7.10457	0.0816571
bk4-c200g4	bk4-c200	4	10.40690	0.0830019
bk4-f200g1	bk4-f200	1	18.43080	0.0295088
bk4-f200g2	bk4-f200	2	9.68684	0.0572150
bk4-f200g3	bk4-f200	3	7.10457	0.0816571
bk4-f200g4	bk4-f200	4	10.40690	0.0830019

3.17.2 Blending classification models

Let's assume we have classification models $f_{g,i}$, i=1, 2, …, N and g=1, 2, 3 or 4. We create a combined classification model F_g using a linear combination of the logit.

$$F_g = \sigma\left(\sum_{i=1}^{N} b_{g,i}\sigma^{-1}(f_{g,i})\right) \tag{74}$$

The values of $b_{g,i}$ are selected to minimize the RMSE of the classification error over the probe set. The process is entirely automated using the Nelder-Mead Simplex Method.

One interesting point is that we can easily include Decomposed RBMs or Split RBMs into the mix.

The following variants are included in the solution:

Variant	g	*Components*							
blend2-gb1	1	bk3-200g1	bk3-100ga1	bk3-100g1	frbm2-100g1	bk4-c200g1	bk4-f200g1	frbm300g1	drbm160-640g1
blend2-gb2	2	bk3-200g2	bk3-100ga2	bk3-100g2	frbm2-100g2	bk4-c200g2	bk4-f200g2	frbm300g2	drbm160-640g2
blend2-gb3	3	bk3-200g3	bk3-100ga3	bk3-100g3	frbm2-100g3	bk4-c200g3	bk4-f200g3	frbm300g3	drbm160-640g3
blend2-gb4	4	bk3-200g4	bk3-100ga4	bk3-100g4	frbm2-100g4	bk4-c200g4	bk4-f200g4	frbm300g4	drbm160-640g4
blend2-ga1	1	bk3-200g1	bk3-100ga1	bk3-100g1	frbm2-100g1				
blend2-ga2	2	bk3-200g2	bk3-100ga2	bk3-100g2	frbm2-100g2				
blend2-ga3	3	bk3-200g3	bk3-100ga3	bk3-100g3	frbm2-100g3				
blend2-ga4	4	bk3-200g4	bk3-100ga4	bk3-100g4	frbm2-100g4				

3.17.3 Making predictions using classification

We already discussed that a prediction can be obtained by summing the layers:

$$\hat{r}_0 = 1 + \sum_{g=1}^{4} f_g \tag{75}$$

Instead, we used a different expression that yields slightly more accurate results. The expression evaluates a linear regression of the different layers.

$$\hat{r} = a_0 + \sum_{g=1}^{4} a_g f_g \tag{76}$$

The values of *a* are computed using a linear regression to minimize the error over the training set. No regularization is used.

The following variants are included in the solution:

Variant	Components			
bk3-200g	bk3-200g1	bk3-200g2	bk3-200g3	bk3-200g4
bk3-100ga	bk3-100ga1	bk3-100ga2	bk3-100ga3	bk3-100ga4
bk4-c200g	bk4-c200g1	bk4-c200g2	bk4-c200g3	bk4-c200g4

3.18 Per-user linear regression

We developed a method where we blend a small number of prediction sets using a linear regression model specific to each user. Given that there are 480189 users in the Netflix dataset, we compute 480189 such linear regressions.

Initially, we compute a simple linear regression of the basic predictions over the probe set. This is a single linear regression (R_0) computed over the 1408395 samples from the probe set. We then train a linear model specific to each user using ridge regression, which minimizes the expression:

$$\min_{w} \sum_{m \text{ rated by } u} \left[\left(r_m - l(m) - \sum_i w_i \, p_i(m) \right)^2 + \sum_i \lambda_i w_i^2 \right] \tag{77}$$

Where:

- *u* is the user and *m* the movie being predicted;
- *l(m)* is the prediction of R_0 (the global linear regression) for movie *m*;
- p_i is the prediction set *i*;
- w_i is the weight given predictor *i;*
- λ_i is the ridge regression coefficient, computed as a function of the number of ratings N_u made by *u*.

$$\lambda_i = \frac{\alpha_i}{N_u} + \beta_i \tag{78}$$

The parameters $\alpha_i$and β_i are the same for all users and selected by validation on the probe set, using the Nelder-Mead Simplex Method. The final prediction is:

$$\hat{r}(m) = l(m) + \sum_i w_i \, p_i(m) \tag{79}$$

The following variants are included in the solution:

Variant	Components				
blend2b	frbm300	integ80-80-0TZM			
blend3	bk1-b1000x	bk3-d200z	frbm100-mf27		
blend5	bk1-b1000x	bk3-d200z	frbm100-mf27	bk1-a200	brismf760n

Interestingly, this method can also be used to combine together different layers of a classification model. The following are included in the blend:

Variant	Components			
bk3-100gx	bk3-100g1	bk3-100g2	bk3-100g3	bk3-100g4
bk3-200gx	bk3-200g1	bk3-200g2	bk3-200g3	bk3-200g4
bk4-c200gx	bk4-c200g1	bk4-c200g2	bk4-c200g3	bk4-c200g4
bk4-f200gx	bk4-f200g1	bk4-f200g2	bk4-f200g3	bk4-f200g4
drbm160-640gx	drbm160-640g1	drbm160-640g2	drbm160-640g3	drbm160-640g4
frbm2-100gx	frbm2-100g1	frbm2-100g2	frbm2-100g3	frbm2-100g4
frbm300gx	frbm300g1	frbm300g2	frbm300g3	frbm300g4

3.19 KNN1

This section describes a series of movie Nearest-Neighbour models in which various proximity measures are used both to select neighbours and to compute the weight for each neighbour.

3.19.1 Proximity measures

When predicting the rating for a specific movie, the first step of a neighbourhood model consists of selecting a set of similar movies. We propose five methods to select neighbours. In each case, we compute a proximity *z* between the movie *m* being rated and a neighbour *i* rated by user *u*.

3.19.1.1 Common Support

The first method evaluates *z as*:

$$z(m,i) = \left(\frac{n_{m,i}U}{n_m n_i}\right)\left(\frac{1}{1+\tau\Delta t}\right)\left(\frac{n_{m,i}}{n_{m,i}+\alpha}\right) \tag{80}$$

Where:

- *z* is the proximity;
- $n_{m,i}$ is the number of users having rated both *m* and *i*;
- n_m is the number of users having rated *m;*
- n_i is the number of users having rated *i;*
- *U* is the total number of users (480189);
- Δt is the number of days between the rating of *m* and *i*;
- α and τ are parameters learned through validation on the probe set.

The first factor of the equation measures the preference of *m* and *i* to appear together. If *m* and *i* are independent, the value is one. Like the Set Correlation suggested in [4] Section 12, this measure is independent from the ratings and depends only on who rated what. However, the expression above gives better results than [4] when n_m and n_i are widely different. Note that this is usually evaluated over all movies rated by the user, including those for which we don't know the rating. In some instances, only the known ratings were used.

The second factor decreases the proximity as the time interval between the ratings increase. Such decrease in proximity for movies rated at different dates has been proposed by many authors, usually in the form $e^{-\tau\Delta t}$. Our experiments show that equation (80) gives more accurate results. It follows the intuition that while two movies rated on the same day are much better indicators than movies rated one year apart for example, movies rated respectively five and six years apart should have more similar weights.

The third factor penalizes movie pairs with sparse data.

3.19.1.2 Pearson's correlation

The second method uses a proximity based on Pearson's correlation coefficient:

$$z(m,i) = \left(\frac{\varphi^2}{1-\varphi^2}\right)\left(\frac{1}{1+\tau\Delta t}\right) \tag{81}$$

$$\varphi = \rho_{m,i}\left(\frac{n_{m,i}}{n_{m,i}+\alpha}\right) \tag{82}$$

Where:

- *z* is the proximity;
- $\rho_{m,i}$ is Pearson's correlation coefficient between movies *m* and *i* ratings, measured over users having rated both, which can be pre-computed for each movie pair;
- φ is Pearson's correlation coefficient penalized for sparse data;
- $n_{m,i}$ is the number of users having rated both *m* and *i*;
- Δt is the number of days between the rating of *m* and *i*;
- α and τ are parameters learned through validation on the probe set.

The first factor was suggested in [7]. The second factor is identical to the one in the first method above.

3.19.1.3 Mean Square Error

The third method uses a proximity based on the Mean Square Error (MSE) between the ratings:

$$z(m,i) = \left(\frac{1}{1+\tau\Delta t}\right)\left(\frac{n_{m,i}+\alpha}{n_{m,i}\varepsilon^2 + a\bar{\varepsilon}^2}\right) \tag{83}$$

Where:

- *z* is the proximity;

- ε^2 is the MSE between ratings from movie *m* and *i* measured over users having rated both, which can be pre-computed for each movie pair;
- $\bar{\varepsilon}^2$ is the mean MSE average over all movie pairs in the training set;
- Δt is the number of days between the rating of *m* and *i*;
- α and τ are parameters learned through validation on the probe set.

The first factor is the same time factor used in the first two methods. The second factor is the inverse of the MSE, shrunk toward the global MSE.

3.19.1.4 Common Support vs. user support

The fourth method is similar to the first one, except that it penalizes movie pairs for users that have high support. The logic here is that if a user has only made a few ratings, the probability of correlation between the movies he has rated is stronger than for a user that has rated a high number of movies.

The proximity measure is defined as:

$$z(m,i) = \left(\frac{n_{m,i}U}{n_m n_i}\right)\left(\frac{1}{1+\tau\Delta t}\right)\left(\frac{n_{m,i}}{n_{m,i}+\alpha}\right) \tag{84}$$

$$n_{m,i} = \sum_{u \in N(m,i)} \frac{1}{s_u} \tag{85}$$

$$n_x = \sum_{u \in N(x)} \frac{1}{s_u} \tag{86}$$

Where:

- *z* is the proximity;
- *N(m, i)* is the subset of users which have rated movies *m* and *i*;
- s_u is the number of movies that user *u* has rated (including the ratings for which we don't have a value)*;*
- *N(x)* is the subset of users that have rated movie *x*;
- *U* is the total number of users (480189);
- Δt is the number of days between the rating of *m* and *i*;
- α and τ are parameters learned through validation on the probe set.

Here, the different factors of *z* are the same as in the first method. α and τ are parameters learned through validation on the probe set.

3.19.1.5 Latent feature distance

The fifth method uses the latent features from an integrated model to determine the correlation between movies. In this case, the BK5 model was used with *n*=100 to produce a set of latent features. All the other parameters and the training procedure are as described in Section 3.4 BK5 SVD++ model. Then, the sum of the square of the difference between the latent feature values is computed for all movie pairs. Finally, we calculate *z* as follows:

$$z(m,i) = \left(\frac{1}{1+\tau\Delta t}\right)\left(\frac{1}{\omega(m,i)^2+\alpha}\right) \quad (87)$$

$$\omega(m,i) = \sum_{j}^{n}\left(q_j(m) - q_j(i)\right)^2 \quad (88)$$

Where:

- z is the proximity;
- $q(m)$ is the vector of movie latent factors for movie m;
- Δt is the number of days between the rating of *m* and *i*;
- α and τ are parameters learned through validation on the probe set.

The first factor of *z* is the same time factor used in all the above methods. The second term is the squared inverse of the latent feature square-error.

3.19.2 Weight measure

Once the proximity measure is computed for all movies rated by a user, the neighbours with the top K proximity values are kept. K is chosen through validation on the probe set. Once the neighbours are selected, weights must be chosen for each neighbour.

In the kNN1 series of models, weights are derived directly from the proximity measure as suggested in [4] Section 12.3:

$$w(i) = \sigma(\gamma + \delta z) \quad (89)$$

$$\sigma(x) = \frac{1}{1+e^{-x}} \quad (90)$$

$$\hat{r} = \frac{\sum_{i\ in\ top\ K} w(i) r_i}{\sum_{i\ in\ top\ K} w(i)} \quad (91)$$

Where:

- z is the proximity;
- $w(i)$ is the weight given each rating in the chosen nearest K movies;
- $\sigma(x)$ is the sigmoid function;
- γ and δ are selected through validation on the probe set;
- $\hat{r}$ is the predicted score for movie *m.* More exactly, it is the predicted residual error for movie *m*, since the model is computed on residual errors.

3.19.3 Selected combinations

The basic name of this model is *knn1*. The second digit in the predictor name indicates the selected proximity measurement in the order described above (e.g. *knn1-1* uses the common support measurement; *knn1-2* uses Pearson's correlation; etc.). Note that the proximity measurement calculated with equation (80) but computed without the movies with unknown ratings is called *knn1-0*. This model is always run on the residual errors of another model. The various parameters, including K, are optimized automatically using the Nelder-Mead approach. The parameters are either selected to optimize the probe set score or the blending improvement on a linear regression of our selected

predictors. The parameters are seeded with different values depending on the proximity measure used. The following table indicates the initial parameter values for each method.

Proximity method	K			α			τ			γ			δ		
	val	min	max	val	min	max	val	min	max	val	min	max	val	min	max
Common support (1)	80	10	n/a	15000	1	n/a	0.02	0.0	n/a	-4	n/a	n/a	1	0	n/a
Pearson's correlation (2)	80	10	n/a	200	1	n/a	0.02	0.0	n/a	-3	n/a	n/a	20	0	n/a
Mean Square Error (3)	80	10	n/a	20	1	n/a	0.005	0.0	n/a	-5	n/a	n/a	4	0	n/a
Common support w/ user (4)	80	10	n/a	15000	1	n/a	0.02	0.0	n/a	-4	n/a	n/a	1	0	n/a
Latent feature distance (5)	80	10	n/a	30	0	n/a	0.05	0.0	n/a	-5	n/a	n/a	250	0	n/a

The following tables indicate all the instances of this model that were used in the solution. The first column identifies the predictor and the second column indicates if the parameters were optimized on the probe set (P) or on the blend (B).

3.19.3.1 Common support

Variant	Opt.	Variant	Opt.	Variant	Opt.
bk1-a200-knn1-1	P	bk4-e200-knn1-1B170	B	frbm200-mf27-flip20-knn1-1B067	B
bk1-a200x-knn1-1	P	bk4-e50a-knn1-1B188	B	frbm300-bk4-knn1-1B045	B
bk1-a50-2-knn1-1	P	bk4-f200gx-knn1-1B223	B	frbm300-knn1-1B068	B
bk1-a50-2-knn1-1B059	B	bk4-f200z4-nlpp1-knn1-1B204	B	frbm300gx-knn1-1B177	B
bk1-a50-knn1-1B058	B	bk5-b200-knn1-1B095	B	gte14b-knn1-1B048	B
bk1-b200-1-knn1-1	P	bk5-b200B089-knn1-1B091	B	integ0-0-0TZ-grbm200-knn1-1B056	B
bk1-b200-5-knn1-1B060	B	blend2-gax-knn1-1B052	B	integ0-0-0TZ-knn1-1B046	B
bk1-b200-6x-knn1-1B054	B	blend2-gb-knn1-1B213	B	integ0-100-100TZ-knn1-1B069	B
bk1-b200-6x-knn1-1B061	B	blend2-gb1-knn1-1B196	B	integ0-200-200TZ-knn1-1	P
bk1-c200-knn1-1B062	B	blend2-gb2-knn1-1B203	B	integ0-200-200TZ-knn1-1B070	B
bk1-c200x-knn1-1B063	B	blend2-gb3-knn1-1B205	B	integ40-200-0ST-knn1-1B071	B
bk2-b200hz-knn1-1	P	blend2-gb4-knn1-1B244	B	mf27-20-knn1-1B072	B
bk3-100ga-knn1-1B064	B	blend5-knn1-1B047	B	mfw31-05-asym3v250-knn1-1B073	B
bk3-200gx-knn1-1B065	B	brismf760n-knn1-1	P	mfw31-10-milestone0-150-knn1-1B053	B
bk3-d200-knn1-1B049	B	brismf760n-knn1-1B050	B	mfw31-10-milestone5-150-knn1-1B074	B
bk4-b200-knn1-1B100	B	drbm160-640-bk4-knn1-1B055	B	mfw31-60-10-120-knn1-1B057	B
bk4-c200gx-knn1-1B162	B	drbm160-640-knn1-1B066	B	ssvd-31-00-asym1-20-knn1-1B075	B

3.19.3.2 Pearson's correlation

Variant	Opt.	Variant	Opt.	Variant	Opt.
bk4-f200gx-knn1-2B216	B	blend2-gb-knn1-2B238	B	blend2-gb3-knn1-2B240	B
bk4-f200z4-knn1-2B234	B	blend2-gb1-knn1-2B194	B	blend2-gb4-knn1-2B230	B
bk4-f200z4-nlpp1-knn1-2B202	B	blend2-gb2-knn1-2B215	B		

3.19.3.3 Mean Square Error

Variant	Opt.	Variant	Opt.	Variant	Opt.
bk1-a1000x-knn1-3	P	bk4-e50a-knn1-3B166	B	frbm200-mf27-flip20-knn1-3B036	B
bk1-a200-knn1-3	P	bk4-f200gx-knn1-3B190	B	frbm300-bk4-knn1-3B014	B

bk1-a50-2-knn1-3B028	B	bk4-f200z4-knn1-3B242	B	frbm300-knn1-3B037	B
bk1-a50-knn1-3B027	B	bk5-b200-knn1-3B094	B	frbm300gx-knn1-3B160	B
bk1-b200-5-knn1-3B029	B	bk5-b200B089-knn1-3B090	B	gte14b-knn1-3B017	B
bk1-b200-5x-knn1-3	P	blend2-gax-knn1-3B021	B	integ0-0-0TZ-grbm200-knn1-3B025	B
bk1-b200-6x-knn1-3B030	B	blend2-gb-knn1-3B212	B	integ0-0-0TZ-knn1-3B015	B
bk1-c200-knn1-3B031	B	blend2-gb1-knn1-3B233	B	integ0-100-100TZ-knn1-3B038	B
bk1-c200x-knn1-3B032	B	blend2-gb2-knn1-3B206	B	integ0-200-200TZ-knn1-3	P
bk3-a50-knn1-3	P	blend2-gb3-knn1-3B208	B	integ0-200-200TZ-knn1-3B039	B
bk3-b200-knn1-3	P	blend2-gb4-knn1-3B231	B	integ40-200-0ST-knn1-3B040	B
bk3-d200-knn1-3B018	B	blend5-knn1-3B016	B	mf27-20-knn1-3B041	B
bk4-b200-knn1-3B099	B	brismf760n-knn1-3	P	mfw31-10-milestone0-150-knn1-3B022	B
bk4-c200z-knn1-3B176	B	brismf760n-knn1-3B019	B	mfw31-10-milestone5-150-knn1-3B043	B
bk4-c500-knn1-3B126	B	drbm160-640-bk4-knn1-3B024	B	ssvd-31-00-asym1-20-knn1-3B044	B
bk4-e200-knn1-3B163	B	drbm160-640-knn1-3B035	B	urbm20-1000-knn1-3	P
urbm20-1000-knn1-3B020	B				

3.19.3.4 Common support vs. User support

Variant	Opt.	Variant	Opt.	Variant	Opt.
bk1-a1000x-knn1-4B145	B	bk4-f200z4-nlpp1-knn1-4B221	B	drbm160-640-bk4-knn1-4B150	B
bk3-d200z-knn1-4B146	B	bk5-b200-knn1-4B147	B	drbm160-640-knn1-4B149	B
bk4-c500-knn1-4B143	B	blend2-gb-knn1-4B210	B	frbm300-bk4-knn1-4B152	B
bk4-d500-knn1-4B144	B	blend2-gb1-knn1-4B237	B	frbm300-knn1-4B151	B
bk4-f200gx-knn1-4B241	B	blend2-gb3-knn1-4B195	B	integ0-200-200NT-knn1-4B154	B
bk4-f200z4-knn1-4B224	B	brismf760n-knn1-4B148	B	integ80-80-0TZM-knn1-4B155	B

3.19.3.5 Latent feature distance

Variant	Opt.	Variant	Opt.	Variant	Opt.
bk1-a1000x-knn1-5B131	B	bk4-f200z4-nlpp1-knn1-5B207	B	drbm160-640gx-knn1-5B174	B
bk4-c200gx-knn1-5B164	B	bk5-b200-knn1-5B133	B	frbm300-knn1-5B137	B
bk4-c200z-knn1-5B182	B	blend2-gb-knn1-5B214	B	frbm300gx-knn1-5B184	B
bk4-c500-knn1-5B129	B	blend2-gb1-knn1-5B232	B	integ0-0-0TZmilestone6-200-knn1-5B139	B
bk4-d500-knn1-5B130	B	blend2-gb3-knn1-5B245	B	integ0-200-200NT-knn1-5B140	B
bk4-e200-knn1-5B180	B	blend2-gb4-knn1-5B229	B	integ80-80-0TZM-knn1-5B141	B
bk4-e50a-knn1-5B159	B	brismf760n-knn1-5B134	B	mfw31-10-grbm200-knn1-5B142	B
bk4-f200gx-knn1-5B198	B	drbm160-640-bk4-knn1-5B136	B		
bk4-f200z4-knn1-5B197	B	drbm160-640-knn1-5B135	B		

3.20 KNN2-5

This section describes a series of movie Nearest-Neighbour models where the same proximity measures (as described in the previous section) are used to select the list of K neighbours, but where the weight measure is obtained by using a regression model, as described in [1]. This approach yields more accurate results than the ad-hoc transformation of the proximity measure used in *kNN1*.

3.20.1 Weight computation

In [1], the vector of weights *w* is computed as:

$$\hat{A}w = \hat{b} \tag{92}$$

Where $\hat{A}$ and $\hat{b}$ are defined in [1]. In our implementation, we modified this equation to:

$$\left(x\widehat{A_1} + (1-x)\widehat{A_2}\right)w = x\widehat{b_1} + (1-x)\widehat{b_2} \quad (93)$$

$\widehat{A_1}$ and $\widehat{b_1}$ are computed similarly to [1], except that the residual rating error is used instead of the raw ratings. $\widehat{A_2}$ and $\widehat{b_2}$ are also computed similarly to [1], except that the value r' is used instead of the raw ratings, and r' is defined by:

$$r' = r - baseline_1(u, m) \quad (94)$$

In other words, r' is the raw rating from which we have subtracted the bias value $baseline_1$ described in the Common Concepts section. The values of *w* are obtained by solving the linear system subject to non-negativity constraint like in [1]. Shrinkage toward the mean is used for the diagonal elements and other elements are shrunk toward zero.

By mixing weight selection between the residual error and a simple bias model, we are able to significantly improve the accuracy of the model. While weights from the residual errors seem the best approach in theory, artefacts from the base model cause inaccuracy in the weight evaluation. Weights from the simple bias model are immune to this, but do not reflect the actual task which is to estimate the residual error. The compromise between the two is controlled by *x*, which is selected by validation on the probe set.

Note that the meta-parameters for models kNN2 to kNN5 are automatically tuned using the Nelder-Mead method (described in the Common Concepts section) for each base model. This model specific optimization is key to the achieved accuracy.

3.20.2 KNN2

The basic implementation of the method described above is called the *knn2* model. In this model, the number of neighbours (K) is fixed, but selected through validation on the probe set.

Again, for the sake of conciseness only the seed values of the meta-parameters are provided and not the final values for each base model. The following table shows these seed values (as well as the minimum and maximum values) for the different proximity measures.

Proximity method	K			α			τ			γ			x		
	val	min	max	val	min	max	val	min	max	val	min	max	val	min	max
Common support (1)	40	10	60	15000	1	n/a	0.01	n/a	n/a	500	n/a	n/a	0.9	n/a	1.0
Pearson's correlation (2)	30	10	60	150	1	n/a	0.02	n/a	n/a	500	n/a	n/a	0.9	n/a	1.0

This model is always run on the residual errors of another model. The following table presents the combinations of this model that were used in the final blend. For all of the selected combinations the parameters were trained for probe set accuracy.

Variant	Proximity	Variant	Proximity
bk1-c200-knn2-1	1	frbm2-100gx-knn2-1	1
bk3-200g-knn2-1	1	frbm200-mf27-knn2-2	2
bk3-c50-knn2-1	1	integ0-0-0TZ-flip20-knn2-1	1
blend2-ga-knn2-1	1	integ0-0-0TZmilestone6-200-knn2-1	1
blend3-knn2-1	1	integ0-200-200TZ-knn2-1	1

3.20.3 KNN3

In another variant, the number of neighbours is limited by selecting only the neighbours for which the proximity measure exceeds a certain threshold. This threshold was selected manually through validation on the probe set. Note that we also limit the number of neighbours to a low and high limit. The low limit is also selected through validation on the probe set. The high limit is fixed at 60 to limit running time to about 1 minute on a Core i7-920 processor. Weights and proximity are computed identically to *knn2* and we always use the first proximity measure. Thus, this model is called *knn3-1*. Interestingly, this variant is our most accurate neighbourhood model.

The selected proximity value threshold is 0.25. The following table presents the seed and limit values that are used when optimizing the meta-parameters (using the Nelder-Mead technique).

K_{min}			α			τ			γ			x		
val	min	max	val	min	max	val	min	max	val	min	max	val	min	max
10	1	30	20000	1	n/a	0.01	n/a	n/a	200	n/a	n/a	0.8	n/a	1.0

This model is always run on the residual errors of another model. The following table presents the different combinations that were selected for the solution. The second column indicates if the meta-parameters were selected to optimize the probe accuracy (P) or the blend accuracy (B).

Variant	Opt.	Variant	Opt.	Variant	Opt.
bk1-a200-knn3-1	P	bk4-f200z4-knn3-1B199	B	drbm160-640-knn3-1	P
bk1-b200-6x-knn3-1B010	B	bk4-f200z4-nlpp1-knn3-1	P	drbm160-640gx-knn3-1B168	B
bk3-100ga-knn3-1	P	bk4-f200z4-nlpp1-knn3-1B219	B	frbm300-bk4-knn3-1	P
bk3-c50-knn3-1	P	bk5-b200-knn3-1B096	B	frbm300-bk4-knn3-1B001	B
bk3-d200-knn3-1B005	B	bk5-b200B089-knn3-1B092	B	frbm300-knn3-1	P
bk3-d200z-nlpp1-knn3-1B109	B	blend2-gax-knn3-1B008	B	frbm300gx-knn3-1B179	B
bk4-b200-knn3-1B101	B	blend2-gb-knn3-1B217	B	gte14b-knn3-1B004	B
bk4-c200gx-knn3-1B167	B	blend5-knn3-1	P	integ0-0-0TZ-grbm200-knn3-1B012	B
bk4-c200z-knn3-1B165	B	blend5-knn3-1B003	B	integ0-0-0TZ-knn3-1B002	B
bk4-c500-knn3-1B123	B	brismf760n-knn3-1	P	integ0-200-200TZ-knn3-1	P
bk4-e200-knn3-1B169	B	brismf760n-knn3-1B006	B	mfw31-10-milestone0-150-knn3-1B009	B
bk4-e50a-knn3-1B175	B	drbm160-640-bk4-knn3-1	P	mfw31-60-10-120-knn3-1B013	B
bk4-f200gx-knn3-1B193	B	drbm160-640-bk4-knn3-1B011	B		

3.20.4 KNN4

We have also implemented a variant of *knn2* for which we limit the neighbour list to the movies rated on the same day. Also, in this variant, when evaluating the coefficient in the weight model, we limit the

data to pairs of movies also rated on the same day. While the accuracy of this model is not as good, it does make a nice contribution to the blended results. We called this model *knn4-1*.

The following table presents the seed and limit values that are used when optimizing the meta-parameters (using the Nelder-Mead technique).

α			γ			x		
val	min	max	val	min	max	val	min	max
30000	n/a	n/a	100	1	n/a	0.8	n/a	1.0

This model is always run on the residual errors of another model. The following table presents the different combinations that were selected for the solution. The second column indicates if the meta-parameters were selected to optimize the probe accuracy (P) or the blend accuracy (B).

Variant	Opt.	Variant	Opt.	Variant	Opt.
bk1-b200-6x-knn4-1B085	B	blend5-knn4-1	P	integ0-0-0TZ-grbm200-knn4-1B087	B
bk3-d200-knn4-1B080	B	blend5-knn4-1B078	B	integ0-0-0TZ-knn4-1B077	B
bk3-d200z-knn4-1	P	brismf760n-knn4-1B081	B	mfw31-10-milestone0-150-knn4-1B084	B
bk4-b200-knn4-1B102	B	drbm160-640-bk4-knn4-1B086	B	mfw31-60-10-120-knn4-1B088	B
bk5-b200-knn4-1B097	B	drbm160-640-knn4-1	P	urbm20-1000-knn4-1B082	B
bk5-b200B089-knn4-1B093	B	frbm300-bk4-knn4-1B076	B		
blend2-gax-knn4-1B083	B	gte14b-knn4-1B079	B		

3.20.5 KNN5

Finally, a variant of *knn4* was devised in which the "same day" concept is extended to a set of contiguous days. For example, all movies rated within an 8 day window are considered for neighbour selection and weight computation. The K nearest movies are selected within this subset of movies. K is selected through validation on the probe set. Note that, in this variant, we also changed the definition of r' to use baseline_2 instead of baseline_1 as follows:

$$r' = r - baseline_2(u, m) \tag{95}$$

This model is called *knn5*. The following table presents the seed and limit values that are used when optimizing the meta-parameters (using the Nelder-Mead technique).

K			α			γ			x		
val	min	max	val	min	max	val	min	max	val	min	max
40	10	60	250	n/a	n/a	150	n/a	n/a	0.7	n/a	n/a

This model is always run on the residual errors of another model. The following table presents the different combinations that were selected for the solution. The second column indicates the number of days that were considered for the neighbourhood. Note that the parameters for this model were always optimized for blend improvement.

Variant	Days	Variant	Days	Variant	Days
bk1-a1000x-knn5-8B111	8	bk4-f200z4-knn5-1B218	1	frbm300-bk4-knn5-8B117	8
bk4-c200z-knn5-1B183	1	bk4-f200z4-nlpp1-knn5-1B192	1	frbm300gx-knn5-1B171	1
bk4-c500-knn5-1B125	1	blend2-gb-knn5-1B191	1	integ0-0-0TZmilestone6-200-knn5-8B118	8
bk4-e200-knn5-1B185	1	drbm160-640-bk4-knn5-8B115	8	integ0-200-200NT-knn5-8B119	8
bk4-e50a-knn5-1B161	1	drbm160-640-knn5-8B114	8	integ80-80-0TZM-knn5-8B120	8
bk4-f200gx-knn5-1B228	1	drbm160-640gx-knn5-1B186	1	mfw31-10-grbm200-knn5-8B121	8

3.21 Older movie neighbourhood models

This section describes a number of older movie neighbourhood models because some variants were included in the solution. These methods are superseded by the new movie kNN described earlier.

3.21.1 Movie

This is our earliest neighbourhood model. It is a direct implementation of the model described in [1]. To predict the rating *r(u, m)* of user *u* for movie *m*, we select the *k*=20 movies rated by *u* with the highest similarity score:

$$z(m,i) = \frac{\rho_{m,i} n_{m,i}}{n_{m,i} + \alpha} \tag{96}$$

Where:

- *z(m, i)* is the similarity measure between movies *m* and *i*;
- $\rho_{m,i}$ is the Pearson's correlation coefficient between the raw ratings of movie *m* and movie *i* amongst users having rated both *m* and *i*;
- $n_{m,i}$ is the number of users having rated both *m* and *i*;
- α=400 is the shrinkage coefficient.

The regression model described in [1] is used on the raw ratings to determine the neighbour weight. The regularization coefficient *β=400* is used to build the regression model. While the model described in [1] requires a non-negative least square solver, we used a simple heuristic instead to generate a similar but slightly sub-optimal solution:

1. Solve the regression problem without the non-negative constraint;
2. Pin all negative variables to zero;
3. Re-compute the remaining variables with the pinned variables forced to zero;
4. Repeat 2 and 3 until all variables are non-negative.

This model is applied to the residual error of other models. One variant is present in the solution:

Variant	Baseline
ssvd-31-00-movie	ssvd-31-00

3.21.2 Movie2

This model is very similar to Movie. The first difference is the similarity measure:

$$z(m,i) = \frac{n_{m,i} \sum_{v \in N(m) \cap N(i)} e_{v,m} e_{v,i}}{(n_{m,i} + \alpha)\sqrt{(\sum_{v \in N(m) \cap N(i)} e_{v,m}^2)(\sum_{v \in N(m) \cap N(i)} e_{v,i}^2)}} \quad (97)$$

Where:

- *z(m, i)* is the similarity measure between movies *m* and *i*;
- $e_{v,m}$ ($e_{v,i}$) is the residual error of model *ssvd-04-00* for user *v* and movie *m* (*i*);
- *N(m)* (*N(i)*) is the set of users having rated movie *m* (*i*);
- $n_{m,i} = |N(m) \cap N(i)|$ is the number of users having rated both *m* and *i*;
- α is the shrinkage coefficient.

The *k* neighbours with the highest proximity are kept.

The regression model used to determine weights is computed using the residual of the model *ssvd-04-00*. The weights themselves are then applied to the residual of the actual baseline model.

The values of *α* and *k* were chosen manually around the values giving the best result on average: 500 and 40 respectively.

The following variants are present in our solution.

Variant	Baseline	*α*	*k*
crbm100-movie2	crbm100	400	40
crbm100-ssvd-07-00-movie2	crbm100-ssvd-07-00	400	40
rbm100-ssvd-07-00-movie2	rbm100-ssvd-07-00	500	30
ssvd-07-30-2-movie2	ssvd-07-30-2	600	40
ssvd-31-00-asym3-200-movie2	ssvd-31-00-asym3-200	500	40
ssvd-31-00-asym3v-300-movie2	ssvd-31-00-asym3v-300	500	40
ssvd-31-00-asym3w-200-movie2	ssvd-31-00-asym3w-200	600	30
ssvd-31-20-cluster-movie2	ssvd-31-20-cluster	500	40
ssvd-31-20-movie2	ssvd-31-20	500	40
trbm100-movie2	trbm100	550	35
trbm100-ssvd-31-00-movie2	trbm100-ssvd-31-00	550	35

3.21.3 Movie3

This model is strictly identical to Movie2, except that we implemented a proper non-negative solver (see [12] figure 1 for an example of a non-negative solver), resulting in slightly more accurate result than the heuristic used in Movie and Movie2.

The following variants are present in our solution:

Variant	Baseline	*α*	*k*
mf01-20-movie3	mf01-20	500	40
nmf40-60-10-movie3	nmf40-60-10	500	40
nnmf40-ssvd-07-00-movie3	nnmf40-ssvd-07	500	40

3.21.4 Movie4

This model is strictly identical to Movie3, except that *ssvd-39-00* is used in the residuals that determine the proximity and the weights.

The following variants are present in our solution:

Variant	Baseline	α	k
mf01-40-3-80-movie4	mf01-40-3-80	500	40
mf27-20-movie4	mf27-20	500	40
mf27-20env50-movie4	mf27-20env50	500	40
nmf80-120-20-mf27-movie4	nmf80-120-20-mf27	500	40
nnmf80-ssvd-39-00-movie4	nnmf80-ssvd-39-00	400	50
pmf80-120-20-mf27-movie4	pmf80-120-20-mf27	500	40
ssvd-39-05-movie4	ssvd-39-05	400	30
ssvd-39-10-movie4	ssvd-39-10	400	30
ssvd-63-00-movie4	ssvd-63-00	500	40
ssvd-63-30-movie4	ssvd-63-30	500	40
trbm150-mf27-movie4	trbm150-mf27	500	40
trbm150-ssvd-07-00-movie4	trbm150-ssvd-07-00	500	40

3.21.5 Movie6

This model is similar to Movie3 and Movie4. It uses *mfw31-00* for the residuals that determine the proximity and the weights.

The proximity measure is enhanced to take into account the dates at which the ratings were made. When predicting the rating for user *u* and movie *m*, we consider the proximity for movie *i* to be smaller if *m* and *i* are rated far apart in time.

$$z_0(m,i) = \frac{n_{m,i}\sum_{v\in N(m)\cap N(i)} e_{v,m}e_{v,i}}{(n_{m,i}+\alpha)\sqrt{\left(\sum_{v\in N(m)\cap N(i)} e_{v,m}^2\right)\left(\sum_{v\in N(m)\cap N(i)} e_{v,i}^2\right)}} \qquad (98)$$

$$z(m,i) = \frac{z_0^2(m,i)}{(1+\tau|t_m - t_i|)\left(1-z_0^2(m,i)\right)}$$

Where:

- *z(m, i)* is the similarity measure between movies *m* and *i*;
- z_0*(m, i)* is a component of the similarity measure between movies *m* and *i* (identical to the similarity measure of Movie2, Movie3 and Movie4);
- $e_{v,m}$ ($e_{v,i}$) is the residual error of model *mfw31-00* for user *v* and movie *m* (*i*);
- *N(m)* (*N(i)*) is the set of users having rated movie *m* (*i*);
- $n_{m,i} = |N(m) \cap N(i)|$ is the number of users having rated both *m* and *i*;
- α=500 is the shrinkage coefficient
- t_m and t_i are the dates at which user *u* has rated movies *m* and *i* respectively;
- τ is selected manually as 0.03;
- the neighbourhood size is fixed to *k=40*.

The following variants are included in the solution:

Variant	Baseline
brismf760n-movie6	brismf760n
integ0-0-0TZ-movie6	integ0-0-0TZ
integ0-0-0TZmilestone6-200-movie6	integ0-0-0TZmilestone6-200
integ0-100-100TZ-movie6	integ0-100-100TZ
integ80-80-0TZM-movie6	integ80-80-0TZM
mfw31-00-movie6	mfw31-00
mfw31-00milestone3-100-movie6	mfw31-00milestone3-100
mfw31-00milestone7-100-movie6	mfw31-00milestone7-100
mfw31-00milestone9-100-movie6	mfw31-00milestone9-100
mfw31-05-asym3v250-movie6	mfw31-05-asym3v250
mfw31-60-10-120-movie6	mfw31-60-10-120
ssvd-31-00-asym1-20-movie6	ssvd-31-00-asym1-20
trbm50-asym3v250-movie6	trbm50-asym3v250
trbm50-milestone0-150-movie6	trbm50-milestone0-150

3.21.6 Movie8

This model is identical to Movie6, except that $\tau=0$.

The following variant is included in the solution:

Variant	Baseline
frbm300-movie8	frbm300

3.21.7 Movie5

This model is very different from the *MovieN* models presented so far, as it does not rely on the method described in [1]. Instead, it uses a heuristic function to determine the weights directly from the proximity measure:

$$z(m,i) = \frac{\beta + |N(m) \cap N(i)|}{1.94\beta + \sum_{v \in (N(m_1) \cap N(m_2))} [r(v,m) - r(v,i)]^2} \tag{99}$$

$$w(m,i) = \frac{z^4(m,i)}{1 + \tau |t_m - t_i|} \tag{100}$$

Where:

- z is the proximity measure between movies m and i;
- w(m, i) is the weight given to movie i when predicting movie m;
- $r(v, m)$ is the rating given to movie m by user v, $r(v, i)$ is the rating given to movie i by user v;
- $N(m)$ and $N(i)$ are the sets of users having rated movie m and i respectively;
- $\beta=100$ is a regularization factor, which pulls the expression toward the mean square between arbitrary pairs of movies (1.94);
- t_m and t_i are the dates at which user u has rated movies m and i respectively;
- τ=1/60 is selected manually;
- the neighbourhood includes the $k=50$ neighbours with the highest weights w.

The following variants are included in the solution:

Variant	Baseline
bk1-a50-movie5	bk1-a50
brismf250-movie5	brismf250
brismf40-movie5	brismf40
brismf760-movie5	brismf760
brismf760n-movie5	brismf760n
globalEffect14-movie5	globalEffect14
integ0-200-200TZ-movie5	integ0-200-200TZ
integ60-0-0TS-movie5	integ60-0-0TS
integ80-80-0TZ-movie5	integ80-80-0TZ
mf27-20-movie5	mf27-20
mf27-20env50-movie5	mf27-20env50
mf27-40-3-80-movie5	mf27-40-3-80
mfw31-00milestone4-100-movie5	mfw31-00milestone4-100
mfw31-00milestone5-100-movie5	mfw31-00milestone5-100
mfw31-10-milestone0-150-movie5	mfw31-10-milestone0-150
mfw31-10-milestone5-150-movie5	mfw31-10-milestone5-150
ssvd-31-00-asym4v-200-movie5	ssvd-31-00-asym4v-200
ssvd-31-00-movie5	ssvd-31-00
trbm150-movie5	trbm150
trbm50-asym3v250-mfw27-movie5	trbm50-asym3v250-mfw27
trbm50-asym3v250-movie5	trbm50-asym3v250
trbm50-milestone9-150-movie5	trbm50-milestone9-150

3.22 User neighbourhood models

We implemented two variants of user neighbourhood models. The idea is to identify user pairs having similar taste and rating habits, in order to infer unknown ratings from the ratings of these neighbours. Overall, we found user neighbourhood models much less useful than movie neighbourhood models.

3.22.1 User2 model

This variant is a very naive neighbourhood model. For each rating that we wish to predict, we select the 100 most similar users that have rated the movie being predicted. The prediction is the weighted sum of the 100 similar ratings, where the weight varies linearly between 1.0 and 0.6 according to the rank among the 100 neighbours.

The distance measure used is the mean squared error between pairs of user ratings, computed over the common rated movies. Movies most similar to the movie being predicted are given a greater weight when computing the user distance. The distance measure is computed as:

$$d_u(u_1, u_2, m) = \frac{2\alpha + \sum_{i \in (N(u_1) \cap N(u_2))} w(m,i)[r(u_1,m) - r(u_2,m)]^2}{\alpha + \sum_{i \in (N(u_1) \cap N(u_2))} w(m,i)} \quad (101)$$

$$\alpha = \frac{5 \sum_{i \in (N(u_1) \cap N(u_2))} w(m,i)}{|N(u_1) \cap N(u_2)|} \quad (102)$$

$$w(m,i) = \left[\frac{1.94\beta + \sum_{v \in (N(m) \cap N(i))} [r(v,m) - r(v,i)]^2}{\beta + |N(m) \cap N(i)|}\right]^4 \quad (103)$$

Where:

- d_u is the distance between users u_1 and u_2 when predicting movie *m*;
- α is a regularization factor, which pulls d_u toward 2.0 for pair of users with few common movies, the value of α is chosen as 5.0 times the average weight;
- *w* is the distance measure between two movies, used to weigh the movie when comparing users, the value is pre-computed and kept in computer memory for all movie pairs;
- *r(u, m)* is the rating given to movie *m* by user *u*;
- *N(u)* is the set of movies rated by user *u*;
- *N(m)* is the set of users having rated movie *m*;
- β is a regularization factor, which pulls the expression toward the mean square between arbitrary pairs of movies (1.94), β is chosen as 100.

Computing this expression for all users and all predictions would take an excessive amount of time. To accelerate processing, we use pre-filtering steps that prune the most unlikely candidates, with virtually no impact on the final model accuracy:

1. When searching for the best 100 neighbours of user *u* for movie *m*, we first identify all users *v≠u* having rated *m*;
2. We discard all users *v* having provided few ratings, keeping only the 12288[18] users with the most ratings;
3. We sort the remaining 12288 *v* users using the Euclidean distance between the users latent features of a rank=8 simple matrix factorization model. We then keep the top 1280 candidate *v*.
4. Finally we sort the remaining 1280 candidates using the d_u formula described above and discard all *v* except the top 100.

[18] A nice round number... in hexadecimal!

This model is always computed on the residual error of another model. The following variants are included in the solution:

Variant	Baseline
brismf760-user2	brismf760
brismf760n-user2	brismf760n
drbm100-500-user2	drbm100-500
integ0-200-200NT-user2	integ0-200-200NT
integ0-200-200TZ-user2	integ0-200-200TZ
integ60-0-0TS-user2	integ60-0-0TS
mfc27-60-10-120-user2	mfc27-60-10-120
mfw31-05-asym3v250-user2	mfw31-05-asym3v250
mfw31-80-x-user2	mfw31-80-x
ssvd-31-00-user2	ssvd-31-00
trbm100-ssvd-31-00-user2	trbm100-ssvd-31-00
trbm50-asym3v250-user2	trbm50-asym3v250

3.22.2 Flipped model

The second user neighbourhood model that we implemented is directly inspired from Section 4.1 of [10]. Neighbourhood models are most useful between movies, thus the name *flipped* when applied to users.

In [10], Koren describes a neighbourhood model where the weights are approximated by a low rank matrix factorization. We used the same approach, adding the implicit feedback of movies for which the rating is not known. Also, since we only compute this model on the residual error of another model, the bias terms are unnecessary.

The model uses the following expression:

$$\hat{e}(u,m) = \sum_{i=1}^{n} p_i(u) \left[\frac{1}{\sqrt{|R(m)|}} \sum_{v \in R(m)} e(v,m) q_i(v) + \frac{1}{\sqrt{|N(m)|}} \sum_{v \in N(m)} y_i(v) \right] \quad (104)$$

Where:

- $\hat{e}$ is the estimated residual error for user *u* and movie *m*;
- *e(v, m)* is the residual error for the prediction of user *v* and movie *m*;
- *p* is a vector of length *n* representing the latent features of user *u*;
- *q* is a vector of length *n* such that $p^T q$ is the approximation of the weight given to user *v*;
- *y* is a vector of length *n* such that $p^T y$ is the approximation of the implicit feedback given to user *v*;
- *R(m)* is the set of users providing known ratings for movie *m*;
- *N(m)* is the set of users providing ratings (known or not) for movie *m*;

The model is trained using stochastic gradient descent. p and q are trained using a learning rate γ_1 and weight decay λ_1. y is trained using a learning rate γ_2 and weight decay λ_2. γ_1 and γ_2 are reduced by a factor Δ_γ at each iteration. Training stops when maximum accuracy is achieved on the probe set.

The values of γ_1, γ_2, λ_1, λ_2 and Δ_γ are selected automatically using the Nelder-Mead Simplex Method to optimize the probe set accuracy.

The following variants are included in the solution:

Variant	Baseline	n
frbm200-mf27-flip20	frbm200-mf27	20
integ0-0-0TZ-flip20	integ0-0-0TZ	20

4 Blending

During our participation in the Netflix Prize competition, we followed the strategy used by most participants which consists of developing multiple models to predict the ratings, and then combining these multiple predictions into a single solution using various blending methods.

One of the keys to achieving a 10.09% improvement on the quiz set was the complex blending scheme that combined the many results from the individual teams. This important component was handled by our colleagues from Big Chaos. Thus, this part will not be described here. Nevertheless, we will describe our blending techniques, as a few partial blends have been included in the solution.

In general, a blending method involves selecting a number of parameters that determine how the various prediction sets should be combined. Our strategy has been to optimize these parameters on the probe set, using models trained on data excluding the probe set. Then, the same parameters are reused to blend together corresponding models that were trained including the probe set. This methodology has no theoretical grounding because the probe set is not statistically equivalent to the rest of the training data, but it works well in practice. Note that it is critical for the two versions of each model (computed with and without the probe set) to be precisely equivalent.

Finally, when investigating blending methods, an approach we have followed is to split the probe set in two subsets. Then the blending method is trained on one subset, keeping the other subset to obtain an objective evaluation of the success of the method.

4.1 Set selection

Over time, we produced a large number of prediction sets. Unfortunately, most of them are highly collinear with the others and do not contribute to the blend. Furthermore, these collinear sets, if used, are likely to actually degrade the accuracy on the test set, due to overfitting. Consequently, we routinely discarded predictors that showed little or no contribution to the blend result.

We used a simple algorithm to help us in this task, known as backward selection:

1. Compute a linear regression of all prediction sets over the probe set;
2. For each set, compute a linear regression of the collection without the current set;
3. Remove the set with the smallest contribution and repeat from step 1.

This algorithm allowed us to rank predictors from the worst to the best in a greedy fashion, in term of contribution to a linear regression. It was useful to determine which prediction sets to include or not in the blend. Typically we excluded prediction sets that contributed less than 2 or 3×10^{-6} to the blend accuracy.

4.2 Neural network blending

We have generated four neural network blends in the solution, using a strategy directly inspired from [4]. Each neural network consists of a multi-layer perceptron with one hidden layer. Each neuron uses a hyperbolic tangent activation function. Inputs are scaled to zero mean and unit variance.

The input variables are augmented by two values: the log of the user support and the log of the movie support. Weights are learned through gradient descent (back prop). Weight decay is set to $1x10^{-5}$ for the first layer, and zero for the second. Learning rates are set initially to 0.0008 for the first layer, and 0.0003 for the second layer. Learning rates are decreased to zero linearly over 500 iterations. Between 12 and 20 runs are executed with different seeds, and the most accurate is kept.

The following variants are included in the solution:

Variant	Number of sets	Number of hidden nodes
nnblend-top212	212	9
nnblend-top100	100	18
nnblend-top50	50	35
nnblend-bottom95	95	18

The following tables show the list of predictors used in each case:

nnblend-top212			
bk1-a1000x-knn1-3	bk1-a200-knn3-1	bk1-a200x-knn1-1	bk1-a50
bk1-a50-2x	bk1-a50-movie5	bk1-b1000	bk1-b200-1
bk1-b200-1-knn1-0	bk1-b200-1-knn1-1	bk1-b200-2	bk1-b200-5
bk1-b200-6x	bk1-c200-knn2-1	bk1-c200x	bk2-b200hz-knn1-1
bk3-100g1	bk3-100g2	bk3-100g4	bk3-100ga-knn3-1
bk3-100ga4	bk3-100gax	bk3-100gx	bk3-200g1
bk3-200g3	bk3-200g4	bk3-200gx	bk3-a0z
bk3-b200-knn1-3	bk3-c50	bk3-c50-knn2-1	bk3-d200
bk3-d200-knn1-1B049	bk3-d200-knn1-3B018	bk3-d200-knn3-1B005	bk3-d200z
bk3-d200z-knn4-1	bk4-biasZ	blend2-ga-knn2-1	blend2-ga1
blend2-ga4	blend2-gax	blend2-gax-knn1-1B052	blend2-gax-knn1-3B021
blend2b	blend5-knn1-1B047	blend5-knn3-1	blend5-knn4-1
brismf250	brismf250-movie5	brismf40-movie5	brismf760-movie5
brismf760n	brismf760n-knn1-1	brismf760n-knn1-1B050	brismf760n-knn1-3
brismf760n-knn1-3B019	brismf760n-knn3-1	brismf760n-knn4-1B081	crbm100-movie2
crbm100-ssvd-07-00-movie2	crbm100-ssvd-07-20	crbm100x-ssvd-03-00	crbm200
crbm200-ssvd-07-00	drbm100-500	drbm100-500-user2	drbm160-640
drbm160-640-bk4-knn3-1	drbm160-640-bk4-knn3-1B011	drbm160-640-knn3-1	drbm160-640-knn4-1
frbm100-mf27-m	frbm2-100g3	frbm2-100g4	frbm200
frbm200-mf27-flip20	frbm200-mf27-knn2-2	frbm200x	frbm300
frbm300-bk4-knn3-1	frbm300-knn1-1B068	frbm300-knn3-1	frbm300x
integ0-0-0TZ	integ0-0-0TZ-flip20-knn2-1	integ0-0-0TZ-grbm200-knn1-3B025	integ0-0-0TZ-knn1-1B046
integ0-0-0TZ-knn1-3B015	integ0-0-0TZ-knn3-1B002	integ0-0-0TZ-movie6	integ0-0-0TZmilestone6-200-movie6
integ0-100-100TZ	integ0-100-100TZ-movie6	integ0-200-200NT	integ0-200-200NT-user2
integ0-200-200TZ	integ0-200-200TZ-knn1-1	integ0-200-200TZ-knn1-3B039	integ0-200-200TZ-knn3-1
integ0-200-200TZ-movie5	integ0-200-200TZ-user2	integ20-100-100NT	integ40-200-0ST-knn1-1B071
integ40-200-0T	integ60-0-0TS	integ60-0-0TS-movie5	integ60-0-0TS-user2
integ80-80-0TZ-movie5	integ80-80-0TZM	integ80-80-0TZM-movie6	mf01-20-movie3
mf01-40-3-80-movie4	mf27-20	mf27-20-knn1-1B072	mf27-20-movie4
mf27-20-movie5	mf27-20-u	mf27-20env50-m	mf27-20env50-movie4

mf27-20env50-movie5	mf27-40-3-80-movie5	mfc27-60-10-120	mfc27-60-10-120-m
mfc27-60-10-120-user2	mfw31-00-movie6	mfw31-00milestone3-100-movie6	mfw31-00milestone5-100-movie5
mfw31-00milestone6-100	mfw31-00milestone7-100-movie6	mfw31-00milestone9-100-movie6	mfw31-05-asym3v250
mfw31-05-asym3v250-movie6	mfw31-05-m	mfw31-10-milestone0-150	mfw31-10-milestone0-150-knn1-3B022
mfw31-10-milestone0-150-movie5	mfw31-10-milestone5-150	mfw31-10-milestone5-150-movie5	mfw31-40env50
mfw31-40env50-m	mfw31-40env50-m2	mfw31-60-10-120-m	mfw31-60-10-120-movie6
mfw31-60-x	mfw31-80-x	mfw31-80-x-m	mfw31-80-x-user2
mmean	mp5	mskew	nmf40-60-10
nmf40-60-10-movie3	nmf80-120-20-m	nmf80-120-20-mf27-movie4	nnmf40
nnmf40-ssvd-07-00	nnmf40-ssvd-07-00-movie3	nnmf80-ssvd-39-00-movie4	pmf40-60-10-m
pmf80-120-20-mf27-movie4	rbm100-ssvd-07-00-movie2	ssvd-04-00	ssvd-04-10
ssvd-04-10-m	ssvd-04-40-m	ssvd-07-30-2-movie2	ssvd-31-00-asym1-20-movie6
ssvd-31-00-asym3-100	ssvd-31-00-asym3-20	ssvd-31-00-asym3-200	ssvd-31-00-asym3-200-movie2
ssvd-31-00-asym3v-300-movie2	ssvd-31-00-asym3w-200-movie2	ssvd-31-00-asym4-200	ssvd-31-00-asym4v-200-movie5
ssvd-31-00-movie5	ssvd-31-00-user2	ssvd-31-10-m	ssvd-31-20
ssvd-31-20-cluster-movie2	ssvd-31-20-m	ssvd-31-20-movie2	ssvd-31-20-u
ssvd-31-60	ssvd-39-05	ssvd-39-05-movie4	ssvd-39-10
ssvd-39-10-movie4	svd05x	trbm100	trbm100-ssvd-31-00-movie2
trbm150-mf27-m	trbm150-mf27-movie4	trbm150-movie5	trbm150-ssvd-07-00-movie4
trbm150-ssvd-39-00	trbm50-asym3v250-movie5	trbm50-asym3v250-movie6	trbm50-asym3v250-user2
trbm50-mf27	trbm50-milestone0-150-movie6	trbm50-milestone9-150-movie5	trbm50-ssvd-39-00
ucount	up2	up3	up4
up5	urbm20-1000-knn1-3	usermovie	uskew

nnblend-top100			
bk1-a200-knn3-1	bk1-b1000	bk1-b200-1	bk1-b200-5
bk1-b200-6x	bk1-c200-knn2-1	bk1-c200x	bk2-b200hz-knn1-1
bk3-100g1	bk3-100g2	bk3-100g4	bk3-100ga-knn3-1
bk3-100gax	bk3-100gx	bk3-200g3	bk3-200gx
bk3-a0z	bk3-b200-knn1-3	bk3-d200	bk3-d200-knn1-1B049
bk3-d200-knn1-3B018	bk4-biasZ	blend2-gax	blend2-gax-knn1-1B052
blend2-gax-knn1-3B021	blend5-knn1-1B047	blend5-knn3-1	blend5-knn4-1
brismf250	brismf40-movie5	brismf760n	brismf760n-knn1-1
brismf760n-knn1-1B050	brismf760n-knn1-3	brismf760n-knn1-3B019	brismf760n-knn3-1
brismf760n-knn4-1B081	crbm100x-ssvd-03-00	drbm100-500	drbm100-500-user2
drbm160-640-bk4-knn3-1	drbm160-640-bk4-knn3-1B011	frbm100-mf27-m	frbm2-100g4
frbm200-mf27-flip20	frbm300	frbm300-knn3-1	frbm300x
integ0-0-0TZ	integ0-0-0TZ-grbm200-knn1-3B025	integ0-0-0TZ-knn1-1B046	integ0-0-0TZ-knn1-3B015
integ0-0-0TZ-knn3-1B002	integ0-200-200TZ-knn1-1	integ0-200-200TZ-knn1-3B039	integ0-200-200TZ-movie5
integ40-200-0ST-knn1-1B071	integ60-0-0TS-user2	mf01-20-movie3	mf27-20
mf27-20-knn1-1B072	mf27-20-u	mf27-40-3-80-movie5	mfc27-60-10-120
mfc27-60-10-120-user2	mfw31-00milestone3-100-movie6	mfw31-00milestone9-100-movie6	mfw31-05-asym3v250
mfw31-05-asym3v250-movie6	mfw31-10-milestone0-150-knn1-3B022	mfw31-10-milestone0-150-movie5	mfw31-10-milestone5-150-movie5
mfw31-40env50	mfw31-60-10-120-m	mfw31-60-10-120-movie6	mfw31-80-x
mfw31-80-x-user2	mmean	mp5	nnmf40-ssvd-07-00-movie3
nnmf80-ssvd-39-00-movie4	rbm100-ssvd-07-00-movie2	ssvd-04-00	ssvd-04-40-m
ssvd-31-00-asym1-20-movie6	ssvd-31-00-asym3-200-movie2	ssvd-31-00-asym3w-200-movie2	ssvd-31-00-asym4v-200-movie5
ssvd-31-00-movie5	ssvd-31-00-user2	ssvd-31-20	ssvd-31-20-cluster-movie2
ssvd-31-20-u	ssvd-39-05-movie4	trbm100-ssvd-31-00-movie2	trbm150-ssvd-07-00-movie4
trbm50-asym3v250-movie6	trbm50-milestone0-150-movie6	urbm20-1000-knn1-3	usermovie

nnblend-top50			
bk1-b1000	bk1-b200-5	bk3-100g2	bk3-100g4
bk3-100gax	bk3-100gx	bk3-200gx	bk3-a0z
bk3-d200-knn3-1B005	bk3-d200z	bk3-d200z-knn4-1	bk4-a50

bk4-b200-knn3-1B101	bk4-biasZ	bk5-b200-knn1-1B095	bk5-b200B089
blend2-gax	blend2-gax-knn1-1B052	blend5-knn3-1	brismf760n-knn1-1
brismf760n-knn1-1B050	drbm160-640-bk4-knn3-1	drbm160-640-bk4-knn3-1B011	frbm2-100g4
frbm200-mf27-flip20	frbm300	frbm300-knn3-1	frbm300x
integ0-0-0TZ	integ0-0-0TZ-grbm200-knn1-3B025	integ0-0-0TZ-knn1-3B015	integ0-0-0TZ-knn3-1B002
integ0-200-200TZ-knn1-3B039	integ0-200-200TZ-movie5	integ40-200-0ST-knn1-1B071	mf27-20
mf27-20-knn1-1B072	mf27-20-u	mfc27-60-10-120	mfw31-05-asym3v250-movie6
mfw31-10-milestone0-150-movie5	mfw31-10-milestone5-150-movie5	mfw31-60-10-120-m	nnmf80-ssvd-39-00-movie4
ssvd-04-00	ssvd-04-40-m	ssvd-31-00-asym1-20-movie6	trbm50-asym3v250-movie6
urbm20-1000-knn1-3	usermovie		

nnblend-bottom95			
bk1-a200-knn1-1	bk1-a200-knn3-1	bk1-a200x-knn1-1	bk1-a50-2-knn1-1
bk1-b1000	bk1-b200-1	bk1-b200-2	bk1-b200-5x-knn1-3
bk1-b200-6	bk3-100g1	bk3-200g3	bk3-200gx-nlpp1B105
bk3-a50-knn1-3	bk3-c50	bk3-c50x	bk4-b200-knn4-1B102
bk5-b200B089-nlpp1B108	blend2-ga1	blend2-ga4	blend2b
blend3-knn2-1	brismf250-movie5	brismf760-movie5	brismf760-user2
brismf760n-knn1-3B019	brismf760n-user2	crbm100	crbm100-ssvd-07-00
crbm200	crbm200-ssvd-07-00	drbm100-500-user2	frbm200-mf27-knn2-2
frbm300-bk4-knn3-1-X-bk3-b200-knn1-3	frbm300-knn1-1B068	frbm300-movie8	globalEffect14-movie5
gte14b	integ0-0-0TZ-flip20-knn2-1	integ0-0-0TZmilestone6-200	integ0-0-0TZmilestone6-200-knn2-1
integ0-0-0TZmilestone6-200-movie6	integ0-200-200TZ	integ0-200-200TZ-knn1-3	integ0-200-200TZ-knn3-1
integ0-200-200TZ-user2	integ20-100-100NT	integ60-0-0TS-movie5	integ80-80-0TZM
integ80-80-0TZM-movie6	mf27-20-movie4	mf27-20-movie5	mf27-20env50-m
mf27-20env50-movie4	mf27-20env50-movie5	mfc27-60-10-120-m	mfc27-60-10-120-user2
mfw31-00	mfw31-00-movie6	mfw31-05-asym3v250	mfw31-05-asym3v250-user2
mfw31-10-milestone5-150	mfw31-40env50-m2	mfw31-60-x	mfw31-80-x
mskew	nmf40-60-10	nmf40-60-10-movie3	nmf80-120-20-m
nmf80-120-20-mf27-movie4	nnmf40-ssvd-07-00	pmf40-60-10-m	pmf80-120-20-mf27-movie4
ssvd-04-00	ssvd-07-30-2-movie2	ssvd-31-00-asym3-200	ssvd-31-00-movie
ssvd-31-10-m	ssvd-31-20-m	ssvd-31-60	ssvd-39-05
ssvd-39-05-movie4	ssvd-39-10	ssvd-39-10-movie4	svd02x
svd05x	trbm100-movie2	trbm100-ssvd-31-00	trbm100-ssvd-31-00-movie2
trbm100-ssvd-31-00-user2	trbm150-mf27-movie4	trbm50-asym3v250	trbm50-asym3v250-mfw27-movie5
trbm50-asym3v250-movie5	trbm50-milestone9-150-movie5	urbm20-1000-knn1-3B020	

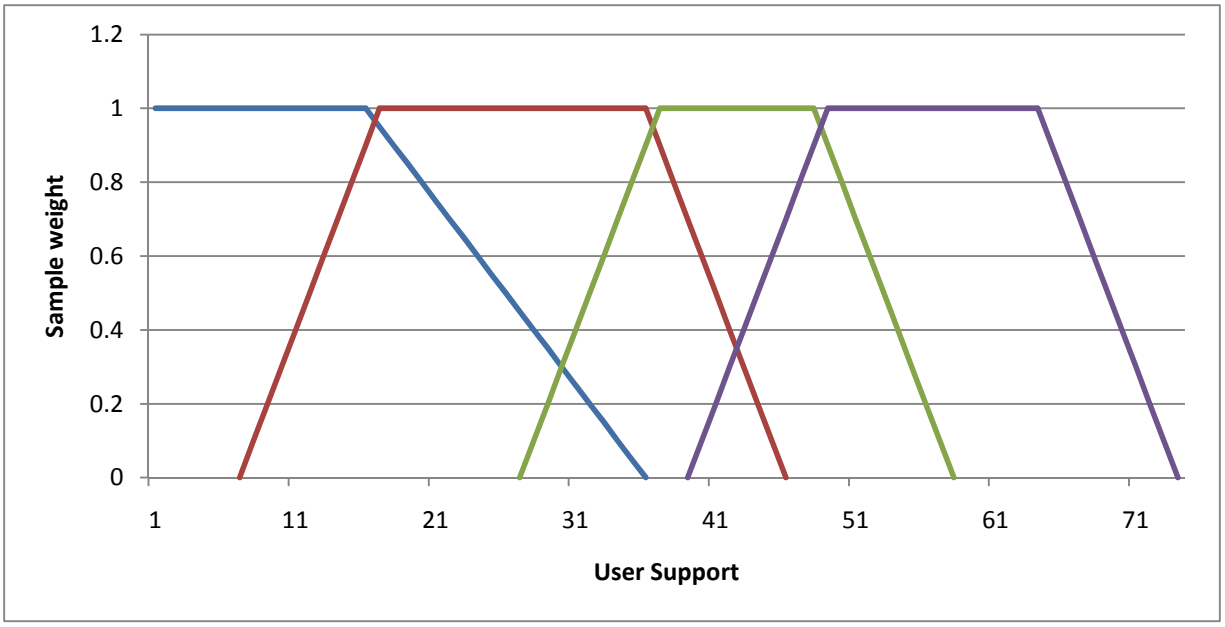
4.3 Multi-stage blending

Prior to implementing the complete and efficient Neural Network mechanism described above (which yields better results) we had devised a number of blending techniques to try to capture various aspects of the data and improve the overall result. These classifiers were used in a stack; each one being trained on the residual error of the previous with the resulting predictor being the sum of the output of each classifier. The following subsections describe each classifier (or stage) in this complex blending mechanism. The final subsection presents the details of the variants that were included in the solution.

4.3.1 Multi-linear classifier

This classifier is actually a set of simple linear classifiers trained on a subset of the probe set. The probe set is divided into *n* buckets of equal size (number of samples) using user support. The prediction for each user-movie pair is the output of the linear classifier corresponding to that user's support.

To refine the training of each classifier, data from neighbouring buckets on both sides is used. If insufficient buckets are available on one side (for buckets near the start of the end), extra buckets are taken from the other side to have a constant number of buckets. The samples are given a weight value which decays linearly with the distance of the sample user's support to that of the trained bucket support. This is illustrated in the example chart below, which shows the sample weight per user support for each bucket. Notice how the bucket starting at 1 has a different weight slope, as all extra buckets are taken to the right. The number of buckets (N) as well as the width of the base of each bucket (i.e. % of the total samples used for the training of each bucket) was optimized through validation on the probe set.



4.3.2 Per-movie Linear classifier

This classifier is another set of simple linear classifiers. This time one classifier is trained for each movie, using the residual errors from the previous step. First, we compute the regularized mean of each movie over the probe set as follows:

$$\mu_m = \frac{\sum_{i \in P(m)} r(i) + \alpha G}{|P(m)| + \alpha}$$

Where:

- $P(m)$ is the set of ratings for movie m in the probe set;
- $r(i)$ is the rating for item *i*;

- α is a regularization parameter found through validation on the probe set;
- G is the global mean, 3.6043.

Then, the linear equation is solved through Ridge Regression, with each sample being the difference between the predicted output of the previous stage and μ_m; optimized towards the difference between the actual probe rating and μ_m. Sparseness being an issue, the proper Ridge Regression regularization is necessary and found manually through validation on the probe set.

4.3.3 Neural-Network classifier

We experimented with using a Neural Network to estimate the residual error after the previous blending steps. This Neural Network is very similar to the one described in Section 4.2 Neural network blending, with the following differences:

- Like in [4], a neural network with 12 hidden nodes, and one with 13 hidden nodes are chosen and then linearly combined;
- The output layer has a linear activation function;
- Weight decay is 0.00012 for both layers;
- Initial learning rate is 0.00012 for both layers;
- Learning rates are decreased linearly to zero over 1000 iterations.

4.3.4 Tree (or GAM) classifier

Prior to implementing the more complex Neural-Network step, we had implemented a Generalized Additive Model (GAM) classifier to capture non-linearities in the predictor sets. Even after having implemented the Neural-Network classifier, this classifier still proved useful when trained on the residual error of that stage.

We define a classifier step function as follows:

$$\hat{r}(u,m) = r_{res}(u,m) + \alpha \begin{cases} \mu_a \; if \; p_i\,(u,m) < x \\ \mu_b \; if \; p_i\,(u,m) \geq x \end{cases} \tag{105}$$

Where:

- u is the user and m the movie being predicted;
- $r_{res}(u,m)$ is the residual error of the previous classifier for that user-movie pair;
- $p_i\,(u,m)$ is the value of predictor i for that user-movie pair;
- μ_a is the average of all samples in the probe set for which $p_i < x$;
- μ_b is the average of all samples in the probe set for which $p_i \geq x$;
- α is a correction ratio optimized through validation on the probe set.

We find the optimal x and i to minimize the residual error on the probe set. We can generalize this function to a tree function with two levels:

$$\hat{r}(u,m) = r_{res}(u,m) + \alpha \begin{cases} \mu_a \, if \; p_i(u,m) < x \; and \; p_j(u,m) < y \\ \mu_b \, if \; p_i(u,m) < x \; and \; p_j(u,m) \geq y \\ \mu_c \, if \; p_i(u,m) \geq x \; and \; p_k(u,m) < z \\ \mu_d if \; p_i(u,m) \geq x \; and \; p_k(u,m) \geq z \end{cases} \quad (106)$$

Where:

- $p_i(u,m)$, $p_j(u,m)$ and $p_k(u,m)$ are the values of predictors $i, j \; and \; k$ for that user-movie pair;
- μ_a is the average of all samples in the probe set for which $p_i < x \; and \; p_j < y$;
- μ_b is the average of all samples in the probe set for which $p_i < x \; and \; p_j \geq y$;
- μ_c is the average of all samples in the probe set for which $p_i \geq x \; and \; p_k < z$;
- μ_d is the average of all samples in the probe set for which $p_i \geq x \; and \; p_k \geq z$;

Because of processing issues, we first find x and i as in the step function above. Then, we find y, z, j and k to minimize the residual error on the probe set.

Finally, we define the GAM classifier as a stacking of *n* of these tree functions trained on the residual error of the previous ones. The value of *n* is optimized through validation on the probe set. Note that to avoid overfitting, the number of samples which fall in a particular quadrant is limited to a minimum value and the correction in each tree function limited by multiplying the output by a certain ratio. Each of these meta-parameters was optimized manually through validation on the probe set.

4.3.5 Clipping

The final step in our multi-stage classifier is the clipping stage. The minimum threshold t_{min} is chosen as the highest value for which the predicted output is lower than the average of the probe set scores that have a predicted output below t_{min}. All values below that threshold are clamped to t_{min}. The maximum clamping threshold t_{max} is found similarly.

4.3.6 Variants

Two predictors produced with this multi-stage blending approach were included in the solution. The following tables indicate the parameters used in these blends. Note that the Neural-Network parameters are described in the corresponding section above.

Variant	Multi-Linear		Movie-Linear			Neural-Network Used?
	N	Base	offset	slope	α	
pragmatictheory-20090316	50	50%	1200	350	1	Y
pragmatictheory-20090527	50	50%	1200	350	1	N

Variant	Tree			Clipping		Number of sets
	nb tree	min samples	corr. ratio	t_{min}	t_{max}	
pragmatictheory-20090316	221	5000	0.10	1.0398	4.9616	233
pragmatictheory-20090527	142	5000	0.25	1.0599	4.9659	208

The following tables show the list of predictors used in each case:

pragmatictheory-20090316			
bk1-a1000x	brismf760n-knn1-1	integ80-80-0TZM	ssvd-31-00-asym1-20-movie6
bk1-a1000x-knn1-3	brismf760n-knn1-3	integ80-80-0TZM-movie6	ssvd-31-00-asym3-100
bk1-a200-knn1-1	brismf760n-knn3-1	integ80-80-0TZ-movie5	ssvd-31-00-asym3-20
bk1-a200-knn1-3	brismf760n-movie5	mf01-20-movie3	ssvd-31-00-asym3-200
bk1-a200-knn3-1	brismf760n-movie6	mf01-40-3-80-movie4	ssvd-31-00-asym3-200-movie2
bk1-a200x-knn1-1	brismf760n-user2	mf27-20	ssvd-31-00-asym3v-300-movie2
bk1-a50	brismf760-user2	mf27-20env50-m	ssvd-31-00-asym3w-200-movie2
bk1-a50-2	crbm100	mf27-20env50-movie4	ssvd-31-00-asym4-200
bk1-a50-2-knn1-1	crbm100-movie2	mf27-20env50-movie5	ssvd-31-00-asym4v-200-movie5
bk1-a50-2x	crbm100-ssvd-07-00	mf27-20-movie4	ssvd-31-00-movie
bk1-a50-movie5	crbm100-ssvd-07-00-movie2	mf27-20-movie5	ssvd-31-00-movie5
bk1-b1000	crbm100-ssvd-07-20	mf27-20-u	ssvd-31-10-m
bk1-b200-1	crbm100x-ssvd-03-00	mf27-40-3-80-movie5	ssvd-31-20
bk1-b200-1-knn1-0	crbm200	mfc27-60-10-120	ssvd-31-20-cluster-movie2
bk1-b200-1-knn1-1	crbm200-ssvd-07-00	mfc27-60-10-120-m	ssvd-31-20-m
bk1-b200-2	drbm100-500	mfc27-60-10-120-user2	ssvd-31-20-movie2
bk1-b200-5	drbm100-500-mfw31-m	mfw31-00	ssvd-31-20-u
bk1-b200-5x	drbm100-500-user2	mfw31-00milestone3-100-movie6	ssvd-31-60
bk1-b200-5x-knn1-3	drbm160-640	mfw31-00milestone4-100-movie5	ssvd-39-05
bk1-b200-6	drbm160-640-knn3-1	mfw31-00milestone5-100-movie5	ssvd-39-05-movie4
bk1-b200-6x	drbm160-640-knn4-1	mfw31-00milestone6-100	ssvd-39-10
bk1-c200-knn2-1	frbm100-mf27-m	mfw31-00milestone7-100-movie6	ssvd-39-10-movie4
bk1-c200x	frbm200	mfw31-00milestone9-100-movie6	ssvd-63-00
bk2-b200hz-knn1-1	frbm200-mf27-flip20	mfw31-00-movie6	ssvd-63-00-movie4
bk3-100g1	frbm200-mf27-knn2-2	mfw31-05-asym3v250	ssvd-63-30
bk3-100g2	frbm200x	mfw31-05-asym3v250-movie6	ssvd-63-30-movie4
bk3-100g4	frbm2-100g3	mfw31-05-asym3v250-user2	svd02x
bk3-100ga1	frbm2-100g4	mfw31-05-m	svd05x
bk3-100ga4	frbm2-100gx-knn2-1	mfw31-10-milestone0-150	trbm100
bk3-100ga-knn3-1	frbm300	mfw31-10-milestone0-150-movie5	trbm100-movie2
bk3-100gax	frbm300-knn3-1	mfw31-10-milestone5-150	trbm100-ssvd-31-00
bk3-100gx	frbm300-movie8	mfw31-10-milestone5-150-movie5	trbm100-ssvd-31-00-movie2
bk3-200g1	frbm300x	mfw31-40env50	trbm100-ssvd-31-00-user2
bk3-200g3	globalEffect14-movie5	mfw31-40env50-m	trbm150-mf27-m
bk3-200g4	gte14b	mfw31-40env50-m2	trbm150-mf27-movie4
bk3-200g-knn2-1	integ0-0-0TZ	mfw31-60-10-120-m	trbm150-movie5
bk3-200gx	integ0-0-0TZ-flip20-knn2-1	mfw31-60-10-120-movie6	trbm150-ssvd-07-00-movie4
bk3-a50-knn1-3	integ0-0-0TZ-grbm200	mfw31-60-x	trbm150-ssvd-39-00
bk3-b200-knn1-3	integ0-0-0TZmilestone6-200	mfw31-80-x	trbm50-asym3v250
bk3-c50	integ0-0-0TZmilestone6-200-knn2-1	mfw31-80-x-m	trbm50-asym3v250-mfw27-movie5
bk3-c50-knn2-1	integ0-0-0TZmilestone6-200-movie6	mfw31-80-x-user2	trbm50-asym3v250-movie5
bk3-c50-knn3-1	integ0-0-0TZ-movie6	mmean	trbm50-asym3v250-movie6
bk3-c50x	integ0-100-100TZ	mp5	trbm50-asym3v250-user2
bk3-d200	integ0-100-100TZ-movie6	mskew	trbm50-mf27
bk3-d200z	integ0-200-200NT	nmf40-60-10	trbm50-milestone0-150-movie6
bk3-d200z-knn4-1	integ0-200-200NT-user2	nmf40-60-10-movie3	trbm50-milestone9-150-movie5
blend2b	integ0-200-200TZ	nmf80-120-20-m	trbm50-ssvd-39-00
blend2-ga1	integ0-200-200TZ-knn1-1	nmf80-120-20-mf27-movie4	ucount
blend2-ga4	integ0-200-200TZ-knn1-3	nnmf40	up2
blend2-ga-knn2-1	integ0-200-200TZ-knn2-1	nnmf40-ssvd-07-00	up3
blend2-gax	integ0-200-200TZ-knn3-1	nnmf40-ssvd-07-00-movie3	up4
blend3-knn2-1	integ0-200-200TZ-movie5	nnmf80-ssvd-39-00-movie4	up5
blend5-knn3-1	integ0-200-200TZ-user2	pmf40-60-10-m	urbm20-1000

blend5-knn4-1	integ20-100-100NT	pmf80-120-20-mf27-movie4	urbm20-1000-knn1-3
brismf250	integ40-200-0ST	rbm100-ssvd-07-00-movie2	usermovie
brismf250-movie5	integ40-200-0T	ssvd-04-00	uskew
brismf40-movie5	integ60-0-0TS	ssvd-04-10-m	
brismf760-movie5	integ60-0-0TS-movie5	ssvd-04-40-m	
brismf760n	integ60-0-0TS-user2	ssvd-07-30-2-movie2	

pragmatictheory-20090527			
bk1-a200-knn3-1	brismf760n-knn1-3B019	integ60-0-0TS-user2	nnmf80-ssvd-39-00-movie4
bk1-a200x-knn1-1	brismf760n-knn3-1	integ80-80-0TZ-movie5	pmf40-60-10-m
bk1-a50	brismf760n-knn4-1B081	integ80-80-0TZM	pmf80-120-20-mf27-movie4
bk1-a50-2-knn1-1	brismf760n-user2	integ80-80-0TZM-movie6	rbm100-ssvd-07-00-movie2
bk1-a50-2x	crbm100	mf01-20-movie3	ssvd-04-00
bk1-a50-movie5	crbm100-movie2	mf01-40-3-80-movie4	ssvd-04-10-m
bk1-b1000	crbm100-ssvd-07-00	mf27-20	ssvd-04-40-m
bk1-b200-1	crbm100-ssvd-07-00-movie2	mf27-20-knn1-1B072	ssvd-07-30-2-movie2
bk1-b200-1-knn1-0	crbm100-ssvd-07-20	mf27-20-movie4	ssvd-31-00-asym1-20-movie6
bk1-b200-1-knn1-1	crbm100x-ssvd-03-00	mf27-20-movie5	ssvd-31-00-asym3-100
bk1-b200-2	crbm200-ssvd-07-00	mf27-20-u	ssvd-31-00-asym3-20
bk1-b200-5	drbm100-500	mf27-20env50-m	ssvd-31-00-asym3-200
bk1-b200-6x	drbm100-500-user2	mf27-20env50-movie4	ssvd-31-00-asym3-200-movie2
bk1-c200-knn2-1	drbm160-640	mf27-20env50-movie5	ssvd-31-00-asym3v-300-movie2
bk1-c200x	drbm160-640-bk4-knn3-1	mf27-40-3-80-movie5	ssvd-31-00-asym3w-200-movie2
bk2-b200hz-knn1-1	drbm160-640-bk4-knn3-1B011	mfc27-60-10-120	ssvd-31-00-asym4-200
bk3-100g1	drbm160-640-knn3-1	mfc27-60-10-120-m	ssvd-31-00-asym4v-200-movie5
bk3-100g2	drbm160-640-knn4-1	mfc27-60-10-120-user2	ssvd-31-00-movie
bk3-100g4	frbm100-mf27-m	mfw31-00	ssvd-31-00-movie5
bk3-100ga-knn3-1	frbm2-100g3	mfw31-00-movie6	ssvd-31-10-m
bk3-100ga4	frbm2-100g4	mfw31-00milestone3-100-movie6	ssvd-31-20
bk3-100gax	frbm200	mfw31-00milestone4-100-movie5	ssvd-31-20-cluster-movie2
bk3-100gx	frbm200-mf27-flip20	mfw31-00milestone5-100-movie5	ssvd-31-20-m
bk3-200g1	frbm200-mf27-knn2-2	mfw31-00milestone6-100	ssvd-31-20-movie2
bk3-200g3	frbm200x	mfw31-00milestone7-100-movie6	ssvd-31-20-u
bk3-200g4	frbm300	mfw31-00milestone9-100-movie6	ssvd-31-60
bk3-200gx	frbm300-bk4-knn3-1	mfw31-05-asym3v250	ssvd-39-05
bk3-a50-knn1-3	frbm300-knn1-1B068	mfw31-05-asym3v250-movie6	ssvd-39-05-movie4
bk3-b200-knn1-3	frbm300-knn3-1	mfw31-05-asym3v250-user2	ssvd-39-10
bk3-c50	frbm300x	mfw31-05-m	ssvd-39-10-movie4
bk3-c50-knn2-1	integ0-0-0TZ	mfw31-10-milestone0-150	svd05x
bk3-d200	integ0-0-0TZ-flip20-knn2-1	mfw31-10-milestone0-150-knn1-3B022	trbm100
bk3-d200z	integ0-0-0TZ-grbm200-knn1-3B025	mfw31-10-milestone0-150-movie5	trbm100-ssvd-31-00-movie2
bk3-d200z-knn4-1	integ0-0-0TZ-knn1-1B046	mfw31-10-milestone5-150	trbm150-mf27-m
bk4-biasZ	integ0-0-0TZ-knn1-3B015	mfw31-10-milestone5-150-movie5	trbm150-mf27-movie4
blend2-ga-knn2-1	integ0-0-0TZ-knn3-1B002	mfw31-40env50	trbm150-ssvd-07-00-movie4
blend2-ga1	integ0-0-0TZ-movie6	mfw31-60-10-120-m	trbm150-ssvd-39-00
blend2-ga4	integ0-0-0TZmilestone6-200	mfw31-60-10-120-movie6	trbm50-asym3v250-movie5
blend2-gax	integ0-0-0TZmilestone6-200-movie6	mfw31-60-x	trbm50-asym3v250-movie6
blend2-gax-knn1-1B052	integ0-100-100TZ	mfw31-80-x	trbm50-asym3v250-user2
blend2b	integ0-100-100TZ-movie6	mfw31-80-x-m	trbm50-mf27
blend5-knn1-1B047	integ0-200-200NT	mfw31-80-x-user2	trbm50-milestone0-150-movie6
blend5-knn3-1	integ0-200-200NT-user2	mmean	trbm50-milestone9-150-movie5
blend5-knn4-1	integ0-200-200TZ	mp5	trbm50-ssvd-39-00
brismf250	integ0-200-200TZ-knn1-1	mskew	ucount

brismf250-movie5	integ0-200-200TZ-knn3-1	nmf40-60-10	up2
brismf40-movie5	integ0-200-200TZ-user2	nmf40-60-10-movie3	up3
brismf760-movie5	integ20-100-100NT	nmf80-120-20-m	up4
brismf760n	integ40-200-0ST	nmf80-120-20-mf27-movie4	up5
brismf760n-knn1-1	integ40-200-0T	nnmf40	urbm20-1000-knn1-3
brismf760n-knn1-1B050	integ60-0-0TS	nnmf40-ssvd-07-00	usermovie
brismf760n-knn1-3	integ60-0-0TS-movie5	nnmf40-ssvd-07-00-movie3	uskew

4.4 Variable multiplications

In the late stages of the competition, we used a final blending step consisting of a linear regression blending done directly on quiz sets, as described in [7]. Since this type of blending is linear in nature, it can only capture certain relationships between predictors. In order to try to capture non-linear aspects, we introduced new predictors which consist of simple arithmetic multiplications of pairs of base predictors. This technique is suggested in [6].

In order to find the best pairs of predictors to multiply, we constructed a baseline using a linear regression of the 233 sets in the pragmatictheory-20090316 blend. Then, we added, in turn, each possible pair of predictors multiplied together and compute the linear blend again. Finally, the pair which improves the blend result the most is selected. This new predictor is added to the baseline and the algorithm is run again. This method is known as forward selection. The following table presents the predictors that were introduced to the solution using this technique.

Name	Base predictor 1	Base predictor 2
bk3-200g3-X-bk3-100ga4	bk3-200g3	bk3-100ga4
nnmf80-ssvd-39-00-movie4-X-blend2-ga4	nnmf80-ssvd-39-00-movie4	blend2-ga4
ucount-X-drbm100-500	ucount	drbm100-500
ucount-X-nmf80-120-20-m	ucount	nmf80-120-20-m
frbm300-bk4-knn3-1-X-bk3-b200-knn1-3	frbm300-bk4-knn3-1	bk3-b200-knn1-3

5 Towards a better recommendation system

The solution presented in this document was exclusively aimed at building a system that would predict subscriber ratings with the highest possible accuracy. Although highly accurate predictions are an important part of a recommender system, these alone cannot ensure good recommendations. Moreover, the solution is based on a huge amount of models and predictors which would not be practical as part of a commercial recommender system. However, this result is a direct consequence of the nature and goal of the competition: obtain the highest possible accuracy at any cost, disregarding completely the complexity of the solution and the execution performance.

On the other hand, we believe that a real life recommender system can benefit from some of the techniques and algorithms proposed in this document. In our opinion, the following are our most worthwhile innovations:

- The use of the frequency measure leads to higher accuracy. Frequency was most useful for movie related coefficients. This variable is especially useful because it can be used even with new customers that have a short history of ratings.
- The classification view of linear models using logistic transformation allows for increased blend accuracy, using the same basic algorithm.
- The non-linear envelope allows greater model accuracy with virtually no increase in model complexity.
- We presented an extension to Koren's integrated model that achieves very high accuracy (bk4-f200z4-nlpp1-knn3-1 has a quiz RMSE of 0.8713). One innovative contribution is to use a low rank matrix factorization to capture time-dependent biases.
- We also presented an extension to Bell and Koren's neighbourhood approach that has been successful at improving virtually any model (kNN3). One innovative contribution was to estimate neighbour weights by combining the residual error and a simple baseline.
- We demonstrated the use of automatic parameter tuning across multiple model types using the Nelder-Mead Simplex Method, which greatly reduces the manual effort required to adjust model meta-parameters.

We believe the methods presented here will allow the creation of higher accuracy recommendation systems or, at the very least, simpler ones with equivalent accuracy. For example, the 2007 Progress Prize was won with an entry consisting of 107 predictors with a quiz set accuracy of 0.8712. Currently, we can achieve accuracy within 0.0001 of this milestone using a single predictor (bk4-f200z4-nlpp1-knn3-1). This illustrates the progress made in collaborative filtering algorithms over the past two years.

6 Acknowledgments

We would like to take this opportunity to acknowledge all the people who assisted us throughout the competition.

First, special thanks are given to Yehuda Koren for his numerous publications that have been an inspiration to us and for his insightful comments.

We also thank Andreas Töscher and Michael Jahrer for their help with Neural Network blending and for their suggestion to optimize model parameters directly on the blended score.

We would also like to express our gratitude to all the teams and individuals who offered their assistance throughout the competition, especially during the hectic last day. In addition, acknowledgments go out to everyone who shared insight through publications, blogs or the Netflix Prize forum during the course of the competition.

Finally, we would like to extend our sincere thanks to Netflix who had the vision and courage to harness the power of the community.

7 List of predictors

The following table lists all predictors that were included in the final quiz blend, sorted alphabetically.

Name	Probe RMSE	Quiz RMSE
bk1-a1000-nlpp1B107	0.8891	0.8815
bk1-a1000x	0.8876	0.8794
bk1-a1000x-knn1-3	0.8855	0.8766
bk1-a1000x-knn1-5B131	0.8888	0.8803
bk1-a200-knn1-1	0.8873	0.8787
bk1-a200-knn1-3	0.8865	0.8776
bk1-a200-knn3-1	0.8847	0.8762
bk1-a200x-knn1-1	0.8861	0.8773
bk1-a50	0.8917	0.8844
bk1-a50-2	0.8917	0.8844
bk1-a50-2-knn1-1	0.8889	0.8803
bk1-a50-2x	0.8898	0.8819
bk1-a50-movie5	0.8887	0.8797
bk1-b1000	0.8863	0.8781
bk1-b200-1	0.8877	0.8792
bk1-b200-1-knn1-0	0.8872	0.8777
bk1-b200-1-knn1-1	0.8871	0.8776
bk1-b200-2	0.8880	0.8795
bk1-b200-5	0.8872	0.8789
bk1-b200-5-knn1-1B060	0.8868	0.8777
bk1-b200-5x	0.8865	0.8778
bk1-b200-5x-knn1-3	0.8858	0.8766
bk1-b200-6	0.8865	0.8784
bk1-b200-6x	0.8854	0.8768
bk1-c200-knn2-1	0.8840	0.8755
bk1-c200x	0.8855	0.8770
bk2-b200h	0.8868	0.8785
bk2-b200hz-knn1-1	0.8851	0.8761
bk3-100g1	1.0426	1.0411
bk3-100g2	0.9725	0.9698
bk3-100g4	0.9717	0.9684
bk3-100ga1	1.0411	1.0395
bk3-100ga4	0.9697	0.9663
bk3-100ga-knn3-1	0.8821	0.8747
bk3-100gax	0.8879	0.8818
bk3-100gx	0.8895	0.8836
bk3-200g1	1.0385	1.0366
bk3-200g3	0.9140	0.9092
bk3-200g3-X-bk3-100ga4	0.9616	0.9577
bk3-200g4	0.9669	0.9631
bk3-200g-knn2-1	0.8812	0.8738
bk3-200gx	0.8851	0.8785
bk3-a0z	0.9610	0.9555
bk3-a50-knn1-3	0.8901	0.8817
bk3-b200-knn1-3	0.8901	0.8814

bk3-c50	0.8895	0.8825
bk3-c50-knn2-1	0.8841	0.8763
bk3-c50-knn3-1	0.8837	0.8758
bk3-c50x	0.8895	0.8826
bk3-d200	0.8850	0.8772
bk3-d200-knn1-1B049	0.8836	0.8749
bk3-d200-knn1-3B018	0.8850	0.8769
bk3-d200-knn3-1B005	0.8822	0.8738
bk3-d200z	0.8849	0.8775
bk3-d200z-knn1-4B146	0.8998	0.8941
bk3-d200z-knn4-1	0.8828	0.8749
bk4-a50	0.8893	0.8823
bk4-b200-knn1-1B100	0.8856	0.8779
bk4-b200-knn3-1B101	0.8817	0.8736
bk4-b200-knn4-1B102	0.8854	0.8778
bk4-bias	0.9525	0.9469
bk4-biasZ	0.9520	0.9464
bk4-c200g	0.8938	0.8875
bk4-c50	0.8875	0.8805
bk4-c500-knn5-1B125	0.8842	0.8765
bk4-d50	0.8848	0.8768
bk4-d500-knn1-5B130	0.8861	0.8775
bk4-d50B128	0.8850	0.8770
bk4-e50a	0.8873	0.8803
bk4-f200z4	0.8837	0.8760
bk4-f200z4-nlpp1-knn3-1	0.8798	0.8713
bk5-b200B089	0.9070	0.9013
bk5-b200B089-knn1-3B090	0.9047	0.8973
bk5-b200-knn1-1B095	0.8953	0.8882
bk5-b200-knn1-5B133	0.8979	0.8914
bk5-b200-knn4-1B097	0.8945	0.8881
blend2b	0.8897	0.8840
blend2-ga1	1.0353	1.0333
blend2-ga4	0.9663	0.9624
blend2-ga-knn2-1	0.8810	0.8736
blend2-gax	0.8844	0.8778
blend2-gax-knn1-1B052	0.8895	0.8823
blend2-gax-knn1-3B021	0.8842	0.8771
blend2-gb	0.8823	0.8761
blend3-knn2-1	0.8766	0.8682
blend5-knn1-1B047	0.8777	0.8692
blend5-knn3-1	0.8762	0.8678
blend5-knn4-1	0.8773	0.8691
brismf250	0.9012	0.8945
brismf250-movie5	0.8998	0.8915
brismf40-movie5	0.9012	0.8930
brismf760-movie5	0.8995	0.8913
brismf760n	0.8992	0.8926
brismf760n-knn1-1	0.8967	0.8886
brismf760n-knn1-1B050	0.8973	0.8895

brismf760n-knn1-3	0.8960	0.8877
brismf760n-knn1-3B019	0.8970	0.8892
brismf760n-knn3-1	0.8953	0.8877
brismf760n-knn4-1B081	0.8973	0.8900
brismf760n-movie5	0.8972	0.8888
brismf760n-movie6	0.9000	0.8932
brismf760n-user2	0.8987	0.8917
brismf760-user2	0.8998	0.8926
crbm100	0.9096	0.9051
crbm100-movie2	0.8963	0.8895
crbm100-ssvd-07-00	0.9038	0.8986
crbm100-ssvd-07-00-movie2	0.8937	0.8864
crbm100-ssvd-07-20	0.9033	0.8978
crbm100x-ssvd-03-00	0.9048	0.8996
crbm200	0.9067	0.9020
crbm200-ssvd-07-00	0.9013	0.8961
drbm100-500	0.9073	0.9045
drbm100-500-mfw31-m	0.9018	0.8974
drbm100-500-user2	0.9039	0.8997
drbm160-640	0.9069	0.9027
drbm160-640-bk4-knn3-1	0.8885	0.8811
drbm160-640-bk4-knn3-1B011	0.8899	0.8822
drbm160-640-knn3-1	0.8936	0.8869
drbm160-640-knn4-1	0.9024	0.8966
frbm100-mf27-m	0.8999	0.8940
frbm200	0.9056	0.9002
frbm200-mf27-flip20	0.8965	0.8904
frbm200-mf27-knn2-2	0.8886	0.8811
frbm200x	0.9048	0.8995
frbm2-100g3	0.9809	0.9785
frbm2-100g4	1.0041	1.0005
frbm2-100gx-knn2-1	0.8991	0.8911
frbm300	0.9050	0.9001
frbm300-bk4-knn3-1	0.8871	0.8788
frbm300-knn1-1B068	0.8941	0.8856
frbm300-knn3-1	0.8913	0.8841
frbm300-movie8	0.8942	0.8875
frbm300x	0.9039	0.8992
globalEffect14-movie5	0.9327	0.9227
gte14b	0.9537	0.9471
integ0-0-0TZ	0.9592	0.9551
integ0-0-0TZ-flip20-knn2-1	0.9034	0.8957
integ0-0-0TZ-grbm200	0.9192	0.9137
integ0-0-0TZ-grbm200-knn1-3B025	0.9212	0.9153
integ0-0-0TZ-knn1-1B046	0.9459	0.9404
integ0-0-0TZ-knn1-3B015	0.9311	0.9235
integ0-0-0TZ-knn3-1B002	0.9131	0.9052
integ0-0-0TZmilestone6-200	0.9211	0.9162
integ0-0-0TZmilestone6-200-knn2-1	0.8920	0.8839
integ0-0-0TZmilestone6-200-movie6	0.8919	0.8840

integ0-0-0TZ-movie6	0.9079	0.9003
integ0-100-100TZ	0.8981	0.8921
integ0-100-100TZ-movie6	0.8911	0.8837
integ0-200-200NT	0.9057	0.8996
integ0-200-200NT-user2	0.9037	0.8972
integ0-200-200TZ	0.8969	0.8906
integ0-200-200TZ-knn1-1	0.8931	0.8845
integ0-200-200TZ-knn1-3	0.8924	0.8836
integ0-200-200TZ-knn1-3B039	0.8949	0.8869
integ0-200-200TZ-knn2-1	0.8891	0.8814
integ0-200-200TZ-knn3-1	0.8889	0.8811
integ0-200-200TZ-movie5	0.8920	0.8829
integ0-200-200TZ-user2	0.8963	0.8895
integ20-100-100NT	0.9003	0.8946
integ40-200-0ST	0.8995	0.8939
integ40-200-0ST-knn1-1B071	0.8982	0.8914
integ40-200-0T	0.8999	0.8936
integ60-0-0TS	0.9048	0.8988
integ60-0-0TS-movie5	0.8998	0.8912
integ60-0-0TS-user2	0.9014	0.8948
integ80-80-0TZM	0.8989	0.8927
integ80-80-0TZM-movie6	0.8909	0.8831
integ80-80-0TZ-movie5	0.8907	0.8817
mf01-20-movie3	0.9038	0.8974
mf01-40-3-80-movie4	0.9044	0.8984
mf27-20	0.9078	0.9020
mf27-20env50-m	0.8984	0.8915
mf27-20env50-movie4	0.8988	0.8919
mf27-20env50-movie5	0.8985	0.8904
mf27-20-knn1-1B072	0.9064	0.8993
mf27-20-movie4	0.8995	0.8922
mf27-20-movie5	0.9005	0.8917
mf27-20-u	0.9071	0.9014
mf27-40-3-80-movie5	0.8990	0.8900
mfc27-60-10-120	0.9072	0.9017
mfc27-60-10-120-m	0.8996	0.8918
mfc27-60-10-120-user2	0.9051	0.8991
mfw31-00	0.9708	0.9665
mfw31-00milestone3-100-movie6	0.8961	0.8876
mfw31-00milestone4-100-movie5	0.9036	0.8935
mfw31-00milestone5-100-movie5	0.9031	0.8932
mfw31-00milestone6-100	0.9289	0.9235
mfw31-00milestone7-100-movie6	0.8954	0.8869
mfw31-00milestone9-100-movie6	0.8954	0.8869
mfw31-00-movie6	0.9124	0.9040
mfw31-05-asym3v250	0.9083	0.9024
mfw31-05-asym3v250-movie6	0.8919	0.8838
mfw31-05-asym3v250-user2	0.9031	0.8963
mfw31-05-m	0.9176	0.9091
mfw31-10-milestone0-150	0.9055	0.8994

mfw31-10-milestone0-150-knn1-3B022	0.9049	0.8977
mfw31-10-milestone0-150-movie5	0.8972	0.8874
mfw31-10-milestone5-150	0.9049	0.8989
mfw31-10-milestone5-150-movie5	0.8966	0.8869
mfw31-40env50	0.9017	0.8966
mfw31-40env50-m	0.8961	0.8887
mfw31-40env50-m2	0.8948	0.8874
mfw31-60-10-120-m	0.8965	0.8883
mfw31-60-10-120-movie6	0.8978	0.8906
mfw31-60-x	0.9027	0.8968
mfw31-80-x	0.9031	0.8973
mfw31-80-x-m	0.8965	0.8890
mfw31-80-x-user2	0.9012	0.8948
mmean	1.0516	1.0520
mp5	1.0648	1.0657
mskew	1.0687	1.0691
nmf40-60-10	0.9138	0.9089
nmf40-60-10-movie3	0.9068	0.9005
nmf80-120-20-m	0.9069	0.8997
nmf80-120-20-mf27-movie4	0.9012	0.8942
nnmf40	0.9156	0.9088
nnmf40-ssvd-07-00	0.9122	0.9050
nnmf40-ssvd-07-00-movie3	0.9056	0.8971
nnmf80-ssvd-39-00-movie4	0.9019	0.8933
nnmf80-ssvd-39-00-movie4-X-blend2-ga4	0.9639	0.9597
pmf40-60-10-m	0.9102	0.9033
pmf80-120-20-mf27-movie4	0.9024	0.8957
rbm100-ssvd-07-00-movie2	0.8949	0.8878
ssvd-04-00	0.9891	0.9872
ssvd-04-10	0.9278	0.9236
ssvd-04-10-m	0.9160	0.9099
ssvd-04-40-m	0.9136	0.9073
ssvd-07-30-2-movie2	0.9263	0.9197
ssvd-31-00-asym1-20-movie6	0.9063	0.8987
ssvd-31-00-asym3-100	0.9279	0.9227
ssvd-31-00-asym3-20	0.9311	0.9262
ssvd-31-00-asym3-200	0.9275	0.9223
ssvd-31-00-asym3-200-movie2	0.8990	0.8909
ssvd-31-00-asym3v-300-movie2	0.8984	0.8902
ssvd-31-00-asym3w-200-movie2	0.8986	0.8905
ssvd-31-00-asym4-200	0.9254	0.9201
ssvd-31-00-asym4v-200-movie5	0.9055	0.8959
ssvd-31-00-movie	0.9166	0.9091
ssvd-31-00-movie5	0.9323	0.9234
ssvd-31-00-user2	0.9290	0.9230
ssvd-31-10-m	0.9124	0.9049
ssvd-31-20	0.9185	0.9124
ssvd-31-20-cluster-movie2	0.9047	0.8971
ssvd-31-20-m	0.9100	0.9020
ssvd-31-20-movie2	0.9059	0.8982

ssvd-31-20-u	0.9172	0.9112
ssvd-31-60	0.9165	0.9102
ssvd-39-05	0.9282	0.9239
ssvd-39-05-movie4	0.9061	0.8991
ssvd-39-10	0.9215	0.9169
ssvd-39-10-movie4	0.9053	0.8985
ssvd-63-00	0.9593	0.9545
ssvd-63-00-movie4	0.9138	0.9058
ssvd-63-30	0.9148	0.9086
ssvd-63-30-movie4	0.9046	0.8971
svd02x	0.9496	0.9461
svd05x	0.9376	0.9336
trbm100	0.9087	0.9036
trbm100-movie2	0.8956	0.8884
trbm100-ssvd-31-00	0.9043	0.8989
trbm100-ssvd-31-00-movie2	0.8948	0.8874
trbm100-ssvd-31-00-user2	0.9017	0.8955
trbm150-mf27-m	0.8986	0.8926
trbm150-mf27-movie4	0.8921	0.8849
trbm150-movie5	0.8944	0.8860
trbm150-ssvd-07-00-movie4	0.8939	0.8869
trbm150-ssvd-39-00	0.9003	0.8946
trbm50-asym3v250	0.9112	0.9072
trbm50-asym3v250-mfw27-movie5	0.8931	0.8845
trbm50-asym3v250-movie5	0.8957	0.8872
trbm50-asym3v250-movie6	0.8929	0.8863
trbm50-asym3v250-user2	0.9073	0.9021
trbm50-mf27	0.9077	0.9026
trbm50-milestone0-150-movie6	0.8936	0.8869
trbm50-milestone9-150-movie5	0.8955	0.8871
trbm50-ssvd-39-00	0.9079	0.9030
ucount	1.1264	1.1277
ucount-X-drbm100-500	1.1266	1.1279
ucount-X-nmf80-120-20-m	1.1264	1.1276
up2	1.0947	1.0936
up3	1.1004	1.1001
up4	1.1271	1.1285
up5	1.0813	1.0799
urbm20-1000	0.9366	0.9311
urbm20-1000-knn1-3	0.9253	0.9167
usermovie	0.9823	0.9802
uskew	1.1059	1.1050

The following table lists additional predictors that were not included as part of the final quiz blend, but were included in the solution through probe-based blending. The quiz RMSE for these predictors is not available.

Name	Probe RMSE
bk1-a1000x-knn1-4B145	0.8875
bk1-a1000x-knn5-8B111	0.8865

bk1-a50-2-knn1-1B059	0.8897
bk1-a50-2-knn1-3B028	0.8909
bk1-a50-knn1-1B058	0.8902
bk1-a50-knn1-3B027	0.8926
bk1-b200-5-knn1-3B029	0.8863
bk1-b200-6x-knn1-1B054	0.8849
bk1-b200-6x-knn1-1B061	0.8850
bk1-b200-6x-knn1-3B030	0.8848
bk1-b200-6x-knn3-1B010	0.8839
bk1-b200-6x-knn4-1B085	0.8850
bk1-c200-knn1-1B062	0.8864
bk1-c200-knn1-3B031	0.8864
bk1-c200x-knn1-1B063	0.8847
bk1-c200x-knn1-3B032	0.8848
bk3-100ga-knn1-1B064	0.8892
bk3-200gx-knn1-1B065	0.8833
bk3-d200-knn4-1B080	0.8836
bk3-d200z-nlpp1-knn3-1B109	0.8807
bk4-b200-knn1-3B099	0.8852
bk4-c200gx-knn1-1B162	0.8905
bk4-c200gx-knn1-5B164	0.8910
bk4-c200gx-knn3-1B167	0.8843
bk4-c200z-knn1-1B172	0.8854
bk4-c200z-knn1-3B176	0.8859
bk4-c200z-knn1-5B182	0.8865
bk4-c200z-knn3-1B165	0.8817
bk4-c200z-knn5-1B183	0.8838
bk4-c500-knn1-3B126	0.8859
bk4-c500-knn1-4B143	0.8864
bk4-c500-knn1-5B129	0.8876
bk4-c500-knn3-1B123	0.8816
bk4-d500-knn1-4B144	0.8849
bk4-e200-knn1-1B170	0.8861
bk4-e200-knn1-3B163	0.8863
bk4-e200-knn1-5B180	0.8865
bk4-e200-knn3-1B169	0.8816
bk4-e200-knn5-1B185	0.8838
bk4-e50a-knn1-1B188	0.8858
bk4-e50a-knn1-3B166	0.8871
bk4-e50a-knn1-5B159	0.8869
bk4-e50a-knn3-1B175	0.8818
bk4-e50a-knn5-1B161	0.8845
bk4-f200gx-knn1-1B223	0.8884
bk4-f200gx-knn1-2B216	0.8889
bk4-f200gx-knn1-3B190	0.8878
bk4-f200gx-knn1-4B241	0.8889
bk4-f200gx-knn1-5B198	0.8885
bk4-f200gx-knn3-1B193	0.8836
bk4-f200gx-knn5-1B228	0.8871
bk4-f200z4-knn1-2B234	0.8832

bk4-f200z4-knn1-3B242	0.8833
bk4-f200z4-knn1-4B224	0.8845
bk4-f200z4-knn1-5B197	0.8850
bk4-f200z4-knn3-1B199	0.8803
bk4-f200z4-knn5-1B218	0.8824
bk4-f200z4-nlpp1-knn1-1B204	0.8839
bk4-f200z4-nlpp1-knn1-2B202	0.8831
bk4-f200z4-nlpp1-knn1-4B221	0.8832
bk4-f200z4-nlpp1-knn1-5B207	0.8834
bk4-f200z4-nlpp1-knn3-1B219	0.8802
bk4-f200z4-nlpp1-knn5-1B192	0.8820
bk5-b200-knn1-3B094	0.8959
bk5-b200-knn1-4B147	0.8949
bk5-b200-knn3-1B096	0.8924
bk5-b200B089-knn1-1B091	0.9065
bk5-b200B089-knn3-1B092	0.9047
bk5-b200B089-knn4-1B093	0.9056
blend2-gax-knn3-1B008	0.8796
blend2-gax-knn4-1B083	0.8834
blend2-gb-knn1-1B213	0.8816
blend2-gb-knn1-2B238	0.8790
blend2-gb-knn1-3B212	0.8788
blend2-gb-knn1-4B210	0.8801
blend2-gb-knn1-5B214	0.8818
blend2-gb-knn3-1B217	0.8772
blend2-gb-knn5-1B191	0.8793
blend2-gb1-knn1-1B196	1.0023
blend2-gb1-knn1-2B194	1.0063
blend2-gb1-knn1-3B233	0.9850
blend2-gb1-knn1-4B237	0.9974
blend2-gb1-knn1-5B232	1.0091
blend2-gb2-knn1-1B203	0.9645
blend2-gb2-knn1-2B215	0.9524
blend2-gb2-knn1-3B206	0.9513
blend2-gb3-knn1-1B205	0.9392
blend2-gb3-knn1-2B240	0.9265
blend2-gb3-knn1-3B208	0.9274
blend2-gb3-knn1-4B195	0.9370
blend2-gb3-knn1-5B245	0.9432
blend2-gb4-knn1-1B244	0.9716
blend2-gb4-knn1-2B230	0.9486
blend2-gb4-knn1-3B231	0.9414
blend2-gb4-knn1-5B229	0.9654
blend5-knn1-3B016	0.8859
blend5-knn3-1B003	0.8765
blend5-knn4-1B078	0.8773
brismf760n-knn1-4B148	0.9011
brismf760n-knn1-5B134	0.9148
brismf760n-knn3-1B006	0.8963
drbm160-640-bk4-knn1-1B055	0.8971

drbm160-640-bk4-knn1-3B024	0.8983
drbm160-640-bk4-knn1-4B150	0.9012
drbm160-640-bk4-knn1-5B136	0.9011
drbm160-640-bk4-knn4-1B086	0.8962
drbm160-640-bk4-knn5-8B115	0.8949
drbm160-640-knn1-1B066	0.9101
drbm160-640-knn1-3B035	0.9072
drbm160-640-knn1-4B149	0.9046
drbm160-640-knn1-5B135	0.9041
drbm160-640-knn5-8B114	0.9007
drbm160-640gx-knn1-1B181	0.9029
drbm160-640gx-knn1-5B174	0.8969
drbm160-640gx-knn3-1B168	0.8918
drbm160-640gx-knn5-1B186	0.8976
frbm200-mf27-flip20-knn1-1B067	0.8925
frbm200-mf27-flip20-knn1-3B036	0.8924
frbm300-bk4-knn1-1B045	0.8933
frbm300-bk4-knn1-3B014	0.8928
frbm300-bk4-knn1-4B152	0.8962
frbm300-bk4-knn3-1B001	0.8888
frbm300-bk4-knn4-1B076	0.8948
frbm300-bk4-knn5-8B117	0.8925
frbm300-knn1-3B037	0.8967
frbm300-knn1-4B151	0.8961
frbm300-knn1-5B137	0.8998
frbm300gx-knn1-1B177	0.8951
frbm300gx-knn1-3B160	0.9006
frbm300gx-knn1-5B184	0.8971
frbm300gx-knn3-1B179	0.8915
frbm300gx-knn5-1B171	0.8979
gte14b-knn1-1B048	0.9394
gte14b-knn1-3B017	0.9411
gte14b-knn3-1B004	0.9167
gte14b-knn4-1B079	0.9459
integ0-0-0TZ-grbm200-knn1-1B056	0.9173
integ0-0-0TZ-grbm200-knn3-1B012	0.9022
integ0-0-0TZ-grbm200-knn4-1B087	0.9140
integ0-0-0TZ-knn4-1B077	0.9467
integ0-0-0TZmilestone6-200-knn1-5B139	0.9142
integ0-0-0TZmilestone6-200-knn5-8B118	0.9146
integ0-100-100TZ-knn1-1B069	0.8972
integ0-100-100TZ-knn1-3B038	0.8964
integ0-200-200NT-knn1-4B154	0.9003
integ0-200-200NT-knn1-5B140	0.9020
integ0-200-200NT-knn5-8B119	0.8989
integ0-200-200TZ-knn1-1B070	0.8964
integ40-200-0ST-knn1-3B040	0.8972
integ80-80-0TZM-knn1-4B155	0.8957
integ80-80-0TZM-knn1-5B141	0.8957
integ80-80-0TZM-knn5-8B120	0.8923

mf27-20-knn1-3B041	0.9018
mfw31-05-asym3v250-knn1-1B073	0.9032
mfw31-10-grbm200-knn1-5B142	0.9083
mfw31-10-grbm200-knn5-8B121	0.9026
mfw31-10-milestone0-150-knn1-1B053	0.9019
mfw31-10-milestone0-150-knn3-1B009	0.8926
mfw31-10-milestone0-150-knn4-1B084	0.9005
mfw31-10-milestone5-150-knn1-1B074	0.8996
mfw31-10-milestone5-150-knn1-3B043	0.8995
mfw31-60-10-120-knn1-1B057	0.9030
mfw31-60-10-120-knn3-1B013	0.8956
mfw31-60-10-120-knn4-1B088	0.8995
ssvd-31-00-asym1-20-knn1-1B075	0.9213
ssvd-31-00-asym1-20-knn1-3B044	0.9157
urbm20-1000-knn1-1B051	0.9286
urbm20-1000-knn4-1B082	0.9332

Finally, the following table lists the probe blends that were included as part of the final quiz blend. The second column reports the quiz set RMSE.

Name	Quiz RMSE
nnblend-top50	0.8602
nnblend-top100	0.8597
nnblend-top212	0.8598
nnblend-bottom95	0.8617
pragmatictheory-20090316	0.8599
pragmatictheory-20090527	0.8599

8 References

[1] Bell, R., Koren, Y., Scalable Collaborative Filtering with Jointly Derived Neighborhood Interpolation Weights, IEEE International Conference on Data Mining (ICDM 2007), 2007.

[2] Bell, R., Koren, Y., Volinsky, C., The BellKor 2008 Solution to the Netflix Prize, 2008.

[3] Salakhutdinov, R., Mnih, A., Bayesian Probabilistic Matrix Factorization using Markov Chain Monte Carlo, Proceedings of the 25th international conference on Machine learning (ICML 2008), 2008.

[4] Töscher, A., Jahrer, M., The BigChaos Solution to the Netflix Prize 2008, 2008.

[5] Salakhutdinov, R., Mnih, A., Hinton, G., Restricted Boltzmann Machines for Collaborative Filtering, Proceedings of the 24th international conference on Machine learning (ICML 2007), 2007.

[6] Paterek, A., Improving regularized singular value decomposition for collaborative filtering, KDD Cup and Workshop 2007.

[7] Bell, R., Koren, Y., Volinsky, C., The BellKor solution to the Netflix Prize, 2007.

[8] Koren, Y., Factorization Meets the Neighborhood: a Multifaceted Collaborative Filtering Model, ACM Int. Conference on Knowledge Discovery and Data Mining (KDD'08), 2008.

[9] Koren, Y., Collaborative Filtering with Temporal Dynamics, Proceedings of the 15th ACM SIGKDD international conference on Knowledge discovery and data mining (KDD 2009), 2009.

[10] Koren, Y., Factor in the Neighbors: Scalable and Accurate Collaborative Filtering, Transactions on Knowledge Discovery from Data (TKDD), 2009.

[11] Takacs, G., Pilasky, I., Nemeth, B., Investigation of Various Matrix Factorization Methods for Large Recommender Systems, Proceedings of the 2008 IEEE International Conference on Data Mining Workshops (ICDMW 2008),2008

[12] R. Bell, Y. Koren and C. Volinsky, Modeling Relationships at Multiple Scales to Improve Accuracy of Large Recommender Systems, Proceedings of the 13th ACM Int. Conference on Knowledge Discovery and Data Mining (KDD 2007), 2007.

[13] "Nelder-Mead method." *Wikipedia, The Free Encyclopedia*. 29 Jul 2009, 14:19 UTC. 29 Jul 2009 <http://en.wikipedia.org/w/index.php?title=Nelder-Mead_method&oldid=304884132>.

[14] Saša Singer and John Nelder (2009), Nelder-Mead algorithm. Scholarpedia, 4(7):2928

[15] "Logistic regression." *Wikipedia, The Free Encyclopedia*. 23 Jul 2009, 11:57 UTC. 23 Jul 2009 <http://en.wikipedia.org/w/index.php?title=Logistic_regression&oldid=303721280>