

# A Brief Survey of Multi-Processor Scheduling For Hard Real-Time Systems

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## Abstract

In class, both of scheduling algorithms [1] and priority inheritance protocols [2] in the context of a single processor were examined in details. Nevertheless, the emergence and popularity of distributed computing system gave rise to the need to solve multi-processor scheduling and priority inheritance problems. As the supplementary study, this paper surveys existing scheduling algorithms in the context of multiple processors. The very first section outlines the background of multi-processor scheduling problems, as well as system models, terminology, and the metrics of scheduling algorithms. After that, partitioned scheduling and global scheduling, as the primary objects of our research, will be fully explored. Moreover, we will also give brief sketch to the hybrid approaches of partitioned scheduling and global scheduling.

*Keywords:* System, Scheduling Algorithm, Task Management

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## 1. Introduction

### 1.1. Problem Defintion

### 1.2. Preview Of Related works

### 1.3. Paper Organization

#### 0. Background and introduction

#### 1. System Models

#### 2. Partitioned Scheduling

#### 3. Global Scheduling

#### 4. Hybrid Approach

#### 5. Conclusion and Discussion

## 11 **2. System Models**

12 *2.1.*

## 13 **3. Partitioned Scheduling**

14 In this section, we will review some partitioned approaches to multipro-  
15 cessor real-time scheduling.

16 *3.1. Characteristic of Partitioned Scheduling*

17 *3.2. RMNF*

18 *3.3. RMFF*

19 *3.4. EDF-FF*

20 *3.5. EDF-BF*

21 *3.6. Comparision*

## 22 4. Global Scheduling

23 In this section, we will dive into another branch of scheduling strategies  
24 – global approaches to multiprocessor real-time scheduling.

### 25 4.1. Overview

26 Global scheduling algorithms, as its name suggests, globally schedules  
27 any feasible periodic task set. In contrast to partitioned scheduling, global  
28 scheduling schedules jobs and tasks in one single shared queue instead of  
29 multiple local, dedicated queues. By this means, the automatic load balancing  
30 and lower average response time can be achieved by global approaches.  
31 In addition to this, global approaches also take advantages of the simpler  
32 implementations and the existence of optimal schedulers.

33 Global scheduling strategies include Global Fixed-Job-Priority Scheduling,  
34 Global Fixed-Task-Priority Scheduling, and Global Dynamic Priority  
35 Scheduling. Although there are various categories of global scheduling algorithms,  
36 the focus of this paper is on the Global Dynamic Priority Scheduling.  
37 In the following subsection, it will be characterized in details.

### 38 4.2. Global Dynamic Priority Scheduling

39 In this subsection, we will present our in-depth exploration to the track  
40 of global dynamic priority scheduling algorithm. To the best of our knowledge,  
41 a number of global dynamic priority scheduling algorithms are optimal  
42 for periodic tasksets with explicit or implicit deadlines. For example, Proportionate  
43 Fairness algorithm and its variants including PD, PD<sup>2</sup>, ERFair, BF, SA [3],  
44 and LLREF [4] as well, are all optimal for offline environment.  
45 Nevertheless, no algorithms until now are optimal to cope with online preemptive  
46 scheduling problem, where tasksets are sporadic and multi-processor environments  
47 are enforced. On the other hand, despite of its optimality and dominance in theory,  
48 the usage of global dynamic priority algorithms are limited in practice. This is  
49 because the existence of frequent preemption and migration between tasks gives  
50 rise to excessive overheads in potential.

51 The following part of this subsection will provide brief summary of three  
52 classic global dynamic priority scheduling algorithms. They are respectively  
53 Proportionate Fairness Algorithm (PFair), and Largest Local Remaining Execution  
54 First (LLREF).

#### 4.2.1. PFair

Baruah et al [5] introduced Proportionate Fairness Algorithm. The Pfair class of algorithms that allow full migration and fully dynamic priorities have been shown to be theoretically optimal – i.e., they achieve a schedulable utilization bound (below which all tasks meet their deadlines) that equals the total capacity of all processors. Here are some fundamental principles of Proportionate Fairness algorithms:

1. Timeline is divided into equal length slots.
2. Task period and execution time are multiples of the slot size.
3. Each task receives amount of slots proportional to its task utilization.

The essential part of PFair algorithms is the quantum-based optimization defined over the lag of each task  $lag(\tau_i, t)$ , with the goal of minimizing the maximum lags of all tasks  $\max_t |lag(\tau_i, t)|$ .

$$\underbrace{lag(\tau_i, t)}_{error} = \underbrace{t \cdot \left(\frac{C_i}{T_i}\right)}_{fluid\ execution\ in[0,t)} - \underbrace{allocated(\tau_i, t)}_{real\ execution\ in[0,t)} \quad (1)$$

The generation of an optimal schedule is based on the above definition of  $lag$ . PFair algorithm does execute all urgent tasks with  $lag(\tau_i, t) > 0$  and  $lag(\tau_i, t + 1) \geq 0$  if  $\tau_i$  executes. On top of that, PFair algorithm does not execute tnegru tasks, for which  $lag(\tau_i, t) < 0$  and  $lag(\tau_i, t + 1) \leq 0$  if  $\tau_i$  does not execute. Besides, for other tasks, only those that have the least  $t$  such that  $lag(\tau_i, t) > 0$  are executed.

The PFair algorithm will assign priorities to tasks at every time slot, which indicated itself as one of job-level dynamic priority scheduling policies. However, this characteristic gives rise to some issues, for example, frequent preemptions and frequent migrations.

#### 4.2.2. LLREF

LLREF was firstly introduced by Cho et al [4]. LLREF was designed based on a novel abstraction for reasoning over task execution behavior on multiprocessors, called Time and Local Execution Time Domain Plane (T-L Plane).

### 83 4.3. Summary

84 The global scheduling paradigm has advantages over the partitioned ap-  
85 proach. First of all, if tasks can join and leave the system at run-time,  
86 then it may be necessary to reallocate tasks to processors in the partitioned  
87 approach. In addition, the partitioned approach cannot produce optimal  
88 real-time schedules – one that meets all task deadlines when task utiliza-  
89 tion demand does not exceed the total processor capacity – for periodic task  
90 sets, since the partitioning problem is analogous to the bin-packing problem  
91 which is known to be NP-hard in the strong sense. On top of that, in some  
92 embedded processor architectures with no cache and simpler structures, the  
93 overhead of migration has a lower impact on the performance. Finally, global  
94 scheduling can theoretically contribute to an increased understanding of the  
95 properties and behaviors of real-time scheduling algorithms for multiproces-  
96 sors.

97 However, the global scheduling paradigm has also several disadvantages.  
98 Firstly, global scheduling strategies are much more complicated to implement  
99 than partitioned scheduling. In other words, for the partitioned approach,  
100 once a set of tasks are allocated to processors, the multiprocessor real-time  
101 scheduling problem becomes a collection of single processor real-time schedul-  
102 ing problems. The ease of programming partitioned scheduling is obvious  
103 since the single processor scheduling problem has already been well-studied  
104 and optimal algorithms with easy implementations already exist. Secondly,  
105 migrating tasks at run-time means more runtime overhead in that migrating  
106 tasks may suffer cache misses on the newly assigned processor. If the task set  
107 is fixed and known in advanced, it is obvious that the partitioned approach  
108 provides more appropriate solutions.

## 109 5. Hybrid Approaches

## 110 6. Conclusions

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