RESEARCH PAPERS

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First Paper

An improved tabu search algorithm for solving heterogeneous fixed fleet open vehicle routing problem with time windows.

Introduction

The heterogeneous fixed fleet open vehicle routing problem with time windows is a very significant type of the vehicle routing problem (VRP) that aims to find the minimum fixed and variable cost of transportation for a heterogeneous fleet with a fixed number in which the capacity of every vehicle and usage of the vehicles should not be ignored.

VRP and distance constrained VRP. In the VRP, a fleet of vehicles in a warehouse must service a group of customers located around the warehouse, while ensuring that the total demand of the customers allocated to each vehicle does not exceed a fixed capacity (Q), and each customer is only serviced by one vehicle. In addition, all the vehicles must be the same and deliver only one kind of item to the customers.

We will not come back to the warehouse after finishing their jobs, but end their paths in the End-customers. This problem is called the open VRP (OVRP), wherein Hamilton routes of every vehicle are open, unlike the VRP that are closed.

This problem has been considered by several researchers and many methods have been presented to solve it, some of which are genetic algorithm, iterated local search tabu search , and variable neighborhood search.

Considering the open routes due to the rental of vehicles and the fleet is heterogeneous fixed fleet so it becomes heterogeneous fixed fleet open vehicles routing problem (HFFOVRP).

To provide service with minimum cost we follow the following conditions:

- The vehicles are different in regards of their capacity, fixed cost (hiring cost, maintenance cost) and variable cost (per distance unit cost) and there is a fixed and specific number of every type.
- Each vehicle starts its route from the warehouse and finishes its route in the last visited customer.
- The total amount of request for goods by the customers is less than the capacity of the assigned vehicle.
- The amount of acceptable time for each vehicle is less than the maximum specified time for that vehicle.

And we also use the time window to it becomes HFFOVRP with time window.

Since the OVRP and the HFFVRP are NP- hard, and the HFFOVRPTW is also NP-hard that means, the problem is very difficult to solve using exact methods. So, one can go for the heuristic/metaheuristic algorithms and tabu search (TS) is one of the good heuristic algorithms.

So, we used heuristic algorithms like hybrid particle swarm optimization using tabu search and we aim to improve TS (ITS) to solve HFFOVRPTW.

The Objective of this study are as follows:

- To develop a mixed integer linear programming (MILP) model for the HFFOVRPTW.
- To solve several instances of the problem by an exact method.
- To develop an ITS algorithm for solving the problem.
- To carry out a comparative study among ITS, an exact method, basic TS and simulated annealing (SA) algorithms on some benchmark problem instances.
- To prove the efficiency of the proposed ITS algorithm.

Literature review:

In the transportation industry heterogeneous fleet VRP (HFVRP) has numerous service and industrial applications.
In this kind of fleet problems characteristics of the vehicles may be heterogeneous having different capacities, the number of vehicles in a limited and specified way, the fixed cost for using each vehicle and the variable cost for using any vehicle in the distance unit.

In 2005, Chu examination of new version of the HFVRP.

Mathematical modeling:

as follows:

$$Min \sum_{\mathrm{k}=0}^{\mathrm{K}} F_k \sum_{j=1}^{n} x_{0j}^k + \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^k c \qquad (1)$$

Subject to

$$\sum_{k=1}^{K} \sum_{i=1}^{n} x_{ij}^k = 1; i = 1,2,..,n \qquad (2)$$

$$\sum_{k=1}^{K} \sum_{i=1}^{n} x_{ij}^k \le 1; \mathrm{i} = 1,2,..,\mathrm{n} \qquad (3)$$

$$0 \le \sum_{i=1}^{n} x_{ij}^{k} - \sum_{i=1}^{n} x_{ij}^{k} \le 1; j = 1,2,..,n; k = 1,2,..,K \tag{4}$$

$$\sum_{j=1}^{n} x_{0j}^{k} \le n_k ; k = 1,2,..,K \tag{5}$$

$$\sum_{k=1}^{K} \sum_{i=1}^{n} y_{ij}^{k} - \sum_{k=1}^{K} \sum_{i=1}^{n} y_{ij}^{k} = q_j; j = 1,2,..,n \tag{6}$$

$$q_j x_{ij}^{k} \le y_{ij}^{k} \le (Q_k - q_i) x_{ij}^{k}; i,j = 0,1,\dots,n, i \ne j; k = 1,2,..,K \tag{7}$$

$$\sum_{i=1}^{n} x_{i0}^{k} = 0; k = 1,2,..,K \tag{8}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}^{k} (t_{ij}^{k} + f_i^{k} + w_i^{k}) \le \mathrm{r}; \mathrm{fork} \in \{1,2,..,\mathrm{K}\}; \mathrm{i} \ne j \tag{9}$$

$$t_0^k = f_0^k = w_0^k = 0 \tag{10}$$

$$\sum\nolimits_{i=1}^{n} x_{ij}^{k} (t_i^k + t_{ij}^{k} + f_i^{k} + w_i^{k}) \le t_j^k; for\ j \in \{1,2,..,\mathrm{n}\}; k \in \{1,2,..,\mathrm{K}\};\ \mathrm{i} \ne j \tag{11}$$

$$e_i \le \left(t_i^k + w_i^k\right) \le l_i; for\ \mathrm{i} \in \{1,2,..,\mathrm{n}) and k \in \{1,2,..,\mathrm{K}\} \tag{12}$$

$$x_{ij}^{k} \in \{0,1\}; \mathrm{i,j} = 0,1,..,\mathrm{n}; \mathrm{k} = 1,2,..,\mathrm{K} \tag{13}$$

$$y_{ij}^{k} \ge 0 ;i,j=0,1,..,n;k=1,2,..,K \tag{14}$$

In the objective function (1), the first sum shows the total fixed costs of vehicles, and the second sum shows the total variable costs based on the routes travelled by all vehicles. Constraints (2) and (3) show that only one vehicle is imported to each customer and a maximum of one vehicle is returned from each customer respectively. Also, the relationship (4) causes the route of each vehicle is continuous from depot to a final customer in which the amount of difference in limit (4) for middle customers in the route is zero, but if these customers are the end nodes, these values are one unit more than the number of exit vehicles. The relationships (5) indicates that the number of vehicles used in the kth type should be at most equal to n_k and relationships (6) ensures that each customer should be addressed only in one visit and by a vehicle. Relationship (7) shows that the capacity limitation of each vehicle is observed, and relationships (8) cause no paths exist to the warehouse. The relationship (9) expresses the limitation of maximum travel time and the constraints (10–12) apply the time window limits for each customer. Finally, relationships (13) and (14) determine the range of variables.

The proposed algorithm:

Max: The defined maximum number of iteration.
X_{best}: An initial solution obtained by another algorithm.
Tlist: The considered tabu list.
N(x): The neighborhood set for solution X.
AC: The aspiration criteria.
Cset: The Candidate set. I=1; ***While*** *i<=Max*
Cset=N(X_{best})-Tlist+AC

Find the best solution belonging to Cset called $X_{current}$.

If *f(Xcurrent)<f(Xbest)*

Xbest= Xcurrent.
Update Tlist with FIFO policy.

End

Step 1: Get the value of polar coordinates for each customer in relation to the warehouse. To get the value of this angle uniquely, consider it relative to the positive axis X and between -pi to+pi.

Step 2: Start from the customer with the smallest angle and meet the customers one by one until the largest amount until the requests of the desired cluster do not exceed the amount of the capacity of the allocated customers' vehicles.

Step 3:If it is not possible to add a client to the desired cluster, generate a new cluster starting from step 2.

Step 4: Continue steps 2 and 3 until all clients are assigned to the clusters.

then he used tabu search with PSO and found that he can improve it with ITs algorithms.

Conclusion:

The study proposes a mixed integer linear programming model for a transportation problem and solves several standard problem instances using an improved tabu search algorithm. The results show that the proposed algorithm is effective in finding better solutions compared to traditional algorithms and the intensification mechanism has a significant impact on the obtained solutions. The simulated annealing algorithm is found to be the worst algorithm for both medium and large-sized problem instances.

Second Paper

Hybrid particle swarm optimization with genetic algorithm for solving capacitated vehicle routing problem with fuzzy demand – A case study on garbage collection system.

Introduction:

The passage discusses Indonesia's waste disposal issue and suggests modelling it with the capacitated vehicle routing problem with fuzzy demand (CVRPFD). A CVRP variant known as CVRPFD uses fuzzy sets theory to account for uncertain elements such as the amount of trash at makeshift landfills. The paragraph also refers to earlier work on CVRPFD, including a differential evolution algorithm and a hybrid intelligence algorithm. The approach that is suggested in this paper, known as HPSOGA, combines the advantages of both the particle swarm optimisation (PSO) and genetic algorithm (GA) approaches to address the CVRPFD problem. The suggested method is then tested on a real-world Indonesian trash collection system using a CVRP benchmark dataset that has been transformed into a CVRPFD dataset.

Background:

This section briefly presents the necessary background regarding CVRP, fuzzy credibility measurement theory, and two meta-heuristic methods used as basic algorithm in this study, named PSO, and GA.

1.CVRP

Finding the optimum vehicle route that departs from a depot, meets every client demand, and then heads back to the depot is the main goal of the vehicle routing problem (VRP). A variation of the vehicle routing problem (VRP) that takes into consideration vehicle capacity is known as the capacitated vehicle routing problem (CVRP). To service client demand, CVRP is concerned with determining the best routes for vehicles while taking into account a number of restrictions. Depending on the goals of the problem, the goal of CVRP may be to reduce the amount of time, money, or vehicles utilized for travel. Each truck has a predetermined capacity, and the depot has a specific number of cars to meet customer demand. All vehicles must return to the depot at the conclusion of the operations.

- A complete graph $G = (V, E)$, where $V = \{0, 1, . . .,n\}$ is the vertex set and E is the edge set.
- Vertices $i = 1,2,. . .,n$ correspond to the customers. Vertex 0 corresponds to the depot and n is number of customers.
- Each customer i has non-negative demand di that must be delivered to the depot.
- A set of vehicles $H = \{1, 2, . . .,h\}$, where every vehicle has capacity C, is available at the depot and must return to depot after finishing their deliveries.

- A non-negative cost cij is associated with each edge (i, j) 2 E and represents distance from vertex i to vertex j for all i ≠j.

The CVRP is modeled in Eqs. (1) – (10).

Decision variables

$$x_{ijk} = \begin{cases} 1, & \text{if vehicle } k \text{ passed route from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$y_{ik} = \begin{cases} 1, & \text{if customer } i \text{ is visited by vehicle } k \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Minimize

$$\sum_{k=1}^{h} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ijk} \tag{3}$$

Subject to,

$$\sum_{i=0}^{n} x_{i0k} - \sum_{j=0}^{n} x_{0jk} = 0,\ \forall k = 1, \dots, h \tag{4}$$

$$\sum_{i=0}^{n} \sum_{k=1}^{h} x_{ijk} = 1,\ \forall j = 1,2, \dots, n \tag{5}$$

$$\sum_{j=0}^{n} \sum_{k=1}^{h} x_{ijk} = 1,\ \forall i = 1,2, \dots, n \tag{6}$$

$$\sum_{j=1}^{n} x_{ojk} \leqslant 1, \forall k = 1,2, \dots, h \tag{7}$$

$$\sum_{i=0}^{n} x_{ijk} = y_{jk}, \forall j = 0,1, \dots, n; k = 1,2, \dots, h \tag{8}$$

$$\sum_{j=0}^{n} x_{ijk} = y_{ik}, \forall i = 0,1, \dots, n; k = 1,2, \dots, h \tag{9}$$

$$\sum_{i=1}^{n} d_i y_{ik} \leqslant C, \forall k = 1,2, \dots, h \tag{10}$$

The objective function in Eq. (3) is to minimize total travel distance, while the constraint in Eq. (4) guarantees that number of vehicles that arrive at and depart from depot is the same. Constraints in Eqs. (5) and (6) ensure that each customer is visited exactly once. The constraint in Eq. (7) defines that at most h vehicles are used. Constraints in Eqs. (8) and (9) express the relation between two decision variables. Finally, the constraint in Eq. (10) guarantees that vehicle capacity is not exceeded.

2. Fuzzy credibility measure theory

Zadeh's fuzzy set theory introduced the idea of sets with fuzzier edges and elements with varying degrees of membership. Zadeh created the possibility measure theory of fuzzy variables to quantify fuzzy events.

Later, Liu created the credibility theory, which utilizes comparisons between two fuzzy variables or a fuzzy variable and a crisp variable to solve the Capacitated Vehicle Routing Problem with Fuzzy Data (CVRPFD). Probability, necessity, and believability are the fundamental notions in fuzzy comparison.

Definition 1. Let Θ be a non-empty set, and $P(\Theta)$ be the power set of Θ. For each $A \in P(\Theta)$, there is a non-negative num $\operatorname{Pos}(A)$, which is its corresponding possibility, defined as follows:

$$
\begin{aligned}
&\operatorname{Pos}\{\emptyset\}=0 \\
&\operatorname{Pos}\{\Theta\}=1 \\
&\operatorname{Pos}\{\cup_k A_k\}=\mid \sup_k \cdot \operatorname{Pos}(A_k),\ \forall\{A_k\} \in P(\Theta)
\end{aligned}
$$

The triplet $(\Theta, P(\Theta), \text{Pos})$ is called a possibility space, and the function Pos is referred to as a possibility measure.

Definition 2. A fuzzy variable is defined as a function from the possibility space $(\Theta, P(\Theta), Pos)$ to the real line.

Definition 3. Let ξ be a fuzzy variable on a possibility space $(\Theta, P(\Theta), \text{Pos})$. Then its membership function is derived from the possibility measure Pos in Eq. (12).

$$
\mu(x)=\operatorname{Pos}\{\theta \in \Theta \mid \xi(\theta)=x\}, x \in \Re
$$

Definition 4. Let ξ be a fuzzy variable on a possibility space $(\Theta, P(\Theta), Pos)$, and then set $\xi_\alpha=\{\xi(\theta) \mid \theta \in \Theta, P0s\{\theta\} \geqslant \alpha\}$. This is called the α-level set of ξ.

Definition 5. Let $(\Theta, P(\Theta), \text{Pos})$ be a possibility space, and A be a set in $P(\Theta)$. Then the necessity measure of A is defined in Eq. (13).

$$\text{Nec}\{A\} = 1 - \text{Pos}\{A^c\}$$

Definition 6. Let $(\Theta, P(\Theta), \text{Pos})$ be a possibility space, and A be a set in $P(\Theta)$. Then the credibility measure of A is defined in Eq. (14).

$$\text{Cr}\{A\} = \frac{1}{2}(\text{Pos}\{A\} + \text{Nec}\{A\})$$

If the membership function $\mu(u)$ of ξ is given as μ, then the possibility, necessity, credibility of the fuzzy event $\{\xi \geqslant r\}$ can be represented as in Eqs. (15)-(17), respectively.

$$\text{Pos}\{\xi \geqslant r\} = \sup_{u \geqslant r} \mu(u)$$
$$\text{Nec}\{\xi \geqslant r\} = 1 - \sup_{u \geqslant r} \mu(u)$$
$$\text{Cr}\{\xi \geqslant r\} = \frac{1}{2}[\text{Pos}\{\xi \geqslant r\} + \text{Nec}\{\xi \geqslant r\}]$$

The credibility of a fuzzy event is defined as the average of its possibility and necessity. The fuzzy event must hold if its credibility is 1 , and fail if its credibility is 0 . For the triangular fuzzy variable $\tilde{\xi} = (r_1, r_2, r_3)$ the possibility, necessity and credibility are shown in Eqs. (18)-(20), respectively.

$$\text{Pos}\{\tilde{\xi} \geqslant r\} = \begin{cases} 1, & \text{if } r < r_2 \\ \dfrac{r_3 - r}{r_3 - r_2}, & \text{if } r_2 \leqslant r < r_3 \\ 0, & \text{if } r \geqslant r_3 \end{cases}$$

$$\text{Nec}\{\tilde{\xi} \geqslant r\} = \begin{cases} 1, & \text{if } r < r_1 \\ \dfrac{r_2 - r}{r_2 - r_1}, & \text{if } r_1 \leqslant r < r_2 \\ 0, & \text{if } r \geqslant r_2 \end{cases}$$

$$\text{Cr}\{\tilde{\xi} \geqslant r\} = \begin{cases} 1, & \text{if } r < r_1 \\ \dfrac{2r_2 - r_1 - r}{2(r_2 - r_1)}, & \text{if } r_1 \leqslant r > r_2 \\ \dfrac{r_3 - r}{2(r_3 - r_2)}, & \text{if } r_2 \leqslant r < r_3 \\ 0, & \text{if } r \geqslant r_3 \end{cases}$$

3. Methods for solving CVRP

There have been meta-heuristic methods developed for solving CVRP, for instance tabu search, ant colony optimization, simulated annealing, genetic algorithm (GA), particle swarm optimization (PSO), etc. In this research, GA and PSO are used for basic algorithms, and these methods are discussed in this section.

2.3.2. PSO for CVRP

Similar to GA, PSO is a population-based search algorithm. In PSO, individuals, referred to as particles, are "flown" in the search space with their own velocities. Their velocities are dynamically adjusted according to their historical behaviors, and so the particles will fly toward better search areas. The velocity of each particle is calculated according to formula shown in Eq.

$$v_i^{t+1} = w \times v_i^t + c_1 \times r_1 \times \left(\text{pbest}_i - x_i^t \right) + c_2 \times r_2 \times (\text{gbest} - x_i^t)$$

Methodology

This section presents the CVRPFD model employed for representing garbage collection system problem and the algorithm of proposed HPSOGA.

1. CVRPFD model development

Some studies have used a change-constraint program (CCP) to construct a model of CVRP using fuzzy variables since fuzzy variables are used to deal with unclear parameters. A VRP with fuzzy travelling time and a CCP model with credibility measurement were created by Zheng and Liu [40]. The same approach was utilized by Erbao and Mingyong [16,41] to model CVRPFD and Open CVRPFD. To resolve the fuzzy variable in this study, CCP mode with credibility measurement is used.

The dispatcher preference index Cr^* expresses the dispatcher attitude toward risk], and the value of Cr^* is de mined subjectively in the range of $[0,1]$. The capacity constraint is satisfied if $\mathrm{Cr}\left\{\tilde{L}_k\right\} \geqslant \mathrm{Cr}^*$. Lower values of of parameter Cr indicate that the dispatcher endeavors to use the vehicle capacity as much as possible. Whereas a greater Cr^* value indic the dispatcher is risk averse.
The CVRPFD is modeled in Eqs.
Minimize

$$\sum_{k=1}^{h} \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} x_{ijk}$$

Subject to,

$$\sum_{i=0}^{n} x_{i0k} - \sum_{j=0}^{n} x_{0jk} = 0,\ \forall k = 1, \dots, h$$

$$\sum_{i=0}^{n} \sum_{k=1}^{h} x_{ijk} = 1,\ \forall j = 1,2, \dots, n$$

$$\sum_{j=0}^{n} \sum_{k=1}^{h} x_{ijk} = 1,\ \forall i = 1,2, \dots, n$$

$$\sum_{j=1}^{n} x_{0jk} \leqslant 1,\ \forall k = 1,2, \dots, h$$

$$\sum_{i=0}^{n} x_{ijk} = y_{jk},\ \forall j = 0,1, \dots, n; k = 1,2, \dots, h$$

$$\sum_{j=0}^{n} x_{ijk} = y_{ik},\ \forall i = 0,1, \dots, n; k = 1,2, \dots, h$$

$$\mathrm{Cr}\left\{\sum_{i=1}^{n} \tilde{d}_i y_{ik} \leqslant C\right\} \geqslant Cr^*,\ \forall k = 1,2, \dots, h$$

The objective function in Eq. (23) intends to minimize the total travel distance. The constraint in Eq. (24) guarantees that the number of vehicles that arrive at and depart from the depot is same. Constraints in Eqs. (25) and (26) ensure that each customer is visited exactly once. The constraint in Eq. (27) defines that at most h vehicles are used. Constraints in Eqs. (28) and (29) express the relation between two decision variables. Finally, the constraint in Eq. (30) guarantees that all customers are visited within vehicle capacity with a confidence level Cr^*.

2. Hybrid particle swarm optimization with genetic algorithm

This paper suggests a novel hybrid approach to tackle the Capacitated Vehicle Routing Problem with Fuzzy Data (CVRPFD), which combines Particle Swarm Optimization (PSO) and Genetic Algorithm (GA). The drawback of utilizing PSO to solve discrete issues like CVRP served as the inspiration for the improvement. Chromosomes rather than binary integers are a more appropriate representation of the CVRP solution, which correlates to the order of cities to be visited. To make sure the solution satisfies the capacity constraint, two different types of chromosomes were created. The hybrid approach combines the advantages of GA and PSO to enhance the speed and accuracy of solving CVRPFD. Chromosomes are used to represent solutions because they guarantee that the capacity constraint is met, which is important for solving CVRP.

The length of temporary chromosome length should be equal to number of customers, n. Then, to construct particle's solution, we determine the sub-route according to capacity constraint. Steps to form sub-route with fuzzy demand is described as follows.

(1) Let customer i is a customer represented in bit j of X_p.

(2) Initialize current load of vehicle k which serving sub-route k is $\tilde{L}_k = (l_{k1}, l_{k2}, l_{k3}) = (0,0,0)$ and $j = 1$.

(3) Calculate current load $\tilde{L}_k = \tilde{L}_k + \tilde{d}_1$ where,

$$\begin{aligned} l_{k1} &= l_{k1} + d_{i1} \\ l_{k2} &= l_{k2} + d_{i2} \\ l_{k3} &= l_{k3} - d_{i3} \end{aligned}$$

(4) Calculate current credibility,

- If $C < l_{k1}$, then $Cr = 0$
- If $l_{k1} \leqslant C < l_{k2}$, then $Cr = 1 - \frac{2l_{kk} - l_{k1} - C}{2(l_{k2} - l_{k1})}$
- If $l_{k2} \leqslant C < l_{k3}$, then $Cr = 1 - \frac{l_{k3} - C}{2(l_{k3} - l_{21})}$
- Otherwise $\mathrm{Cr} = 0$.

(5) Determine solution

- If $\mathrm{Cr} > \mathrm{Cr}$, insert customer i to sub-route k.
- Otherwise, update $k = k + 1$, insert customer i to sub-route k, update $\tilde{L}_k = (0,0,0)$.

(6) Update $j = j + 1$. Go back to step 3 until $j = n$.

Conclusions

The study proposed a hybrid method based on Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) to solve the Capacitated Vehicle Routing Problem with Flexible Delivery (CVRPFD).

The method combined the particle's best and social best solutions in PSO with crossover and mutation of GA and modified the decoding and encoding scheme of Discrete PSO to ensure feasible solutions.

The proposed method was verified on nine modified CVRP instances and compared with DPSO and GA, showing better performance in terms of solution quality, stability, and convergence speed.

The proposed method was also applied to solve a garbage collection system in Palembang City, Indonesia, with better results than DPSO and GA. Future research should consider other factors such as travel time, number of customers, and customer locations, and explore other meta-heuristic methods such as ant colony system and artificial immune system.

Third Paper

The school bus routing problem: a case study

Introduction:

In Hong Kong, where there is currently no scientific method for allocating students to buses and planning bus routes, the project seeks to create a computer system for school bus services. The study focuses on creating methods for arranging school bus routes, particularly early in the day. The essay offers a survey of the literature on vehicle routing and school bus routing, formulates the issue of school bus routing in Hong Kong, and suggests an algorithm to resolve the issue. The algorithm's efficacy is evaluated using information from a Hong Kong kindergarten, and the results are shown in the experimental results section.

Literature review:

The school bus routing problem is a particular kind of vehicle routing problem (VRP), which deals with the effective utilization of a fleet of vehicles to pick up and deliver clients or goods while lowering overall costs and meeting limitations. The VRP has been extensively explored in operational research journals, and most of the study focuses on creating heuristic algorithms because it is impractical to always discover the best solution. In the last two decades, the issue of school bus routing has drawn attention, and there are numerous, situation-specific options. In Braca et al.13, a survey of several of these strategies is presented.

Table 1 Literature review on the school bus routing problem

Referenc e	Proble m type	Objective	Constraint
Bennett and Gazis [6]	Single school	Minimise total student travel time	Bus capacity
Dulac et al[7]	Single school	Min {total distance × number of routes }	1. Bus capacity 2. The number of stops 3. The length of a route
Chen and Kallsen [8]	Single school	1. Minimise the number of buses requir 2. Minimise fleet travel time 3. Balance the bus loads	1. Bus capacity 2. Student riding time 3. School time window

Bowerman et at	Single school	1. Minimise the number of routes (buse 2. Minimise total bus route length 3. Balance bus loads and route lengths 4. Minimise student walking distance	1. Bus capacity 2. Travel time on each route 3. The total travel time
Newton and Thomas [10]	Multi-school	1. Minimise total bus travel time 2. Minimise the number of routes requi	1. Bus capacity 2. Student riding time
Angel et al^{11}	Multi-school	1. Minimise the number of routes 2. Minimise total bus travel time	1. Bus capacity 2. Specified route time limit
Bodin and Bermen [12]	Multi-school	Minimise total bus travel time	1. Bus capacity 2. Allowable student travel time
Braca et al^{13}	Multi-school	Minimise the number of buses needed	1. Upper and lower bounds on bus capa 2. Student riding distance 3. School time window 4. Earliest pick-up time

Although many papers mention multiple objectives to be considered, only Bowerman et al9 claim that their method examines the school bus routing problem from a multi-objective viewpoint. As will be seen in the next section, our problem is different from those previously reported in several aspects. The objectives considered include minimizing the number of buses required, total travel time spent by pupils at all points, total bus travel time, and balancing the bus loads and travel times. The main constraint is the bus capacity.

Problem representation:

Notation K is the number of buses available for the school bus service.

C_k is the capacity of bus k, which may be the same or different for each bus.
n is the total number of pick-up points.
M is the total number of pupils to be served.
$p_1, p_2, \ldots, p_n$, are the n pick-up points. These are ordered by decreasing distance from the school. p_{n+1} denotes the school.
t_{ij} is the travel time from p_i to p_j.

f_i is the number of pupils to be picked up at p_i.
L is the average pick-up time at pick-up points.

$$x_{ijk} = \begin{cases} 1 & \text{if bus } k \text{ travels directly from } p_i \text{ to } p_j \\ 0 & \text{otherwise.} \end{cases}$$
$$z_{ik} = \begin{cases} 1 & \text{if bus } k \text{ picks up pupils at } p_i \\ 0 & \text{otherwise.} \end{cases}$$

y_{ik} is the number of pupils picked up by bus k at p_i.

Objectives:

To evaluate school bus routes, multiple criteria need to be considered simultaneously, including efficiency, effectiveness, and equity. Each criterion has its own set of objectives to meet, but they are all interrelated when assessing service provision. The objectives for the school bus routing problem are listed and discussed, and then classified according to the relevant criteria.

(1) Minimize the total number of buses required. This is an objective related to the service cost. The minimum number of buses K (assume one bus for one route only) required to serve all points for a school can be determined by

$$K = \min(q) \text{ such that } \sum_{k=1}^{q} k \geqslant M$$

Schools usually do operate with only K buses for the service owing to the consideration of cost.

(2) Minimize the total travel time spent by pupils at all points. This objective is what the school and parents are concerned about most. To minimize this objective is to ensure higher quality service. It can be formulated as

$$\min \sum_{k=1}^{K} \left\{ \sum_{i=1}^{n} \left[\sum_{j=1}^{n+1} t_{ij} x_{ijk} \left(\sum_{l=1}^{i} z_{lk} \right) + L z_{ik} \left(\sum_{l=1}^{i} z_{lk} \right) \right] \right\}$$

Let di denote the shortest travel time from pi to the school. The school and parents will compare the actual travel times spent with di to evaluate the service quality. Therefore

$$T = \sum_{i=1}^{n} d_i$$

is the lower bound of this objective.

(3) Minimize the total bus travel time. This is another objective related to the service cost. It consists of two parts: minimizing the total bus loaded travel time (from route origins to the school) and minimizing the total bus vacant travel (deadheading) time (from parking places to the route origins).
The former can be formulated as

$$\min \sum_{k=1}^{K} \left[\sum_{i=1}^{n} \left(\sum_{j=1}^{n+1} t_{ij} x_{ijk} + L z_{ik} \right) \right]$$

A solution resulting in the improvement of objective (2) usually leads to the improvement of this part of objective (3).

In the case of all buses with the same capacity, the latter can be formulated as the following assignment problem.

$$\text{Minimise} \quad \sum_{i=1}^{K} \sum_{j=1}^{K} c_{ij} u_{ij}$$

$$\text{subject to} \quad \sum_{i=1}^{K} u_{ij} = 1 \; j = 1, \dots, K$$

$$\sum_{j=1}^{K} u_{ij} = 1 \; i = 1, \dots, K$$

$$u_{ij} = 0 \text{ or } 1 \; i, j = 1, \dots, K$$

where c_{ij} is the travel time from parking place of bus i to the origin of route j.

$$u_{ij} = \begin{cases} 1 & \text{if bus } i \text{ is assigned to route } j \\ 0 & \text{otherwise.} \end{cases}$$

Otherwise, this assignment, or part of it, is given by the route planner.

(4) Balance the loads and travel times between buses. It is very important in practice to do so. The acceptable level of load and travel time balance depends on the route planner, parents, and bus drivers.

The priority of objectives is the same order as above, ie giving first priority to objective (1), second to objective (2), then objective (3), and finally objective (4).

The classification of the objectives considered is given below.

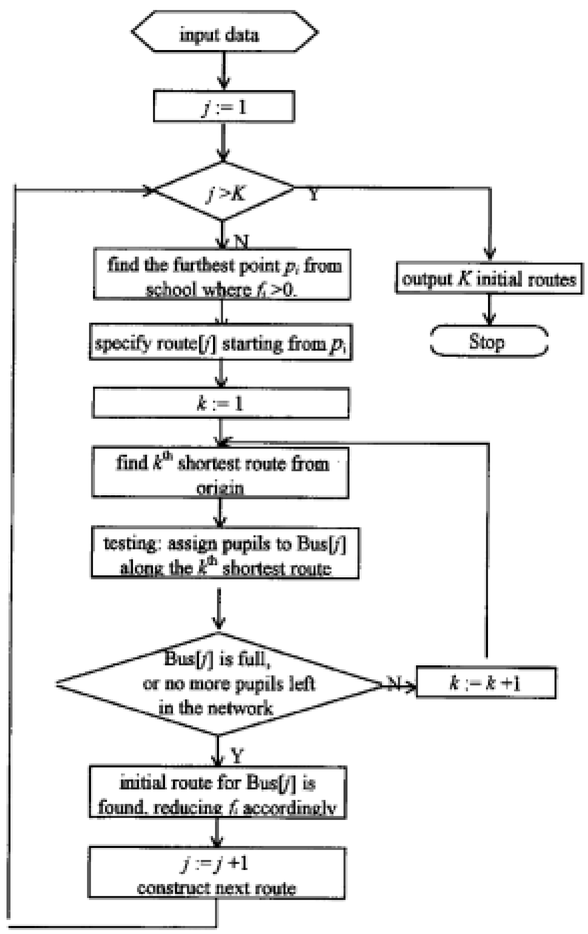
A measure of efficiency is the ratio of the level of a service compared to the cost of the resources required to provide the service. Objectives (1) and (3) belong to the efficiency criterion.

Solution algorithm:

Stage 1. Find the optimal solution K for objective (1), where.

$$K = \min(q) \text{ such that } \sum_{k=1}^{q} C_k \geqslant M$$

Stage 2. At this stage the solution building strategy is applied to find an initial feasible solution by constructing the routes one by one. The flowchart of stage 2 is shown in Figure 1. If the bus capacities in the fleet are different, the route planner may determine randomly or according to own preference which bus to serve this route interactively and hence its capacity is known.

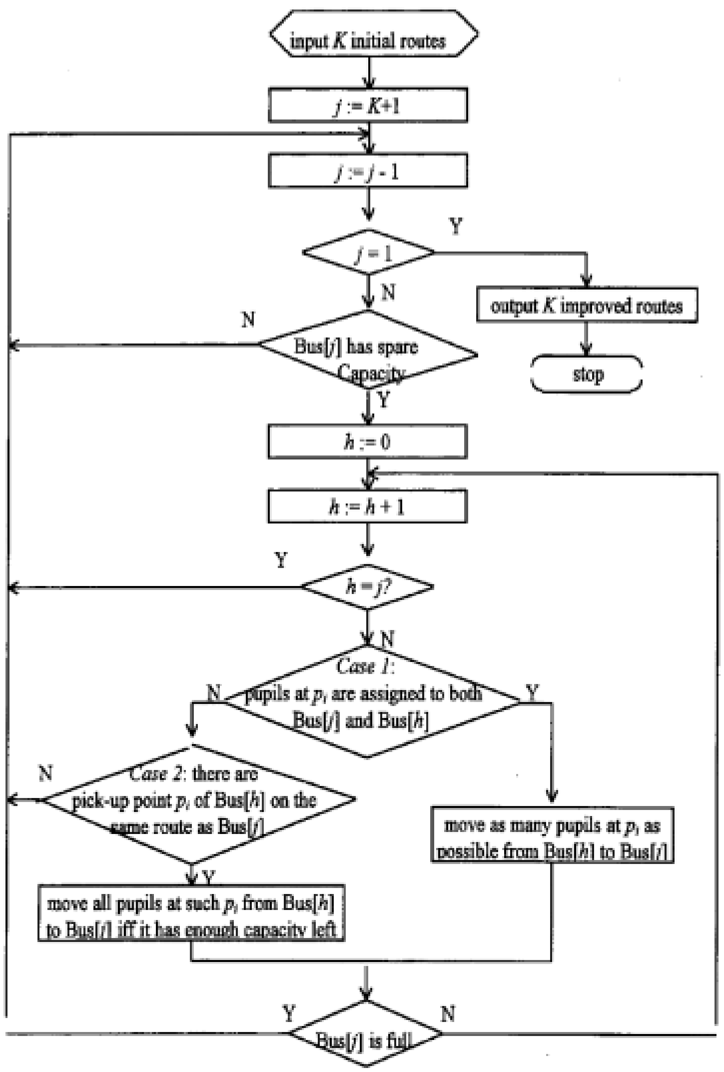


Stage 3. The improvement strategy is adopted at this stage.
to improve the initial solution.

Experimental results:

The algorithm is applied to a kindergarten in Hong Kong to test its effectiveness. The existing school bus routes of the kindergarten are planned manually.

Data input
The following (see Table 2) are the data used to find the solution. The total number of pupils to be served is 86 . All buses have the same capacity, 36. Then $K = 3$. Average pick-up time $L = 25$



(s). There are 54 pick-up points, where each has 1-8 pupils. The shortest distances from every pick-up point to the school are found by using the program codes of Dijkstra's algorithm in Syslo et al. [18] All pick-up points p_i are ordered by decreasing distance to the school. There are both one-way streets and two-way streets on the street network.

Result analysis

The algorithm has been programmed using Turbo Pascal. After inputting the above data, the algorithm requires only a few seconds on a PC to output the results. The results are shown in Table 3.

In this example the load of bus 3 is much less than that of buses 1 and 2. The reason is that the improvement of load balance in Stage 3 is limited owing to priorities been given to objectives (2) and (3). However, one may obtain the desirable level of load balance simply by decreasing the bus capacity to a suitable level interactively and forcing the algorithm to get a result with desirable balance loads. The closer to the average bus load the bus capacity, the more balanced the actual bus loads are. In this case, however, the values for objectives (2) and (3) will possibly increase, resulting in lower service quality. This is a trade-off to be made.

A comparison of objective (2) (excluding pick-up time) is shown in Table 4. The result is just 17.4% above the lower bound $T = \sum_{i=1}^{n} d_i$. Notice that the lower bound is just a very loose one. The actual gap between the optimum and the solution found by the algorithm should be much narrower. The new routes produced by the algorithm save 29% total pupil travel time in comparison with that of the existing routes. A comparison of the bus loaded-travel times (objective (3)) (including pick-up times) is shown in Table 5. 13.2%, 32.5% and 23.4% are shortened respectively for each route.

Conclusion:

The paper presents a case study of the school bus routing problem, which is formulated as a multi-objective combinatorial optimization problem. An efficient heuristic algorithm, combining various optimization methods, is proposed for its solution. The objectives of the problem include minimizing the total number of buses required and the total travel time spent by pupils at all pick-up points, which are important concerns for the school and parents. The algorithm runs efficiently on a personal computer.