

Work and Energy

Work is a measure of energy transfer. In the absence of friction, when positive work is done on an object, there will be an increase in its kinetic or potential energy. In order to do work on an object, it is necessary to apply a force along or against the direction of the object's motion. If the force is constant, work can be calculated using the vector dot product

$$W = F \times s$$

where F is the constant force and s the displacement of the object. If the force is not constant, we divide the overall displacement into short segments, Δs , the force is nearly constant during each segment. The work done during that segment can be calculated using the previous expression. The total work for the overall displacement is the sum of the work done over each individual segment:

$$W = \sum F(s)\Delta s$$

This sum can be determined graphically as the area under the plot of force vs. distance.¹

These equations for work can be easily evaluated using a force sensor and a Motion Detector. In either case, the work-energy theorem relates the work done to the change in energy as

$$W_{NC} = \Delta U + \Delta K$$

where W_{NC} is the work done (by non-conservative forces), ΔU is the change in potential energy, and ΔK the change in kinetic energy.

In this experiment you will investigate the relationship between work, potential energy, and kinetic energy.

OBJECTIVES

- Use a Motion Detector and a force sensor to measure the position and force on a hanging mass, a spring, and a dynamics cart.
- Determine the work done on an object using a force vs. distance graph.
- Use the Motion Detector to measure velocity and calculate kinetic energy.
- Compare the work done on a cart to its change of mechanical energy.
- Test the Conservation of Mechanical Energy Principle.

MATERIALS





Power Macintosh or Windows PC	dynamics cart
Vernier Motion Detector	masses (200 g and 500 g)
Vernier Force Sensor	spring with a low spring constant (~10 N/m)
Universal Lab Interface	Tape and rubber band
Logger Pro	wire basket (to protect Motion Detector)

¹ If you know calculus, you may recognize this sum as leading to the integral $W = \int_{s_{\text{initial}}}^{s_{\text{final}}} F(s)ds$.

PROCEDURE

Part I Work When The Force Is Constant

In this part you will measure the work needed to lift an object straight upward at constant speed. The force you apply will balance the weight of the object, and so is constant. The work can be calculated using the displacement and the average force, and also by finding the area under the force vs. distance graph. Take data to three significant digits. Later you will be asked to justify the proper number of sig. figs.

1. Connect the Vernier Motion Detector to PORT 2 and the Vernier Force Sensor to DIN 1 (PORT 1 if using ULI Force Probe) of the Universal Lab Interface.
2. Open “Exp18DRA” from the *Physics with Computers* experiment files of *Logger Pro*. Set the range switch on the sensor to 10 N. Three graphs will appear on the screen: distance vs. time, force vs. time, and force vs. distance. Data will be collected for 5 s.
3. Hold the Force Sensor with the hook pointing downward, but with no mass hanging from it. Click  and then  to set the Force Sensor to zero.
4. Place the Motion Detector on the floor, away from table legs and other obstacles. Place a wire basket over it as protection from falling weights.
5. Hang a 200-g mass from the Force Sensor.
6. Hold the Force Sensor and mass about 0.5 m above the Motion Detector. Click  to begin data collection. Wait about 1.0 s after the clicking sound starts, and then slowly raise the Force Sensor and mass about 0.5 m straight upward. Then hold the sensor and mass still until the data collection stops at 5 s.
7. Examine the graphs by clicking the *Examine* button, . Identify when the weight started to move upward at a constant speed and when it stopped moving upwards. Record the initial and final heights in the data table.

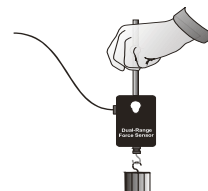




Figure 1

8. Determine the average force exerted while you were lifting the mass. Do this by selecting the portion of the force vs. time graph corresponding to the time you were lifting. Do not include the brief periods when the up motion was starting and stopping. Click the *Statistics* button, , to calculate the average force. Record the value in your data table.
9. On the force vs. distance graph select the region corresponding to the upward motion of the weight. (Click and hold the mouse button at the starting distance, then drag the mouse to the stopping distance and release the button.) Click the *Integrate* button, , to determine the area under the force vs. distance curve during the lift. Record this area in the data table.

Part II Work Done To Stretch A Spring

In Part II you will measure the work needed to stretch a spring. Unlike the work needed to lift a mass, the work done in stretching a spring is not a constant. The work can still be calculated using the area under the force vs. distance graph.

11. Open “Exp18DRB” from the Physics with Computers experiment files of Logger Pro. You will have the same set-up as in the previous experiment.
12. Attach one end of the spring to a rigid support. Attach the Force Sensor hook to the other end. Rest the Force Sensor on the table with the spring extended but relaxed, so that the *spring applies no force to the Force Sensor*.
13. Place the Motion Detector about one meter from the Force Sensor, along the line of the spring. Be sure there are no nearby objects to interfere with the distance measurement.

$$W = F \times s$$

Figure 2

14. The starting point is when the spring is in a *relaxed state*. Hold the end of the Force Sensor as shown in Figure 3. The Motion Detector will measure the distance to your hand, not the Force Sensor. With the rest of your arm out of the way of the Motion Detector beam, click . On the dialog box that appears, click . Logger Pro will now use a coordinate system which is positive *towards* the Motion Detector with the origin at the Force Sensor.

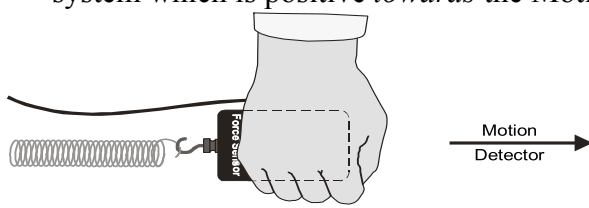



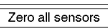


Figure 3

15. Click to begin data collection. Within the limits of the spring, move the Force Sensor and slowly stretch the spring about 50 cm over several seconds. Hold the sensor still until data collection stops. Do not get any closer than 40 cm to the Motion Detector
16. Examine the graphs. Identify when you started to pulling the spring and when you stopped.
17. Examine the force vs. distance graph. The force *should start at zero and increase linearly* with distance. If it doesn't, you should *redo* the experiment until you get a good example of Hooke's law by making sure that the spring is unstretched when you start pulling.
18. Click the force vs. distance graph, then click the *Linear Regression* button, , to determine the slope of the force vs. distance graph. The slope is the spring constant, k . Record the slope.
19. The area under the force vs. distance graph is the work done to stretch the spring. On the force vs. distance graph select the region corresponding to the first 10 cm stretch of the spring. (“Click, hold and drag the mouse”). Click the *Integrate* button, , to determine the area under the force vs. distance curve during the stretch. Record this area in the data table.
20. Now select the portion of the graph corresponding to the first 20 cm of stretch (twice the stretch). Find the work done to stretch the spring 20 cm. Record the value in the data table. Repeat using the maximum stretch of the spring.

Experiment 18

Part III Work Done To Accelerate A Cart

In Part III you will push on the cart with the Force Sensor, causing the cart to accelerate. The Motion Detector allows you to measure the initial and final velocities; along with the Force Sensor, you can measure the work you do on the cart to accelerate it.

22. Open “Exp18DRC” from the Physics with Computers experiment files of Logger Pro. Same set-up as before.
23. Remove the spring and support. Determine the mass of the cart and force sensor. Record in the data table.
24. Place the cart at rest about 1.5 m from the Motion Detector, ready to roll toward the detector. Attach the Force Sensor to the top of the cart with the hook pointing away for the detector.
25. Click . On the dialog box that appears, click . Logger Pro will now use a coordinate system which is positive towards the Motion Detector with the origin at the cart.
26. Click  to begin data collection. When you hear the Motion Detector begin clicking, gently accelerate the cart *horizontally* toward the detector by *pushing only on the hook* of the Force Sensor. The push should last about half a second and it should be *horizontal* (no up or down components.) Let the cart roll toward the Motion Detector, but catch it before it strikes the detector.
27. Examine the graphs. Identify when you started to push the cart and when you stopped pushing the cart.
28. Determine the velocity of the cart *just after you stopped pushing* (don’t look at the velocities after the cart starts to slow down). Use the *Tangent* tool to determine this final velocity and record it in the data table. The initial speed of the cart was, of course, zero.
29. From the force vs. distance graph, determine the work you did to accelerate the cart. To do this, select the region *corresponding to the push* (but no more). Click the *Integrate* button, , to measure the area under the curve. Record the value in the data table.

Part IV Conservation of Mechanical Energy

In Part IV you will allow a cart to accelerate down a ramp. The motion detector lets you measure the positions and velocities at different times. You can then calculate the kinetic and potential energies at various points and compare the sum total of both energies.

31. Open “Exp6” from the Physics with Computers experiment files of Logger Pro. Change the graph scales to a maximum of 2 m for distance, 1 m/s for velocity and 0.5 m/s² for acceleration. Then open the “Experiment” menu and change the *sampling rate* to “10” and (if possible) the *averaging rate* to “3”.
32. Tilt the ramp slightly and place the motion detector at the bottom of the ramp facing up.
33. Hold the cart about 0.5 m from the detector. Click “Collect” and when the clicking begins release the cart and let it roll to the end of the track, but don’t let it crash into the detector!
34. Look at the graphs. Pick *four random points* about equally spaced. Record the positions and velocities for these four points.
35. Select a section of the acceleration graph where the acceleration is fairly constant and use the “Stat” tool to determine the average value of the acceleration. Since acceleration down a

frictionless ramp is equal to “ $g\sin\theta$ ”, we can use the acceleration to determine the angle of the incline (actually it is “ $\sin\theta$ ” that you really need!).

36. Record the mass of the cart.

37. Fill in the table of data and results below. The last column of the table will allow to compare the total mechanical energy at the chosen points.

Data Table *In this experiment we will not measure uncertainties but simply assume that three significant digits is a reasonable number to consider. You will notice that the computer generally reads out more digits than are significant and you should simply round off.*

Part I	
Starting height (m)	
Stopping height (m)	

Average force (N)	
Work done by the average force ($N\cdot m=J$)	
Integral (during lift) of force vs. distance graph (J)	
% comparison between Work by F_{average} and Integral	
Change in potential energy, $\Delta U=mg\Delta y$ (J)	
% difference between Work and ΔU	

Part II	
Spring Constant (N/m)	

	Stretch		
	0->10 cm	0->20 cm	0->Maximum
Integral (during pull) ($N\cdot m=J$)			
Change in potential energy, $\Delta U=ks^2/2$ (J)			
% difference between Work and ΔU			

Part III	
Mass of cart and sensor (kg)	

Experiment 18

Final velocity (m/s)	
Integral during push ($N \cdot m = J$)	
Change in kinetic energy, $\Delta K = mv^2/2$ (J)	
% difference between net Work and ΔK	

Part IV	
Mass of cart (kg)	
Acceleration (m/s^2)	
$a/g = \sin \theta$	

Point	Position on ramp (m)	Velocity (m/s)	Vertical height (m)	Potential Energy (J)	Kinetic Energy (J)	Total Mechanical Energy (J)	% Difference between points	
1							1-2	
2							2-3	
3							3-4	
4							4-1	

ANALYSIS & QUESTIONS

- In parts I through III compare the *work done* with the *change in the energy* associated with that type of work. In part IV, compare the *total energy from one point to the next*? Were the expected agreements satisfactory to three significant digits?...to two significant digits? If we consider agreement to two or three significant digits a success, how successful was this experiment overall?
- In Part II, did the spring follow Hooke's law ($F = -kx$)? Explain why it is important that the force be zero at the origin of the spring force graph.
- Predict the appearance of the graph if you continued to stretch the spring beyond its elastic limit so that it became permanently deformed. Explain the implications of exceeding the elastic limit.
- In Part III explain why it is important that you push the cart *horizontally*. What error would result if you pushed a little up or down when attempting to push the cart horizontally?
- In Part IV, how would the individual potential, kinetic, and total energies differ if the detector had been placed at the top of the ramp? Would the total energy still be conserved?
- In Part IV, if the total amount of energy decreased steadily (more than would be justified by measurement error) it could indicate that the resistive forces were not negligible in the experiment. Is there evidence of this in your results? How could you tell whether the effect was due to surface friction or to air resistance?