

## Experiment 6

### Uniform Circular Motion

A body is suspended by a string from a horizontal crossbar which is spun around a vertical axis. The body is prevented from swinging outward by a horizontal spring attached to the axis of rotation. The centripetal force exerted by the spring on the body is determined. The system is then brought to rest and a static force pulls the body to the position the body occupied in the dynamic case. This force is compared with the centripetal force.

#### Theory, Part I

For circular motion, the *centripetal* force is given by

$$F = \frac{mv^2}{R} .$$

Where the speed is tangential around the circular path. If the speed is constant, then its value may be determined by using the number of revolutions,  $N$ , that occur during a time interval  $T$  (" $T$ " here represents the time for  $N$  revolutions not one revolution), that is,

$$v = \frac{2\pi RN}{T} .$$

Substituting (2) into (1), the centripetal force is then

$$F = 4\pi^2 m \frac{RN^2}{T^2} .$$

#### Theory, Part II

In the experiment, the value of  $m$  and  $N$  are known. However, due to uncertainties in the measurements of  $R$  and  $T$ , there will be an uncertainty in the computed value of  $F$ . (This is referred to as propagation of error.) For measurements  $R \pm \delta R$  and  $T \pm \delta T$  (where  $\delta R$  and  $\delta T$  represent the uncertainties in these values and are assumed to be positive). Because only an approximate value of  $\delta F$  is required, the fractional uncertainty of  $F$  is approximately the sum of the fractional uncertainties of  $R$  and  $T$  in which each is multiplied by the power of the variable that occurs in (3):

$$\frac{\delta F}{F} = \frac{\delta R}{R} + \frac{2\delta T}{T} ,$$

when  $\delta R / R \ll 1$  and  $\delta T / T \ll 1$ . A general proof of the fractional uncertainty formula appears in the instruction sheet for the *Measurements and Uncertainty* lab.

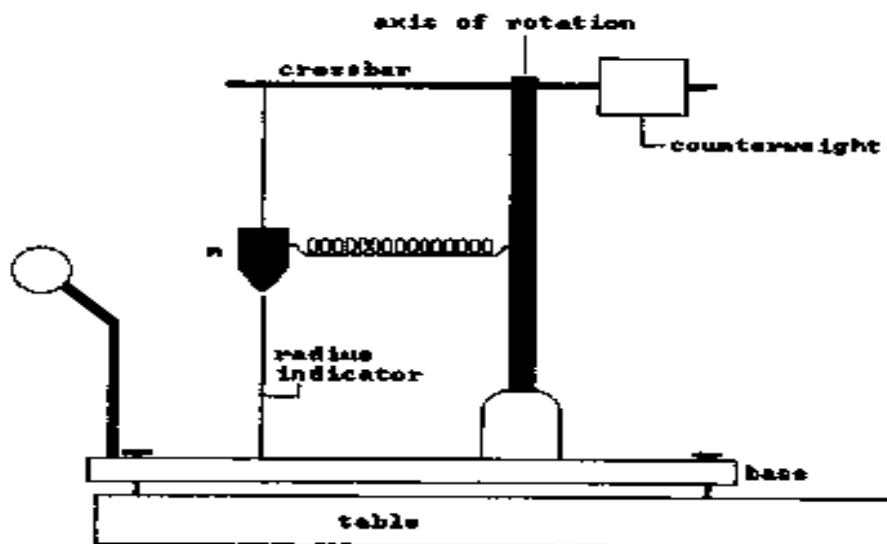
## Apparatus

- o centripetal force apparatus
- o stop watch,  $\pm 0.02$  sec
- o metric ruler,  $\pm 0.05$  cm
- o bubble level
- o 50 gram weight hanger
- o assortment of slotted weights
- o string
- o double pan balance

The apparatus consists of a heavy mass supported by a string tied to a crossbar that is attached to a vertical shaft. The shaft is supported in a metal housing on a wooden base and has bearings that allow the shaft to be rotated easily. The crossbar is counterbalanced. An adjustable vertical rod mounted on the base serves as a radius indicator. A pulley mounted on a rod near one end of the base is used when measuring the static force exerted on the spring by a hanging mass.

### *Dynamic case:*

In the dynamic situation, the vertical shaft is turned by hand until the pointer on the hanging body passes directly over the radius indicator. (Refer to Figure 2.)



### *Static Case:*

In the static situation, the hanging body is pulled outward by a mass,  $M$ , attached to the

axis of rotation

body by a string passing over the pulley. The amount of mass,  $M$ , is adjusted until the pointer on

Figure 2. Centripetal force apparatus in the configuration it has when operated dynamically.

the hanging body is directly over the radius indicator. (Refer to Figure 3).

Figure 3. Centripetal force apparatus in the configuration it has when operated statically.

## Procedure

- 1) Position the apparatus so that the pulley overhangs the edge of the table as shown in Figures 2 and 3. Level the apparatus. Record the mass of the hanging body,  $m$ .
- 2) Detach the spring and allow the hanging body to hang freely. Adjust the position of the crossbar to some convenient position on the vertical shaft. Make sure that the crossbar is tightened securely to the shaft and that the counterweight is also tightened securely to the crossbar.
- 3) Adjust the position of the radius indicator until the pointer on the hanging body is directly over the tip of the radius indicator. This position then represents the radius that the hanging body must be at in both the dynamic and the static determination of the forces. Measure the distance and its uncertainty from the tip of the radius indicator horizontally to the center of the vertical shaft. This distance is the radius,  $R \pm \delta R$ .
- 4) Attach the spring to the vertical shaft and to the hanging body. Begin to rotate the system by turning the vertical shaft with your fingers. Slowly increase the rate of rotation until the pointer on the hanging body passes directly over the tip of the radius indicator. (You should practice this procedure.) Maintain this speed of rotation. A piece of white paper used as a background may make it easier to see the tip of the hanging mass as it passes over the radius indicator.
- 5) While one student maintains the speed of rotation, a second student determines the time,  $T$ , for  $N$  rotations. A convenient value of  $N$  is 50. *Repeat this determination a total of at least three times.* Partners should perform different functions from one trial to the next to help eliminate systematic error due to personal bias. Use the maximum and minimum time values to

estimate  $T \pm \delta T$ .

6) Stop the rotation of the system. Hook one end of a piece of string to the hanging body, pass the string over the pulley, and attach the other end to a weight hanger. (Refer to Figure 3.) Add weight to the weight hanger until the pointer on the hanging body is directly over the tip of the radius indicator. Estimate the range over which the mass can vary while still maintaining the hanging body over the radius indicator. Record the total mass attached to the string and its uncertainty,  $M \pm \delta M$ .

7) Add 100 g to the rotating bob and repeat the experiment. If time allows, repeat with an additional 100g on the rotating bob.

### Analysis, Part I

Calculate the centripetal force in the dynamic case from (3) for each mass trial. Use the average value of  $T$  in the calculations. Use the SI system of units.

Calculate the weight needed to stretch the spring in the static case. This value is

$$W = Mg ,$$

where  $M$  is the mass stretching the spring. Use the SI system of units.

### Analysis, Part II

Use (5) to compute the uncertainty,  $\delta F$ , of  $F$  for each trial. Also compute the uncertainty of  $W$  for each, which follows from (6):

$$\delta W = (\delta M) g ,$$

Clearly display the quantities  $F \pm \delta F$  and  $W \pm \delta W$  for each trial in a *neat results table*. On a single one-dimensional graph, plot and label the intervals for each tension. Use points and error bars. Separate the intervals by slightly displacing them vertically from each other.

### Conclusions and Questions

- 1) Do the forces derived from the static and dynamic cases agree to the degree of uncertainty predicted by the fractional uncertainty formula? Is there evidence of *systematic* errors? Can you think of some possible systematic errors that could account for the differences?
- 2) Our expectation that " $F = W$ " (since they both measure the force of the stretched spring) assumes the mass of the spring is negligible compared to the mass  $m$ . Explain why ignoring the mass of

the spring in your data is a *systematic* error, which would make  $F < W$  in (3). Can this error be corrected by simply adding the spring mass to “m” in the formula? Explain. Did you see evidence of this error in your results?

3) Explain why it is reasonable to neglect air resistance in this experiment.

4) Explain why the 100 g additional mass on the bob does not change the value of  $F$  or  $W$ .

5) Why is it important that the spring be horizontal? If not, how would it affect the results?

6) How could you determine the spring constant of the spring? Be specific.