Experiment 13

Rigid Body Equilibrium

A static equilibrium condition is first studied experimentally and then theoretically. A meter stick, with small holes drilled at specific locations, is the rigid body and five forces are applied in order to produce a static condition that satisfies given constraints. Three of these forces have their magnitudes, directions, and points of application specified. The point of application the fourth force is variable and both the magnitude and direction of the fifth force are variable. These variable quantities are to be determined both experimentally and theoretically, and the results compared.

Theory

The conditions for a rigid body to be in equilibrium are that the sum of the external forces acting on a rigid body must be equal to zero,
known as the First Condition of Equilibrium; and that the sum of the external torques acting on a

$$\sum \vec{F} = 0 \, , \qquad (1)$$

rigid body must be equal to zero,

$$\sum \vec{\tau} = 0 \, , \qquad (2)$$

known as the Second Condition of Equilibrium. When the forces are confined to the *x*-*y* plane, (1) and (2) become

$$\sum F_x = 0 \, , \; \sum F_y = 0 \, , \; \sum \tau_z = 0 \, , \qquad (3)$$

where F_x 1 and F_y 2 are the forces in the x 3 and y 4 directions and τ_z 5 is the torque in the z 6 direction (i.e., "around" the z-axis).

In this experiment the forces acting on the rigid body are coplanar. According to (3), then, there exist three independent equations that relate the quantities m_1 7, m_2 8, m_3 9, m_4 10, x 11, θ 12, and m_5 13. (Refer to Figure 1.) Therefore, specification of all but three of the quantities allows the unspecified quantities to be uniquely determined. These variables, or unknowns, are chosen to be x 14, θ 15, and m_5 16.

When the conditions for static equilibrium are applied to any system, the location and orientation of the axes around which the torques are calculated are arbitrary. It is advantageous, then, to locate the origin at a position that will reduce the number of terms in the torque equation. For example, if the origin in Figure 1 is positioned at the 1.0 cm mark on the meter stick, the torques caused by m_1 17 and m_2 18 will be eliminated as well as the torque produced by the horizontal component of m_5 19.

Figure 1. Geometry of the equilibrium problem. The masses $m_1, m_2, m_3,$ 20 and m_4 21 will be specified; the quantities $x, \theta,$ 22 and m_5 23 are unknown.

When the above equations are solved for the variable quantities, the results are

$$x = \frac{40\,m_3 + 90\,m_4 - 89\,m_2}{m_4}, \tag{4}$$

$$\theta = \tan^{-1} \frac{m_3 + m_4 - m_2}{m_1}, \tag{5}$$

and

$$m_5 = m_1 \sqrt{1 + \left(\frac{m_3 + m_4 - m_2}{m_1}\right)^2}, \tag{6}$$

where all quantities are in cgs units.

Apparatus

- o meter stick with holes at the 1 cm and 90 cm marks
- o string
- o 3 50-gram weight hangers

- o 3 rod-mounted pulleys
- o 2 support rods
- o 500 gram weight with hook
- o plastic meter stick
- o double pan balance
- o 3 right angle clamps
- o 2 table clamps
- o assorted slotted weights
- o protractor

Procedure

1) Measure and record the mass of the meter stick, m_3 24.

2) Cut three pieces of string approximately one meter long. Thread two pieces through the hole at the 1.0 cm mark on the meter stick and the third piece through the hole at the 90.0 cm mark.

3) Clamp the table clamp to the ends of the table and attach the support rods, right angle clamps, and rod pulleys as shown in Figure 1.

4) Pass the strings that are attached to the meter stick over the pulleys and hang from each the mass appropriate to your group. (Refer to Figure 2.)

Group Number	m_1 25 (grams)	m_2 26 (grams)	m_4 27 (grams)
1	120	300	500
2	130	300	500
3	130	350	500
4	140	300	500
5	140	350	500
6	150	350	500
7	150	400	500
8	160	350	500
9	160	400	500
10	170	400	500

Figure 2. List of masses to be used in the experiment.

5) Adjust the position x 28 of m_4 29, the amount of mass m_5 30, and the angle θ 31 until the following conditions are satisfied:

a) the meter stick is horizontal

b) the string attaching m_1 32 to the meter stick is horizontal

c) the string attaching m_2 33 to the meter stick is vertical

If the table top is horizontal, the height measurement from the table top to the meter stick will determine whether or not the meter stick is horizontal. The rod pulleys supporting masses m_2 34 and m_5 35 can be moved back and forth to vary θ 36.

6) When you think that the above conditions are satisfied, ask the instructor to approve your setup. Once it is approved, record the position x 37 of m_4 38, record the mass m_5 39, and measure the angle θ 40 with the protractor and record.

7) Due primarily to friction in the pulleys, uncertainties exist in the measurements. Use the following procedure to estimate those uncertainties, while maintaining the meter stick in the specified equilibrium position:

- slide m_4 41 back and forth, and record the range over which m_4 42 can be moved.

- slide the pulley that supports m_5 43 back and forth, and measure and record the range over which θ 44 can vary.

- add and remove mass from m_5 45, and record the range over which m_5 46 can vary.

Analysis

Substitute the values of m_1 47, m_2 48, m_3 49, and m_4 50 in (4), (5), and (6) and find the theoretical values of x 51, θ 52, and m_5 53. Report these values together with the experimentally determined values in a results table. Also include the range over which the experimental values can vary if step (7) in the Procedure is done.

Conclusions

Indicate what the major sources of error are in the experiment and how they affect the values of x 54, θ 55, and m_5 56. Indicate also whether or not the results reflect the presence of any of these sources of error.

Questions

1) Take the origin at the 1.0 cm position and write the two equations for the First Condition of Equilibrium and the equation for the Second Condition of Equilibrium.

2) Show that (4), (5), and (6) arise from the equations in Problem 1.

3) Choose the origin at the 90 cm position on the meter stick, write the equations for static equilibrium, and solve them to show that (4), (5), and (6) are valid.