Experiment 3

Projectile Motion

A mounted spring gun is used to verify the predicted ranges of a projectile. The projectile is fired at various angles of inclination, while its initial height above the ground is held constant.

Theory

The equations of motion for constant acceleration in two dimensions are expressed as

$$x = x_o + v_{ox}t + \frac{1}{2}a_x t^2$$

$$y = y_o + v_{oy}t + \frac{1}{2}a_y t^2 \tag{1}$$

In the case of the gravitational field near the surface of the earth, $a_x = 0$ and $a_y = -g$, where the positive y axis is chosen to be upward.
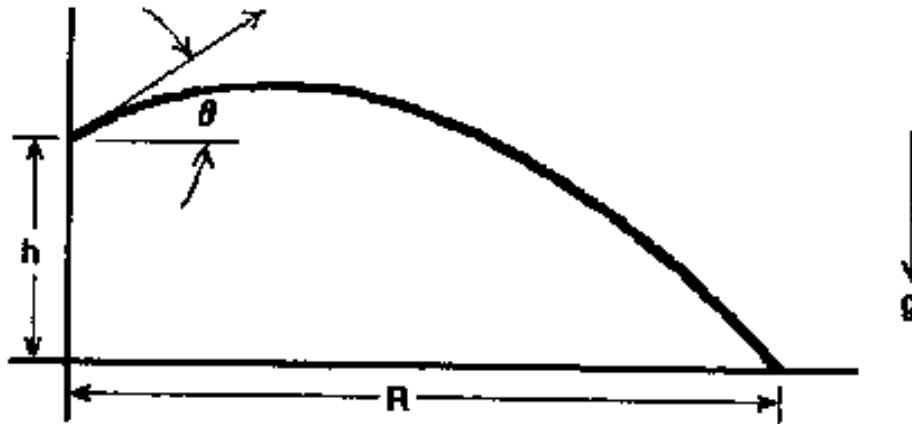

Figure 1. path of a projectile fired with an initial speed v_o.

By substituting the accelerations and the initial positions, $x_o = 0$ and $y_o = h$, the range in given by

$$R(\theta) = \frac{v_o^2 \cos \theta}{g} \left[\sin \theta + \sqrt{\sin^2 \theta + \frac{2gh}{v_o^2}} \right]$$

(2)

for $-90° \leq \theta \leq +90°$. For $\theta = 0°$, (2) yields

$$R(0°) = v_o \sqrt{\frac{2h}{g}} \;,$$

or

$$v_o = R(0°) \sqrt{\frac{g}{2h}} \;.$$

(3)

For $\theta \neq 0°$, (2) may be expressed as

$$R(\theta) = \frac{v_o^2 \sin 2\theta}{2g} \left[1 + \sqrt{1 + \frac{2gh}{v_o^2 \sin^2 \theta}} \right],$$

(4)

for $0° < \theta \leq 90°$. Equations (3) and (4) are the "working equations" for this experiment. The most compact form has been chosen in order to minimize computations.

The angle, $\overline{\theta}$, that maximizes the range is given by

$$\overline{\theta} = \cot^{-1} \sqrt{1 + \frac{2gh}{v_o^2}} \;.$$

(5)

The maximum range is then

$$R(\overline{\theta}) = \frac{v_o^2}{g} \sqrt{1 + \frac{2gh}{v_o^2}} \;.$$

(6)

A tabulation of values of (5) and (6) is given in Appendix II.

Apparatus

  o ballistic pendulum apparatus   o masking tape
  o table clamp and support rod   o inclinometer
  o two right-angle clamps   o tape measure
  o small metal rod used as axle   o plumb line
  o two meter stick   o paper

  The apparatus consists of a spring gun mounted on a platform so that the initial height of the projectile is constant with respect to variations of the angle of inclination, θ. The projectile is a cored steel ball which slips onto the shaft of the spring gun. An inclinometer is used to measure the angle θ.

Procedure

1) Adjust the apparatus such that the initial height of the projectile (when the spring is not compressed) remains fixed as the angle of the inclination, θ, is varied. This is accomplished by having the axle (the small metal rod supporting the ballistic pendulum apparatus) and the center of the projectile on a common line. Before continuing, have the apparatus checked by the instructor.

2) Tape paper to a small section of the floor underneath the apparatus. With the aid of the plumb line, label the floor at the point directly beneath the projectile when the spring is not compressed. The ranges will be measured from this point.

3) With the two meter stick, measure the height, h, from the floor to the <u>bottom</u> of the projectile.

4) Tape paper over the impact area of the floor.

5) Set the apparatus at $0°$. Fire the projectile and circle the impression made by the projectile. Check the angle of inclination prior to each trial and alternate trials between partners for a total of at least five.

6) Repeat at $15°, 30°, 45°, 60°,$ and $75°$. Perform at least five trials for each angle. Support the front of the apparatus so that a constant θ is maintained.

7) Return to $0°$ and perform at least five more trials. (The $\theta = 0°$ data must be taken with special care because the average value of the range will be used to compute the initial velocity of the projectile, v_o.)

8) Place the end of the tape measure at the point directly beneath the apparatus and measure all the $0°$ range values. For the remaining range values, measure only the

maximum and minimum values of the ranges (R_{max} and R_{min}) for each angle of projection.

Extended Procedure (optional): gather data at negative angles. It may be necessary to use a finger to lightly restrain the projectile prior to firing so that it does not fall off the shaft.

Analysis

Determine the average value of all range at $\theta = 0°$, and compute the value of v_o from (3). Insert the value of v_o into (4) and compute and tabulate the theoretical range values, $R(\theta)$, for $\theta = 0°, 15°, 30°, 45°, 60°,$ and $75°$. The range at $\theta = 0°$ is identical to the average experimental value and does not need to be computed. Using (5) and (6), of the table in Appendix II, determine $\overline{\theta}$ and $R(\overline{\theta})$. Clearly display the values in the table of theoretical range data.

Plot the theoretical range data with θ along the abscissa and $R(\theta)$ along the ordinate. Draw a smooth curve through the points.

From the experimental data for each $\theta \neq 0°$, find the range, R, where

$$R = \frac{1}{2}(R_{max} + R_{min}),$$

and R_{max} and R_{min} are the measured values of the maximum and minimum ranges. Find also a measure of the uncertainty, ΔR, by computing the quantity

$$\Delta R = \frac{1}{2}(R_{max} - R_{min}).$$

Include these values in the above table and label them as the experimental values of the ranges and their corresponding uncertainties.

On the graph of the theoretical values of the ranges, plot the experimental ranges, R, and their uncertainties, ΔR. Use points and error bars. Do not draw a curve through the experimental data.

Extended Analysis (optional): Compute and plot $R(\theta)$ from $\theta = -90°$ to $\theta = +90°$. Note: Equation (4) is not valid for negative θ; for these values a negative sign must be placed in front of the radical. Equation (2) is valid for all angles.

Conclusions

Indicate the sources of error in the experiment, and explain how the experimental values are affected by the one source of error that contributes most to the error. Explain whether or not the results show the effects of the sources of error.

Questions

1. Derive (2) from the basic equations of motion, (1). Note: The derivation of (2) involves the solution of a quadratic equation in which two roots emerge. Explain why the negative root is discarded and what it represents.

2. Derive (4) from (2), assuming $\theta > 0°$.

3. Derive (5). Hint: Using (2), find θ such that $dR/d\theta = 0$. Or, it is easier to let $\sin^2 \theta = q$ and find q such that $dR/dq = 0$. Note that $\cos^2 \theta = 1 - \sin^2 \theta = 1 - q$.

4. According to the theoretical range curve, the maximum does not occur at $\theta = 45°$. Why not?

5. Due to the component of the gravitational force along the shaft of the gun, the initial speed of the projectile actually varies according to θ. Why? Give a qualitative sketch of the effect this has upon a plot of the theoretical ranges. Does the actual plot exhibit such a deviation?