

The Ballistic Pendulum

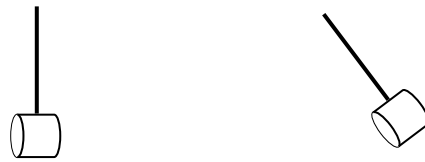
A spring-loaded gun is used to shoot a projectile horizontally into the “bob” of a pendulum. The initial velocity of the projectile can be determined from the vertical rise of the ballistic pendulum using momentum and energy conservation principles. As a check, it is also possible to find the speed of the ball using simple kinematics with the help of a photo-gate timer.

Theory

Method I:

The experiment involves the use of a ballistic pendulum, which immediately catches the ball in a pendulum bob and registers the maximum height attained by the total mass. The ball is given an initial velocity by a spring-loaded gun. During the collision, the momentum of the projectile is transferred to the combined bob-and-projectile system. Momentum conservation can be used to determine the speed of the bob *just after* it catches the projectile. After the impact, the pendulum rises and, if friction is negligible, its kinetic energy changes to potential energy. Working backwards from the height, the initial speed of the projectile can be found. Assuming that the entire mass of the pendulum resides in the bob and that the projectile and bob are point masses, the initial velocity (referred to as “ v_1 ” for this method) is given by

$$v_1 = \left(1 + \frac{M}{m} \right) \sqrt{2gh} ,$$



where m = mass of the projectile, M = mass of the pendulum, and h = the vertical rise of the center of mass of the system. (The details of the derivation can be found in Example 8-8 of the textbook.)

Method II:

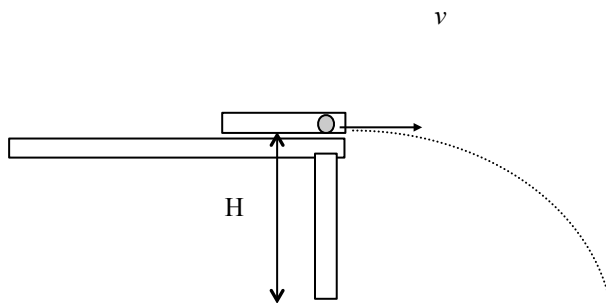
To confirm the results obtained with Method I, we can attach a photo-gate timer at the exit point of the spring-loaded gun and measure the time it takes the projectile to traverse the photo-gate beam. If the photo-gate time interval is small, the instantaneous initial velocity of the projectile (referred to as “ v_2 ” for this method) can be approximated with the average speed formula, so that where D is the diameter of the projectile, and T is the photo-gate time interval.

$$v_2 = D / T ,$$

Method III:

If a photo-gate timer is not available, it is still possible to confirm the results obtained with Method I by firing the ball horizontally into the air and letting it fall to the ground. Then we can use projectile motion kinematics to determine the initial speed of the ball.

With this method the speed of the ball is the horizontal speed of the projectile. The time the projectile spends in the air is determined by the acceleration due to gravity. Therefore,



$$v = R / \Delta t \quad (3)$$

$$\Delta t = \sqrt{2H / g} \quad (4)$$

where R is the horizontal range (Δx) of the projectile, Δt is the time the ball spend in the air, and H is the initial vertical height (Δy) of the ball.

Combining (3) and (4) allows us to calculate the velocity (referred to as “ v_3 ” for this method)from the following formula:

$$v_3 = R\sqrt{g / 2H} \quad (5)$$

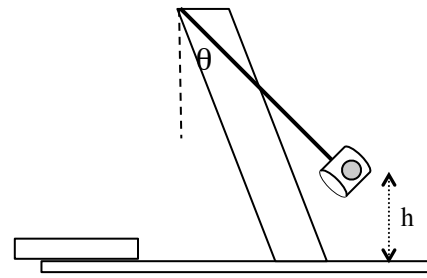
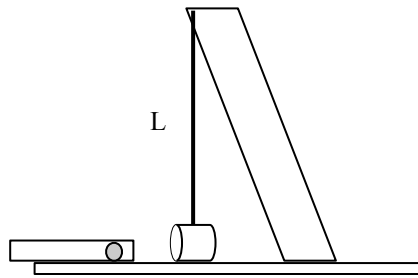
Apparatus

- | | |
|--------------------------------|--------------------|
| o ballistic pendulum apparatus | o photo-gate timer |
| o balance | o masking tape |
| o large table clamp | o meter stick |
| o level | o ruler |

The apparatus is a combination of a ballistic pendulum and a spring gun. The pendulum consists of a massive hollow cylindrical bob that is suspended by a pivoting rod. The projectile is a rubber ball that, when shot into the pendulum bob, is held there by a spring. When the projectile is shot into the bob, the pendulum swings upwards and an indicator reveals the maximum angle reached by the pendulum. Below is a sketch of the apparatus:

Before firing

After firing



The height reached by the bob can be determined from the swing angle θ and the length of the pendulum L . You should be able to prove that

$$h = L(1 - \cos \theta) \quad (6)$$

Procedure

- 1) Measure the mass of the projectile, m , and record its diameter D . Measure the mass of the pendulum, M . Also measure the length of pendulum L from the pivot to the center of mass of the bob.
- 2) Set the apparatus along the edge of the table, and clamp the apparatus. Make sure the apparatus is level.
- 3) Fire the projectile at least 10 times into the ballistic pendulum. Make sure to use the same setting of the spring-loaded gun each time and always pull steadily when releasing the spring.
- 4) After each firing, record the angle of elevation of the pendulum. Make sure the angle indicator started from zero or make an appropriate correction.
- 5) Attach the photo-gate to measure the exit motion of the projectile. Again fire the projectile ten times.
- 6) After each firing, record the time interval of the photo-gate. Use the "Gate" setting and 0.1 ms range.
- 7) To find v by the projectile method, use a plumb bob to mark the point of the ground directly below the center of mass of the ball when it is released.
- 8) Measure the vertical height H that the ball will fall.
- 9) Lift the pendulum out of the way of the projectile then fire the projectile into the air and note where it lands. Place a piece of paper below the landing site.

- 10) Repeat the firing ten times and measure the range for each firing.
- 11) If time permits, put a rubber band around the pendulum bob and fire the ball so that it will bounce back. Observe the height of the swing in this case.

Analysis, Method I

Calculate the average velocities for each method. Compare methods I and II, and also methods I and III if you use the projectile method. Do this by calculating the percentage difference between the averages in each pair of methods. Remember the percentage difference formula is:

Use the maximum and minimum velocity values to roughly estimate the uncertainties in the velocity averages. Put all the results in a *final Results Table* with proper significant digits. Also

$$\frac{|v_1 - v_2|}{\frac{1}{2}(v_1 + v_2)} \times 100\%.$$

include an error bar graph of your results.

Analysis, Part II

A better way of analyzing the data in this experiment is to use statistical methods. In addition to the *average* value, one can determine what is called the “*standard deviation*” using a well-known formula (derived using statistical mathematical methods). The *standard deviation* calculates how a set of data points is distributed about the average value. We can use the *standard deviation* as an indication of *precision* in the results. Precision tells you how consistent or repeatable an experimental result is. This is different from *accuracy* which tells how close your measurement is to the correct value.

For a set of “N” values of “x”, the standard deviation is calculated with the formula given below, but most scientific calculators have a programmed function that will calculate it automatically after you enter a set of data points (check your calculator!).

Determine the standard deviation of the velocities in each method. Indicate the standard deviations in the error graph of results. Do the velocity values agree to one standard deviation?...

$$SD = \sqrt{\frac{\sum_{i=1}^{i=N} (x_i - \bar{x})^2}{N - 1}}$$

two standard deviations? Add the standard deviations to your “Table of Results” and draw the standard deviations in your error bar graph.

Conclusions and Questions

- 1) Given the estimated uncertainties, was the experiment a success? Which method was

more *precise*, which method do you think was more *accurate*? Justify your answers.

- 2) Make sure you derived equation (1) in your theory section, using momentum and energy conservation principles. If you need help, check text or notes. Be sure to *explain* your derivation. Also justify formula (6) for determining h .
- 3) The initial kinetic energy of the projectile is $(mv^2/2)$. The final potential energy of the system is $(m+M)gh$. However these two values are not equal, even under ideal conditions, in this experiment. Explain what happened to the missing mechanical energy and why this “missing energy” doesn’t affect the results at all.
- 4) Resistive forces are present in three distinct places in this experiment. (i) The bob exerts a frictional force on the ball to bring it to rest relative to the bob during the collision. (ii) There is friction in the pendulum support. (iii) There is air resistance as the pendulum swings.
Compare and contrast these three different resistive forces. What effect would each of these forces have on the results? Justify your answers.
- 5) Compare and contrast the three different methods of determining the speed of the projectile. What is the major *source of error* in each of the methods used to determine the speed of the projectile? Did these methods *underestimate* or *overestimate* the true value of v ? Explain.
- 6) If the projectile were to bounce off the bob, would the pendulum swing higher or lower? Explain your answer.