PROBLEM 1 (KRANE # 23)

COPPER HAS ONE VALENCE ELECTRON. THIS MEANS IT HAS ONE ELECTRON AVAILABLE FOR CONDUCTION PER ATOM. GOOGLE GIVES THE DENSITY OF COPPER AS $8.96 \ g/cm^3$ & 1 COPPER ATOM HAS MASS $1.0552 \cdot 10^{-22}$ g/atom THIS GIVES

$$\frac{8.96}{1.0552 \cdot 10^{-22}} \frac{g \cdot atom}{g \cdot cm^3} = 8.49 \cdot 10^{22} \text{ atoms per } cm^3$$

OR, IF $N = \#$ electrons, $\frac{N}{V} = 8.49 \cdot 10^{22}$.

NOW, EQUATION (10.50) GIVES

$$E_F = \frac{h^2}{2m}\left(\frac{3}{8\pi} \cdot \frac{N}{V}\right)^{2/3} = \frac{h^2}{2m}\left(\frac{3}{8\pi} \cdot 8.49 \cdot 10^{22}\right)^{2/3}$$

$$= \frac{(1240 \ \cancel{eV} \cdot nm)^2}{2(.511 \times 10^6 \ \cancel{eV})}\left(\frac{3}{8\pi} \cdot 8.49 \cdot 10^{22} \ cm^{-3}\right)^{2/3} = 7.0456 \cdot 10^{14} \ \cancel{nm^2} \cdot \cancel{cm^{-2}} \cdot eV$$

$$= \boxed{7.0456 \ eV}$$

$$\Rightarrow E_m = \frac{3}{5} E_F = \boxed{4.23 \ eV}$$

PROBLEM 2 (KRANE #35)

$$f_{FD}(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

$$\Rightarrow e^{(E-E_F)/kT} = \frac{1}{f_{FD}(E)} - 1$$

$$\Rightarrow E = kT\left[\ln\left(\frac{1}{f_{FD}(E)} - 1\right)\right] + E_F$$

$$\Rightarrow E_{0.1} - E_{0.9} = kT\left[\ln\left(\frac{1}{f_{FD}(E_{0.1})} - 1\right)\right] + \cancel{E_F} - \left\{kT\left[\ln\left(\frac{1}{f_{FD}(E_{0.9})} - 1\right)\right] + \cancel{E_F}\right\}$$

$$= kT\left[\ln\left(\left(\frac{1}{f_{FD}(E_{0.1})}\right) - 1\right) - \ln\left(\left(\frac{1}{f_{FD}(E_{0.9})}\right) - 1\right)\right].$$

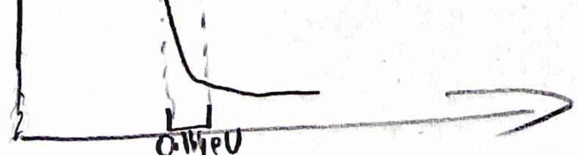
WITH

$$= kT\left[\ln(9) - \ln(.11)\right] = kT\left[\ln(81.81)\right]$$

$$= 4.4 \cdot kT = 4.4(8.617\times10^{-5}\ eV\ K^{-1})(293K)$$

$$= \boxed{.111\ eV}\ .$$

THIS SHOULD MAKE FOR A SHARP DISTRIBUTION WITH THE PROBABILITY DROPPING IN A SMALL, 0.111eV, INTERVAL.



PROBLEM 3

LET SOME VOLUME OF BOSONS HAVE

DENSITY $0.125\, g/cm^3$ AND $\text{mass} = m = 4u = 4\cdot(931{,}500000\ eV)/c^2$

$= 3.726\cdot 10^9\, eV/c^2$

$= 6.64\cdot 10^{-24}\ g/\text{atom}$

$$\Rightarrow \frac{N}{V} = \frac{0.125}{6.64\cdot 10^{-24}} \frac{\text{atoms}\ g}{g\cdot cm^3}$$

$$= 1.88\cdot 10^{22}\ cm^{-3} = \eta$$

FROM CLASS NOTES EQUATION (10).

$$T_c = \frac{h^2}{2mk_B}\left(\frac{\eta}{2\pi(2.315)}\right)^{2/3} = \frac{h^2}{2(3.726\cdot 10^9\, eV/c^2)(k_B)}\left(\frac{\eta}{2\pi(2.315)}\right)^{2/3}$$

$$= \frac{(hc)^2}{2(3.726\cdot 10^9\, eV)(k_B)}\left(\frac{\eta}{2\pi(2.315)}\right)^{2/3} = \frac{(1240\, eV\cdot nm)^2}{2(3.726\cdot 10^9)(8.617\cdot 10^{-5})eV^2 K^{-1}}\left(\frac{1.88\cdot 10^{22}\, cm^{-3}}{2\pi(2.315)}\right)^{2/3}$$

$$= 2.39\, nm^2 K \left(1.29\cdot 10^{21}\ cm^{-3}\right)^{2/3} = 2.83\cdot 10^{14}\ nm^2\, cm^{-2}\ K$$

$$= 2.83\cdot 10^{14}\ (10^{-7} cm)^2 cm^{-2}\ K$$

$$= 2.83\cdot 10^{14}\cdot 10^{-14}\ K = \boxed{2.83\ K.}$$

THIS IS CLOSE TO THE ACTUAL T_c.

Problem 4

a.) $\frac{N}{V} = \frac{1.9 \cdot 10^4}{3.27 \cdot 10^{-25}} \frac{\text{atms} \; \text{kg}}{\text{kg} \cdot m^3} = 5.81 \cdot 10^{28} m^{-3}$

$\Rightarrow E_F = \frac{h^2}{2m_e}\left(\frac{3}{4\pi} \cdot 5.81 \cdot 10^{28} m^{-3}\right)^{2/3}$

$= \frac{(1240 \text{eV nm})^2}{2(.511 \cdot 10^6 \text{eV})}\left(3.64 \cdot 10^{18} m^{-2}\right)$

$= 5.48 \cdot 10^{18} \text{eV nm}^2 m^{-2} = \boxed{5.48 \text{ eV}}$

b.) $\Rightarrow E_m = \frac{3}{5} E_F = \boxed{3.288 \text{ eV}}$

c.) For a (MB) gas

$\langle E \rangle = \frac{3 k_B T}{2} = \frac{3(.02525) eV}{2} = \boxed{0.0388 \text{ eV}}$

The low energy states are guarded by the Pauli principle, a stipulation that isn't accounted for in (MB) statistics.

PROBLEM 5

a.)

FROM CLASS NOTES, ASSUMING THE SOMMERFELD MODEL GIVES THE DENSITY OF STATES AS

$$g(E) = \frac{\sqrt{2m^3}}{\pi^2 \hbar^3} V E^{1/2} .$$

THEREFORE,

$$n(E) = \frac{\frac{\sqrt{2m^3}}{\pi^2 \hbar^3} V E^{1/2} \cdot N \cdot \exp\left\{-\frac{E}{k_B T}\right\}}{\frac{m^{3/2} V}{\sqrt{2\pi^3}\hbar^3} (k_B T)^{3/2}}$$

$$= \frac{2\sqrt{\pi} \cdot \sqrt{E} \cdot N \cdot \exp\left\{-\frac{E}{k_B T}\right\}}{(k_B T)^{3/2}}$$

FROM PROBLEM 4,

$$\frac{N}{V} = 5.81 \cdot 10^{28} m^{-3} = 5.81 \cdot 10^{28} \cdot (10^{2} cm)^{-3} = 5.81 \cdot 10^{22} cm^{-3}$$

$$\Rightarrow N = 5.81 \cdot 10^{22} cm^{-3} \cdot V$$

WHERE $V = 1 cm^3$ $\Rightarrow N = 5.8 \cdot 10^{22}$ AT ROOM TEMP

$k_B T = .02525 \, eV$.

PLUGGING THOSE INTO OUR EQUATION ABOVE

GIVES

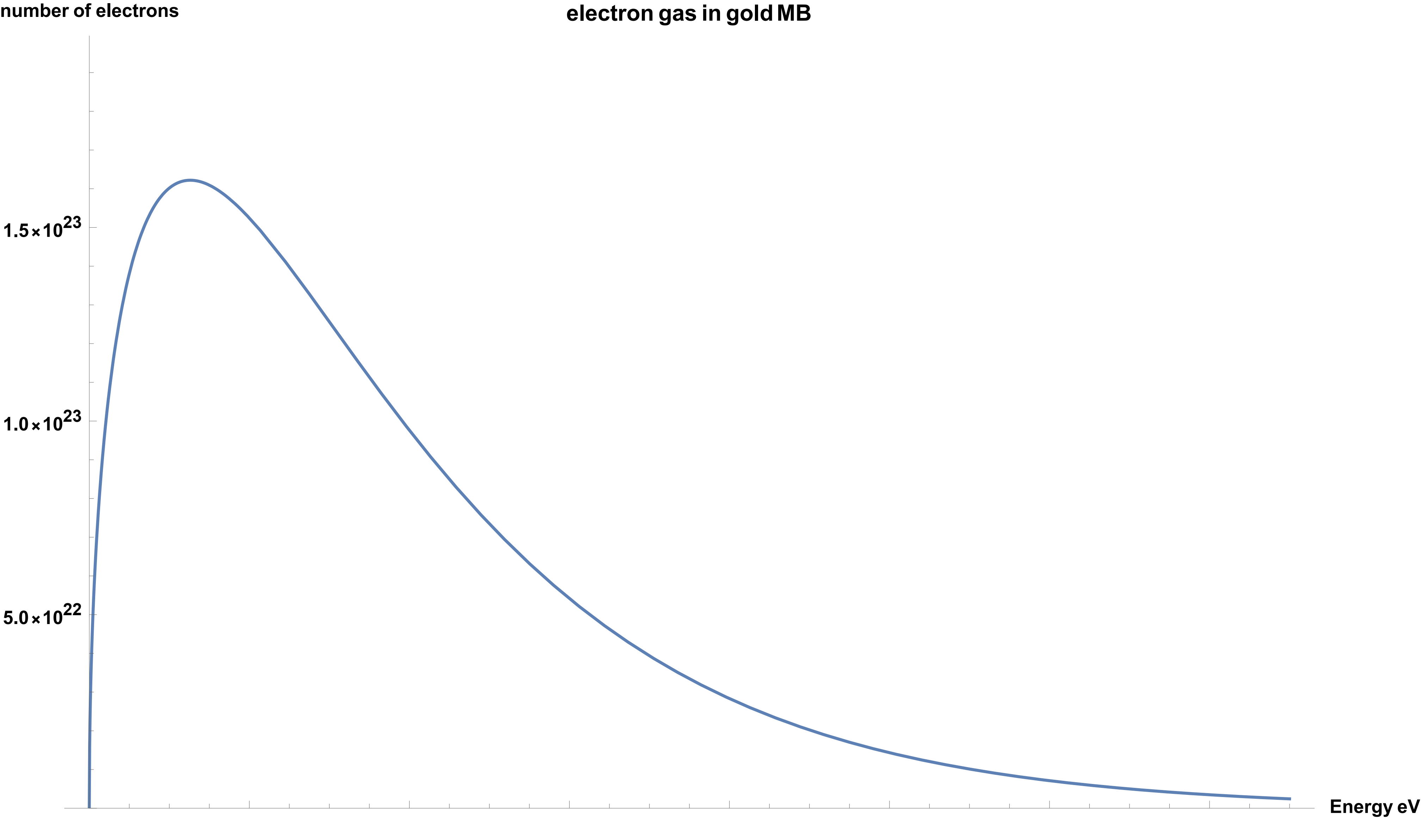
$$n(E) = 2.38 \cdot 10^{24} \cdot \sqrt{E} \exp\left\{ -\frac{E}{.02525\,eV} \right\}$$

FOR MB. FOR FD, WE HAVE

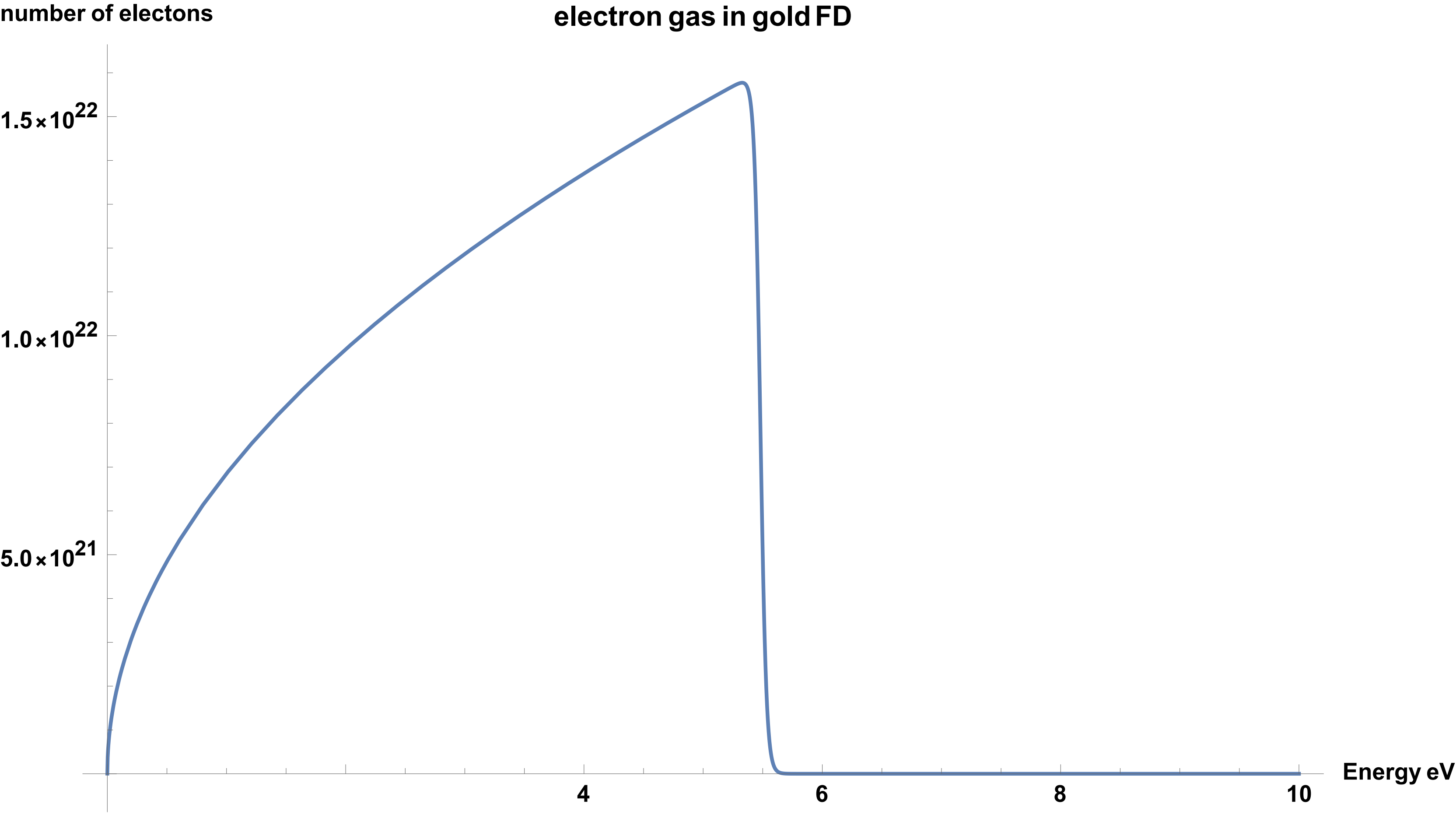
$$n(E) = G(E) f_{FD}(E) = \frac{1}{\exp\left\{ \frac{E - 5.48\,eV}{0.02525\,eV} \right\} + 1} \cdot \frac{\sqrt{2m^3}}{\pi^2 \hbar^3} E^{1/2}$$

$$= \frac{6.85 \cdot 10^{21}\,cm^{-3} \sqrt{E}}{\exp\left\{ \frac{E - 5.48\,eV}{0.02525} \right\} + 1}$$

PLOTTED WITH MATHEMATICA, THEY DO NOT HAVE THE SAME SHAPE. THEY DO NOT PEAK AT THE SAME ENERGY, AS SUSPECTED FROM PROBLEM 4. THEY AGREE AT VERY HIGH ENERGY WHERE BOTH GO TO ZERO OCCUPANCY.



electron gas in gold MB
number of electrons
1.5×10^23
1.0×10^23
5.0×10^22
Energy eV



number of electons
electron gas in gold FD
1.5×10^22
1.0×10^22
5.0×10^21
Energy eV
4
6
8
10

Problem 6

a.) $n(E) = g(E) \cdot f_{BE}(E)$

$$= \frac{V 8\pi E^2}{(hc)^3 \left(e^{E/kT} - 1\right)}$$

$$\Rightarrow \frac{E\, n(E)}{V} = \frac{8\pi E^3}{(hc)^3 \left(e^{E/kT} - 1\right)} = u(E)$$

b.) $$u(\lambda) = u(E) \frac{dE}{d\lambda} = \frac{\frac{dE}{d\lambda} E^3 \cdot 8\pi}{(hc)^3 \left(e^{E/kT} - 1\right)}$$

$$= \frac{(hc) \cdot \lambda^{-2} \cdot \left(\frac{hc}{\lambda}\right)^3}{(hc)^3 \left(e^{E/kT} - 1\right)} = \frac{8\pi hc}{\lambda^5 \left(e^{E/kT} - 1\right)}$$

c.) $$I(\lambda) = \frac{c}{4} \cdot u(\lambda) = \frac{2\pi hc^2}{\lambda^5 \left(e^{E/kT} - 1\right)}$$