PROBLEM 1 (KRANE 18)

WIEN LAW GIVES

$$\lambda_{max} = \frac{2.8978 \cdot 10^{-3}\, m \cdot K}{6000\, K} = 482\, nm.$$

GOOGLE GIVES PEAK SENSITIVITY OF THE HUMAN EYE AS 555nm. THIS, IS UN SUPRISINGLY VERY CLOSE TO λ_{max}.

PROBLEM 2

STEFAN'S LAW GIVES $I = 5.67037 \cdot 10^{-8}\ W/m^2 \cdot K^4 \cdot (6000\ K)^4$.

IF $r = 6.96 \cdot 10^8\ m$ IS THE RADIUS OF THE SUN,

THE AREA THAT IS EARTH ADJACENT IS

$$\frac{SA}{2} = \frac{4\pi\,(6.96 \cdot 10^8\ m)^2}{2} = 2\pi\,(6.96 \cdot 10^8\ m)^2$$ THE

THE POWER OF THE SUN IS

$$P = \frac{I\,SA}{2} = \left(5.67037 \cdot 10^{-8}\ W\cancel{m^{-2}}\cancel{K^{-4}} \cdot (6000\cancel{K})^4\right) \cdot 2\pi\left(6.96 \cdot 10^8\ \cancel{m}\right)^2$$

$$= \boxed{2.24 \cdot 10^{26}\ \text{watts.}}$$

Problem 3

Wien's law gives

$$\lambda_{max} = \frac{2.8978 \cdot 10^{-3}\, m \cdot K}{2.7K} = 1mm$$

The energy of a photon with this wavelength is

$$E = \frac{h(c)}{\lambda_{max}} = \frac{(4.136 \cdot 10^{-15}\, eVs)(2.998 \cdot 10^{8}\, m s^{-1})}{1mm}$$

$$= \boxed{1.24 \cdot 10^{-3}\, eV}.$$

Wikipedia has $\lambda = 1mm \iff 1.24 meV$ in the FIR (Far Infrared) region of the spectrum, which is on the cusp of the microwave region.

PROBLEM 4

a.) WIEN LAW GIVES

$$\lambda_{max} = \frac{2.8978 \cdot 10^{-3}\, m \cdot K}{2800K} = 1.03 \cdot 10^{-6}\, m$$

$$\approx \boxed{1 \text{ micron}}$$

WIKI PUTS THIS IN THE NUV (Near ULTRA VIOLET), SO THE UPPER END OF THE VISIBLE SPECTRUM.

b.) THE TOTAL INTENSITY IN A 15nm NIEGHBORHOOD AROUND $\lambda = 300nm$ CAN BE ESTIMATED AS FOLLOWS:

$$\int_{292.5nm}^{307.5nm} I(\lambda) \approx 15_{nm} \cdot \left(I(300nm) \right) = 15_{nm} \cdot \frac{2\pi h c^2}{(300nm)^5} \cdot \frac{1}{\exp\left\{\frac{hc}{300nm\, k (2800K)}\right\} - 1}$$

$$\frac{1.5 \cdot 10^{-8}_{m} \cdot 2\pi \left(2.998 \cdot 10^{8}\, m\, s^{-1} \right)^2 \left(6.626 \cdot 10^{-34}\, J \cdot s \right)}{(3 \cdot 10^{-7}_{m})^5 \left[\exp\left\{ \frac{(6.626 \cdot 10^{-34}\, \cancel{J} \cdot \cancel{s}) \cdot (2.998 \cdot 10^{8}\, \cancel{s^{-1}})}{(3 \cdot 10^{-7}) (1.381 \cdot 10^{-23}\, \cancel{J}\, \cancel{K^{-1}}) (2800\cancel{K})} \right\} - 1 \right]} = \boxed{84.5\, J\, s^{-1}\, m^{-2}}$$

c.) USING THE EQUATION FROM PART b.)

WITH $T = 6000K$ GIVES

$$\underline{I} = \boxed{5.21 \cdot 10^{13} \, J s^{-1} m^{-2}} .$$

PROBLEM 5 (KRANE 15)

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$$

Product rule $\Rightarrow$

$$I'(\lambda) = 2\pi hc^2 \left[\left(\lambda^{-5}\right)' \cdot \frac{1}{e^{hc/\lambda kT} - 1} + \frac{1}{\lambda^5} \cdot \left(\frac{1}{e^{hc/\lambda kT} - 1}\right)' \right]$$

$$= 2\pi hc^2 \left[-5\lambda^{-6} \cdot \frac{1}{e^{hc/\lambda kT} - 1} + \lambda^{-5} \cdot \left(\frac{hc}{kT\lambda^2}\right)\left(e^{hc/\lambda kT} - 1\right)^{-2} \cdot e^{hc/kT\lambda} \right]$$

$$= 2\pi hc^2 \left[-5\lambda^{-6}\left(e^{hc/\lambda kT} - 1\right)^{-1} + \lambda^{-7} \cdot \frac{hc}{kT}\left(e^{hc/\lambda kT} - 1\right)^{-2} \cdot e^{hc/kT\lambda} \right]$$

$$= \underbrace{2\pi hc^2 \left(\lambda^{-6}\left(e^{hc/\lambda kT} - 1\right)^{-1}\right)}_{>0} \underbrace{\left[-5 + \frac{hc}{\lambda kT}\left(e^{hc/\lambda kT} - 1\right)^{-1} \cdot e^{hc/kT\lambda} \right]}$$

$\lambda^{-6} > 0$ & $e^{hc/\lambda kT} > 1$ $\Rightarrow$

$$\left[\left(e^{hc/\lambda kT} - 1\right)^{-1}\right]' = -\left(e^{hc/\lambda kT} - 1\right)^{-2} \cdot \left[e^{hc/k\lambda T}\right]'$$

$$= \left(e^{hc/\lambda kT} - 1\right)^{-2} \cdot \frac{hc}{kT\lambda^2} \cdot e^{hc/k\lambda T}$$

LETTING $\frac{hc}{\lambda kT} = x$ AND USING MATHEMATICA NSOLVE ON THE PURPLE BRACKET FINDS THAT $I'(\lambda) = 0$ WHEN

$$x = 4.96511 = \frac{hc}{\lambda kT}$$

$$\Rightarrow \boxed{\lambda} = \frac{hc}{k(4.96511)} \cdot \frac{1}{T}$$

$$= \boxed{\frac{.002897\ m \cdot k}{T}}$$

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In[39]:= NSolve[x*Exp[x]*(Exp[x]-1)^-1 - 5 == 0, x, Reals]

Out[39]= {{x → 4.96511}}
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Problem 6 (Krane 16)

$$\int_0^\infty I(\lambda)\,d\lambda = \int_0^\infty \frac{2\pi hc^2}{\lambda^5}\cdot\frac{1}{e^{hc/\lambda kT}-1}\,d\lambda$$

$$= 2\pi c\int_0^\infty \frac{hc}{\lambda^5}\cdot\frac{1}{e^{hc/\lambda kT}-1}\,d\lambda$$

$$= -2\pi c\cdot\left(\frac{k^4T^4}{h^3c^3}\right)\int_{\cancel{0}}^{\cancel{\infty}} \frac{h^3c^3}{\lambda^3k^3T^3}\cdot\frac{1}{e^{hc/\lambda kT}-1}\cdot -\frac{hc}{\lambda^2kT}\,d\lambda$$

$$= \cancel{-}\frac{2\pi k^4T^4}{h^3c^2}\int_{\cancel{\infty}}^{\cancel{0}} x^3\,\frac{1}{e^x-1}\,dx$$

$$x = \frac{hc}{\lambda kT}$$

$$\frac{dx}{d\lambda} = -\lambda^{-2}\cdot\frac{hc}{kT}$$

$$dx = -\frac{hc}{\lambda^2kT}\,d\lambda$$

$$x^3 = \frac{h^3c^3}{\lambda^3k^3T^3}$$

$$= \frac{2\pi k^4T^4}{h^3c^2}\cdot\frac{\pi^4}{15} = I_{total} \Rightarrow I_{total} = \sigma T^4$$

WHERE $\sigma = \dfrac{2\pi^5k^4}{15h^3c^2}$.