HOMEWORK 10

PROBLEM 1 (KRANE #3)

a.) $S_z = m_s \hbar$.

$S = 1 \Rightarrow m_s \in \{-1, 0, 1\}$

$S = \frac{1}{2} \Rightarrow m_s \in \{-\frac{1}{2}, \frac{1}{2}\}$.

THIS SHOULD ALLOW FOR $3 \cdot 2 = 6$ MICROSTATES.

b.) THE ASSOCIATED TOTAL SYSTEM SPIN IN THE Z DIRECTION FOR THESE 6 MICROSTATES ARE

$-\frac{3}{2}, \underbrace{-\frac{1}{2}, -\frac{1}{2}}, \underbrace{\frac{1}{2}, \frac{1}{2}}, \frac{3}{2}$.

OR 4 TOTAL MACROSTATES.

c.)

MACRO	MICRO	# of micro
$-\frac{3}{2}$	$-1-\frac{1}{2}$	1
$-\frac{1}{2}$	$0-\frac{1}{2}, -1+\frac{1}{2}$	2
$\frac{1}{2}$	$0+\frac{1}{2}, 1-\frac{1}{2}$	2
$\frac{3}{2}$	$1+\frac{1}{2}$	1

PROBLEM 2 (KRANE #13)

LET $E = 0\,eV$, $E_1 = 0.045\,eV$, $E_2 = 0.135\,eV$. WE HAVE NONDEGENERACY, $T = 650\,K$, EQUATION (5) FROM CLASS NOTES GIVES

$$n_0 = \frac{N e^{-0}}{Z}, \quad n_1 = \frac{N e^{-\frac{.045\,eV}{.056\,eV}}}{Z} = \frac{N}{Z} \cdot (\cancel{2.23}) \; 0.45$$

$$n_2 = \frac{N e^{-\frac{0.135\,eV}{.056\,eV}}}{Z} = \frac{N}{Z} \cdot (\cancel{11.14}) \; 0.09$$

$$\Rightarrow \frac{n_1}{n_0} = \frac{1}{2.23} = \boxed{.45} \quad + \quad \frac{n_2}{n_0} = \frac{1}{11.14} = \boxed{.09}.$$

PROBLEM 3

THE AVERAGE SHOULD BE

$$\frac{1.0 + (.45)(.045) + (.09)(.135)}{3} = \boxed{0.0108\,eV}$$

PROBLEM 4

$$\frac{n_2}{n_1} = \frac{1}{10\text{ million}} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}}. \quad \text{FOR HYDROGEN,}$$

$$E_2 = -3.4\,eV \;\&\; E_1 = -13.6\,eV.$$

$$\Rightarrow \quad \frac{1}{10\text{ million}} = e^{\frac{1}{kT}(-3.4\,eV + 13.6\,eV)} = e^{-\frac{1}{kT}(-10.2)}$$

$$\Rightarrow \quad \ln\left(\frac{1}{10\text{ mil}}\right) = \frac{1}{kT}\cdot(-10.2\,eV)$$

$$\Rightarrow \quad kT = \frac{-10.2\,eV}{\ln\left(\frac{1}{10\text{ mil}}\right)} \Rightarrow T = \frac{-10.2\,eV}{k\cdot\ln\left(\frac{1}{10\text{ mil}}\right)}$$

$$= \frac{-10.2\,eV}{(8.617\,eV/K\cdot 10^{-5})\left(\ln\left(\frac{1}{10\text{ mil}}\right)\right)}$$

$$= \boxed{7343.96\text{ K}}.$$

PROBLEM 5

a.) $$\frac{n_0}{n_1} = \frac{e^{-\frac{\hbar\omega}{2kT}}}{e^{-\frac{3\hbar\omega}{2kT}}} = e^{\frac{\hbar\omega}{kT}}$$

b.) $$\frac{n_0}{n_1} = e^{\frac{.01eV}{.026eV}} = 1.47$$

c.) $$\frac{n_0}{n_2} = \frac{e^{-\frac{\hbar\omega}{2kT}}}{e^{-\frac{5\hbar\omega}{2kT}}} = e^{\frac{2\hbar\omega}{kT}} = \left(e^{\frac{\hbar\omega}{kT}}\right)^2 = (1.47)^2 = 2.161$$

PROBLEM 6

a.) WE HAVE $E_n = \hbar\omega\left(n+\frac{1}{2}\right)$. FROM EQUATION (6) FROM CLASS NOTES, WE HAVE

$$Z = \sum_{0}^{\infty} \exp\left\{-\hbar\omega\left(n+\frac{1}{2}\right)/kT\right\}$$

$$= \sum_{0}^{\infty} \exp\left\{-n\frac{\hbar\omega}{kT} - \frac{\hbar\omega}{2kT}\right\}$$

$$= e^{-\frac{\hbar\omega}{2kT}} \sum_{0}^{\infty} e^{-n\frac{\hbar\omega}{kT}} = e^{-\frac{\hbar\omega}{2kT}} \sum_{n=0}^{\infty} \left(e^{-\frac{\hbar\omega}{kT}}\right)^n$$

$$= e^{-\frac{\hbar\omega}{2kT}} \left(\frac{1}{1-e^{-\frac{\hbar\omega}{kT}}}\right)$$

FROM THE HINT

b.) $\langle E \rangle = \sum_{n=0}^{\infty} E_n P(E_n)$ WHERE $P(E_n)$ IS GIVEN BY EQUATION (5) CLASS NOTES DIVIDED BY THE TOTAL PARTICLES N, SUCH THAT

$$P(E_n) = \frac{e^{-E_n/kT}}{Z}.$$

$$\Rightarrow P(E_n) = \frac{\left(1-e^{-\frac{\hbar\omega}{kT}}\right)\cdot e^{-E_n/kT}}{e^{-\frac{\hbar\omega}{2kT}}}$$

$$= e^{\frac{\hbar\omega}{2kT}}\left(1-e^{-\frac{\hbar\omega}{kT}}\right)\cdot e^{-\frac{\hbar\omega\left(n+\frac{1}{2}\right)}{kT}}$$

$$= e^{\frac{\hbar\omega}{kT}\left(\frac{1}{2}-n-\frac{1}{2}\right)}\cdot\left(1-e^{-\frac{\hbar\omega}{kT}}\right) = e^{-\frac{\hbar\omega}{kT}\cdot n}\left(1-e^{-\frac{\hbar\omega}{kT}}\right)$$

$$\Rightarrow \langle E\rangle = \sum_{0}^{\infty}\hbar\omega\left(n+\frac{1}{2}\right)\left[\left(e^{-\frac{\hbar\omega}{kT}}\right)^n\cdot\left(1-e^{-\frac{\hbar\omega}{2kT}}\right)\right]$$

$$= \hbar\omega\left(1-e^{-\frac{\hbar\omega}{kT}}\right)\sum_{0}^{\infty}\left(n+\frac{1}{2}\right)\left(e^{-\frac{\hbar\omega}{kT}}\right)^n$$

$$= \hbar\omega\left(1-e^{-\frac{\hbar\omega}{kT}}\right)\left[\sum_{0}^{\infty} n\left(e^{-\frac{\hbar\omega}{kT}}\right)^n + \frac{1}{2}\sum\left(e^{-\frac{\hbar\omega}{kT}}\right)^n\right]$$

$$= \hbar\omega\,\cancel{\left(1-e^{-\frac{\hbar\omega}{kT}}\right)}\left[\frac{e^{-\frac{\hbar\omega}{kT}}}{\left(1-e^{-\frac{\hbar\omega}{kT}}\right)^{\cancel{2}}} + \frac{1}{2}\cdot\frac{1}{\cancel{\left(1-e^{-\frac{\hbar\omega}{kT}}\right)}}\right]$$

$$= \hbar\omega\left[\frac{1}{2} + \frac{e^{-\frac{\hbar\omega}{kT}}}{1-e^{-\frac{\hbar\omega}{kT}}}\right]$$

$\boxed{\text{HINT}}$

c.) THERFORE

$$\langle E \rangle_{solid} = 3N \cdot \langle E \rangle$$

$$= 3N \left[\frac{1}{2} + \frac{e^{-\frac{\hbar\omega}{kT}}}{1 - e^{-\frac{\hbar\omega}{kT}}} \right].$$

PROBLEM 7

a.) $KE = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{p}{m}\right)^2 = \frac{p^2}{2m} = \frac{\left(\frac{h}{\lambda}\right)^2}{2m}$

$$= \frac{h^2}{2\lambda^2 m} = \frac{3}{2} k_B T$$

$$\Rightarrow \lambda = \sqrt{\frac{h^2}{3mk_B T}} = \frac{h}{\sqrt{3mk_B T}}$$

b.)

$$\lambda << d$$

$$\Rightarrow \frac{h}{\sqrt{3mk_B T}} << d$$

$$\Rightarrow \frac{\sqrt{3mk_B T}}{h} >> \frac{1}{d} \Rightarrow \frac{\left(\sqrt{3mk_B T}\right)^3}{h^3} >> \frac{1}{d^3} = \frac{N}{V}$$

$$\Rightarrow 1 >> \frac{h^3}{\left(\sqrt{3mk_B T}\right)^3} \cdot \frac{N}{V}$$

c.) FOR A GAS, $\frac{N}{V} = \frac{P}{kT} = \frac{1 \text{ mole}}{1 \text{ cm}^3} = \frac{10^{23}}{10^{-6} \text{ m}^3} = 10^{29} \text{ m}^{-3}$

$m_{argon} = 6.63 \cdot 10^{-23} \text{ g}$

$$\frac{h^3}{(3mk_BT)^{3/2}} \quad \frac{N}{V} = \frac{N}{V} \cdot \frac{(6.626 \cdot 10^{-34} J s)^3}{\left[3(6.63 \cdot 10^{-23} g)(1.381 J \cdot K^{-1} \cdot 10^{-23})(293 K)\right]^{3/2}}$$

$$= \frac{N}{V}\left(4 \cdot 10^{-37}\right) \approx 10^{-14} << 1 .$$

MB IS VALID FOR IDEAL GAS.

SILVER HAS $10.49 g/cm^3 = 10.49 \cdot \left(\frac{1}{108} \frac{mole}{g}\right) g/cm^3 \doteq \frac{10.49}{108} \frac{mole}{cm^3}$

$$= \frac{10.49}{108} \cdot 10^{23} cm^{-3}$$

ASSUMING EACH ATOM CONTRIBUTES 2 electrons

WE HAVE ROUGHLY 10^{22} electrons per cm^3 or $10^{28} m^{-3}$. NOW

$$10^{28} \cdot \frac{h^3}{(3mk_BT)^{3/2}} \approx 10^3$$ WHICH IS NOT $<< 1$.

MB DOES NOT MODEL THIS WELL.