Homework 8

Problem 1 (#4 Krane)

Let $\ell = 3$. From equations 7.5, 7.6, 7.7 we have

a.) $|\vec{L}| = \sqrt{3(3+1)}\hbar = \boxed{\sqrt{12}\hbar} = 3.653 \cdot 10^{-34}$ J·s

b.) $L_z \in \{-3\hbar, -2\hbar, -1\hbar, 0, \hbar, 2\hbar, 3\hbar\}$

$\Rightarrow$ 7 possible z components.

c.)

$$\theta \in \left\{ \cos^{-1}\left(\frac{-3}{\sqrt{12}}\right), \cos^{-1}\left(\frac{-2}{\sqrt{12}}\right), \cos^{-1}\left(\frac{-1}{\sqrt{12}}\right), \cos^{-1}(0), \right.$$

$$\left. \cos^{-1}\left(\frac{1}{\sqrt{12}}\right), \cos^{-1}\left(\frac{2}{\sqrt{12}}\right), \cos^{-1}\left(\frac{3}{\sqrt{12}}\right) \right\}$$

Problem 2 (Krane # 5)

$$l=2 \Rightarrow L_z \in \{-2\hbar, -1\hbar, 0, 1\hbar, 2\hbar\}$$

$$+ \quad |\vec{L}| = \sqrt{2(3)}\cdot\hbar = \boxed{\sqrt{6}\cdot\hbar}$$

$$\Rightarrow \theta \in \left\{\cos^{-1}\left(\frac{-2}{\sqrt{6}}\right), \cos^{-1}\left(\frac{-1}{\sqrt{6}}\right), \cos^{-1}(0), \cos^{-1}\left(\frac{1}{\sqrt{6}}\right), \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)\right\}.$$

Problem 3 (Krane #12)

If $n=1$ & $l=0$, $R(r)=\dfrac{2}{a_0^{3/2}}e^{-r/a_0}$.

$$\Rightarrow \quad P(r)=r^2\cdot\left|\frac{2}{a_0^{3/2}}e^{-r/a_0}\right|^2$$

$$\Rightarrow \text{If } 0=P'(r)=\frac{8re^{-\frac{2r}{a}}\underline{(a_0-r)}}{a_0^4}$$

$$\Rightarrow \quad a_0-r=0 \quad \Rightarrow r=a_0.$$

PROBLEM 4 (KRANE #18)

THE DEGENERACY AT $n=5$ SHOULD BE a.)

$2 \cdot 5^2 = \boxed{50}$ TO CONFIRM,

b.) $l=4 \Rightarrow m_l$ 9 values times 2 spin values = 18

$l=3 \Rightarrow m_l$ 7 values times 2 spin values = 14

$l=2 \Rightarrow m_l$ 5 * 2 = 10

$l=1 \Rightarrow m_l$ has 3 * 2 = 6

$l=0 \Rightarrow m_l$ 1 x 2 = 2

||

$\boxed{50}$

PROBLEM 5

A COMPLETE ALIGNMENT OF THE ANGULAR MOMENTUM VECTOR WOULD IMPLY THAT WE KNOW ALL THREE COMPONENTS OF THE VECTOR. THIS WOULD VIOLATE THE UNCERTAINTY PRINCIPLE.

Problem 6

a.) If $n=1$, $l=1$, $m_l=1$ we have

$$|\vec{L}| = \sqrt{1(2)}\,\hbar = \sqrt{2}\,\hbar \quad \& \quad L_z = \hbar \Rightarrow$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right). \quad \text{Now;}$$

$$\tau = \frac{e}{m_e} B|\vec{L}|\sin\theta = \frac{1.602\cdot10^{-19}\,C}{9.109\cdot10^{-31}\,kg}\cdot\left(\cancel{2}T\right)\left(\frac{1}{\cancel{\sqrt{2}}}\hbar\right)\left(\frac{1}{\cancel{\sqrt{2}}}\right)$$

$$= 1.86\cdot10^{-23}\,Am^2\cdot\overbrace{1\,kg\cdot s^{-2}\cdot A^{-1}}^{Tesla}$$

$$= 1.86\cdot10^{-23}\,kg\,m^2 s^{-2} = \boxed{1.86\cdot10^{-23}\,J}.$$

b.) I believe the point here is that the torque applied is independent of spin magnetic moment, $\vec{\mu}_s$, and the external field only acts on $\vec{\mu}_L$. So, an external magnetic field can tug on the spin moment, as we saw with silver atoms, but it can not apply a torque within the atom or system? $\boxed{0}$

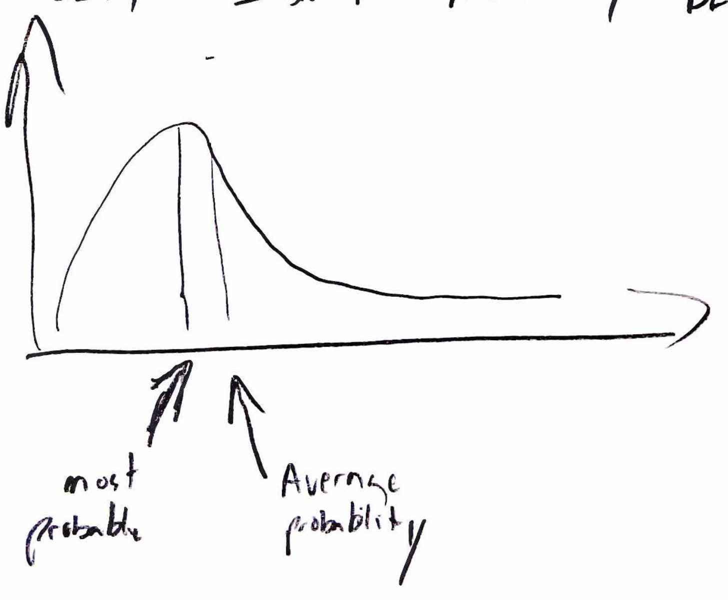
Problem 7

$$\langle r \rangle = \frac{4}{a_0^3}\int_0^\infty r^3 e^{-2r/a_0} = \frac{4}{a_0^3}\cdot\frac{3a_0^4}{8}$$

Wolfram Alpha

$$= \boxed{\frac{3}{2}a_0}$$

THE AVERAGE VALUE FOR r IS SKEWED TO THE RIGHT OF THE MOST PROBABLE VALUE OF r BECAUSE THIS PROBABILITY ISN'T NORMALLY DISTRIBUTED.



Problem 8

Let $Y_{21} = -\frac{1}{2}\sqrt{\frac{15}{2\pi}} \sin\theta \cos\theta e^{i\phi}$. We have

$$|Y_{21}|^2 = \frac{1}{4}\cdot\frac{15}{2\pi}\sin^2\theta\cos^2\theta e^{i\phi} e^{-i\phi} = \frac{15}{8\pi}\sin^2\theta\cos^2\theta .$$

a.) From equation 11, class notes # 13, we have that

$$\int_0^{2\pi}\int_0^{\pi}|Y_{21}|^2 \sin\theta \, d\theta \, d\phi = \frac{15}{8\pi}\int_0^{2\pi}\int_0^{\pi}\sin^3\theta\cos^2\theta \, d\theta \, d\phi$$

$$= \frac{15}{8\pi}\int_0^{2\pi}\left(\frac{4}{15}\right)d\phi$$

Wolfram Alpha

$$= \frac{1}{2\pi}\left[\phi\right]_0^{2\pi} = \boxed{1.}$$

b.) The probability of finding the particle in the cone defined by $0 \le \theta \le \pi/4$ + $0 \le \phi \le 2\pi$ is calculated as follows:

$$\int_0^{2\pi}\int_0^{\pi/4}|Y_{21}|^2 \sin\theta \, d\theta \, d\phi = \frac{15}{8\pi}\int_0^{2\pi}(.0508)\,d\phi = \frac{15}{4}\cdot(.0508) = \boxed{0.1905}.$$