PROBLEM 1

WITH $8eV = E > U_0 = 5$ WE CAN EMPLOY

EQUATIONS 1, 2, 8 + 9 FROM CLASS NOTES. FIRST

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{2mc^2 E}{(\hbar c)^2}} = \sqrt{\frac{2(511{,}000\,eV)(8\,eV)}{(197\,eV \cdot nm)^2}} = 14.5\,nm^{-1}$$

$$k_2 = \sqrt{\frac{2m(E-U_0)}{\hbar^2}} = \sqrt{\frac{2mc^2(E-U_0)}{(\hbar c)^2}} = \sqrt{\frac{2(511{,}000\,eV)(3\,eV)}{(197\,eV \cdot nm)^2}} = 8.89\,nm^{-1}$$

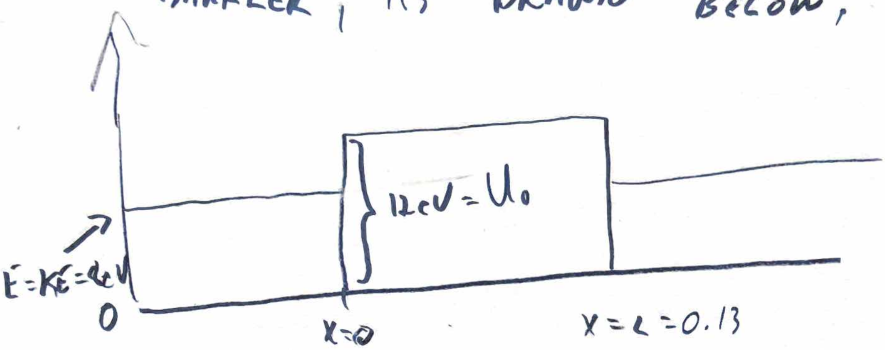
NOW,

$$T = \frac{4k_1k_2}{(k_1+k_2)^2} = \frac{4(14.5)(8.89)}{(14.5+8.89)^2} = \boxed{.94}$$

+ $$R = \frac{(k_1-k_2)^2}{(k_1+k_2)^2} = \boxed{.06}$$

Problem 2

Assuming $KE = 8eV = E$ before the barrier, as drawn below,



We can use equations 10, 14 for the case where $E = KE < U_0$. Therefore,

$$q = \sqrt{\frac{2m(U_0-E)}{\hbar^2}} = \sqrt{\frac{2mc^2(U_0-E)}{(\hbar c)^2}} = \sqrt{\frac{2(511{,}000)(4\,eV)}{(197\,eV\cdot nm)^2}} = 10.26\,nm^{-1}$$

& $$T = \left(1 + \frac{U_0^2}{4E(U_0-E)}\sinh^2(qL)\right)^{-1} = \boxed{0.22} \Rightarrow R = 1 - 0.22 = \boxed{.78}.$$

This seems like a large T, but I suppose 13 Å is an extremely skinny barrier.

PROBLEM 3

$E_0 = 1.24\,eV = \frac{1}{2}\hbar\omega_0 \Rightarrow \omega_0 = \frac{2.48\,eV}{\hbar}$

Equation 5.52

$$\Rightarrow E_2 = \left(\frac{4}{2} + \frac{1}{2}\right)\hbar\left(\frac{2.48\,eV}{\hbar}\right) = 6.2\,eV$$

$$\& \quad E_4 = \left(\frac{8}{2} + \frac{1}{2}\right)\left(2.49\,eV\right) = 11.16\,eV$$

$$\Rightarrow E_2 - E_0 = \boxed{4.96\,eV}$$

$$\& \quad E_4 - E_0 = \boxed{9.92\,eV}$$

ARE THE NECESSARY ENERGIES TO MOVE AN ELECTRON FROM THE GROUND STATE TO THE 2nd & 4th EXCITED STATE, RESPECTIVELY.

PROBLEM 4

MODELING THIS AS A HARMONIC OSCILLATOR, WE HAVE THAT

$$E_{n+1} - E_n = \left((n+1)+\tfrac{1}{2}\right)\hbar\omega_0 - \left(n+\tfrac{1}{2}\right)\hbar\omega_0$$

$$= \hbar\omega_0 .$$

THE PHOTON RELEASED HAS ENERGY

$$E_p = \frac{hc}{\lambda} = \frac{1240 \cdot nm}{561\, nm} = 2.21 eV .$$

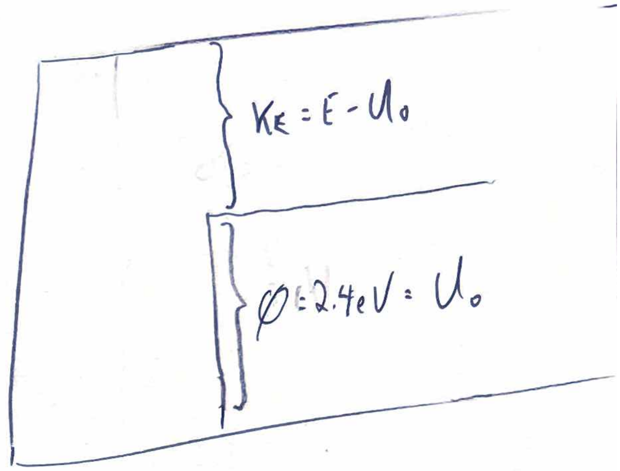
THIS MUST MATCH THE ENERY LOST FROM THE TRANSITION. THAT IS

$$2.2 eV = \hbar\omega_0 \implies \omega_0 = \frac{2.21 eV}{\hbar} = \boxed{3.36 \cdot 10^{15}\, s^{-1}} .$$

Problem 5

Let $\phi = 2.4\,eV$ be the work function of some metal surface.

A laser with power $0.1\,Js^{-1}$ & $\lambda = 350nm$ is shown on the surface.



$$\lambda = 350nm \Rightarrow E_p = \frac{hc}{\lambda} = \frac{1240\,eV\cdot nm}{350nm} = 3.54\,eV.$$

The problem states that all energy of the photon is absorbed by the electron. Therefore,

$$KE = E_p - U_0 = 3.54eV - 2.4eV = 1.14eV.$$

We are given that this is a plane wave so the proportion of electrons ejected to photons incident can be calculated with class notes equation 1, 2, & 8 as follows:

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{2mc^2E}{(\hbar c)^2}} = \sqrt{\frac{2(511,000eV)(3.54eV)}{(197\,eV\cdot nm)^2}} = 9.65\,nm^{-1}$$

&

$$k_2 = \sqrt{\frac{2(511,000eV)(1.14eV)}{(197\,eV\cdot nm)^2}} = 5.48\,nm^{-1}$$

$$\Rightarrow T = \frac{4k_1k_2}{(k_1+k_2)^2} = .924.$$

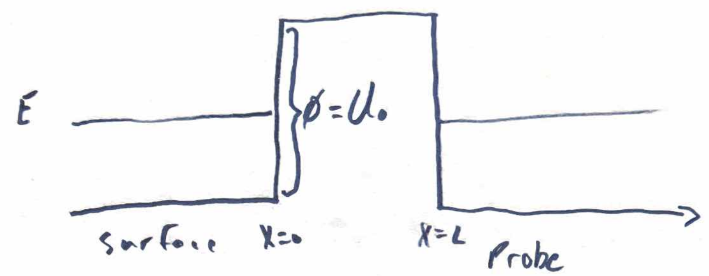
THE POWER OF THE LASER GIVES US HOW MANY TIMES PER SECOND AN ELECTRON WILL HAVE A $T = .924$ CHANCE OF EJECTION.

THAT IS, $\dfrac{0.1\,J\,s^{-1}}{3.54\,eV} = \dfrac{6.24\cdot 10^{17}\,eV\,s^{-1}}{3.5\,eV} = 1.78\cdot 10^{17}\,s^{-1}$ PHOTONS INCIDENT PER SECOND. EACH ONE PROVIDING A 0.924 CHANCE OF EJECTION GIVES

$$1.78\cdot(.924)\cdot 10^{17} = \boxed{1.65\cdot 10^{17} \text{ EJECTED ELECTRONS PER SECOND.}}$$

Problem 6

We can model STM as a potential barrier with schematic drawn below.



a.) Let $U_0 = \phi = 1.5\,eV$ & $L = 1.2\,nm$ & $E = 0.026\,eV$.

With $E < U_0$ we have (from class notes again) that

$$q = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} = \sqrt{\frac{2(511{,}000\,eV)(1.497\,eV)}{(197\,eV.nm)^2}} = 6.279\,nm^{-1}$$

$$\& \quad T = \left(1 + \frac{U_0^2}{4E(U_0 - E)}\sinh^2(qL)\right) = \boxed{7.897 \cdot 10^{-8}}$$

b.) With $T = 7.897 \cdot 10^{-8}$ we have

$$T \cdot 1.4 \cdot 10^{14} = 7.897 \cdot (1.4) \cdot 10^{14} \cdot 10^{-8} = 1.1 \cdot 10^{7}$$

electrons tunneling per second. That's

$$\frac{1.1 \cdot 10^{7}}{6.24 \cdot 10^{18}}\,A = \boxed{1.76 \cdot 10^{-12}\,Amp}$$

c.) REPLACING L IN OUR FORMULA FOR PART a.) GIVES $T = .000148$ WHICH MEANS DECREASING THE WIDTH OF THE BARRIER (SPACE BETWEEN PROBE & SURFACE) BY dx INCREASES TUNNELLING BY

$$\frac{.000148}{7.897 \cdot 10^{-8}} = \boxed{18{,}741.3 \text{ TIMES.}}$$

PROBLEM 7

a.) $\Psi(x) = Ae^{ikx} + Be^{-ikx} = A\left(\cos(kx) + i\sin(kx)\right) + B\left(\cos(kx) - i\sin(kx)\right)$

$$= (A+B)\left(\cos(kx)\right) + (A-B)i\sin(kx).$$

$$\Psi(0) = 0 = (A+B)\left(\cos(0)\right) + (A-B)\sin(0) = A+B$$

$\Rightarrow A = -B.$

$$\Rightarrow \Psi(x) = (A+B)\left(\cos(kx)\right) + (A-B)i\sin(kx)$$

$$= \left(A + (-A)\right)\left(\cos(kx)\right) + \left(A - (-A)\right)i\sin(kx)$$

$$= 2Ai\sin(kx).$$

$\boxed{\text{Ⅲ}}$

b.) LET $\Psi(L) = 0 = 2Ai\sin(kL)$

$\Rightarrow \sin(kL) = 0$ WHICH IS ONLY TRUE

IF $kL = n\pi$ where $n = 1, 2, 3.$

$$\Rightarrow k_n = \frac{n\pi}{L}.$$

$\boxed{\text{Ⅲ}}$

Problem 8

A Normalized Wave Function Implies

that

$$1 = \int_0^L |\Psi(x)|^2 dx = \int_0^L \Psi(x)\Psi^*(x)\,dx = \int_0^L \left(2Ai\sin\left(\frac{n\pi x}{L}\right)\right)\left(-2Ai\sin\left(\frac{n\pi x}{L}\right)\right)dx$$

$$= -4(i^2)A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right)dx$$

$$= 4A^2\left[\frac{x}{2} - \frac{L\sin\left(\frac{2\pi n x}{L}\right)}{4\pi n}\right]_0^L$$

$$= 4A^2\left[\frac{L}{2} - \frac{L\sin(2\pi n)^{=0}}{4\pi n} - \left(0 - \sin(0)\right)\right]$$

$$= 2A^2L \implies A = \sqrt{\frac{1}{2L}}$$

$$\implies \Psi(x) = i2\sqrt{\frac{1}{2L}}\sin\left(\frac{n\pi x}{L}\right) = i\sqrt{\frac{4}{2L}}\sin\left(\frac{n\pi x}{L}\right)$$

$$= i\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right).$$

From what I gather, this is correct in function but not in form. I have attached a better solution below.

Problem 8

Let $A' = 2Ai$ such that

$$\psi(x) = A' \sin(kx).$$

Now, normalization is accomplished as follows

$$1 = \int_0^L (A'A'^*) \sin^2(kx)\, dx$$

$$= \int_0^L |A'|^2 \sin^2(kx)\, dx$$

$$= |A'|^2 \left[\frac{1}{2}\left(x - \frac{L}{2\pi n} \sin\left(\frac{2\pi n x}{L}\right)\right) \right]_0^L$$

$$= |A'|^2 \left[\frac{L}{2} - \underbrace{\frac{L}{2\pi n} \sin(2\pi n)}_{=0} - \big(0 - \underbrace{\sin(0)}_{=0}\big) \right]$$

$$= |A'|^2 \cdot \frac{L}{2} \implies |A'| = \sqrt{\frac{2}{L}} = 2Ai$$

$$\boxed{|A'|^2 = \left(\sqrt{0^2 + (2Ai)^2}\right)^2 = (2Ai)^2 \implies |A'| = 2Ai}$$

$$\implies \psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right).$$