PROBLEM 1

FROM SECTION 5.4 WE HAVE THAT

$$E_n = \frac{h^2 n^2}{8mL^2} \quad \text{where} \quad n = 1, 2, 3, \ldots$$

WE ARE GIVEN $E_1 = 4.4\,eV = \frac{h^2}{8mL^2}$. WHERE $L \to 2L$ WE HAVE $E_1 \to \frac{4.4\,eV}{2^2} = \boxed{1.1\,eV}$.

PROBLEM 2

FROM EQUATION 5.1,

$$\lambda_1 = \frac{2 \cdot (0.12\,nm)}{1} = \boxed{0.24\,nm}$$

$$\lambda_2 = \frac{2(0.12\,nm)}{2} = \boxed{0.12\,nm}$$

$$\lambda_3 = \frac{2(0.12\,nm)}{3} = \boxed{0.080\,nm}$$

Problem 3

a.) $P(N_i) = \boxed{\frac{1}{6}} \quad \forall i$

b.) $\langle N \rangle = \sum_{i=1}^{N} N_i P(N_i)$

$= 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$

$= \frac{1}{6}(21) = \boxed{3.5}$

c.) $\langle N^2 \rangle = \sum_{i=1}^{N} N_i^2 P(N_i) = \frac{1}{6} \sum_{1}^{6} N_i^2$

$= \frac{1}{6}\left(\frac{\cancel{6}(6+1)(2(6)+1)}{\cancel{6}}\right) = \boxed{\frac{91}{6}}$

$\Rightarrow \quad \sigma = \sqrt{\langle N^2 \rangle - \langle N \rangle^2}$

$= \sqrt{\frac{91}{6} - 3.5^2} \approx \boxed{1.71}$

PROBLEM 4

a.) IF $\boxed{P(r) = A\exp\{-2r\}}$ WE CAN NORMALIZE

OVER $0 \le r \le \infty$ AS FOLLOWS:

$$\int_0^\infty P(r) = 1 \Rightarrow \int_0^\infty A\exp\{-2r\}\,dr = 1$$

$$\Rightarrow A\cdot\left(\frac{1}{2}\right) = 1 \quad \boxed{\Rightarrow A = 2.}$$

b.) $2\int_0^1 \exp\{-2r\}\,dr = 2(0.43) = \boxed{0.86} = P(0 \le r \le 1)$

c.) $\langle r \rangle = \int_0^\infty r\cdot\left(2\exp\{-2r\}\right)dr = \boxed{0.5.}$

d.) $\langle r^2 \rangle = \int_0^\infty r^2\cdot\left(2\exp\{-2r\}\right)dr = \boxed{0.5}$

$$\Rightarrow \sigma = \sqrt{.5 - .5^2} = \boxed{.5.}$$

e.) $\langle \sin(r) \rangle = \int_{-\infty}^\infty \sin(r)\left(2\exp\{-2r\}\right)dr = \boxed{0.4.}$

PROBLEM 5

a.) BEFORE WEIGHTING WE HAD

$$P(N_i) = \frac{1}{6} \quad \forall i \quad . \text{ IF } P(N_i) = \frac{4}{6},$$

$$\sum_i P(N_i) = 5\left(\frac{1}{6}\right) + \frac{4}{6} = \frac{3}{2}.$$

NORMALIZING GIVES $P(N_i) = \frac{2}{3} \cdot \frac{1}{6} = \frac{1}{9} \quad 0 < i \leq 5$

$$P(N_i) = \frac{2}{3} \cdot \frac{4}{6} = \frac{4}{9} \quad \text{where } i = 6$$

b.) $$\langle N_i \rangle = \sum_{i=1}^{N} N_i P(N_i)$$

$$= 1\left(\frac{1}{9}\right) + 2\left(\frac{1}{9}\right) + 3\left(\frac{1}{9}\right) + 4\left(\frac{1}{9}\right) + 5\left(\frac{1}{9}\right) + 6\left(\frac{4}{9}\right)$$

$$= \boxed{4.3}.$$

c.) $$\langle N_i^2 \rangle = \sum_{i=1}^{N} N_i^2 P(N_i)$$

$$= 1^2\left(\frac{1}{9}\right) + \dots + 5^2\left(\frac{1}{9}\right) + 6^2\left(\frac{4}{9}\right) = 22.11$$

$$\Rightarrow \sigma = \sqrt{22.11 - 4.3^2} = \boxed{1.9}.$$

PROBLEM 6

ASSUMING A RADIATIVE TRANSITION FROM $n=2 \rightarrow n=1$ RESULTS IN A A PHOTON OF $\lambda = 600\,nm$ $\left(E = \frac{1240\,eV \cdot nm}{600} = 2.07\,eV\right)$ WE CAN USE EQUATION 5.30 TO FIND L AS FOLLOWS:

$$2.07\,eV = E_2 - E_1 = \frac{h^2 2^2}{8mL^2} - \frac{h^2}{8mL^2}$$

$$\Rightarrow \frac{1}{L^2} = 2.07\,eV \left(\frac{h^2 2^2}{8m} - \frac{h^2}{8m}\right)^{-1}$$

$$\Rightarrow L^2 = \frac{h^2}{8m}(3) \cdot \frac{1}{2.07\,eV}$$

$$\Rightarrow L = \sqrt{\frac{3h^2}{8m \cdot 2.07\,eV}} = \sqrt{\frac{3(hc)^2}{8mc^2 \cdot 2.07\,eV}}$$

$$= \sqrt{\frac{3 \cdot (1240\,eV \cdot nm)^2}{8(511{,}000\,eV)(2.07\,eV)}} = \boxed{.74\,nm}$$

PROBLEM 7

a.) $\int_{-\infty}^{\infty} P(x)\,dx = \int_{-\infty}^{\infty} \exp\left(\frac{-ax^2}{k_BT}\right)dx = 1$. Let $b = \frac{a}{k_BT}$.

Now $\int_{-\infty}^{\infty} \exp\left(-bx^2\right)dx = 1$

Formula sheet $\Longrightarrow$ $\sqrt{\frac{\pi}{b}} = 1 \Rightarrow \sqrt{\frac{k_BT\pi}{a}} = 1$

$\Longrightarrow$ $a = k_BT\pi$ FOR A NORMALIZED

$P(x)$.

b.) $\langle ax^2\rangle = \int_{-\infty}^{\infty} ax^2 \exp\left(-\frac{ax^2}{k_BT}\right)dx$

$= a\int_{-\infty}^{\infty} x^2 \exp\left(-bx^2\right)dx$

formula sheet

$= a \cdot \frac{1}{2}\sqrt{\frac{\pi}{b^3}} = k_BT\pi \cdot \frac{1}{2}\sqrt{\frac{\pi}{\frac{a^3}{(k_BT)^3}}}$

$= k_BT\pi \cdot \frac{1}{2}\sqrt{\frac{\pi (k_BT)^3}{\pi^3 \cancel{(k_BT)^3}}} = \frac{k_BT\pi}{2}\sqrt{\frac{1}{\pi^2}} = \boxed{\frac{k_BT}{2}}$.