PROBLEM 7

WE HAVE THAT $\Psi(x)$, $\frac{\partial \Psi}{\partial x} \to 0$ AS $x \to a, -a$.

THEREFORE, FROM PROBLEM 8,

$$\langle p \rangle = -i\hbar \int_{-a}^{a} \Psi^* \frac{\partial \Psi}{\partial x} dx = -i\hbar \int_{-a}^{a} \Psi \frac{\partial \Psi}{\partial x} dx = 0$$

BECAUSE $\Psi(x)$ AN ODD POLYNOMIAL

$$\Rightarrow \frac{\partial \Psi(x)}{\partial x} \text{ AN EVEN POLYNOMIAL}$$

$$\Rightarrow \frac{\Psi(x)}{\partial x} \cdot \Psi(x) \text{ AN ODD POLYNOMIAL}$$

$$\Rightarrow \int_{-a}^{a} \Psi(x) \cdot \frac{\Psi(x)}{dx} = 0.$$

IF $\langle p \rangle = \int_{-a}^{a} \Psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \Psi \, dx$ THEN

$$\langle p^2 \rangle = \int_{-a}^{a} \Psi^* \left(-i\hbar \frac{\partial}{\partial x}\right)^2 \Psi \, dx$$

$$= -\hbar^2 \int_{-a}^{a} \Psi^* \frac{\partial^2 \Psi}{\partial x^2} dx$$

WHERE

$$\frac{\partial^2 \psi}{\partial x^2} = -2\left(\sqrt{\frac{1}{1.07a^5}}\right) \quad \psi^* = \sqrt{\frac{1}{1.07a^5}}\left(a^2 - x^2\right)$$

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$$\langle p^2 \rangle = -\hbar^2 \int_{-a}^{a} \left(\sqrt{\frac{1}{1.07a^5}}\right)\left(a^2 - x^2\right)\left(2\sqrt{\frac{1}{1.07a}}\right) dx$$

$$= \frac{2\hbar^2}{1.07a^5} \int_{-a}^{a} \left(a^2 - x^2\right) dx$$

$$= \frac{2\hbar^2}{1.07a^5} \cdot \frac{4}{3} a^3 = \frac{2.49\hbar^2}{a^2}$$

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$$\Rightarrow \sigma_p = \sqrt{\frac{2.49\hbar^2}{a^2}} = \frac{1.58\hbar}{a}$$