

MATH 414/514 HOMEWORK 0

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Exercise 1. What is the definition of $\lim_{x \rightarrow c} f(x) = L$, using ϵ and δ ?

Answer.

Let $f(x) : E \rightarrow \mathbb{R}$ be a function with domain E and suppose that c is a point of accumulation of E . Then we write

$$\lim_{x \rightarrow c} f(x) = L$$

if $\forall \epsilon > 0 \exists \delta > 0$ s.t.

$$|f(x) - L| < \epsilon$$

when x is a point of E that is not c and $|x - c| < \delta$.

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Exercise 2. The derivative of x^2 is $2x$.

Answer. Gibbs energy is defined as

$$(2.1) \quad G = H - TS.$$

Gibbs energy of a gas is a function of temperature and pressure. More precisely, H can be expressed with $H(T)$ and S with $S(T, P)$ given its standard entropy S_{298}° .

For a diatomic gas, its enthalpy is given as

$$(2.2) \quad H = C_P T = \frac{7}{2} RT.$$

It is somewhat more complicated to calculate entropy. Consider the differential of entropy:

$$\begin{aligned} dS &= \frac{C_P}{T} dT - V \alpha dP \\ &= \frac{7R}{2T} dT - \frac{R}{P} dP. \end{aligned}$$

Therefore, we can find entropy with integration.

$$S(T, P) - S_{298}^\circ = \int_{298}^T \frac{7R}{2T} dT - \int_1^P \frac{R}{P} dP.$$

$$(2.3) \quad S = S_{298}^\circ + R \left(\frac{7}{2} \ln \frac{T}{298} - \ln \frac{P}{1} \right)$$

Combine equation (2.1), (2.2), and (2.3):

$$G = \frac{7}{2} RT - S_{298}^\circ T + RT \left(\ln \frac{P}{1} - \frac{7}{2} \ln \frac{T}{298} \right)$$

Plot this function given $S_{298}^\circ = 130.57$ and $R = 8.314$. This binary function, whose plot is a surface in a 3-D space, is preferably plotted with a computer program. \diamond

Proof. Using the limit definition of derivative, we have

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} &= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \rightarrow 0} 2x + h = 2x\end{aligned}$$

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Exercise 3. Some students write quotation marks like this:

"quotation marks" or like this: "quotation marks".

Why are these bad, and what is the right way to write quotation marks in L^AT_EX?

Answer.

"The proper way to create quotations is by using the key to the left of the number '1' key for the opening quotes, and to use the apostrophe for the closing quotes. " - James Rushing.

The aforementioned methods are bad because L^AT_EX always treats '"' as closing quotations.

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