MATH 414/514 HOMEWORK 2

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Exercise 1. (8.3.2)

If f is continuous, show that there is a point ξ in (a,b) s.t.

$$\int_a^b f(x)dx = f(\xi)(a-b).$$

Answer.

By 8.8 we have a function $F(x)$ s.t. $F'(x) = f(x)$ for $x \in [a,b]$. By the MVT we have a value ξ s.t.

$$F'(\xi) = \frac{F(b)-F(a)}{b-a}$$
$$\implies F'(\xi)(b-a) = F(b)-F(a).$$

Then, using 8.9 gives us

$$F'(\xi)(b-a) = F(b)-F(a) = \int_a^b F'(x)dx = \int_a^b f(x)dx.$$

$\diamond$

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Exercise 2. (8.3.4)

Show that if f is continuous and non-negative on $[a, b]$ and

$$\int_a^b f(x)dx = 0$$

show that $f(x) = 0$.

Proof. By 8.8 we have a function $F(x) = \int_a^x f(t)dt$ s.t. that $F'(x) = f(x)$. Because $f(x)$ is non-negative, we observe that F(x) is an increasing function. However, because $\int_a^b f(x)dx = 0$, we know that F(a) and F(b) both equal 0. Therefore $F(x) = 0$ for all $x \in [a, b]$ or $f(x) = 0$ for all $x \in [a, b]$. $\square$

Exercise 3. (8.3.6)

If f is continuous on $[a,b]$ and $\int_a^b f(x)g(x)dx = 0$ for every continuous g on $[a,b]$, then $f(x) = 0$ for $x \in [a,b]$

Proof. If the conclusion holds for all continuous functions $g(x)$ then it holds where $g(x) = f(x)$. This gives us

$$\int_a^b (f(x))^2 dx = 0$$

Using 8.3.4, where $p(x) = (f(x))^2$ is non-negative we have

$$\begin{aligned} \int_a^b p(x)dx = 0 &\implies p(x) = (f(x))^2 = 0 \\ &\implies f(x) = 0 \quad \forall x \in [a,b] \end{aligned}$$

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