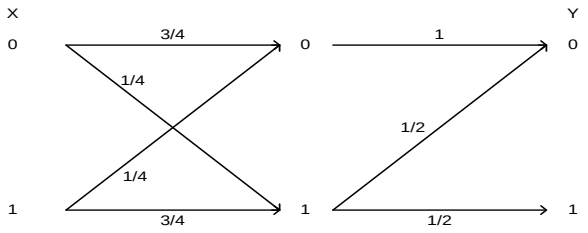
CS 527 / ECE599 Error Correcting Codes - Midterm _____ *November 4, 2015*

NAME: _____

1. (5 points) Let (X, Y) have the following joint distribution. Find $I(X, Y)$.

X \ Y	1	2	3	4
1	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{32}$
2	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{32}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
4	$\frac{1}{4}$	0	0	0

2. (5 points) Consider a cascade of Binary Symmetric Channel (B.S.C) and Z-channel as shown below. Find the mutual information between the input random variable, X, and the output random variable, Y. Assume that $P(X = 0) = \frac{1}{2}$ and $P(X = 1) = \frac{1}{2}$.



3. (4 points) A fair coin is tossed. What is the mutual information between the Top and Bottom faces?

4. (6 points) An additive channel has input X = {0, 1}, error noise Z = {a} and output Y = X + Z. Assuming $Pr(a/0) = Pr(a/1) = 1/2$ find the capacity of this channel. Assume that Z is independent of X. Note that the capacity depends on the value of a.

5. (5 points)

(a) For the following cases, explain whether or not it is possible to have a uniquely decodable binary variable length code. If *yes* give a code and if not, explain why it is not possible.

i. lengths 2, 2, 3, 3, 3, 3 and 4?

ii. lengths 2, 2, 3, 3 and 3 ?

(b) Find the 4-ary Huffman code for $p = \left(\frac{8}{36}, \frac{7}{36}, \frac{6}{36}, \frac{5}{36}, \frac{4}{36}, \frac{3}{36}, \frac{2}{36}, \frac{1}{36}\right)$.