1. First, we have $\begin{cases} T^{ik} = \rho u^i u^k, \\ T^{ik}_{;k} = \left(\rho u^i u^k\right)_{;k} = \left(\rho u^k\right)_{;k} u^i + \rho u^k u^i_{;k} = 0. \end{cases}$ As $u^i u_i = 1$, then

$$\begin{cases} u_i T^{ik}_{;k} = \left(\rho u^k\right)_{;k} + \rho u^k u_i u^i_{;k} = 0, \\ \left(u_i u^i\right)_{;k} = u_i u^i_{;k} + u_{i,k} u^i = 2u_i u^i_{;k} = 0, \end{cases}$$

then

$$\left(\rho u^k\right)_{;k} = 0,\ T^{ik}_{;k} = \rho u^k u^i_{;k} = \rho u^k \left(u^i_{,k} + \Gamma^i_{mk} u^m\right) = 0,\ u^i_{,k} u^k + \Gamma^i_{mk} u^m u^k = \frac{\partial u^i}{\partial x^k}\frac{dx^k}{ds} + \Gamma^i_{mk} u^m u^k = \frac{d^2 x^i}{ds^2} + \Gamma^i_{mk} u^m u^k = 0.$$

2. In the Newtonian gravitation theory, the motion of a photon of mass m is $u \equiv \frac{1}{r} = \frac{1+e\cos\theta}{l}$, where $l = \frac{h^2}{GM}, h = r^2\dot{\theta} = \text{constant}$, G is the gravitational constant, M is the mass of the sun, and e is a parameter.

At perihelion $\theta = 0$ and the distance $b \equiv r_{\min}$. We then could have

$$\begin{cases} \frac{1}{r_{\min}} = \frac{1}{b} = \frac{1+e\cos 0}{l} = \frac{1+e}{l} \\ h = r^2\dot{\theta} = r_{\min}^2\dot{\theta}_0 = bc, l = \frac{b^2c^2}{GM} \end{cases}.$$

Then $e = \frac{l}{b} - 1 = \frac{bc^2}{GM} - 1, u = \frac{1+e\cos\theta}{l} = \frac{1+\left(\frac{bc^2}{GM}-1\right)\cos\theta}{b^2c^2/GM}$.

When $r \to \infty, u = \frac{1+\left(\frac{bc^2}{GM}-1\right)\cos\theta}{b^2c^2/GM} \to 0$, then $\cos\theta_0 = \frac{1}{1-\frac{bc^2}{GM}}, \theta_0 \equiv \lim_{r\to\infty}\theta$. As shown in the picture above, $\varphi_0 = \theta_0 - \frac{\pi}{2}$, $\sin\varphi_0 = -\cos\left(\varphi_0 + \frac{\pi}{2}\right) = -\cos\theta_0 = \frac{1}{\frac{bc^2}{GM}-1}$. It is easy to see that $bc^2 \gg Gm$ (by more than 34 orders of magnitude). Then $\sin\varphi_0 = \frac{1}{\frac{bc^2}{GM}-1}$ must be an infinitesimal that $\varphi_0 \approx \sin\varphi_0 = \frac{1}{\frac{bc^2}{GM}-1} \approx \frac{GM}{bc^2}$. And that the light deflection is

$$\Delta\varphi = 2\varphi_0 = \frac{2GM}{bc^2}.$$

In the GR theory, we know that the light deflection is $\Delta\varphi' = \frac{4GM}{bc^2}$. Then

$$\Delta\varphi = \frac{\Delta\varphi'}{2}.$$

3. $g_{kl}=\frac{1}{\sinh^2 t}\begin{bmatrix} 1/c^2 & 0 \\ 0 & -1 \end{bmatrix}, g^{kl}=\frac{1}{g}\frac{\partial g}{\partial g_{ik}}=\sinh^2 t\begin{bmatrix} c^2 & 0 \\ 0 & -1 \end{bmatrix}$, where the determinant g is made up from the components of the tensor g_{kl}, and the coefficient $\frac{\partial g}{\partial g_{kl}}$ is the corresponding minor of g.

The given particle moves along a geodesic, $\frac{d^2x^i}{ds^2}+\Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds}=0,$ where $ds^2=g_{kl}dx^k dx^l$. Besides, from

$$\begin{cases} \Gamma^i_{ki}=\frac{g^{im}}{2}\left(\frac{\partial g_{mk}}{\partial x^i}+\frac{\partial g_{mi}}{\partial x^k}-\frac{\partial g_{ki}}{\partial x^m}\right)=\frac{g^{im}}{2}\frac{\partial g_{mi}}{\partial x^k} \\ \Gamma^i_{ik}=\frac{g^{im}}{2}\left(\frac{\partial g_{mi}}{\partial x^k}+\frac{\partial g_{mk}}{\partial x^i}-\frac{\partial g_{ik}}{\partial x^m}\right)=\frac{g^{im}}{2}\frac{\partial g_{mk}}{\partial x^i} \\ \Gamma^k_{ii}=\frac{g^{km}}{2}\left(\frac{\partial g_{mi}}{\partial x^i}+\frac{\partial g_{mi}}{\partial x^i}-\frac{\partial g_{ii}}{\partial x^m}\right)=\frac{g^{km}}{2}\left(2\frac{\partial g_{mi}}{\partial x^i}-\frac{\partial g_{ii}}{\partial x^m}\right) \end{cases}, i,k,m=0,1,$$

we could have:

$$\Gamma^0_{00}=\frac{g^{0m}}{2}\frac{\partial g_{m0}}{\partial x^0}=\frac{g^{00}}{2}\frac{\partial g_{00}}{\partial x^0}=\frac{\sinh^2 t}{2}\frac{\partial}{\partial(ct)}\left(\frac{1}{\sinh^2 t}\right)=-\frac{1}{c}\frac{e^t+e^{-t}}{e^t-e^{-t}},$$

$$\Gamma^1_{11}=\frac{g^{11}}{2}\frac{\partial g_{11}}{\partial x^1}=\frac{\sinh^2 t}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sinh^2 t}\right)=0,$$

$$\Gamma^1_{00}=\frac{g^{1m}}{2}\left(2\frac{\partial g_{m0}}{\partial x^0}-\frac{\partial g_{00}}{\partial x^m}\right)=\frac{g^{11}}{2}\left(2\frac{\partial g_{10}}{\partial x^0}-\frac{\partial g_{00}}{\partial x^1}\right)=-\frac{g^{11}}{2}\frac{\partial g_{00}}{\partial x^1}=0,$$

$$\Gamma^0_{11}=-\frac{g^{00}}{2}\frac{\partial g_{11}}{\partial x^0}=\frac{c\sinh^2 t}{2}\frac{\partial}{\partial t}\left(\frac{1}{\sinh^2 t}\right)=-c\frac{e^t+e^{-t}}{e^t-e^{-t}},$$

$$\Gamma^1_{01}=\frac{g^{1m}}{2}\frac{\partial g_{m1}}{\partial x^0}=\frac{g^{11}}{2}\frac{\partial g_{11}}{\partial x^0}=c^3\frac{e^t+e^{-t}}{e^t-e^{-t}}, \Gamma^1_{10}=\frac{g^{11}}{2}\frac{\partial g_{10}}{\partial x^1}=0,$$

$$\Gamma^0_{10}=\frac{g^{0m}}{2}\frac{\partial g_{m0}}{\partial x^1}=\frac{g^{00}}{2}\frac{\partial g_{00}}{\partial x^1}=0, \Gamma^0_{01}=\frac{g^{00}}{2}\frac{\partial g_{01}}{\partial x^0}=0,$$

Among these 8 Christoffel symbols, only four of them is non-zero. Then the equations of the trajectory of the given particle:

$$\begin{cases} \frac{d^2x^0}{ds^2}+\Gamma^0_{00}\frac{dx^0}{ds}\frac{dx^0}{ds}+\Gamma^1_{11}\frac{dx^1}{ds}\frac{dx^1}{ds}=0 \\ \frac{d^2x^1}{ds^2}+2\Gamma^1_{01}\frac{dx^0}{ds}\frac{dx^1}{ds}=0 \end{cases}.$$