1. The four-velocity of a particle is $u^i = \frac{dx^i}{ds}(ds = \frac{cdt}{\gamma}, \gamma = \frac{1}{\sqrt{1-\beta^2}}, \beta = \frac{v}{c})$, or $u^i = \gamma\left(1, \frac{\mathbf{v}}{c}\right)$. Similarly, we have $u_i = \frac{dx_i}{ds} = \gamma\left(1, -\frac{\mathbf{v}}{c}\right)$.

For $S = -m_0 c \int_a^b ds$, we have

$$\delta S = -m_0 c\delta \int_a^b ds = -m_0 c\delta \int_a^b \sqrt{dx_i dx^i} = -m_0 c \int_a^b \frac{\delta(dx_i)dx^i + dx_i \delta(dx^i)}{2\sqrt{dx_i dx^i}} = -m_0 c \int_a^b \frac{dx_i \delta(dx^i)}{\sqrt{dx_i dx^i}}$$

$$= -m_0 c \int_a^b \frac{dx_i d(\delta x^i)}{\sqrt{dx_i dx^i}} = -m_0 c \int_a^b u_i \delta(dx^i) = -m_0 c u_i \delta x^i \Big|_a^b + m_0 c\delta \int_a^b \delta x^i \frac{du_i}{ds} ds.$$

If $\frac{du_i}{ds} = 0$, then $\delta S = -m_0 c u_i \delta x^i \Big|_a^b = m_0 c u_i \left[(\delta x^i)_a - (\delta x^i)_b\right]$. Supposing that the point a is fixed, then $(\delta x^i)_a = 0$. In place of $(\delta x^i)_b$, we may write simply δx^i and thus obtain $\delta S = -m_0 c u_i \delta x^i$.

Then the momentum four-vector $p_i = -\frac{\partial S}{\partial x^i} = m_0 c u_i = \left(\frac{\mathcal{E}}{c}, -\mathbf{p}\right)$. With the general formulas for the Lorentz transformations of four-vectors, we have

$$\mathcal{E} = \gamma(\mathcal{E}' - \beta c p'_x), p_x = \gamma(p'_x - \beta \mathcal{E}'/c), p_y = p'_y, p_z = p'_z.$$

2. Noting that $dx_i dx^i = ds^2$, we have $u_i u^i = 1$. Then

$$p_i p^i = (m_0 c u_i)(m_0 c u^i) = m^2{}_0 c^2 u_i u^i = m^2{}_0 c^2.$$

3. Considering Λ as the density of the Lagrangian of a system: $L = \int \Lambda dV$, we have $T_0^0 = \dot{q}\frac{\partial \Lambda}{\partial \dot{q}} - \Lambda, \dot{q} \equiv \frac{\partial q}{\partial t}$.

In the limiting case that the state of the system can be described in terms of the time and space coordinates only, $q = q(x,y,z;t)$. Or for simplicity, $q = x$. And the relativistic Lagrangian for a one-particle system (again, for simplicity) is $L = -m_0 c^2 \sqrt{1-\frac{v^2}{c^2}} = -m_0 c^2 \sqrt{1-\frac{\dot{x}^2}{c^2}}$, where v is is the velocity of the material particle. Then

$$\int T_0^0\, dV = \int \left(\dot{x}\frac{\partial \Lambda}{\partial \dot{x}} - \Lambda\right) dV = \dot{x}\frac{\partial L}{\partial \dot{x}} - L = \frac{m_0 \dot{x}^2}{\sqrt{1-\frac{\dot{x}^2}{c^2}}} - \left(-m_0 c^2 \sqrt{1-\frac{\dot{x}^2}{c^2}}\right) = \frac{m_0 c^2}{\sqrt{1-\frac{\dot{x}^2}{c^2}}} = \mathcal{E}.$$

that is, $\int T_0^0\, dV = \mathcal{E}$. And T_0^0 is the density of the energy of the system.

For the non-relativistic Lagrangian, $L = T - V = \frac{1}{2}m_0 \dot{x}^2 - V(x;t)$, where T is the total kinetic energy of the system and V is the potential energy of the system, we have

$$\int T_0^0\,dV = \dot{x}\frac{\partial L}{\partial \dot{x}} - L = m_0\dot{x}^2 - \left(\frac{1}{2}m_0\dot{x}^2 - V\right) = \frac{1}{2}m_0\dot{x}^2 + V = \mathcal{E}.$$

Thus, T_0^0 is the density of the mechanical energy of the system.

4. We now use Greek indices to refer to the spatial coordinates alone. Then the components of the four-velocities are $u^0 = u'^0 = \gamma, u^\alpha = \frac{\gamma v^\alpha}{c}, u'^\alpha = \frac{\gamma v'^\alpha}{c}$.

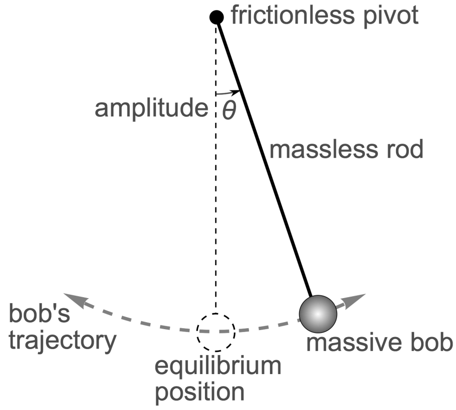
Under the Lorentz transformations, $u^0 = \gamma(u'^0 + \beta u'^1), u^1 = \gamma(u'^1 + \beta u'^0), u^2 = u'^2$, $u^3 = u'^3$. Hence,

$$\begin{cases} \dfrac{u^1}{u^0} = \dfrac{v^1}{c} = \dfrac{\gamma(u'^1 + \beta u'^0)}{\gamma(u'^0 + \beta u'^1)} = \dfrac{u'^1/u'^0 + \beta}{1 + \beta u'^1/u'^0} = \dfrac{v'^1/c + \beta}{1 + \beta v'^1/c}, \\ u^\alpha = \dfrac{\gamma v^\alpha}{c} = u'^\alpha = \dfrac{\gamma v'^\alpha}{c}, \alpha = 2, 3, \end{cases}$$

that is,

$$\begin{cases} v^1 = \dfrac{v'^1 + \beta c}{1 + \beta v'^1/c}, \\ v^2 = v'^2, v^3 = v'^3. \end{cases}$$

5. The equation of motion of a pendulum is $m_i a = m_i l\ddot{\theta} = m_G g\sin\theta$, where m_i is the inertial mass and m_G is the gravitational mass.



Thus, the motion of the pendulum is $\theta = \theta_0\cos(\frac{2\pi}{T} + \varphi)$, where θ_0 is the amplitude (the maximum angle that the pendulum swings away from vertical) and $T = 2\pi\sqrt{\frac{m_i l}{m_G g}}(1 + \frac{1}{16}\theta_0 + \frac{11}{3072}\theta_0^2 + \cdots)$ is the period. For small swings ($\theta_0 \ll 1$) the pendulum approximates a harmonic oscillator, and $T = 2\pi\sqrt{\frac{m_i l}{m_G g}}$.

If we could tell from high-precision experiments that $T = 2\pi\sqrt{\frac{m_i l}{m_G g}}$ is constant, then $\frac{m_i}{m_G}$ must be constant. And if this ratio is a universal constant, it must take the value one (the standard body takes this value by definition). Therefore, we have $m_i = m_G$.