Leetcode 887: Super Egg Drop

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Problem

- You are given k identical eggs and you have access to a building with n floors labeled from 1 to n.
- You know that there exists a floor f where 0 <= f <= n such that any egg dropped at a floor higher than f will break, and any egg dropped at or below floor f will not break.
- Each move, you may take an unbroken egg and drop it from any floor x (where 1 <= x <= n). If the egg breaks, you can no longer use it. However, if the egg does not break, you may reuse it in future moves.
- Return the minimum number of moves that you need to determine with certainty what the value of f is.

Solution 1 (DP)

- k eggs, n floor
- Drop egg from i floor.
  - Broken => test floors under i (k-1 eggs remain)
  - Unbroken => test floors above i (n-i floors in total, k eggs remain)

Recurrence Relation

$$k \text{ eggs}, \quad n \text{ floors}$$

$$\text{Base case} \quad \begin{cases} k & = & 0 & \text{return 0} \\ k & = & 1 & \text{return n} \\ n & <= & 1 & \text{return n} \end{cases}$$

$$ans = min(ans, 1 + max(dp(k-1, i-1), dp(k, n-i)))$$

$$\text{Broken} => dp(k-1, i-1) \quad => (k-1) \quad \text{eggs and} \quad (i-1) \quad \text{floors to check.}$$

$$\text{Unbroken} => dp(k, n-i) \quad => (k) \quad \text{eggs and} \quad (n-i) \quad \text{floors to check.}$$

$\overset{k\ \ n}{D(2,6)} = 3$

① $i=1$

$1+\mathrm{Max}(\underset{=0}{D(1,0)}, \underset{=3}{D(2,5)}) = 4$

⑯ $i=2$

$1+\mathrm{Max}(\underset{=1}{D(1,1)}, \underset{=3}{D(2,4)}) = 4$

⑰ $i=3$

$1+\mathrm{Max}(\underset{=2}{D(1,2)}, \underset{=2}{D(2,3)}) = 3$

⑱ $i=4$

$1+\mathrm{Max}(\underset{=3}{D(1,3)}, \underset{=2}{D(2,2)}) = 4$

⑲ $i=5$

$1+\mathrm{Max}(\underset{=4}{D(1,4)}, \underset{=1}{D(2,1)}) = 5$

⑳ $i=6$

$1+\mathrm{Max}(\underset{=5}{D(1,5)}, \underset{=0}{D(2,0)}) = 6$

$D(2,5) = 3$

② $i=1$

$1+\mathrm{Max}(\underset{=0}{D(1,0)}, \underset{=3}{D(2,4)}) = 4$

⑫ $i=2$

$1+\mathrm{Max}(\underset{=1}{D(1,1)}, \underset{=2}{D(2,3)}) = 3$

⑬ $i=3$

$1+\mathrm{Max}(\underset{=2}{D(1,2)}, \underset{=2}{D(2,2)}) = 3$

⑭ $i=4$

$1+\mathrm{Max}(\underset{=3}{D(1,3)}, \underset{=1}{D(2,1)}) = 4$

⑮ $i=5$

$1+\mathrm{Max}(\underset{=4}{D(1,4)}, \underset{=0}{D(2,0)}) = 5$

$D(2,4) = 3$

③ $i=1$

$1+\mathrm{Max}(\underset{=0}{D(1,0)}, \underset{=2}{D(2,3)}) = 3$

⑨ $i=2$

$1+\mathrm{Max}(\underset{=1}{D(1,1)}, \underset{=2}{D(2,2)}) = 3$

⑩ $i=3$

$1+\mathrm{Max}(\underset{=2}{D(1,2)}, \underset{=1}{D(2,1)}) = 3$

⑪ $i=4$

$1+\mathrm{Max}(\underset{=3}{D(1,3)}, \underset{=0}{D(2,0)}) = 4$

$D(2,3) = 2$

④ $i=1$

$1+\mathrm{Max}(\underset{=0}{D(1,0)}, \underset{=2}{D(2,2)}) = 3$

⑦ $i=2$

$1+\mathrm{Max}(\underset{=1}{D(1,1)}, \underset{=1}{D(2,1)}) = 2$

⑧ $i=3$

$1+\mathrm{Max}(\underset{=2}{D(1,2)}, \underset{=0}{D(2,0)}) = 3$

$D(2,2) = 2$

⑤ $i=1$

$1+\mathrm{Max}(\underset{=0}{D(1,0)}, \underset{=1}{D(2,1)}) = 2$

⑥ $i=2$

$1+\mathrm{Max}(\underset{=1}{D(1,1)}, \underset{=0}{D(2,0)}) = 2$

Code

```
int superEggDrop(int K, int N) {
    // Init table for record
    vector<vector<int>> table(K+1, vector<int>(N+1, INT_MAX));
    // Recursion (lambda expression)
    function<int(int, int)> dp = [&](int k, int n) {
        // Base case
        if(k == 0) return 0;
        else if(k == 1) return n;
        else if(n <= 1) return n;
        int& ans = table[k][n];
        if(ans != INT_MAX) return ans;  // dp(k, n) has done
        for(int i=1; i<=n; i++) {   // Drop from i floor
            ans = min(ans, 1 + max(dp(k-1, i-1), dp(k, n-i)));
        }
        return ans;
    };
    return dp(K, N);
}
```

Complexity (TLE)

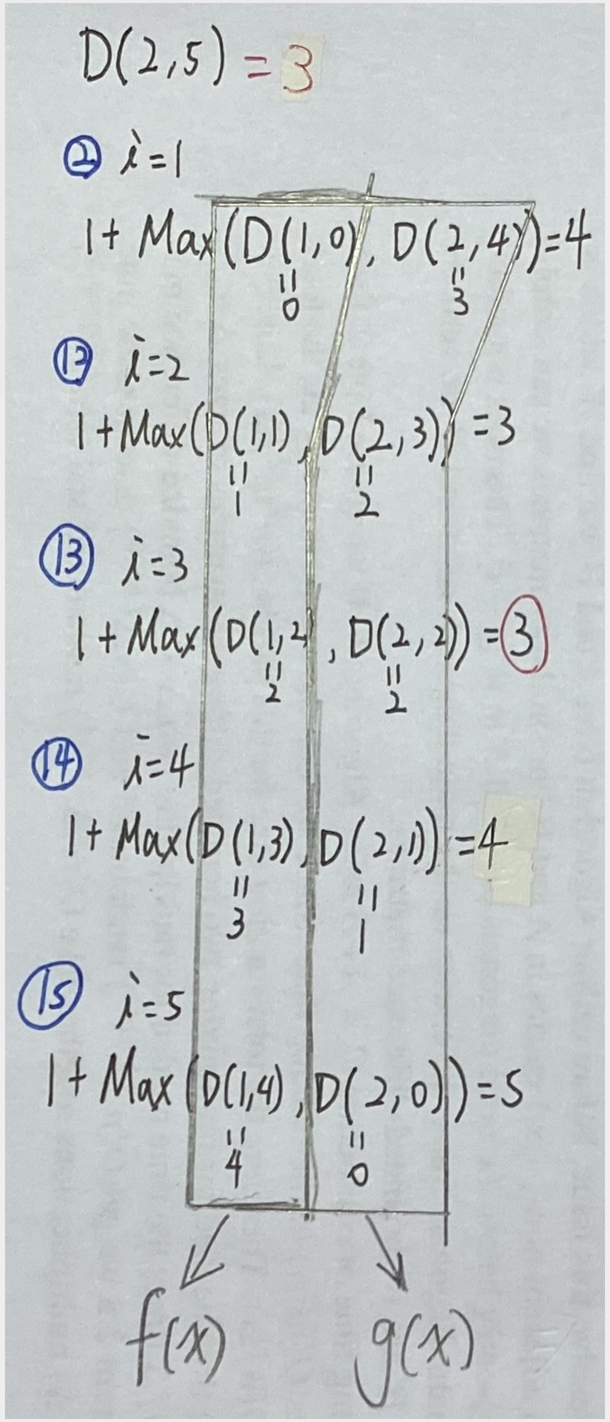
$\#$ of subproblems $O(KN)$, complexity of each take $O(N)$

Time complexity $O(KN^2)$

Space complexity $O(KN)$

Solution 2 (DP)

- f(x) is monotonic increase.
- g(x) is monotonic decrease.
- Binary search.



Recurrence Relation

$$k \text{ eggs}, \quad n \text{ floors}$$

$$\text{Base case} \quad \begin{cases} k & = & 0 & \text{return 0} \\ k & = & 1 & \text{return n} \\ n & <= & 1 & \text{return n} \end{cases}$$

$$\text{Search i with binary search.}$$

$$ans = min(ans, 1 + max(dp(k-1, i-1), dp(k, n-i)))$$

Code

```
int superEggDrop(int K, int N) {
    vector<vector<int>> table(K+1, vector<int>(N+1, INT_MAX));
    function<int(int, int)> dp = [&](int k, int n) {
        if(k == 0) return 0;
        else if(k == 1) return n;
        else if(n <= 1) return n;
        int& ans = table[k][n];
        if(ans != INT_MAX) return ans;  // dp(k, n) has done
        int left = 1, right = n + 1;
        while(left < right) {   // Binary search
            int i = left + (right - left) / 2;    // Drop from i floor
            if(dp(k-1, i-1) >= dp(k, n-i))  right = i;
            else                            left = i + 1;
        }
        ans = 1 + max(dp(k-1, left-1), dp(k, n-left));
        return ans;
    };
    return dp(K, N);
}
```

Complexity

$\#$ of subproblems $O(KN)$, complexity of each take $O(\log N)$

Time complexity $O(KN \log N)$

Space complexity $O(KN)$

More on binary search ...

k n

$D(2,6)=3$

① $i=1$

$1+Max(D(1,0), D(2,5))=4$ with $D(1,0)=0$, $D(2,5)=3$

⑯ $i=2$

$1+Max(D(1,1), D(2,4))=4$ with $D(1,1)=1$, $D(2,4)=3$

⑰ $i=3$

$1+Max(D(1,2), D(2,3))=3$ with $D(1,2)=2$, $D(2,3)=2$

⑱ $i=4$

$1+Max(D(1,3), D(2,2))=4$ with $D(1,3)=3$, $D(2,2)=2$

⑲ $i=5$

$1+Max(D(1,4), D(2,1))=5$ with $D(1,4)=4$, $D(2,1)=1$

⑳ $i=6$

$1+Max(D(1,5), D(2,0))=6$ with $D(1,5)=5$, $D(2,0)=0$

$D(2,5)=3$

② $i=1$

$1+Max(D(1,0), D(2,4))=4$ with $D(1,0)=0$, $D(2,4)=3$

⑫ $i=2$

$1+Max(D(1,1), D(2,3))=3$ with $D(1,1)=1$, $D(2,3)=2$

⑬ $i=3$

$1+Max(D(1,2), D(2,2))=3$ with $D(1,2)=2$, $D(2,2)=2$

⑭ $i=4$

$1+Max(D(1,3), D(2,1))=4$ with $D(1,3)=3$, $D(2,1)=1$

⑮ $i=5$

$1+Max(D(1,4), D(2,0))=5$ with $D(1,4)=4$, $D(2,0)=0$

$D(2,4)=3$

③ $i=1$

$1+Max(D(1,0), D(2,3))=3$ with $D(1,0)=0$, $D(2,3)=2$

⑨ $i=2$

$1+Max(D(1,1), D(2,2))=3$ with $D(1,1)=1$, $D(2,2)=2$

⑩ $i=3$

$1+Max(D(1,2), D(2,1))=3$ with $D(1,2)=2$, $D(2,1)=1$

⑪ $i=4$

$1+Max(D(1,3), D(2,0))=4$ with $D(1,3)=3$, $D(2,0)=0$

$D(2,3)=2$

④ $i=1$

$1+Max(D(1,0), D(2,2))=3$ with $D(1,0)=0$, $D(2,2)=2$

⑦ $i=2$

$1+Max(D(1,1), D(2,1))=2$ with $D(1,1)=1$, $D(2,1)=1$

⑧ $i=3$

$1+Max(D(1,2), D(2,0))=3$ with $D(1,2)=2$, $D(2,0)=0$

$D(2,2)=2$

⑤ $i=1$

$1+Max(D(1,0), D(2,1))=2$ with $D(1,0)=0$, $D(2,1)=1$

⑥ $i=2$

$1+Max(D(1,1), D(2,0))=2$ with $D(1,1)=1$, $D(2,0)=0$

Solution 3 (DP)

- Inverted problem.
- dp(i) is the number of floors that can be measured with i eggs.
- Maintain a counter to count the number of moves.
- Test i from k to 1.

Recurrence Relation

$$res = \text{number of all moves now}$$
$$for(i = k\ to\ 1):$$
$$dp[i] = dp[i-1] + dp[i] + 1$$

$$\text{Broken} => dp(i-1) \quad => \text{Use} \quad (i-1) \quad \text{eggs and check floor in} \quad (res-1) \quad \text{moves.}$$
$$\text{Unbroken} => dp(i) \quad => \text{Use} \quad (i) \quad \text{eggs and check floor in} \quad (res-1) \quad \text{moves.}$$
$$\text{Current floor} => +1$$

Using '+' instead of 'max' is because dp() stands for "moves" not "floors".

Code

```
int superEggDrop(int K, int N) {
    // Init table for record
    vector<int> dp(K + 1);

    int res = 0;
    for (; dp[K] < N; ++res) {  // Max floor smaller than given => drop
        for (int i = K; i > 0; --i) {    // Use
            dp[i] = dp[i] + dp[i - 1] + 1;
        }
    }

    return res;
}
```

Complexity

$\#$ of subproblems $O(logN)$, complexity of each take $O(\log K)$

Time complexity $\qquad O(K \log N)$

Space complexity $\qquad O(K)$

Solution 4 (Mathematical Thought)

- If we have K eggs on hand, and we can move n times.
- Then set the maximum number of floors we can test to fun(k,n), and we can calculate the fun(k,n) from the following formula:

$$fun(k,n) = n + n(n-1)/2! + n(n-1)(n-2)/3! + \ldots\ldots + n(n-1)(n-2)\ldots\ldots(n-k)/k!$$

We can slowly increase n, or we can do a binary search between n and N to find The smallest n, so that fun(k,n) is greater than or equal to N.

reference

Code

```
class Solution {
public:
    int superEggDrop(int k, int N) {
        if (k == 1 || N<3) return N;
        //n is the number of eggs we need when we do binary search
        double n = log(N) / log(2);
        // If the number of eggs is not enough to support our binary search,
        // slowly increase the number of moves n until the return value of the fun function is
        // greater than or equal to the target floor N
        if (k < n++)
            while (fun(k, n) < N) ++n;
        return n;
    }
private:
    //In the worst case,
    // the maximum number of floors that can be tested if
    // the number of moves is n and the number of eggs is k
    int fun(int k, int n) {
        int i=1,temp = 1, maxNumOfF = 0;
        while (i <= k) {
            temp = temp*(n--) / (i++);
            maxNumOfF += temp;
        }
        return maxNumOfF;
    }
```