# EKF

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设系统状态$x$与观测值$z$之前有函数关系：

$$z = h(x) + v, x \sim \mathcal{N}(\mu, P), v \sim \mathcal{N}(0, R) \tag{1}$$

现有此刻系统状态的估计值$x_0$，则对观测值的估计为：

$$z_0 = h(x_0) \tag{2}$$

又有此刻的观测值$z$，欲求$x$。通过泰勒展开线性化式(1)：

$$z = z_0 + h'(x_0)\,(x - x_0) + O((x - x_0)^2) + v \tag{3}$$

令$\delta z = z - z_0, \delta x = x - x_0$，且令$H = h'(x_0) = \frac{dz}{dx}|_{x=x_0}$为$h\colon x \to z$的雅可比矩阵，则有：

$$\delta z \approx H\delta x + v, \delta x \sim \mathcal{N}(0, P), v \sim \mathcal{N}(0, R) \tag{4}$$

解出Kalman增益$K = PH^T(HPH^T + R)^{-1}$，则$\delta x$的最优后验估计为：

$$\widehat{\delta x} = 0 + K(\delta z - 0) = K\delta z \tag{5}$$

相应地：

$$\hat{x} = x_0 + \widehat{\delta x} = x_0 + K\,(z - h(x_0)) \tag{6}$$

由于$x$与$\delta x$的方差相同，更新$x$的方差估计为：

$$\hat{P} = (I - KH)P \tag{7}$$