1.

设 $P(x)$ 表示 x 是无知的

设 $Q(x)$ 表示 x 是教授

设 $R(x)$ 表示 x 是虚荣的

(1)$\forall x(Q(x) \to \neg P(x))$

(2)$\forall x(P(x) \to R(x))$

(3)$\forall x(Q(x) \to \neg R(x))$

2.

(1)

$$(1) A \to \exists vB, A \vdash A \quad (\in)$$
$$(2) A \to \exists vB, A \vdash A \to \exists vB \quad (\in)$$
$$(3) A \to \exists vB, A \vdash \exists vB \quad (1)(2)(\to -)$$
$$(4) A \to \exists vB, A, B \vdash B \quad (\in)$$
$$(5) A \to \exists vB, A \vdash B \quad (3)(4)(\text{定理}10)$$
$$(6) A \to \exists vB \vdash A \to B \quad (5)(\to +)$$
$$(7) \vdash (A \to \exists vB) \to (A \to B) \quad (6)(\to +)$$
$$(8) (A \to B) \to \exists v(A \to B) \quad (\text{定理}2)$$
$$(9) \vdash (A \to \exists vB) \to \exists v(A \to B) \quad (7)(8)(PC\text{定理}8)$$

(2)

$$(1) \exists v(A \to B), A \vdash \exists v(A \to B) \quad (\in)$$
$$(2) \exists v(A \to B), A, A \to B \vdash A \quad (\in)$$
$$(3) \exists v(A \to B), A, A \to B \vdash A \to B \quad (\in)$$
$$(4) \exists v(A \to B), A, A \to B \vdash B \quad (2)(3)(\to -)$$
$$(5) \exists v(A \to B), A \vdash B \quad (1)(4)(\text{定理}10)$$
$$(6) B \to \exists vB \quad (\text{定理}2)$$
$$(7) \exists v(A \to B), A \vdash \exists vB \quad (5)(6)(\to -)$$
$$(8) \exists v(A \to B) \vdash A \to \exists vB \quad (7)(\to -)$$
$$(9) \vdash \exists v(A \to B) \to (A \to \exists vB) \quad (8)(\to -)$$

(3)

$$(1)\forall vB \to A, B \vdash B \quad (\in)$$

$$(2)\forall vB \to A, B \vdash \forall vB \quad (\text{定理}4)$$

$$(3)\forall vB \to A, B \vdash \forall vB \to A \quad (\in)$$

$$(4)\forall vB \to A, B \vdash A \quad (2)(3)(\to -)$$

$$(5)\forall vB \to A \vdash B \to A \quad (4)(\to +)$$

$$(6) \vdash (\forall vB \to A) \to (B \to A) \quad (4)(\to +)$$

$$(7)(B \to A) \to \exists v(B \to A) \quad (\text{定理}2)$$

$$(8) \vdash (\forall vB \to A) \to \exists v(B \to A) \quad (6)(7)(PC\text{定理}8)$$

(4)

$$(1)\exists v(B \to A), \forall vB \vdash \exists v(B \to A) \quad (\in)$$

$$(2)\exists v(B \to A), \forall vB, B \to A \vdash \forall vB \quad (\in)$$

$$(3)\exists v(B \to A), \forall vB, B \to A \vdash B \quad (2)(\text{定理}1)$$

$$(4)\exists v(B \to A), \forall vB, B \to A \vdash B \to A \quad (\in)$$

$$(5)\exists v(B \to A), \forall vB, B \to A \vdash A \quad (3)(4)(\to -)$$

$$(6)\exists v(B \to A), \forall vB \vdash A \quad (1)(5)(\text{定理}10)$$

$$(7)\exists v(B \to A) \vdash \forall vB \to A \quad (6)(\to +)$$

$$(8) \vdash \exists v(B \to A) \to (\forall vB \to A) \quad (7)(\to +)$$

3.

(1)

$$(1)\forall x(A\to B),A\vdash\forall x(A\to B)\quad(\in)$$

$$(2)\forall x(A\to B)\to(\forall xA\to\forall xB)\quad(A_5)$$

$$(3)\forall x(A\to B),A\vdash\forall xA\to\forall xB\quad(1)(2)(\to -)$$

$$(4)\forall x(A\to B),A\vdash A\quad(\in)$$

$$(5)A\to\forall xA\quad(A_6)$$

$$(6)\forall x(A\to B),A\vdash\forall xA\quad(4)(5)(\to -)$$

$$(7)\forall x(A\to B),A\vdash\forall xB\quad(3)(6)(\to -)$$

$$(8)\forall x(A\to B)\vdash(A\to\forall xB)\quad(7)(\to +)$$

$$(9)A\to\forall xB,A\vdash A\quad(\in)$$

$$(10)A\to\forall xB,A\vdash A\to\forall xB\quad(\in)$$

$$(11)A\to\forall xB,A\vdash\forall xB\quad(9)(10)(\to -)$$

$$(12)\forall xB\to B\quad(\text{定理}1)$$

$$(13)A\to\forall xB,A\vdash B\quad(11)(12)(\to -)$$

$$(14)A\to\forall xB\vdash A\to B\quad(13)(\to +)$$

$$(15)A\to\forall xB\vdash\forall x(A\to B)\quad(14)(\text{定理 }4)$$

$$(16)\forall x(A\to B)\vdash\dashv(A\to\forall xB)\quad(8)(15)(\vdash\dashv +)$$

(2)

$$(1)\forall x(A\to B),\exists xA\vdash\exists xA\quad(\in)$$

$$(2)\forall x(A\to B),\exists xA,A\vdash\forall x(A\to B)\quad(\in)$$

$$(3)\forall x(A\to B)\to(A\to B)\quad(\text{定理}1)$$

$$(4)\forall x(A\to B),\exists xA,A\vdash A\to B\quad(2)(3)(\to-)$$

$$(5)\forall x(A\to B),\exists xA,A\vdash A\quad(\in)$$

$$(6)\forall x(A\to B),\exists xA,A\vdash B\quad(4)(5)(\to-)$$

$$(7)\forall x(A\to B),\exists xA\vdash B\quad(1)(6)(\text{定理}10)$$

$$(8)\forall x(A\to B)\vdash\exists xA\to B\quad(7)(\to+)$$

$$(9)\exists xA\to B,A\vdash A\quad(\in)$$

$$(10)\exists xA\to B,A\vdash\exists xA\quad(9)(\text{定理}2)$$

$$(11)\exists xA\to B,A\vdash\exists xA\to B\quad(\in)$$

$$(12)\exists xA\to B,A\vdash B\quad(10)(11)(\to-)$$

$$(13)\exists xA\to B\vdash A\to B\quad(12)(\to+)$$

$$(14)\exists xA\to B\vdash\forall x(A\to B)\quad(13)(\text{定理}4)$$

$$(15)\forall x(A\to B)\vdash\dashv\exists xA\to B\quad(8)(14)(\vdash\dashv+)$$

(3)

$$(1)\forall x(A\wedge B)\vdash\forall x(A\wedge B)\quad(\in)$$

$$(2)\forall x(A\wedge B)\to(A\wedge B)\quad(\text{定理}1)$$

$$(3)\forall x(A\wedge B)\vdash A\wedge B\quad(1)(2)(\to -)$$

$$(4)\forall x(A\wedge B)\vdash A\quad(3)(\wedge -)$$

$$(5)\forall x(A\wedge B)\vdash B\quad(3)(\wedge -)$$

$$(6)\forall x(A\wedge B)\vdash\forall xA\quad(4)(\text{定理}5)$$

$$(7)\forall x(A\wedge B)\vdash\forall xB\quad(5)(\text{定理}5)$$

$$(8)\forall x(A\wedge B)\vdash\forall xA\wedge\forall xB\quad(6)(7)(\wedge +)$$

$$(9)\forall xA\wedge\forall xB\vdash\forall xA\wedge\forall xB\quad(\in)$$

$$(10)\forall xA\wedge\forall xB\vdash\forall xA\quad(9)(\wedge -)$$

$$(11)\forall xA\to A\quad(\text{定理}1)$$

$$(12)\forall xA\wedge\forall xB\vdash A\quad(10)(11)(\to -)$$

$$(13)\forall xA\wedge\forall xB\vdash\forall xB\quad(10)(\wedge -)$$

$$(14)\forall xB\to B\quad(\text{定理}1)$$

$$(15)\forall xA\wedge\forall xB\vdash B\quad(13)(14)(\to -)$$

$$(16)\forall xA\wedge\forall xB\vdash A\wedge B\quad(12)(15)(\wedge +)$$

$$(17)\forall xA\wedge\forall xB\vdash\forall x(A\wedge B)\quad(16)(\text{定理}5)$$

$$(18)\forall x(A\wedge B)\vdash\dashv\forall xA\wedge\forall xB\quad(8)(17)(\vdash\dashv +)$$

(4)

$$(1)\exists x(A\lor B)\vdash\exists x(A\lor B)\quad(\in)$$

$$(2)\exists x(A\lor B),A\lor B\vdash A\lor B\quad(\in)$$

$$(3)\exists x(A\lor B),A\lor B,A\vdash A\quad(\in)$$

$$(4)\exists x(A\lor B),A\lor B,A\vdash\exists xA\quad(3)(\text{定理}2)$$

$$(5)\exists x(A\lor B),A\lor B,A\vdash\exists xA\lor\exists xB\quad(4)(\lor+)$$

$$(6)\exists x(A\lor B),A\lor B,B\vdash B\quad(\in)$$

$$(7)\exists x(A\lor B),A\lor B,B\vdash\exists xB\quad(6)(\text{定理}2)$$

$$(8)\exists x(A\lor B),A\lor B,B\vdash\exists xA\lor\exists xB\quad(7)(\lor+)$$

$$(9)\exists x(A\lor B),A\lor B\vdash\exists xA\lor\exists xB\quad(2)(5)(8)(\lor-)$$

$$(10)\exists x(A\lor B)\vdash\exists xA\lor\exists xB\quad(1)(9)(\text{定理}10)$$

$$(11)\exists xA\lor\exists xB\vdash\exists xA\lor\exists xB\quad(\in)$$

$$(12)\exists xA\lor\exists xB,\exists xA\vdash\exists xA\quad(\in)$$

$$(13)\exists xA\lor\exists xB,\exists xA,A\vdash A\quad(\in)$$

$$(14)\exists xA\lor\exists xB,\exists xA,A\vdash A\lor B\quad(13)(\lor+)$$

$$(15)\exists xA\lor\exists xB,\exists xA\vdash A\lor B\quad(12)(14)(\text{定理}10)$$

$$(16)\exists xA\lor\exists xB,\exists xB\vdash\exists xB\quad(\in)$$

$$(17)\exists xA\lor\exists xB,\exists xB,B\vdash B\quad(\in)$$

$$(18)\exists xA\lor\exists xB,\exists xB,B\vdash A\lor B\quad(17)(\lor+)$$

$$(19)\exists xA\lor\exists xB,\exists xB\vdash A\lor B\quad(16)(18)(\text{定理}10)$$

$$(20)\exists xA\lor\exists xB\vdash A\lor B\quad(11)(15)(19)(\lor-)$$

$$(21)\exists xA\lor\exists xB\vdash\exists x(A\lor B)\quad(20)(\text{定理}2)$$

$$(22)\exists x(A\lor B)\vdash\dashv\exists xA\lor\exists xB\quad(10)(21)(\vdash\dashv+)$$