二项式定理

$$(x+y)^n = C_n^0 x^n y^0 + C_n^1 x^{n-1} y^1 + \cdots + C_n^{n-1} x^1 y^{n-1} + C_n^n x^0 y^n$$

通项 $T_{r+1} = C_n^r x^{n-r} y^r$

基本不等式

$a>0,\ b>0,\quad \dfrac{2}{\frac{1}{a}+\frac{1}{b}} \leqslant \sqrt{ab} \leqslant \dfrac{a+b}{2} \leqslant \sqrt{\dfrac{a^2+b^2}{2}}$

当且仅当 $a=b$ 时等号成立.

毕达哥拉斯三角恒等式

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

诱导公式

$\sin(2k\pi+\alpha) = \sin\alpha$	$\cos(2k\pi+\alpha) = \alpha$	$\tan(2k\pi+\alpha) = \alpha$
$\sin(-\alpha) = -\sin\alpha$	$\cos(-\alpha) = \cos\alpha$	$\tan(-\alpha) = -\tan\alpha$
$\sin(\pi+\alpha) = -\sin\alpha$	$\cos(\pi+\alpha) = -\cos\alpha$	$\tan(\pi+\alpha) = \tan\alpha$
$\sin(\pi-\alpha) = \sin\alpha$	$\cos(\pi-\alpha) = -\cos\alpha$	$\tan(\pi-\alpha) = -\tan\alpha$
$\sin(\frac{\pi}{2}+\alpha) = \cos\alpha$	$\cos(\frac{\pi}{2}+\alpha) = -\sin\alpha$	$\tan(\frac{\pi}{2}+\alpha) = -\cot\alpha$
$\sin(\frac{\pi}{2}-\alpha) = \cos\alpha$	$\cos(\frac{\pi}{2}-\alpha) = \sin\alpha$	$\tan(\frac{\pi}{2}-\alpha) = \cot\alpha$

和差公式

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha\pm\beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

二倍角公式

$$\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

降幂公式

$$\sin^2\theta = \frac{1-2\cos\theta}{2}, \quad \cos^2\theta = \frac{1+2\cos\theta}{2}$$

和差化积

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

积化和差

$$\sin\alpha\cos\beta = \frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{2}$$

$$\cos\alpha\sin\beta = \frac{\sin(\alpha+\beta)-\sin(\alpha-\beta)}{2}$$

$$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}$$

$$\sin\alpha\sin\beta = -\frac{\cos(\alpha+\beta)-\cos(\alpha-\beta)}{2}$$

辅助角公式

$$a\sin x + b\sin x = \sqrt{a^2+b^2}\,\sin(x+\varphi),\quad \varphi = \arctan\frac{b}{a}.$$

基本求导公式

$$(C)' = 0$$

$$(x^{\mu})' = \mu x^{\mu-1}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x$$

$$(\cot x)' = -\csc^2 x$$

$$(\sec x)' = \sec x \tan x$$

$$(\csc x)' = -\csc x \cot x$$

$$(a^x)' = a^x \ln a$$

$$(e^x)' = e^x$$

$$(\log_a x)' = \frac{1}{x\ln a}$$

$$(\ln x)' = \frac{1}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

基本积分公式

$$\int 0\,dx = C$$

$$\int 1\,dx = x + C$$

$$\int x^{\mu}\,dx = \frac{1}{\mu+1}x^{\mu+1} + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int a^x \, dx = \frac{1}{\ln a} a^x + C$$

$$\int e^x \, dx = e^x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{dx}{1+x^2} = \arctan x + C$$

$$\int \tan x \, dx = -\ln|\cos x| + C$$

$$\int \cot x \, dx = \ln|\sin x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

$$\int \frac{dx}{x^2+a^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x \pm \sqrt{x^2 \pm a^2}\right| + C$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\arcsin\frac{x}{a} + C$$

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$$

华里士公式

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \begin{cases} \frac{n-1}{n}\cdot \cdots \cdot \frac{1}{2}\cdot\frac{\pi}{2}, & n\text{为偶数} \\ \frac{n-1}{n}\cdot \cdots \cdot \frac{2}{3}, & n\text{为奇数} \end{cases}$$

基本幂级数

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-1)^n x^n + \cdots = \sum_{n=0}^{\infty}(-1)^n x^n,\quad x\in(-1,1)$$

$$\ln(1+x) = 1 - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^n\frac{x^{n+1}}{n+1} + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n+1},\quad x\in[-1,1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty}\frac{x^n}{n!},\quad x\in(-\infty,+\infty)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!},\quad x\in(-\infty,+\infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n\frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!},\quad x\in(-\infty,+\infty)$$

$$(1+x)^\alpha = 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^2 + \cdots \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n + \cdots = \sum_{n=0}^{\infty}\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n$$

$$x\in(-\infty,+\infty)$$