线性规划和整数规划模型

线性规划模型

1.基本模型

$$
\begin{aligned}
&max(or\ min)\ z=c^Tx\\
&s.t\begin{cases}Ax\geq(or\ \ \leq,=)\ b\\ x\geq 0\end{cases}
\end{aligned}
$$

其中c为目标函数的系数向量，又称为价值向量；
x为决策向量；
A为约束方程组的系数矩阵，b为约束方程组的常数向量。

称满足约束条件的解为线性规划问题的可行解，而使目标函数达到最大（或最小）值的解称为最优解。

2.求解方法

使用python的cvxpy库；

例1

求解下列线性规划模型：

$$
\begin{aligned}
&max\ z=70x_1+50x_2+60x_3\\
&s.t.\begin{cases}2x_1+4x_2+3x_3\leq 150,\\ 3x_1+x_2+5x_3\leq 160,\\ 7x_1+3x_2+5x_3\leq 200,\\ x_i\geq 0,\ i=1,2,3\end{cases}
\end{aligned}
$$

解法：

```
from cvxpy as cp
from numpy import np
c = array([70, 50, 60])
a = array([2, 4, 3], [3, 1, 5], [7, 3, 5])
b = array([150, 160, 200])
x = cp.Variable(3, pos = True)    #定义决策变量
obj = cp.Maximize(c @ x)
cons = [a @ x <= b]
```

```
prob = cp.Problem(obj, cons)
prob.solve(slover = 'GLPK_MI')
print('最优解为', x.value)
print('最优值为', prob.value)
```

例2

求解下列线性规划模型：

$$min\ z=\sum_{i=1}^{6}\sum_{j=1}^{8}c_{ij}x_{ij},$$

$$s.t.\begin{cases}\sum_{i=1}^{6}x_{ij}=d_j, \quad j=1,2,\dots,8,\\ \sum_{j=1}^{8}x_{ij}\le e_i, \quad i=1,2,\dots,6,\\ x_{ij}\ge 0, \quad i=1,2,\dots,6;j=1,2,,\dots,8.\end{cases}$$

解法：

```
#数据由Excel读入
import cxvpy as cp
import numpy as np
data = pd.read_excel("data.xlsx", header = None)
data = data.values
c = data[:-1,:-1]
d = data[-1, ;-1]
e = data[:-1, -1]
x = cp.variable((6, 8), pos = True)
obj = cp.Minimize(cp.sum(cp.multiply(c, x)))
con = [cp.sum(x, axis = 0) == d, #axis = 0表示按列方向
           cp.sum(x, axis = 1) <= e] #axis = 1表示按行方向
prob = cp.Problem(obj, con)
prob.solve(slover = 'GLPK_MI')
print("最优解为", x.value)
print("最优值为", prob.value)
xd = pd.DataFrame(x.value)
xd.to_excel("data.xlxs")
```

整数规划

1.基本模型

$$max(or\ min)\ z = \sum_{j=1}^{n} c_j x_j$$

$$s.t. \begin{cases} \sum_{j=1}^{n} a_{ij} x_j \leq (or\ \ =, \geq) b_i, & i = 1, 2, \ldots, m, \\ x_j \geq 0, \quad j = 1, 2, \ldots, n, \\ Some\ or\ all\ of\ x_1, x_2, \ldots, x_n\ are\ integers. \end{cases}$$

2.求解方法

例

求解下列整数规划问题：

$$min\ z = \sum_{i=1}^{10} x_i,$$

$$s.t. \begin{cases} \sum_{i=1}^{10} y_{ij} \geq 1, \quad j = 1, 2, \ldots 10, \\ d_{ij} y_{ij} \leq 10 x_i, \quad i, j = 1, 2, \ldots, 10, \\ \sum_{j=1}^{10} y_{ij} \leq 5, \quad i = 1, 2, \ldots, 10, \\ x_i \geq y_{ij}, \quad i, j = 1, 2, \ldots, 10, \\ x_i = yii, \quad i, j = 1, 2, \ldots, 10, \\ x_i, y_{ij} = 0\ or 1, \quad i, j = 1, 2, \ldots, 10. \end{cases}$$

解法：

```
import cvxpy as cp
import numpy as np
a = np.loadtxt("data.txt")
d = np.zeros((10, 10))
for i in range(10):
        for j in range(10):
                d[i, j] = np.linalg.norm(a[:, i] - a[:, j]) #计算距离
x = cp.Variable(10, integer = True)                  #设置整数模式
y = cp.Variable((10, 10), integer = True)
obj = cp.Minimize(sum(x))
con = [sum(y) >= 1, cp.sum(y, axis = 1) <= 5,
           x >= 0, x <= 1, y >= 0, y <= 1]
for i in range(10):
        con.append(x[i] == y[i, i])
        for j in range(10):
                con.append(d[i, j] * y[i, j] <= 10 * x[i])
                com.append(x[i] >= y[i, j])
prob = cp.Problem(obj, con)
```

```python
prob.solve(solver = 'GLPK_MI')
print("最优值为", prob.value)
print("最优解为", x.value)
print("————————", y.value)
```