Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
C		2418781

A Unique Momentum Research Based On Axiomatic Approach

Tennis is an athletic sport characterized by unique match rules and intense competitions that stimulate curiosity about the patterns of game dynamics.**Momentum**, as a concept deeply embedded in sports philosophy, has long been employed to investigate flow of tennis matches. This paper establishes models to study the flow of the game and the essence of momentum in a rational manner, exploring the relationship between them. We finally unveils the mysterious veil surrounding momentum in the context of tennis.

In the first step, we observed the data in the table, corrected some errors, and extracted key features reflecting the match flow. Subsequently, we established the PwMF model (**P**erformance **w**ith **M**atch **F**low) to describe the on-court dynamics. In the PwMF model, we selected five indicators: **explosiveness, precision of movement, strategic ability, control, and technical performance**. We depicted the athlete's on-court performance using a radar chart, ultimately obtaining an overall performance score. Next, we measured the current state of the athlete using the overall performance score, stamina, whether they are a server, and their psychological state as four indicators, resulting in the athlete's state score. Finally, we visualized the change in the state scores of the two players to illustrate the match situation, presenting it in a visual format.

In the second step, we noticed a phenomenon that a variety of methods for calculating momentum are already existed, and yet the definition of momentum remained ambiguous. In order to bring clarity to the concept of momentum and enhance our understanding of its role, we opted for the **axiomatic modeling approach**. We engaged in thoughtful exploration and examination of the momentum category, aiming to establish an innovative mathematical model to describe momentum. Subsequently, we applied this model to match-1701 and match-1303, providing visual interpretations of the relationship between momentum and the course of the matches. Fortunately, our model effectively captures shifts in match-winning probabilities and exhibits **excellent predictive capabilities** for the intensity of the matches.

In the third step, we established an **XGBoost** classification model to investigate the relationship between momentum and swings in the play. We refuted the notion that swings occur randomly and proceeded to predict swing points in the 31st match. Two momentum features, along with five other match-related data, were selected as inputs to determine whether a particular time period in the game represented a swing in the situation (swings or normal). The average accuracy of the classification model reached **85.09%**. Subsequently, we utilized **SHAP plots** to illustrate the significant contribution of momentum features to the model output, indicating that changes in the game situation are not random. Additionally, we observed that ace, unforced errors, and rally count were the most relevant factors for the predictions.Based on it, we made some suggestions for the players.

Finally, we conducted **sensitivity and robustness analyses** on the model, emphasizing the model's generalization ability. In addition, we provided a comprehensive summary of the model's strengths and weaknesses, along with **future research directions** such as further data collection and exploring individual differences in momentum. Simultaneously, addressing the role of momentum on the court, we compiled a memo offering recommendations to tennis coaches based on our research findings.

Keywords: PwMF Model, Axiomatic modeling approach, XGBoost

Contents

1 Introduction

“Matches between the top players are always very close. But, if you have the mental ability to stay strong, stay patient and confident and just have belief in the right moments, then you get a win, you know. So that’s what makes the difference,” said Djokovic.

1.1 Problem Background

Tennis is a game of momentum. In the 2023 Wimbledon Men’s Singles final, 20-year-old Spanish prodigy Carlos Alcaraz defeated the legendary tennis star Novak Djokovic. People continue to wonder what played a pivotal role in Carlos’s victory. In the realm of tennis, when discussing turning points in a match, the concept of ”momentum” often emerges. Momentum is considered the driving force behind shifts in a tennis match, yet as a psychological effect, it remains an elusive phenomenon that is challenging to observe. Some remain skeptical about whether momentum genuinely influences the outcome of a match. In this report, we attempt to unravel the mystery of momentum through data analysis, aiming to scientifically enhance players’ performance on the court and thereby improve their overall results. Furthermore, momentum is not confined to the tennis domain; it extends to various competitive fields where its effects can be observed. Therefore, the study of momentum holds great significance beyond just tennis.

1.2 Restatement of the Problem

Data is provided for every point from all Wimbledon 2023 men’s matches after the first 2 rounds. After understanding the basic rules of tennis and thorough research, We decide to dive into the following questions:

1. Introduce a model that captures the nuanced dynamics of sporting events, forecasting the enhanced performance of players amidst shifts in play, and systematically measuring the magnitude of their accomplishments.

2. Use our model to assess the claim that ”momentum” hardly plays any role in the match and that the swings in play and runs of success by one player are random. In our words, that is to provide a proof that momentum exists and plays a relatively important role in tennis court.

3. Uncover key indicators facilitating the identification of impending shifts in the momentum of play, signaling a transition from favoring one player to another. Employ our model to forecast these pivotal moments in specific matches and provide strategic advice to players accordingly. This involves elucidating potential relationships, if any, between momentum and swings in the play.

4. Evaluate the predictive capabilities of the model, examine potential factors not yet considered that may impact the model’s performance, and assess the model’s generalization ability.

1.3 Our work

We began by conducting comprehensive exploratory data analysis, complemented by relevant literature, to gain valuable insights into players and matches, including match formats and underlying trends. Additionally, to capture the dynamic flow of the play, we developed a Performance with Match Flow (PwMF) evaluation model to depict player performance and status during matches, and presented the evolving match dynamics using visualizations.

Additionally, we employed a deductive modeling approach to innovatively define momentum and dynamically capture changes in players' momentum on the court through the occurrence of "events." Momentum is defined as the psychological impact influencing player performance, with scoring situations serving as indicators of swings in the play. Subsequently, we labeled each scoring situation in 31 matches and utilized the XGBoost machine learning algorithm to investigate the relationship between changes in momentum and swings in the play.

By examining fluctuations in players' performance and momentum during scoring changes, we identified certain behavioral patterns specific to players, which formed the basis for training recommendations for players in matches.

Finally, we explored the sensitivity and robustness of the model and analyzed its strengths and weaknesses, supplementing potential directions for further research.

Our exploration of player evaluation systems and the concept of momentum in sports psychology also contributes significantly to related research endeavors.

The following is a flow chart of our model framework:

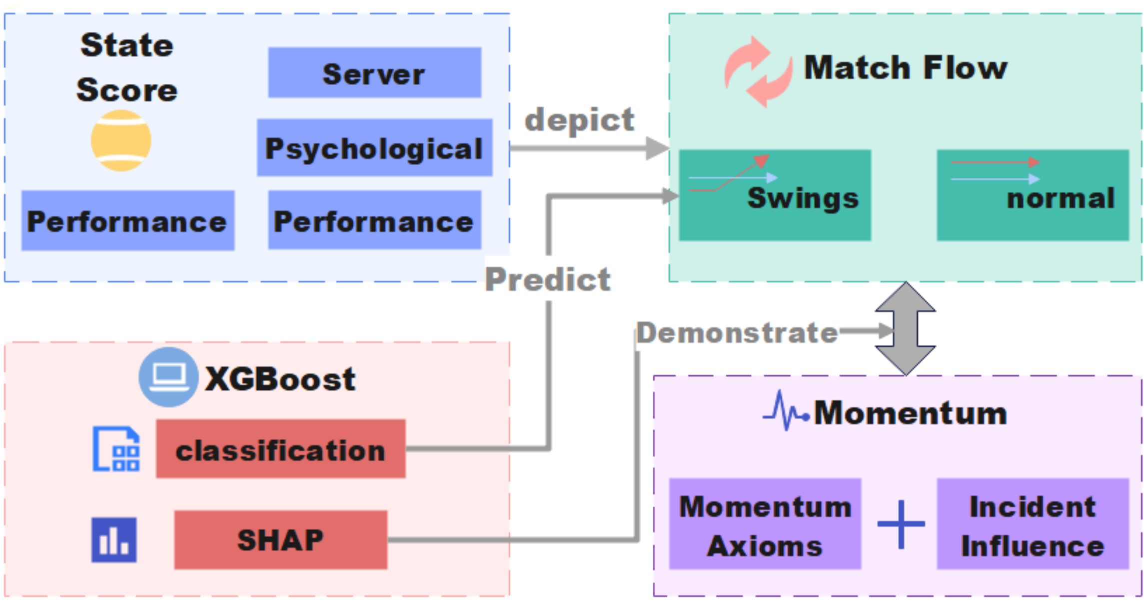


Figure 1: Overall Structure

2 Notations

Symbols	Descriptions
$Expl$	Explosiveness
$Prec$	Precision
$Stra$	Strategic acumen
$Cont$	Control
$Tech$	Technical performance
$Pref$	Overall Performance Rating
$Serv$	To serve or not to serve
Ppw	physical strength
Phb	psychological state
M_i	Momentum of current point
$Exp1$	Expectation of player1 at current point
$Exp2$	Expectation of player2 at current point
$courtExp$	Expectation of the match
PI	number of positive incident happened at current point
NI	number of negative incident happened at current point
η	psychological ability of a specific player
s_i	positive incident when one is serving
r_i	negative incident when one is returning

3 Comprehensive EDA

3.1 Data Processing

To begin with, let's delve into the dataset, which encompasses records from 31 matches featuring a total of 32 players. Notably, Carlos Alcaraz and Novak Djokovic stand out for having participated in the highest number of matches, each boasting 5 appearances. The dataset itself comprises over 7000 entries, with notable instances of missing data found across columns such as 'return_depth,' 'speed_mph,' 'serve_depth,' and 'serve_width,' with counts of 1300+, 752, 54, and 54, respectively. In addressing this, we opted to interpolate the missing values in the 'speed_mph' column using the mean, while all other missing values were uniformly treated as 0.

Turning our attention to anomalies within the dataset, a meticulous review highlighted inaccuracies in the 'p1_points_won,' 'p2_points_won,' and 'rally_count' columns. Specifically, rows 4034-4055 lacked scoring data for both players, necessitating a data correction procedure based on information from other columns. Furthermore, it was observed that all entries in the 'rally_count' column from rows 2187-2674 were recorded as 0, a discrepancy promptly addressed by replacing these values with the mean.

3.2 Data Overview

In our analytical journey, we illustrated the fluctuation of total match points - individual player points through line graphs for each match. This visual analysis facilitated the identification and categorization of match dynamics into three distinct types:

1.The Decisive Type: Characterized by the gradual accumulation of a significant advantage by one side, ultimately leading to victory.

2.The Narrow Margin Type: In these encounters, both sides initially exhibit parity, but one gradually edges ahead following a pivotal moment, securing the win.

3.The Stalemate Type: Here, both sides are locked in a tense battle, with neither able to gain a decisive advantage, resulting in a prolonged deadlock.

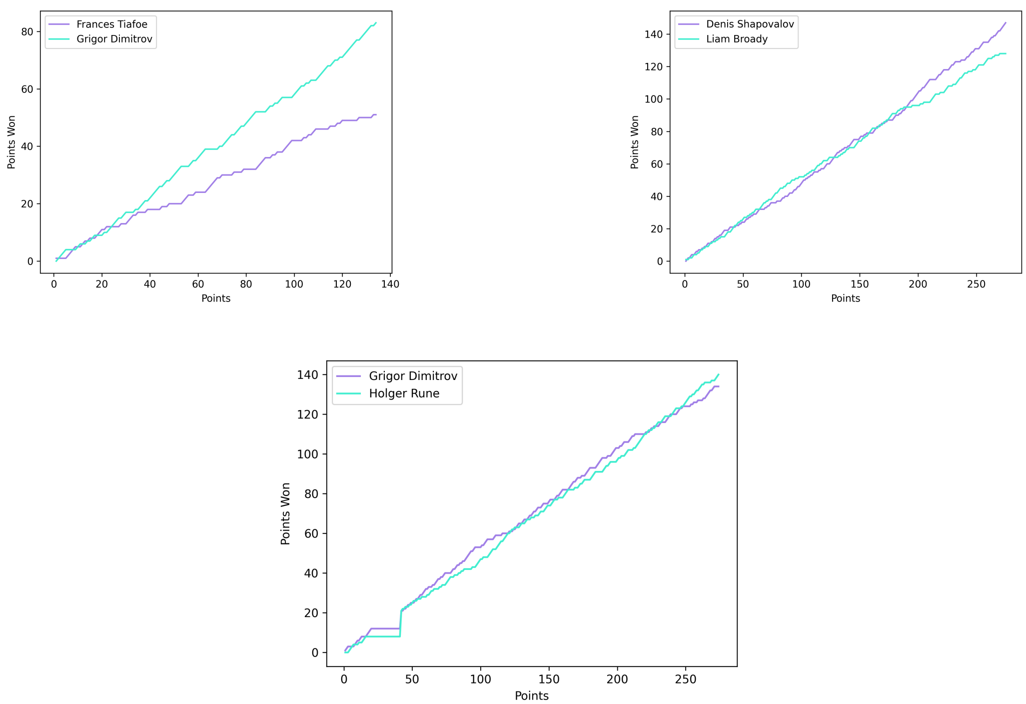


Figure 2: match-point-flow

4 The PwMF Model

In this section, we will establish a PwMF(Perfomance with Match Flow) model to capture the flow of the play and evaluate players' real-time performance. Ultimately, we will utilize a visualization approach, taking the Djokovic vs. Alcaraz match as an example, to depict the real-time dynamics of the match and the performance of the two players.

The following is an overview of the PwMF model:

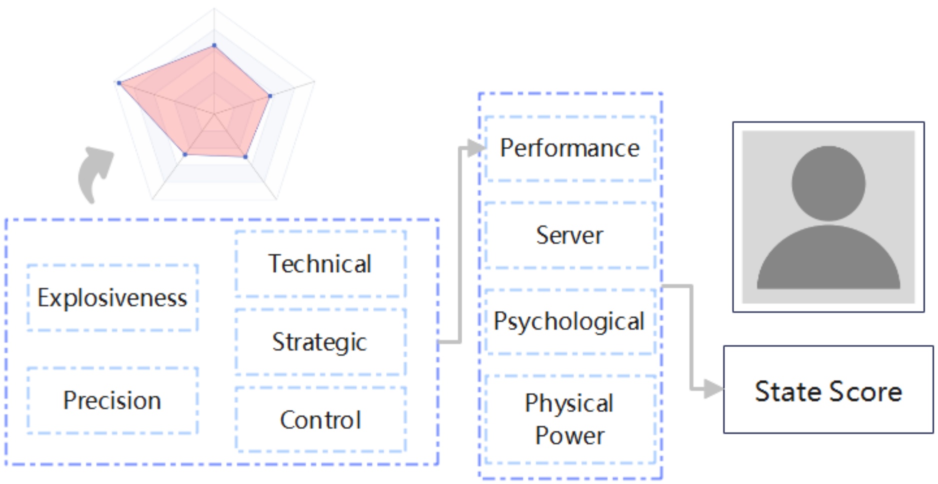


Figure 3: PwMF model

4.1 Establishment of Players Performance Evaluation Model

We initially constructed a model to assess players' real-time performance on the court. This model utilizes indicators across five dimensions to showcase different aspects of the players' performance levels, and it visually presents them through a five-dimensional radar chart. Ultimately, we integrate these five indicators to obtain an overall score for evaluating the players' performance.

4.1.1 Selection of Evaluation Metrics

By thorough research, we discovered that players' performance on the court can be measured using metrics such as explosiveness, precision of movement, strategic capability, control, and technical proficiency. In the following section, we will explain how we will use the data in the table to represent the players' performance in these five aspects.

Explosiveness

We selected serve speed (speed_mph), ace count (ace), and backhand shot rate (winner_shot_type) to measure the explosiveness of the players. These data can reflect the intensity, accuracy, and flexibility of the shots, illustrating the players' level of physical intensity. We will use the average serve speed Spm, ace rate $Acer$, and backhand shot rate Bap over a specific period in the match as variables.

The formula to calculate explosiveness (Expl) is:

$$Expl = k_1 \cdot Spm + k_w \cdot Acer + k_3 \cdot Bap \tag{1}$$

And we set

$$k_1 = 0.1, \quad k_2 = 30, \quad k_3 = 2. \tag{2}$$

Precision of Movement

We chose the average number of serves per point (average of serve_no), the double fault rate (proportion of double faults to total serves), and the non-forced error rate (average of unf_err) to measure the precision of players' movements.

The larger the values of these indicators, the lower the precision of the players' movements, indicating poorer performance. We will use the average number of serves per game Sn, double fault rate Dfp, and non-forced error rate Uep over a specific period in the match as variables.

The formula to calculate precision of movement (Prec) is:

$$Prec = k_4 \cdot (a_1 - Sn) + k_5 \cdot (a_2 - Dfp) + k_3 \cdot (a_3 - Uep) \tag{3}$$

And we set

$$\begin{aligned} &k_4 = 2, \quad k_5 = 15, \quad k_6 = 6, \\ &a_1 = 2, \quad a_2 = 0.2, \quad a_3 = 0.5 \end{aligned} \tag{4}$$

Strategic Capability

The strategic ability of a tennis player is primarily reflected in their ability to identify and exploit their opponent's weaknesses effectively. We calculated the winning percentage for each serving direction (width) and depth during a specific period in the match. The maximum values for each Wid, Dep are selected as variables to assess the player's strategic capability.

The formula to calculate strategic capability (Stra) is:

$$Stra = k_7 \cdot Wid + k_8 \cdot Dep \tag{5}$$

And we set

$$k_7 = 2.5, \quad k_8 = 2.5 \tag{6}$$

Control Power

Control power is reflected in the ability of players to remain calm in crucial situations and seize the opportunity to win key points. The ability to win crucial games (break_pt_won) in the face of returning situations (break_pt) demonstrates a player's ability to control the situation. We use the percentage of players winning games when seizing opportunities while returning serves in the previous period Bpw as a variable to measure control (Cont) power.

Technical Proficiency

Technical proficiency refers to the player's ability to apply various skills flexibly. The player's capability to use appropriate techniques to gain an advantage in crucial moments reflects their technical proficiency. We have selected the net point winning rate $Netw$ during the previous period of matches, and the backhand return rate Bap as variables.

The formula to calculate technical proficiency (Tech) is:

$$Tech = k_{10} \cdot Netw + k_{11} \cdot Bap \tag{7}$$

And we set

$$k_{10} = 3, \quad k_{11} = 2 \tag{8}$$

4.1.2 Visualization of Individual Performances and Calculation of Overall Performance Score

We use a five-dimensional radar chart to represent the performance levels in five dimensions, allowing for a visual comparison of the player's strengths and weaknesses in various aspects. The figure below depicts the performance of a player in a certain match over a specific period :

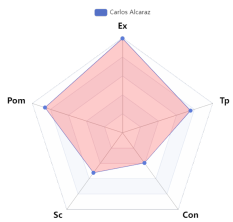


Figure 4: Radar of Alcaraz

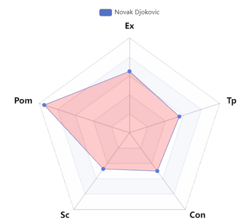


Figure 5: Radar of Djokovic

Finally, we have constructed a formula utilizing the five indicators to calculate a comprehensive score, providing an assessment of the player's real-time performance (Perf):

$$Pref = Expl + Prec + Stra + Cont + Tech \tag{9}$$

And we set the maximum total performance rating score to 60.

4.2 Establishment of Arena Situation Model

In addition to the calculated performance score for each player, we have set three additional indicators to describe the match flow. Ultimately, we integrate these four components to create an index that assesses the current state score of the two players in the matchup, reflecting the present situational status.

4.2.1 Indicators for Assessing Player's Condition

The match flow in tennis are determined by both the performance of the players and various other factors. Among these, three pivotal indicators crucially affect the situation: whether the player is serving $Sevr$, their physical condition Ppw and their psychological state Phb. $Serv$: Based on historical match data, we've observed that servers have a significantly higher probability of winning points compared to receivers. Therefore, we set the value for the player being the server $Serv$ to 5 and 0 if not. Ppw : The player's fatigue level can be measured by the distance covered during a match point. Until the points we need to study are reached, the player with a relatively lower total distance covered is assigned a full score of 25 for stamina. The formula for calculating the stamina value for a player with a relatively higher distance covered is as follows:

$$Ppw = 25 \times (1 - \frac{Dism - Disb}{Disb}) \tag{10}$$

Where $Disb$ represents the distance covered by the player with relatively less running, and $Dism$ represents the distance covered by the player with relatively more running. Phb: The player's psychological state depends on their recent performance in winning matches. Therefore, we set the psychological state score based on the player's win rate in the most recent five matches. The maximum score is 10 points.

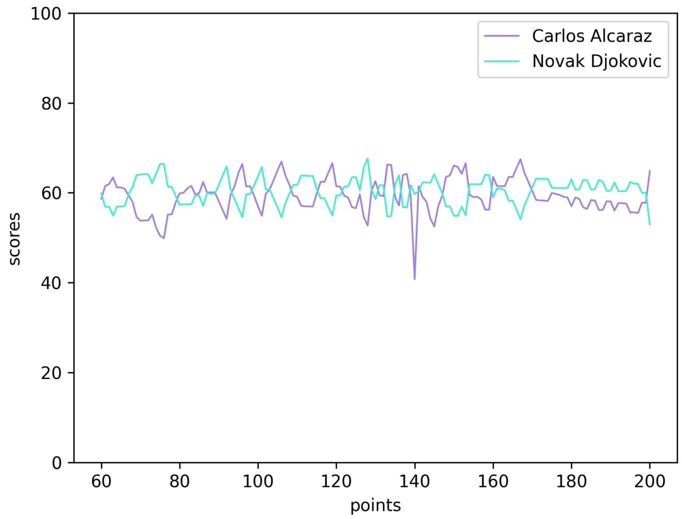


Figure 6: 200_trend

4.2.2 Presentation of the Game Situation

We calculate the current player's state score by summing up the performance score (60 points), serving status (5 points), physical power (25 points), and psychological state (10 points). The match flow are illustrated by comparing the status scores of both players. Figure 6 is the figure depicting the change in state scores for Alcaraz and Djokovic during the 2023 Wimbledon Men's Singles final as the score reached the 200th point in the match.

Ultimately, by combining the five-dimensional radar chart of player performance, the player's points won trend chart, and the state score chart, we effectively present a comprehensive visualization of the match flow, providing a holistic view of the match situation. It can be observed that currently,

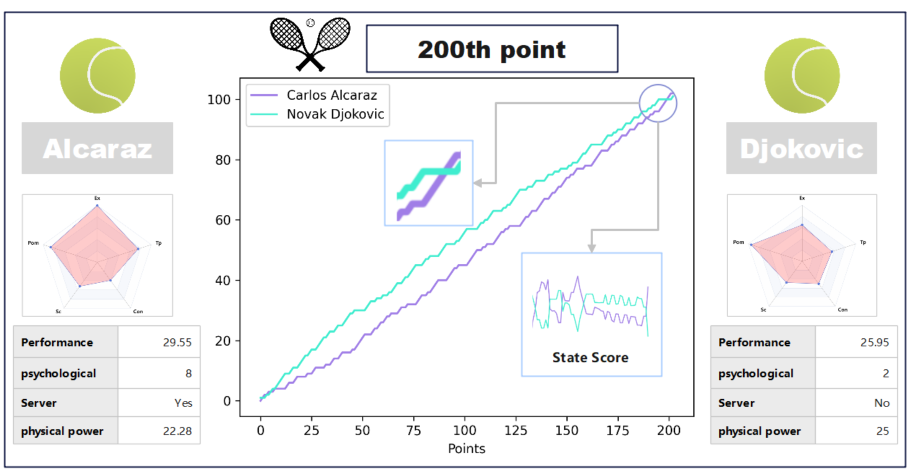


Figure 7: match-trend

Alcaraz's performance score is 29.55, higher than Djokovic's score of 25.95. At the 200th point, the server changed from Djokovic to Alcaraz, indicating that Alcaraz is gaining momentum and potentially turning the match in his favor, while Djokovic is facing a downturn.

5 Axiomatic Momentum Models

For a nuanced concept like momentum, there exist various established research methods. Previous efforts, including those by IBM, have made strides in this field. However, we aspire to define and study momentum from a more foundational perspective. We believe that this approach will aid in a deeper understanding of the role of momentum in tennis. This is the rationale behind adopting the axiomatic modeling approach.

5.1 Axiomatic Approach

The steps of axiomatic approach in modeling construction can be included as the figure below.

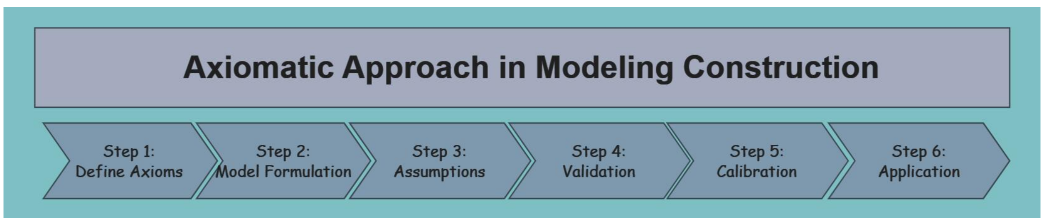


Figure 8: Axiomatic Approach

At first, we come up with fundamental principles for the model, then we use axioms as the basis for constructing the model, and explicitly state any assumptions about the system. One thing that can't be missed is that we need to ensure the model aligns with real-world observations and then we adjust model parameters for better alignment with reality. At last we can implement the model for specific real-world situations.

5.2 Define Axioms

Accordingly, we propose some properties that momentum should possess, including:

1. Momentum is relative, an increase in one side's momentum results in a decrease in the momentum of the other side.

2. Momentum changes with the occurrence of "incidents"

3. The more positive the momentum, the greater the deviation towards the negative side when a negative event occurs.

Now, let's explain why we propose these three axioms. Firstly, we define momentum as the "psychological impact influencing a player's performance in the game." Some potential factors that significantly affect a player's performance include the player's scoring situation on the court and their inherent skill level. We aim to minimize the influence of a player's skill level on the momentum measurement, uncovering factors that help players adjust their mental states and enhance on-court performance.

Therefore, starting from a subjective perspective, we initially define a game property of momentum: an increase in one side's momentum leads to a decrease in the momentum of the other side. This is reasonable as momentum measures the dynamics of the on-court situation, inherently involving one side being in an advantageous position while the other is in a disadvantageous one. Secondly, we posit that momentum changes with the occurrence of "events." When players perceive something happening, it alters their perception of the on-court situation, thereby influencing their behavior. Subsequently, we specify a property of momentum change, based on the empirical understanding that when you feel in control of the game, a detrimental event may significantly impact your confidence.

5.3 Model Formulation

We can extract some useful indicators from the data as "events." Here, we define "events" as "occurrences on the court that reflect a player's performance," encompassing factors such as serving an ace, winning a break point, and so on. The temporal dimension of events corresponds to the scoring situation after each point. The same occurrence can lead to multiple events. For instance, when player1 wins a match point by serving an ace in a game, we consider both the event of securing the match point and the event of serving an ace to have transpired at that moment for player1. We term events that showcase a player's scoring ability as positive incidents and those resulting in losing points as negative incidents. This distinction aids in comprehending the factors on the court that contribute to momentum shifts.

In order to eliminate the influence of a player's inherent skill on the momentum measurement, we conducted an assessment of each player's abilities and devised a model. We refer to the player's perception of their ability to win a match as Expectation. Subsequently, we can define the momentum of player1 on the court as:

$$M_1 = \frac{Exp_1}{Exp_1 + Exp_2} \tag{11}$$

Where M_1 represents the momentum of player 1, while Exp_1 and Exp_2 respectively denote the perception of their own abilities by player 1 and player 2.

Our core logic is that the occurrence of events influences a player's perception of their own abilities, thereby affecting their assessment of relative strengths between the two players in the match. On-court relative strength reflects the inclination of momentum towards a player. The impact of events on momentum can be characterized as:

$$Exp_{i+1} = Exp_i \times [1 + \eta_1 \times (PI - NI)] + \eta_2 \times courtExp \tag{12}$$

Where Exp represents Exp1 or Exp2, indicating the Expectation of player1 or player2, respectively. PI denotes the number of positive incidents occurring at that moment, NI represents the number of negative incidents occurring at that moment. η signifies the player's sensitivity to event perception. courtExp represents historical score data. We assume that the score, on the whole, influences a player's judgment of the on-court situation and is not affected by event occurrences, playing a macro role in the player's assessment of the match situation. An intuitive conceptual diagram is presented below:

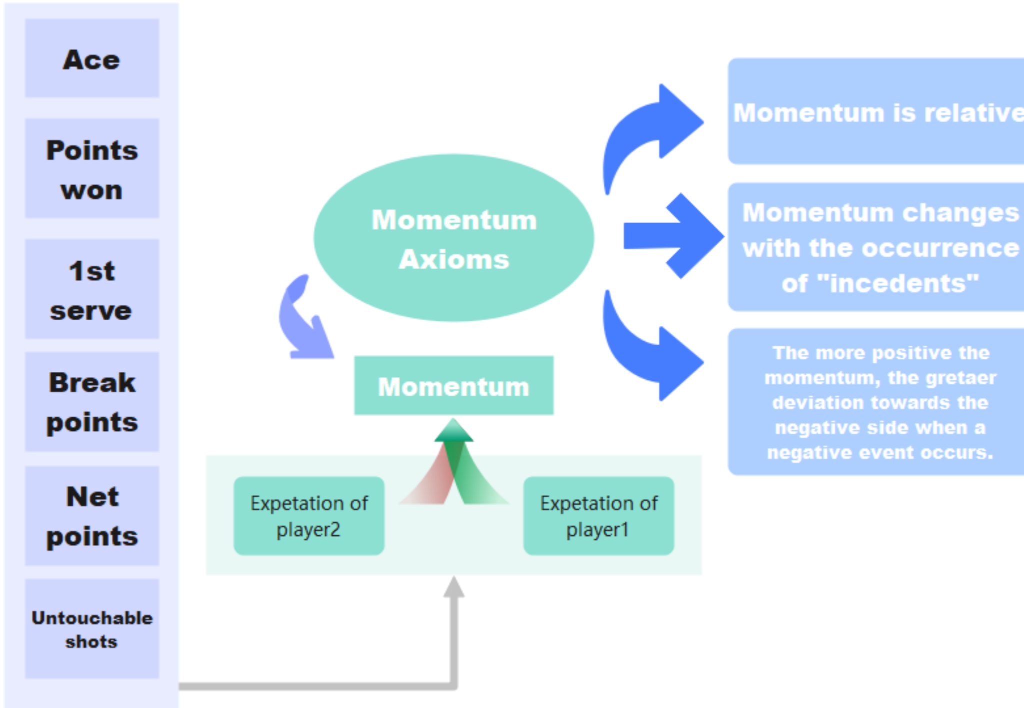


Figure 9: Axiomatic Momentum Model

Considering that the serving side has a higher probability of winning, the expectation of a player while serving should be different from when they are receiving.

Incidents we discover include the following figure 10.

5.4 Assumptions

Our axiomatic model of momentum can be subjective due to the assumptions we make as the followings :

1. The player's perception of the on-court situation is posterior, meaning that the occurrence of events leads to changes in the player's mindset.
2. Changes in the player's mindset will influence their behavior, thereby explaining the impact of momentum on the on-court situation.
3. The player's η is highly personalized.
4. The player's perception of events is indiscriminate across different events.

5.5 Validation and Calibration

Since Properties 1 and 2 are self-evident, we attempt to provide a simple proof approach for Property 3. Firstly, let's consider two cases: one where $Exp1 > Exp2$ and the other where $Exp1 < Exp2$.

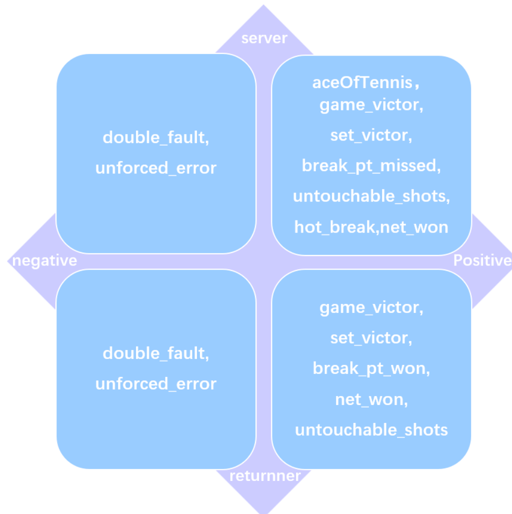


Figure 10: incidents define

Without loss of generality, let's assume a moment in which the score is tied, i.e., $courtExp = 0$. Now, suppose a reverse event occurs for player 1, for example: player 1 is serving in the current game, and after a certain exchange, player 2 scores, i.e., $PI - NI = \sigma < 0$. It can be proven that in these two different scenarios, the degree of change in M_i differs, and the larger $Exp1$ is, the greater the change in M_i after the reverse event occurs.

Different players have different psychological expectations when facing different opponents. Although ideally, we would expect players to have a perfect ratio for regulating their psychological abilities, more precise parameter estimation requires more data support. Here, we set η_1 to be 0.2 and η_2 to be 0.0001. Now, let's try applying the momentum model to some matches to see how it works.

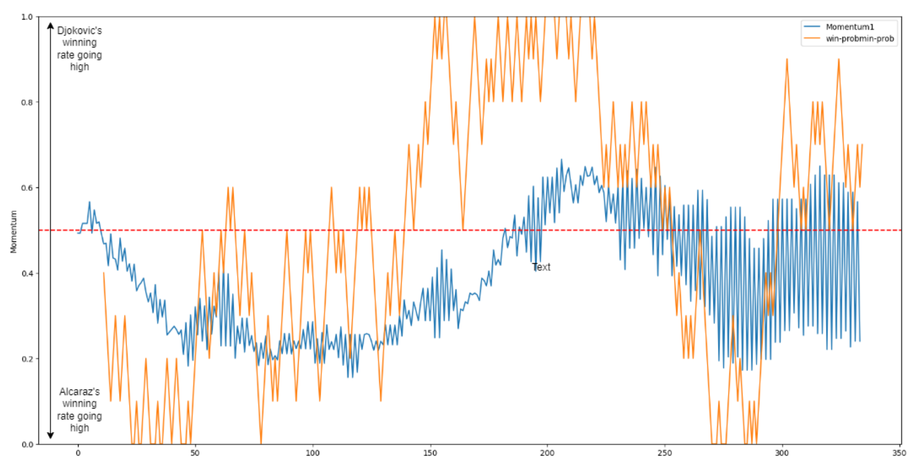


Figure 11: match-1701

5.6 Application

Using the momentum model, we dive into the epic match of Alcaraz and Djokovic and the 3nd match in the data set. And get a result of the following figure:

Figure 11 depicts the dynamic changes in momentum and winning probability for match-1701, while Figure 12 illustrates the dynamic changes in momentum and winning probability for match-1303.

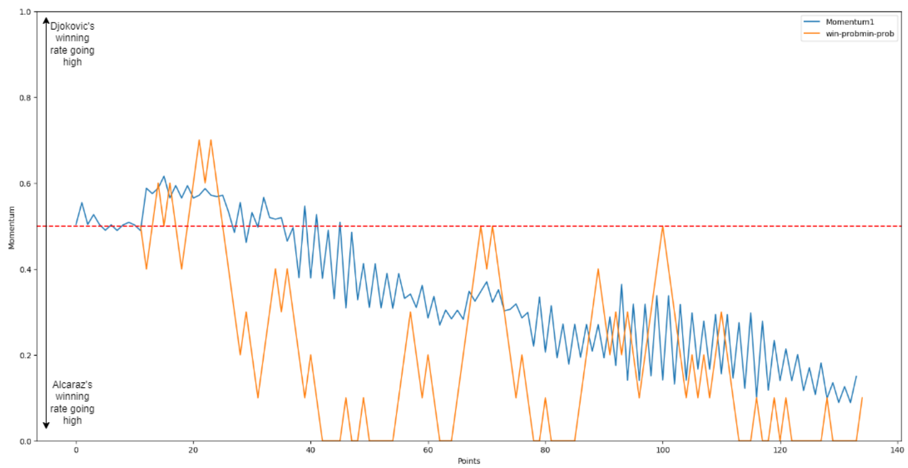


Figure 12: match-1303

We determine the winning probability based on the proportion of individual player scores in the last 10 points. It can be observed that, overall, there is a correlated relationship between our momentum and winning probability, as shown in the figures. Furthermore, for different types of matches, momentum effectively captures the players' perception of the match situation.

In match-1701, during the mid-game, as Carlos's winning probability starts to rise, his momentum also begins to climb. Contrasting with the decisive victory in match-1303, the momentum changes in match-1701 are more pronounced, especially in the later stages of the last set. Here, we observe significant fluctuations in momentum, indicating that the match has reached a critical juncture where both players' enthusiasm has been fully mobilized.

Now let's examine the following questions: Does momentum play any role in a match? And assess the claim that swings in play and runs of success by one player are random.

From the figures, we can observe that momentum effectively reflects the rhythm of the match. When momentum fluctuates significantly, the match becomes more intense. Therefore, momentum can adjust the intensity of the match, fully mobilizing players' drive.

We define swings in play as when a player scores consecutively for more than five points. With this definition, when swings occur, the winning probability tends to be at both ends of the y-axis. As seen in the figures, when the winning probability tends to be at both ends of the y-axis, the balance of momentum also tends to shift in the same direction. This indicates that swings in play are not random. On the contrary, swings in play are related to momentum.

6 Using XGBoost to Reveal the Relationship Between Swings and Momentum

In this section, to demonstrate that the swings in the match are not random but influenced by momentum, we employed an XGBoost machine learning classification model to investigate whether certain features of momentum can help predict the occurrence of swings in the match. Additionally, we introduced other variables as inputs to the classification model, aiming to both emphasize the significance of momentum and facilitate the exploration of other factors influencing the prediction of swings in the match. Finally, SHAP (SHapley Additive exPlanations) plots were used to illustrate the contributions of each input to the classification results.

Before establishing the model, we first need to define the swings in the match. The dynamics of a match can be broadly categorized into two scenarios. In one scenario, both sides engage in a back-and-forth struggle without a significant point differential. The other scenario involves one side gaining a clear advantage, rapidly expanding the points won gap – these are the swings points that we are investigating. In the dataset, intervals where one side consecutively won 5 points or more are labeled as swings points, while intervals where both sides take turns winning points are labeled as normal points. In our dataset of 31 matches, we identified a total of 79 swings points and 76 normal points for subsequent training of the classification model.

Selecting appropriate momentum features as inputs for the model is crucial for training an effective classification model. We chose the maximum difference in momentum between the two sides within the labeled interval as the first input and the type of momentum change between the two sides as the second input(type: oscillation between high and low momentum marked as type 1, one side consistently higher than the other marked as type 2) . Additionally, we selected the total number of aces served (ace), double faults committed (df), points won on the serve (bw), unforced errors (ue), and average duration of rallies (rc) as additional inputs to explore the impact of other factors on situational assessment. In the end, we utilized XGBoost for training the classification model.

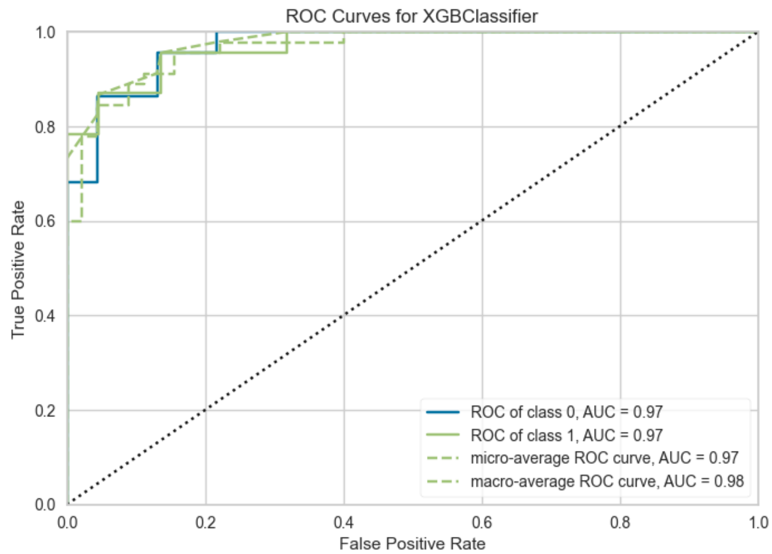
XGBoost (eXtreme Gradient Boosting) is an efficient and flexible machine learning algorithm that belongs to the gradient boosting framework within ensemble learning. It excels in handling structured data and addressing problems such as regression, classification, and ranking. XGBoost models are well-suited for classification tasks on small-scale datasets. Given that our dataset consists of approximately 150 records and involves multiple inputs, employing the XGBoost model is deemed appropriate for classification in this context.

We used the data from the first 30 matches as the training set and compared the prediction results using the data from the 31st match. The obtained predicted results are as follows:

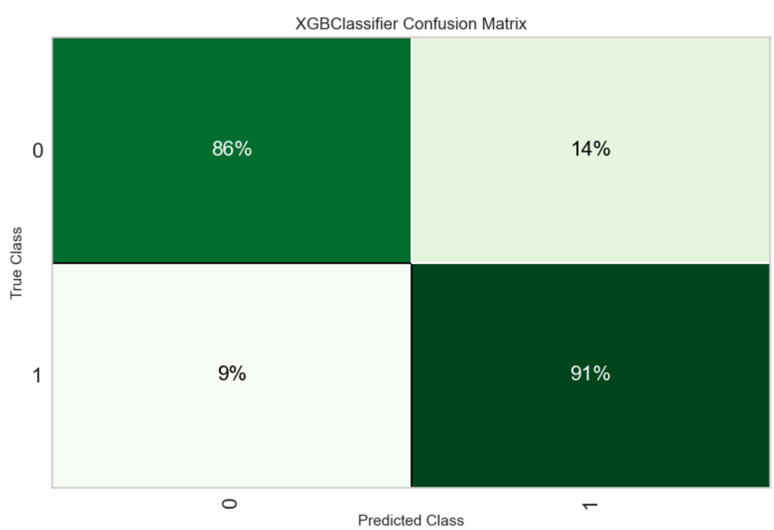
momentum	type	ace	df	bw	ue	rc	prediction label	actual label	prediction scores
0.23	2	0	1	1	4	5	normal	normal	0.9373
0.20	2	1	0	0	2	7	normal	normal	0.9987
0.48	2	0	0	0	1	3	normal	swings	0.5131
0.31	1	0	0	1	4	6	swings	swings	0.8138
0.60	1	1	0	1	1	4	swings	swings	0.9918

The swings points selected for the last three groups in the table are the intervals from 163th to 168th points (Alcaraz narrowing the score gap against Djokovic), 196th to 202th points (Alcaraz overtaking

Djokovic in points), and 295th to 301th points (Alcaraz once again overtaking Djokovic in points). The first two groups represent normal points. It can be observed that, except for the third group where the prediction was incorrect, the predictions for the other four groups were accurate, and the accuracy of the predictions was high. Moreover, the ROC curve has a large area under the curve, and the proportion of true positives and true negatives in the confusion matrix is high, indicating a good performance in classification predictions.



SHAP (SHapley Additive exPlanations) values are a method for interpreting the predictions of machine learning models, based on the concept of Shapley values from cooperative game theory. The goal of SHAP values is to assign a numerical contribution to each feature, explaining the changes in the model's output. SHAP plots are tools used to visualize SHAP values. These plots illustrate the impact of each feature on the model's output.



In a SHAP plot, each feature has a corresponding bar, and the color of the bar represents the magnitude of the feature's value. By observing the SHAP plot, one can intuitively understand how

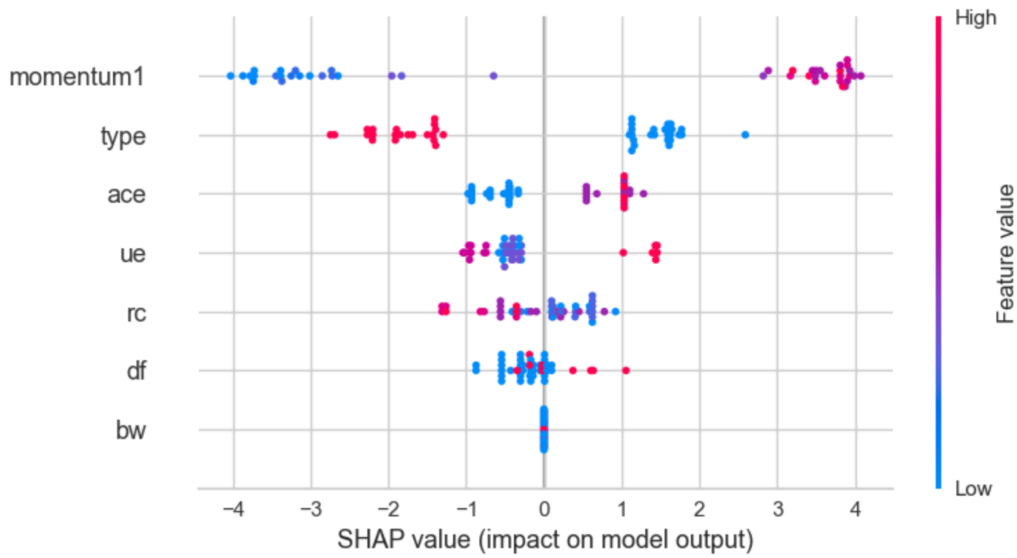


Figure 13: Shap Summary

each feature influences the model's output and whether their contributions to the final prediction are positive or negative.

The plot indicates that the "momentum," representing the maximum difference in momentum, and the "type," representing the change in momentum type, have the highest contribution values. This highlights the significance of momentum itself in predicting the arrival of swings in the match and suggests that the swings in the match dynamics are not random but closely related to momentum.

6.1 Indicators Related to the Prediction

From the SHAP plots mentioned earlier, it is evident that, in addition to momentum, the total number of aces served (ace), unforced errors (ue), and the average duration of rallies (rc) also play a significant role in predicting the arrival of swings in the match. Observations indicate that, during swings points, the side gaining consecutive victories is more likely to score aces, while the opposing side is more prone to committing errors, and the duration of rallies is relatively low. This suggests that the occurrence of swings often originates from one side delivering advantageous shots and entering a favorable state, or the other side experiencing a loss of form due to poor performance. In such cases, within a few rallies, the outcome of the match becomes evident.

6.2 Advise on the Role of Momentum

Through the aforementioned research, we have gained a deeper understanding of the significance of momentum in tennis matches. By consulting relevant literature, we have judiciously summarized the following advise on the role of momentum for a player going into a new match against a different player.:

- Be Aggressive: you can play aggressively by trying harder to hit a ace or simply run faster and hit harder.

- Start Strong: The first few points of a match are crucial, and gaining those points helps to build your momentum throughout the match
- Grasp Your Chance: When there is a chance for break point. Grasp it! our result shows that break point play an important role in both buiding your momentum as well as improve your performance.
- Stay Motivated And Positive: when you are in the downside. Don't be frustrated! Stay motivated and Positive. Seek the chance for comeback. You will eventually be the winner.

7 Model Evaluation

7.1 Sensitivity and Generalization

Sensitivity analysis is used to analyze how different values of a set of independent variables affect a specific dependent variable under certain specific conditions.

For our player performance evaluation model, we have observed that similar metrics are often used to assess player abilities and performances on websites for ATP and other tennis tournaments. This indicates that our work in this area has a certain degree of universality. Through our mechanistic analysis of tennis matches, we have summarized a five-dimensional evaluation system, which holds valuable implications for sports analysis. With more extensive data support, we could further investigate the relationships between these five dimensions and their contributions to overall performance. However, due to limitations in resources and time, we are temporarily concluding our work at this point.

For our momentum model, the psychological factor η plays a significant role in the calculation of momentum. However, in reality, different players may have different η values in various settings, facing different opponents on different courts. Therefore, a crucial feature of our model is to minimize the sensitivity of momentum to η. Fortunately, our model possesses this characteristic, as shown in the following figure:

This is the momentum variation chart obtained by changing parameters in match-1701. From the information in the figure, we can see that the momentum variations overlap to a high degree under different parameters. This indicates that our model successfully captures the general patterns of momentum changes on the tennis court and demonstrates high applicability across different players.

Indeed, for different positive and negative incidents, players' perceptions may vary. A more scientific approach would be to investigate players' psychological factors regarding different events.

Additionally, we haven't considered other factors on the court that may affect players' psychological factors, such as court surface type, distance covered by players, ball speed, etc. These factors impact the model's ability to represent the real world and require further exploration of their influence on the model.

For women's competitions and championships, as well as different playing surfaces, our model remains applicable due to the consistency in the rules and content of the matches. However, adjustments to the model parameters may be necessary due to variations in athletes' physical attributes and external factors. For other types of sports events, such as table tennis, direct application of our model may not be feasible due to differences in fundamental events during the matches. Nevertheless, since the construction of momentum is event-specific and our momentum axiom system possesses broad

Figure 14: Sensitivity Analysis

applicability, we can still employ the same conceptual framework to analyze and construct momentum models for other sports events.

7.2 Robustness

For our XGBoost classification model, we utilize the concept of cross-validation. We divide some matches as training data and others as test data, alternating between them for prediction.

Fold	0	1	2	3	4	Mean
Accuracy	0.9048	0.8095	0.9524	0.8571	0.9048	0.8857

We can observe from the results that across the various training sets selected, the average prediction accuracy reached 88.57%. This indicates that the classification model demonstrates strong adaptability to different data distributions and possesses good robustness.

7.3 Strength and Weakness

7.3.1 Strength

- Innovativeness: We provided a thorough definition of an abstract research object and utilized deductive modeling methods to construct a paradigm for studying momentum.
- Practical significance: Through our established momentum model and player evaluation system, we can effectively reflect the intensity of real matches and the corresponding performance levels of players.

- Strong prospect: Our constructed classifier for swings and momentum can accurately analyze matches, aiding tennis coaches in analyzing match situations and player performance to form more targeted training plans.
- Scalability: Our research and modeling of momentum have implications for using data science to study similar abstract psychological effects.

7.3.2 Weakness

- Subjectivity: The determination of parameters is subjective and lacks validation through data.
- Interpretability of explosive performances: Due to the definition of momentum, our momentum is a posterior indicator based on events, which lacks explanatory power for sudden bursts of aggressive play by players. Here, explosive performances refer to sudden bursts of high-aggression attacks.
- Weak model correlation: There is a lack of connection between the evaluation model and the momentum model. We attempted causal analysis between player evaluation and momentum changes, but the results were not satisfactory, indicating that our model has some biases in reflecting the real world.

8 Conclusion

Tennis is a sport brimming with momentum. In matches, top players strategically leverage this aspect. They may serve quickly when winning or extend the time between serves to disrupt their opponent's rhythm. When facing two break points down in a set, they might concede momentum to conserve energy for the next set. Indeed, the momentum on the court creates a subtle atmosphere of strategy, where every move by players offers psychological cues to themselves and their opponents. Our perception of momentum through data analysis provides only a rough understanding. Yet, it is the mysterious nature of momentum that adds allure to tennis. Fortunately, through our deductive modeling of momentum, we capture the psychological fluctuations of players on the court, allowing us to recognize the intensity of tennis matches and guiding us to delve deeper into enhancing the performance of tennis players.

In summary, our work has achieved the following outcomes:

- Established an evaluation system for player performance.
- Developed a deductive system to describe momentum.
- Explored the relationship between momentum changes and swings in play.
- Identified some noteworthy indicators, based on which training recommendations for tennis have been formulated.

In reality, individuals experiencing tennis matches firsthand may have a deeper understanding of momentum. It serves as an external manifestation of a player's strength and confidence, and while it may not directly correlate with tennis skills and scoring, a player with enough confidence and determination can exhibit momentum. This embodies the spirit of sportsmanship. All we can do is approximate momentum, which can help us understand on-court dynamics and guide training efforts.

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Dear tennis coach:

We feel honored to assist you in addressing issues related to momentum in tennis matches. We hope that our findings and suggestions can help you better understand the flow of tennis matches and the role of momentum. Sincerely wishing that, after reading our memo, you can enhance the training of your athletes and help them achieve better performance.

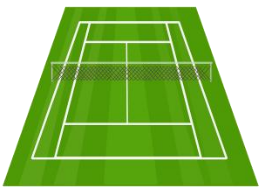
To evaluate the performance of tennis players during the match, we develop a model called PwMF model. One of its advantage is that it can capture the flow of the play by dynamically evaluating the performance of players.

The performance of players is calculated through five dimensions, including explosiveness, precision of movement, strategic capability, control, and technical proficiency. Each of them is measured by data collected from the match such as ace rate, net-point-won rate,etc.

Furthermore, after an extensive review of relevant literature, **we uniquely defined momentum as the psychological impact influencing player performance**. Subsequently, we employed the axiomatic modeling approach to describe momentum. The core logic is that momentum, as a variable measuring the psychological state of competitors on the court, undergoes changes determined by events occurring in the match. In the model results, we found a strong correlation between the defined momentum and match-winning probabilities.

In addition, utilizing the XGBoost machine learning method, we investigated the relationship between momentum and swings in the match. The results indicated that momentum could accurately predict the onset of swings. This leads us to a deeper understanding of momentum, suggesting that it not only changes due to event occurrences but also induces behavioral changes in players, thereby influencing the course of the match. Comparing different matches, we also observed that momentum holds more research and reference value in evenly matched contests, where its impact on the match situation is more pronounced.

With this, we have gained a scientifically in-depth understanding of how momentum affects tennis matches. Based on this, we offer you some reasonable training recommendations, which include:



- **Play Aggressively: Train our players to play more aggressively is a great way of showing high momentum.**
- **Start Strong:Establishing a relatively high momentum from the beginning of the match is more effective than trying to catch up after falling behind.**
- **Grasp key points : A victory on break point can significantly imfluence the level of momentum in the match.**
- **Improve tennis skills : Overall tennis skills still plays a decisive role in the match. We shouldn't rely too much on the momentum.**

Thanks for taking your time to read the letter. We hope these can help you and your players win the match!

Sincerely
Team#2418781