偏微分方程

偏微分方程基本类型

1.椭圆形偏微分方程

泊松方程：

$$\Delta u = \frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y} = f(x,y).$$

拉普拉斯方程（调和方程）：

$$\Delta u = \frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y} = 0.$$

2.抛物型偏微分方程

一维热传导方程：

$$\frac{\partial u}{\partial t} - a\frac{\partial^2 u}{\partial x^2} = 0, \quad a > 0.$$

3.双曲型偏微分方程

一阶双曲型方程：

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0.$$

二阶波动方程：

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}.$$

简单偏微分方程的符号解

求解一阶常系数非齐次偏微分方程

$$a\frac{\partial f(x,y)}{\partial x} + b\frac{\partial f(x,y)}{\partial y} + cf(x,y) = G(x,y)$$

解法：

```
import sympy as sp

f, G = sp.var('f, G', cls=sp.Function)
x, y, a, b, c = sp.var('x, y, a, b, c')
u = f(x, y)
ux = u.diff(x)
uy = u.diff(y)
eq = a * ux + b * uy + c * u - G(x, y)
sp.pprint(eq)
s = sp.pdsolve(eq)
sp.pprint(s)
```