**3. Fuzzy Mediation and Moderated-Mediation Analysis**

In this section, referring to the basic concepts in [1], we introduce the definition of fuzzy numbers by Zadeh [2], and simple fuzzy mediation models with mediators and fuzzy moderated-mediation model introduced by Yoon [3,4].

**3.1 Fuzzy number**

The definition of a fuzzy number in real numbers R, which implies when normalized and convex, is a fuzzy set. An element of a fuzzy set is one that accepts a real value between -1 and 1 as a measure of belonging according to a function known as the membership function. There are no standard rules since the membership function's form might be described in terms of either objective or subjective possibilities. As a result, the LR-fuzzy numbers parametric class of fuzzy numbers is used in this particular situation. A fuzzy number A is referred to be an LR fuzzy number if it meets the conditions listed below.

where L and R are reference functions called left and right shape functions of X and have the following properties : L,R :R→[0,1] are left-continuous and decreasing function with R(0) = L(0) = 1, R(1) = L(1) = 0. And ‘m’ means the mode of the LR-fuzzy number A. ‘l’ and ‘r’ are greater than 0 and mean the width of the left and right sides. We abbreviate the LR-fuzzy number as . And LR-fuzzy number, one of the triangular numbers, has the following two operations.

= (,

.

**3.2 Simple Fuzzy Mediation Model**

Through simple regression analysis, the Baron and Kenny's Simple Mediation Model's mediation analysis method analyzes the causal relationship step-by-step through. A statistical technique called mediation analysis explores ideas on how a causal antecedent variable (X) influences an outcome variable (Y). We suggest the following derivation of the three regression equations of Baron and Kenny: regression between independent variables and dependent variables, regression between independent variables and mediators, and regression between mediators and dependent variables.

The regression constants and are significant in this model. Here, X's estimated "direct effect" on Y is represented by the number "", while X's estimated "indirect effect" on Y through M is represented by the number " " and the number " ", which is the product of the two. Additionally, X's direct and indirect effects are added together to get , which is known as the "total effect" and equal to . It demonstrates that the direct effect () is less significant than the total effect (.

It makes more sense to describe ambiguous concepts as variables with fuzzy numbers than crisp numbers, such as "more," "less," and "happy." The following is the suggested fuzzy mediation model.

In the model as above, is the total effect, is the indirect effect, and is the direct effect. Note that it is easily checked that

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**3.3 Fuzzy Mediation Model for Multiple Mediators**

It is often advisable to utilize a basic fuzzy mediation for various mediators rather than a simple fuzzy mediation model since the real world is much more complicated and multi-causal.

The following is a simple fuzzy mediation model with parameters k (k>1).

where

In the model as above, is the total effect, is the indirect effect through and on , and is the direct effect. In other words, there are k indirect effects.



**3.4 Fuzzy Mediated-Moderation Model**

**3.4.1 Mediatied-Moderation Model**

Moderated-mediation is the mechanism by which the moderating variable (W), the fourth variable in the causal relationship, may adjust the indirect effect from the independent variable (X) to the dependent variable (Y) through the parameter (M). The terms "Adjusted mediating effect" and "conditional indirect effect" are presently used interchangeably and have the same meaning in statistics. (Preacher, Rucker & Hayes, 2007)

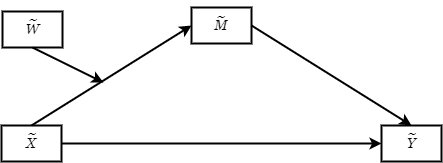
Understanding the conditional character of the mechanism by which one variable effect another variable and testing hypotheses about these conditional effects are the purpose of conditional process analysis, which combines conditioning analysis with mediation analysis. As a result, if the direct and indirect effects of the independent variable (X), which is important in the moderation analysis, are calculated and the influence of the independent variable (X) on the parameter (M), which is organized by the moderating variable (W), then X is not a single number but the effect on M is a function of W.

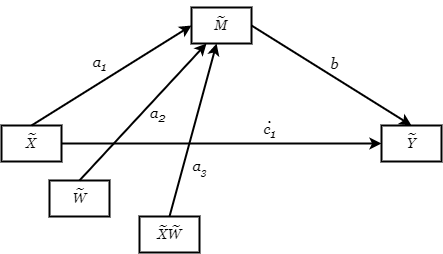
**3.4.2 Fuzzy Mediated-Moderation Model**

Let's say is a fuzzy predictor variable, is a fuzzy response variable, is a fuzzy mediator, and is a fuzzy moderator in a causal relationship involving ambiguous variables. The model shown in the Fig. below is a moderated mediation model, in which controls the route from to but does not influence any other pathways.

Because controls it, the eﬀect of is dependent on . Fig. 1 expresses the following model.

Here, is defined as a function of = and represents the "fuzzy conditional effect" of in . Because of this, it is possible to estimate the "fuzzy conditional indirect impact" from (X) to Y though M ̃ can be estimated as , the effect of the path from to , , which is the product of b and .

****

****

Yoon [3,4] has so far put up a number of fuzzy mediation models . However, no study has used bootstrapping to evaluate this fuzzy model. The initial proposals for bootstrapping-based analysis of parameters are made in Chapter 4 for simple fuzzy parameter analysis and chapter 5's proposals are for analyses of different data types and climatic data.

**4. Bootstrapping for Fuzzy Mediation and Moderated-Mediation Analysis**

**4.1 Estimation for Fuzzy Mediation Analysis**

When using least squares estimation with fuzzy data, it is necessary to have a suitable metric in the fuzzy set spaces. A helpful type of metric can be established through the use of support functions. The support function of any compact, convex set can be represented as which is determined by the following formula for all :

where is the (d-1)-dimensional unit sphere in and represents the scalar product in . It should be noted that for compact and convex sets the support function is uniquely defined. A metric in a fuzzy number set is established through the use of the *-* metric in the space of Lebesgue integrable functions, represented as:

This leads to the definition of an *-* metric for fuzzy numbers as:

A fuzzy regression model was previously introduced in the author's studies [24,25] and is expressed as follows:

.

The variables are represented by and for It is assumed that are the fuzzy random errors that account for the fuzziness. It is worth mentioning that all cases can be covered by defining and as follows:

where represent the left and right spreads of respectively.

The estimators are obtained by minimizing the following objective function:

where *q* is the number of the regression model in this fuzzy mediation analysis and *k=1,2,…,q*,. The objective function is based on the *-*metric, and the *-* distance can be calculated as:

To minimize the above equation, we obtain the normal equation applying

The normal equation has as its solution, and for each value of , the normal equation can be written as follows:

To determine the solution vector, we introduce a *triangular fuzzy matrix* *(t.f.m.)* which is expressed as

,

and abbreviated as , where is a triangular fuzzy number forandAdditionally, we define a triangular fuzzy vector

.

To minimize the objective function mentioned above, we apply the fuzzy operations, fuzzy numbers and estimators defined in our previous studies [26-29]. The fuzzy operations are as follows:

*.*

The following operations are defined for two triangular fuzzy matrices, , , and a crisp matrix :

,

*,*

*,* .

.

The solutions to the normal equation fuzzy estimators are derived for each by using the above operations and algebraic properties, with

where

and , for Note that the solution (16) exists only if .

**4.2 전통적인(기존) 방법을 이용한 Statistical inferences of Fuzzy Mediation Model**

**4.2.1 기존 방법을 이용한 구간 추정(Interval Estimation)**

Fuzzy least squares estimators ( )가 점근적으로 정규분포를 따른다는 가정은 이전 연구[ ]를 통해 알려진 바이다. 모집단의 분산(variance)를 알 수 없는 경우는 t-분포를 적용하고, 데이터의 크기가 큰 경우는 정규분포를 따른다고 가정하여 z값을 통해 총효과와 직접효과에 대한 (1-)100% 신뢰구간을 나타낼 수 있다.

*CI for the total effect :*

*CI for the direct effect*

직접효과와 총효과의 표준 오차(standard error) se는 다음과 같이 정의한다.

표본 평균에 경우 다음과 같은 성질을 적용할 수 있다.[ ]

*,* where is a fuzzy random variable.

간접효과의 경우 “Sobel test” 또는 “delta method” 또는 “product of coefficients method”를 통해 신뢰구간을 추론할 수 있다[ ]. 간접효과 ab는 표본을 기준으로 한 의 추정값이다. 간접효과의 유의 수준은 아래와 같이 매개효과 추정치를 추정치의 표준오차 로 나눈 값을 검정통계량으로 하여 정규분포(Z)를 통해 판정한다.

where (first order standard error estimator)

는 a와 b의 표준 오차를 의미한다.

*CI for the indirect effect*

where .

Sobel 검정의 경우 간접효과의 표본 분포의 정규성을 가정하고 표준 정규 분포를 사용하여 도출한다. 이러한 가정은 표본이 큰 경우에는 합리적이지만, 작은 표본에서는 그렇지 않다. 일반적으로 상황에 따라 달라지는 가정은 낮은 검정력을 산출한다. 아직까지 많은 분야의 전문가들이 sobel 검정을 여전히 사용하고 있다.

**4.3 Statistical inferences of Fuzzy Mediation Model using Bootstrap**

**4.3.1 부트스트랩을 통한 구간 추정(Interval Estimation)**

본 절에서는 3절에서 제안한 모델에 대한 부트스트랩을 이용한 신뢰구간을 통한 통계적 추론을 제안한다.

정규성을 가정하는 매개효과 검정 방법들이 가지는 한계를 극복하기 위해 부트스트랩은 Efron(1979)에 의해 처음 제안되었다. 부트스트랩은 매개효과 추정치의 표집분포에 대한 가정이 필요 없는 비모수적이고 경험적인 재표집 방법이다.

위에서 신뢰 구간을 구하는 고전적인 접근 방식을 검토했다. 다양한 부트스트랩 방법 중 부트스트랩 분포의 표본 백분위수로 신뢰 구간을 추정하는 방식을 제안한다.

원래의 표본에서 재표본을 추출한 후 재표본에 대한 추정치를 산정하고 이 두 단계를 여러 번 반복하여 재표본 추정치의 산포를 파악한다. 주어진 확률분포 , *, ,,*으로부터 복원 추출 방법으로 부트스트랩 표본 , , , 을 얻는다. 이 부트스트랩 표본에 대해 구하고자 하는 통계량을 얻을 수 있으며 이를 통해 추정 통계량 // (여기까지 – 금요일까지 마무리하겠습니다!)

백분위수 부트스트랩 방법(percentile bootstrap method; PB) B개의 부트스트랩 추정량 { ˆθ ∗ 1 , ˆθ ∗ 2 , . . . , ˆθ ∗ B}를 크기순 ˆθ ∗ (1) ≤ ˆθ ∗ (2) ≤ · · · ≤ ˆθ ∗ (B)으로 나열하여 구한 θ에 대한 (1 − 2α) × 100% PB 신뢰구간을 다음과 같다. ( ˆθ ∗ (α×B) , ˆθ ∗ ((1−α)×B) ) .

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