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Fuzzy Multinomial Logistic Regression Analysis and Its Applications to Sovereign Credit Ratings

Jin Hee Yoon[1] · Yoo Young Koo[2] · Wonkyung Lee[3] · Dae Jong Kim[4]

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Abstract In this study, we propose both a multinomial logistic model and a fuzzy multinomial logistic model that utilize the transformation of variable techniques to simplify the expression of the logistic model. For this purpose, least-squares estimation (LSE) and fuzzy least-squares estimation (FLSE) have been employed. Our primary objective is to enhance the predictive capabilities of multi-class classification models, specifically for forecasting sovereign credit ratings. This research is driven by the need for advanced classification methods using multinomial logistic models that offer flexible reference class prediction, addressing the limitations of traditional logistic techniques in achieving consistent accuracy across diverse classes. We introduce four innovative classification methods for the multinomial logistic model and five for the fuzzy multinomial logistic model, applying them to a comprehensive dataset of sovereign credit ratings from 61 countries, spanning 1995 to 2022, sourced from Moody's. Our proposed methods outperform conventional logistic techniques and maintain consistent predictive accuracy across a variety of classes when compared to traditional logistic classification methods and machine learning techniques such as decision trees, random forests, and support vector machines. The significance of our findings lies in the broad applicability of these methods beyond financial data; they can be adapted for any context that requires reliable multi-class classification. This advancement holds potential for improving decision-making processes in fields such as economics, risk management, and policy analysis by providing a more consistent and accurate predictive framework.

Keywords Multinomial logistic regression · Fuzzy multinomial logistic regression · Classification · Machine learning · Credit ratings

✉ Wonkyung Lee
wklee@sejong.ac.kr

Jin Hee Yoon
jin9135@sejong.ac.kr

Yoo Young Koo
yykoo@yonsei.ac.kr

Dae Jong Kim
daejong68@sejong.ac.kr

1 Department of Mathematics and Statistics, Sejong University, Seoul 05006, South Korea

2 University College, Yonsei University, Incheon 21983, South Korea

3 Department of Chinese Trade and Commerce, Sejong University, Seoul 05006, South Korea

4 Department of Business and Administration, Sejong University, Seoul 05006, South Korea

1 Introduction

A sovereign credit rating is a graded evaluation of a country's creditworthiness and debt repayment ability. Major credit-rating agencies (CRAs), S&P, Moody's, and Fitch, evaluate the sovereign credit ratings through various methods with different factors that drive the sovereign's creditworthiness, such as national debt, GDP, inflation rate, exports, imports, per capita income, etc. The rating is important to a country because it can represent the country's ability to meet financial obligations and affects its ability to borrow money, as well as banks' profitability and capital ratios [1]. Although CRAs publish reports identifying the components of sovereign credit ratings, the rating committees also make additional judgmental adjustments. Furthermore, the exact weight assigned to each variable and the manner in which that weighting changes over time remain poorly defined. Kiff et al. [2] contend that the CRAs have flexible criteria for vari-

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ous factors over time, rather than relying on a fixed weighted average. Barta and Johnston [3] also argue that CRAs are likely to evaluate the creditworthiness of countries differently depending on their political ideology.

These credit ratings play a pivotal role in shaping critical decision-making processes for stakeholders such as investors, financial institutions, and policymakers. The insights derived from sovereign credit ratings inform strategies on resource allocation, investment risk management, and economic planning, highlighting the need for robust and interpretable classification methodologies. Therefore, in credit risk management, accurate credit-rating classification is crucial for effective decision making by financial institutions, investors, and policymakers. Decision-making processes often rely on robust classification models that not only predict outcomes but also provide interpretable insights into underlying risks [4–7]. There are several studies analyzing credit rating using decision making [8–10] and classification [11–14].

This paper proposes new multinomial logistic regression methods for classification of credit rating and they have been extended with fuzzy methodologies to better handle uncertainty and imprecision in data, enhancing their applicability in real-world scenarios. By incorporating advanced machine learning classification techniques, this study aims to support more informed and reliable decisions in credit evaluation systems, aligning with the growing emphasis on data-driven strategies in modern finance.

Extensive research has been conducted on the factors determining sovereign credit ratings. Afonso [15] pointed out that per capita income, foreign debt, economic development, default experience, real economic growth rate, and inflation are the most important determinants of credit ratings. Rowland and Torres [16] explained that per capita income and inflation are important factors in determining credit ratings. Butler and Fauver [17] analyzed using qualitative variables such as political systems and legal conditions in addition to macroeconomic variables. They argued that political systems have a strong influence on sovereign credit ratings because they show the willingness of the country to pay its debt. Archer et al. [18] announced the results of their analysis, emphasizing the importance of political variables. He stated that political variables do not affect sovereign credit ratings. However, he argued that default history, inflation, and growth rate are important variables in sovereign credit ratings, as expected. And there have been many studies for credit-rating or credit-scoring analysis using fuzzy theory. Bodur [19] applied fuzzy utility based decision analysis in the credit scoring. Chen and Chiou applied fuzzy theory to credit-rating approach for commercial loans [20]. Jiao et al. [21] used fuzzy adaptive network for modeling credit rating. In addition, Wang et al. used fuzzy support vector machine (SVM) to evaluate credit risk [22], and Luo et al. applied fuzzy cluster in credit scoring [23].

Furthermore, there are studies that have employed machine learning (ML) to analyze and forecast sovereign credit ratings or credit scoring. Especially, Huang et al. [24] studied credit scoring prediction using SVM and neural network. Ye et al. [25] applied multiclass ML approach for credit-rating prediction. Wu et al. [26] applied two-stage credit-rating prediction using ML, and Dai et al. [27] analyzed bank credit rating and risk assessment with some ML technique. Wu et al. [28] predicted credit rating through supply chains with ML approach. Tsai et al. [29] applied a hybrid ML method for credit rating; in addition, Overes and Van der Wel [30] analyzed Sovereign credit ratings with several machine learning techniques. And there are more studies regarding credit rating and ML [26,31–38].

Our paper proposes a new methodology for a multinomial logistic model and a fuzzy multinomial logistic model that transform the dependent variable into a linear function and use LSE and FLSE, instead of maximum likelihood estimation (MLE). We then apply our methodology to predict the credit ratings of 61 countries from 1995 to 2022, using data from Moody's.

For fuzzy logistic regression analysis, many authors have studied applying various methods. Pourahmad et al. [39,40] introduced a fuzzy logistic regression with possibility theory and least-squares approach. Namdari et al. [41] applied least absolute deviation method for fuzzy logistic regression model. In addition, Ahmadini [42] proposed a novel method for parameter estimation in intuitionistic fuzzy logistic regression models.

However, the above studies have focused on binary fuzzy logistic regression models that have only two responses. In this paper we have focused on multinomial logistic model for crisp data, and fuzzy multinomial logistic model that have multiple responses. Moreover, for the proposed multinomial logistic regression model with flexible reference classes, we apply LSE in place of MLE following the transformation of the dependent variable.

Multinomial logistic regression is an extension of binary logistic regression that is used when the dependent variable has more than two categories. It is used to predict the probabilities of the different possible outcomes of a categorically distributed dependent variable, given a set of independent variables. In multinomial logistic regression, the dependent variable Y can take on r different categories, where r is greater than 2. Unlike binary logistic regression, which models the probability of one outcome against the other, multinomial logistic regression models the probability of each possible outcome relative to a reference category. For a dependent variable Y with r categories, we need to select one of these categories as the reference category. Let's denote the categories as $Y = \{1, 2, \cdots, r\}$ with the first category typically chosen as the reference category. The multinomial logistic regression model provides r-1 equations, each comparing one of the r-1 categories to the reference category 1. The model for category k (where $k \in \{2, \cdots, r\}$ is given by: $\log\left(\frac{P(Y=k)}{P(Y=1)}\right) = \beta_{k0} + \beta_{k1}X_1 + \cdots + \beta_{kp}X_p$).

The probabilities for each category are computed from the model equations. For the reference category 1,

$$P(Y = 1) = \frac{1}{1 + \sum_{k=2}^{r} \exp(\beta_{k0} + \beta_{k1}X_1 + \cdots + \beta_{kp}X_p)}$$

For the other categories k,

$$P(Y = k) = \frac{\exp(\beta_{k0} + \beta_{k1}X_1 + \cdots + \beta_{kp}X_p)}{1 + \sum_{k=2}^{r} \exp(\beta_{k0} + \beta_{k1}X_1 + \cdots + \beta_{kp}X_p)}$$

Logistic regression analysis usually uses the maximum likelihood estimation (MLE) because the dependent variable is discrete or categorical. MLE is a probabilistic approach that is naturally suited for modeling categorical outcomes. It seeks to find the parameter values that maximize the likelihood of observing the given data. The likelihood function is directly derived from the assumed probability distribution of the response variable (e.g., Bernoulli distribution for binary outcomes). Logistic regression models the log-odds (logit) of the probability as a linear function of the predictors. This leads to a non-linear relationship between the predictors and the probabilities. However, in this study, we propose a method using the LSE and FLSE with continuous dependent variable after transformation to convert the given model into a linear model.

This paper presents the following contributions:

- *New Estimation Method for Multinomial Logistic Model*: We use LSE instead of MLE after the transformation of the dependent variable for the proposed multinomial logistic regression model.
- *Proposed Classification Methods:* We introduce four innovative classification methods for the estimation of a multinomial logistic model with different reference classes.
- *Application of Fuzzy Theory to Proposed Classification Methods:* We propose fuzzy multinomial logistic models, taking into account the fuzzy nature of economic data, and five classification methods for their estimation.
- *Application of our methods to sovereign credit-rating classification:* We apply our proposed multinomial logistic model and the nine classification methods mentioned above to sovereign credit-rating classification. Our method consistently outperforms conventional classification methods. Furthermore, our classification approach demonstrates the advantage of having less variation in prediction accuracy across classes compared to both conventional classification methods and machine learning methods.
- *Proposed Methods' Expansion to Multi-Class Classification:* This method can be applied to the classification of data with multiple classes.

This paper is structured as follows: Sect. 2 introduces the proposed multinomial logistic (MLog) models based on Least-Squares Estimation (LSE) and four classification criteria, along with the conventional classification criterion. Section 3 presents the proposed fuzzy multinomial logistic (FMLog) models derived from Fuzzy Least-Squares Estimation (FLSE), incorporating fuzzy theory, and proposes five classification criteria. Section 4 explores various ML techniques, including Decision Trees (DT), Random Forests (RF), and Support Vector Machines (SVM), and further proposes the application of fuzzy theory to these techniques for comparative analysis. Section 5 applies the proposed classification methods to classify sovereign credit ratings using data, and compares the performance of these classification methods with conventional classification methods and various machine learning techniques. Finally, Sect. 6 concludes the paper.

2 Multinomial Logistic Regression Based on LSE

2.1 Binomial Logistic Regression Based on LSE

In general, MLE (Maximum Likelihood Estimation) is used for a logistic regression model. But, in this study, we propose a logistic regression analysis using LSE (Least-Squares Estimation). Logistic regression is used to identify the relationship between a probability of binary response variable and one or more explanatory variables. The binary response variable follows a Bernoulli process, taking a value of 1 with a probability of an event μ occurring, or 0 with a probability of no event $1 - \mu$ $(0 \leq \mu \leq 1)$. The relationship between the predictor and μ is not linear, until logit transformation of μ is used:

$$E(Y_i) = \mu = \frac{e^{\beta_0 + \beta_1 X_{1i} + \cdots + \beta_n X_{ni}}}{1 + e^{\beta_0 + \beta_1 X_{1i} + \cdots + \beta_n X_{ni}}} = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X_{1i} + \cdots + \beta_n X_{ni})}} \tag{1}$$

So, $\ln\left(\frac{\mu}{1-\mu}\right) = \beta_0 + \beta_1 X_{1i} + \cdots + \beta_n X_{ni}$, in which the coefficients in this model can be estimated by MLE in general. However, we transform the $\ln\left(\frac{\mu}{1-\mu}\right)$ by Y' to apply LSE. The original response variable Y is a discrete variable, but Y' is a continuous variable. Therefore, LSE can be applied to Y_i'

instead of Y_i.

$$Y_i{}' = \ln\left(\frac{\mu}{1-\mu}\right) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_p X_{pi} + E_i, \quad (i = 1, .., n) \tag{2}$$

in which $\beta_0,\ \beta_1 \ldots, \beta_p$ indicating crisp relationship. μ is the possibility of an event occurring and then logarithm transformation of possibility odds $\frac{\mu}{1-\mu}$ and μ is also considered as the observed outputs.

After obtaining $\hat{\beta}_0,\ \hat{\beta}_1, \ldots, \hat{\beta}_p$ and $\widehat{Y_i'}$ from LSE, we classify the sovereign credit ratings using the criteria described in Sect. 2.2.

2.2 Multinomial Logistic Regression Based on LSE

Multinomial Logistic Regression is a generalized model of binomial logistic regression. Multinomial logistic regression extends the concepts of logistic regression to handle multiple categories in the dependent variable. It is a valuable tool for understanding and predicting outcomes when dealing with scenarios that involve more than two possible categories or options.

Suppose there are r groups or classes. Then we consider the following:

$$Y_i{}' = \ln\left(\frac{\mu_k}{\mu_1}\right) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_p X_{pi} + E_i, \quad (k = 1, \ldots, r,\ i = 1, .., n) \tag{3}$$

Category 1 is taken as the standard category, and the probability of belonging to category from 2 to category k is compared with category 1. That is, we compare r-1 pairs first as follows:

$$(reference\,class,\ class\,to\,be\,compared) = (1, 2),\ (1, 3), \ldots, (1, r)$$

And finally decide that the data belong to the category with the highest probability as follows:

Without loss of generality, we can set the class 1 as a reference class. Then, for the class comparison (1,k), where $k = 1, \ldots, r$, let p_k be the probability that the given data is included in the class k.

Let $C_1, C_2, \ldots, C_r$ be the r classes. Without loss of generality, C_1 is going to be called a reference class.

Here, p_i^k is the probability that ith observation belongs to the kth class, where $i= 1, \cdots, n$, and $k= 1, \cdots, r$.

Criterion 1 below is the conventional classification method, while Criterion 2 through Criterion 5 are the new classification methods we propose.

Criterion 1

Case 1 If $p_i^1 > p_i^k$ for all $k \neq 1$, then the given data is included in class C_1.

Case 2 Otherwise, the given data is included in C_m, where $p_m = \max_{k \neq 1} p_i^k$.

Criterion 2

Step 1 Collect the observations predicted to belong to C_1 from Case 1 of Criterion 1.

Step 2 Construct a new dataset by excluding data classified as belonging to C_1.

Step 3 Using the new dataset, predict which of the $r - 1$ classes of C_2,..., C_r the data belongs to with a new reference class C_2.

Step 4 Then, collect the observations predicted to belong to C_2 and create a new dataset excluding these observations.

Step 5 Repeat the above process updating new reference classes C_3,..., C_{r-1} to keep collecting data until the data are predicted to belong to $C_{r-1}$and C_r, respectively.

Criterion 3

Step 1 Rearrange r class numbers in order of highest number of observations. Let us name the given r classes $C_1,\ C_2, \ldots,\ C_r$ in that order. Then, the classes are newly sorted into $C_1{}', C_2{}', \ldots, C_r{}'$, in order of the number of observations. In other words, if $n\ (C_k)$ is the number of observations of the kth class C_k, then $n\left(C_1'\right) > n\left(C_2'\right) > \ldots > n\left(C_r'\right)$.

Step 2 Apply Criterion 2 to the rearranged classes $C_1',\ C_2', \ldots,\ C_r'$.

Criterion 4

Step 1 Collect the observations predicted to belong to C_1 as in Case 1 of Criterion 1.

Step 2 Construct a new dataset by excluding data classified as belonging to C_1.

Step 3 If $X_j' \quad (j = 1, \cdots, l,\ l < n)$ is the data that are not included in C_1. Let $p_m = \max_{k \neq 1} p_j^k$. If $p_m > p_j^1$ and $p_j^k \leq p_j^1\ (k \neq 1,\ k \neq m)$, then X_j' belongs to C_m.

Step 4 Construct a new dataset by excluding data classified as belonging to C_1 and C_m from Step 1 to Step 3. (Note that we may have more than one C_m.)

Step 5 Using the new dataset with a new reference class C_2 and collect the observations predicted to belong to C_2 as in Case 1 of Criterion 1.

Step 6 Repeat the above process to classify all given data until the data are predicted to belong to $C_{r-1}$and C_r.

Criterion 5

Step 1 Rearrange r class numbers in order of highest number of observations. Let us name the given r classes $C_1,\ C_2, \ldots,\ C_r$ in that order. Then, the classes are newly sorted into $C_1',\ C_2', \ldots,\ C_r'$, in order of the most observations. In other words, if $n\ (C_k)$ is the number of observations of the kth class C_k, then $n\left(C_1'\right) > n\left(C_2'\right) > \ldots > n\left(C_r'\right)$.

Step 2 Apply Criterion 4 to the rearranged classes $C_1^{'}, C_2^{'}, \ldots, C_r^{'}$.

3 Fuzzy Multinomial Logistic Regression Based on FLSE

3.1 Fuzzy Logistic Regression Based on FLSE

In this study, we deal with the fuzzy logistic regression. Logistic regression is used to identify the relationship between a probability of binary response variable and one or more explanatory variables. The binary response variable follows a Bernoulli process, taking a value of 1 with a probability of an event μ occurring, or 0 with a probability of no event $1-\mu$ $(0 \leq \mu \leq 1)$.

The relationship between the predictor and μ is not linear, until logit transformation of μ is used:

$$\mu = \frac{1}{1+e^{\beta_0+\beta_1 X_1+\cdots+\beta_n X_n}} \tag{4}$$

So, $\ln\left(\frac{\mu}{1-\mu}\right) = \beta_0 + \beta_1 X_1 + \cdots + \beta_n X_n$, in which the coefficients in this model can be estimated by maximum likelihood estimation methods.

In the fuzzy logistic regression, $\tilde{Y}$ is replaced with $\ln\left(\frac{\tilde{\mu}}{1-\tilde{\mu}}\right)$, according to Pourahmad et al. [39,40]. The proposed model is

$$\tilde{Y}_i = \ln\left(\frac{\tilde{\mu}}{1-\tilde{\mu}}\right) = \beta_0 \oplus \beta_1 \tilde{X}_{1i} \oplus \beta_2 \tilde{X}_{2i} \oplus \cdots \oplus \beta_p \tilde{X}_{pi} \oplus \tilde{E}_i, \tag{5}$$

in which $\beta_0, \beta_1, \bullet\bullet\bullet, \beta_p$ indicating crisp relationship $(i = 1, \ldots, n)$. $\tilde{\mu}$ is the possibility of an event occurring and then logarithm transformation of possibility odds $\left(\frac{\tilde{\mu}}{1-\tilde{\mu}}\right)$ and $\tilde{\mu}$ is also considered as the observed outputs. In this study, all variables, namely $\tilde{X}_{1i}, \cdots, \tilde{X}_{pi}$, $\tilde{\mu}$ and $\tilde{Y}$ are fuzzy numbers. The membership function of $\tilde{\mu}$ can be found based on well-known Zadeh's extension principle [43] as follows:

$$\tilde{w_i}\,(a) = sup_{\forall x: ln\frac{a}{1-a}=y}\, \tilde{\mu_i}\,(b) \tag{6}$$

in which $f(a) = \ln\frac{a}{1-a}$, $0 < a < 1$ is a one to one function. So, there is one and only one $a \in (0, 1)$ such that $\ln\frac{a}{1-a} = b$. Therefore,

$$\tilde{w_i}\left(b = \ln\frac{a}{1-a}\right) = \tilde{\mu_i}\left(\frac{e^a}{1+e^a}\right) \tag{7}$$

And the above model is estimated by FLSE proposed by Yoon and Choi [44], and its application to logistic regression is proposed by Sohn et al. [45].

3.2 Fuzzy Multinomial Logistic Regression Based on FLSE

In this section, we apply fuzzy theory to the multinomial logistic regression based on the LSE method described in Sect. 2.2.

Suppose there are r groups or classes. Then we consider the following:

$$\tilde{Y}_i = \ln\left(\frac{\tilde{\mu}_k}{\tilde{\mu}_1}\right) = \beta_0 \oplus \beta_1 \tilde{X}_{1i} \oplus \beta_2 \tilde{X}_{2i} \oplus \cdots \oplus \beta_p \tilde{X}_{pi} \oplus \tilde{E}_i, \quad (k = 1, \ldots, r,\ i = 1, .., n) \tag{8}$$

Category 1 is taken as the standard category, and the probability of belonging to category from 2 to category r is compared with category 1. That is, we compare r-1 pairs first as follows:

$$(reference\, class,\ class\, to\, be\, compared) = (1, 2), (1, 3), \ldots, (1, r) \tag{9}$$

And finally decide that the data belong to the category with the highest probability.

The above regression model is estimated by FLSE proposed in author's previous study [44] as follows:

From the model (6), if the fuzzy data are represented by $\tilde{X}_{ij} = \left(l_{x_{ij}}, x_{ij}, r_{x_{ij}}\right)$ and $\tilde{Y}_i = \left(l_{y_i}, y_i, r_{y_i}\right)$ for $i = 1, \ldots, n$, $j = 1, \ldots, p$, it is assumed that $\tilde{E}_i$ are the fuzzy random errors for expressing fuzziness. Note that we can encompass all cases by

$$l_{x_{ij}} = \begin{cases} x_{ij} - \xi_{l_{ij}}, & if\ \beta_j \geq 0, \\ x_{ij} + \xi_{l_{ij}}, & if\ \beta_j < 0, \end{cases} \qquad r_{x_{ij}} = \begin{cases} x_{ij} + \xi_{l_{ij}}, & if\ \beta_j \geq 0, \\ x_{ij} - \xi_{l_{ij}}, & if\ \beta_j < 0, \end{cases} \tag{10}$$

where $\xi_{l_{ij}}$ and $\xi_{r_{ij}}$ are the left and right spreads of $\tilde{X}_{ij}$, respectively. Now the estimators are obtained if we minimize following objective function:

$$Q\left(\beta_{k0}, \beta_{k1}, \ldots, \beta_{kp}\right) = \sum_{i=1}^{n} d^2(\tilde{Y}_i, \sum_{j=0}^{p} \beta_{kj} \tilde{X}_{ij}), \tag{11}$$

For $k = 1, \cdots, h$, where h is the number of the regression model in this fuzzy mediation analysis. And the objective function (14) can be obtained based on the L_2-metric.

To define a fuzzy distance, we consider the support function to derive L_2 distance [45]. A useful type of metric can be defined via support functions. The support function of any compact convex set $A \in \mathbb{R}^d$ is defined as a function $s_A : S^{d-1} \to \mathbb{R}$ given by for all $r \in S^{d-1}$

$$s_A\,(r) = \underset{a \in A}{sub} < r, a >, \tag{12}$$

where S^{d-1} is the (d-1)-dimensional unit sphere in $\mathbb{R}^d$ and $< \cdot, \cdot >$ denotes the scalar product on $\mathbb{R}^d$. Note that for convex and compact $A \in \mathbb{R}^d$ the support function s_A is uniquely determined. A metric on a fuzzy number set is defined by the L_2metric on the space of Lebesgue integrable

$$\delta_2\left(A,\ B\right) = [d\int_0^1\int_{S_0}\left|S_A\left(\alpha, r\right) - S_B\left(\alpha, r\right)\right|^2\mu\left(\mathrm{d}r\right)\mathrm{d}\alpha\,]^{\frac{1}{2}}. \tag{13}$$

In this paper, the distance derived in (16) is generalized as follows:

$$d^2\left(\tilde{X},\ \tilde{Y}\right) = D_2^2\left(Supp\tilde{X},\ Supp\tilde{Y}\right) + \left(m_x - m_y\right)^2, \tag{14}$$

where $\tilde{X} = (l_x,\ x,\ r_x)$ and $\tilde{Y} = (l_y,\ y,\ r_y)$.

Now, the above objective function (14) is defined by (17). To minimize (14), we obtain the normal equation applying

$$\frac{\partial Q}{\partial \beta_{kl}} = 0, \tag{15}$$

And, for each $k = 1, 2, \ldots, h$, the normal equation, which has $\widehat{\beta_{kl}}$ as solutions, can be obtained as follows:

$$\sum_{j=0}^{p}\widehat{\beta_{kj}}\sum_{i=1}^{n}\left(l_{x_{il}}l_{x_{ij}} + x_{il}x_{ij} + r_{x_{il}}r_{x_{ij}}\right) = \sum_{i=1}^{n}\left(l_{x_{il}}l_{y_i} + x_{il}y_i + r_{x_{il}}r_{y_i}\right) \tag{16}$$

To find the solution vector, we define a *triangular fuzzy matrix (t.f.m.)* which is defined by $\check{X} = [\tilde{X}_{ij}]_{n\times(p+1)}$, where $\tilde{X}_{ij}$is a triangular fuzzy number for $i = 1, .., n,\ j = 0, \ldots, p$. And we define a triangular fuzzy vector $\check{\mathbf{y}} = \left[\tilde{Y}_i\right]^t$.

To minimize the above objective function, fuzzy operations fuzzy numbers and estimators have been applied.

$$\begin{aligned}\tilde{X}\diamond\tilde{Y} &= l_x l_y + xy + r_x r_y,\\ \tilde{X}\otimes\tilde{Y} &= (l_x l_y,\ xy,\ r_x r_y),\\ \tilde{X}\circledast\tilde{Y} &= \left(l_{x\circledast y},\ xy,\ r_{x\circledast y}\right),\end{aligned} \tag{17}$$

where

$$l_{x\circledast y} = Inf\,\{l_x l_y,\ l_x r_y,\ r_x l_y,\ r_x r_y\},$$
$$r_{x\circledast y} = Sup\,\{l_x l_y,\ l_x r_y,\ r_x l_y,\ r_x r_y\}.$$

For given two $n\times n$ *t.f.m*'s, $\check{\Gamma} = [\tilde{X}_{ij}]$, $\check{\Lambda} = [\tilde{Y}_{ij}]$, and a crisp matrix $A = \left[a_{ij}\right]$, the operations are defined as follows:

$$\begin{aligned}&\check{\Gamma}\diamond\check{\Lambda} = [\sum_{k=1}^{n}\tilde{X}_{ik}\diamond\tilde{Y}_{kj}], && \check{\Gamma}\otimes\check{\Lambda} = [\oplus_{k=1}^{n}\tilde{X}_{ik}\diamond\tilde{Y}_{kj}],\\ &A\check{\Gamma} = [\oplus_{k=1}^{n}a_{ik}\tilde{X}_{kj}], && k\check{\Gamma} = [\oplus_{k=1}^{n}a_{ik}\tilde{X}_{ij}].\\ &\tilde{X}A = \left[a_{ij}\tilde{X}\right], && \tilde{X}\diamond\check{\Gamma} = \left[\tilde{X}\diamond\tilde{X}_{ij}\right],\\ &\tilde{X}\otimes\check{\Gamma} = [\tilde{X}\otimes X_{ij}].\end{aligned} \tag{18}$$

Using the above operations and algebraic properties, the solutions of normal equation fuzzy estimators are derived for each $k = 1, 2, \ldots, h$ by

$$\hat{\beta}_k = \left(\check{X}^t\diamond\check{X}\right)^{-1}\check{X}^t\diamond\check{\mathbf{y}}, \tag{19}$$

where $\check{X}^t\diamond\check{X}$=[$\sum_{i=1}^{n}(l_{x_{il}}l_{x_{ij}}+x_{il}x_{ij}+r_{x_{il}}r_{x_{ij}})]_{(p+1)\times(p+1)}$ and $\check{X}^t\diamond\check{\mathbf{y}}$=$[\sum_{i=1}^{n}(l_{x_{il}}l_{y_i} + x_{il}y_i + r_{x_{il}}r_{y_i})]_{(p+1)\times1}$,

for $l = 0, 1, \ldots, p$. Note that (19) exists if $det(\check{X}^t\diamond\check{X}) \neq 0$.

For the measurement of goodness of the fit of the fuzzy regression model, fuzzy R^2 and $FRMSE$ (Fuzzy Root Mean Square Error) using distance approach are defined as follows [46]:

$$\text{Fuzzy}R^2 = 1 - \frac{\sum_{i=1}^{n}d^2(\tilde{Y}_i,\ \hat{\tilde{Y}}_i)}{\sum_{i=1}^{n}d^2(\tilde{Y}_i,\ \bar{\tilde{Y}})},$$

$$\text{FRMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}d^2(\tilde{Y}_i,\ \hat{\tilde{Y}}_i)}.$$

3.3 Criteria of Classification for Fuzzy Multinomial Logistic Regression: Fuzzy Ordering

In this section, we modify the definition of Yager's fuzzy ordering [47] and define it as follows:

Definition 1 (*Index for ordering triangular fuzzy numbers*) Let A= (l_a, a, r_a) and B= (l_b, b, r_b) be triangular fuzzy numbers. Then, an index for ordering triangular fuzzy number A is defined by

$$I\left(A\right) = \frac{1}{2}\left(\frac{l_a + r_a}{2} + a\right) = \frac{a}{2} + \frac{l_a + r_a}{4}$$

In addition, for ordering triangular fuzzy numbers

$$\begin{aligned}&\mathrm{A} \succ \mathrm{B} \quad \text{if} \quad I\left(A\right) > I\left(B\right),\\ &\mathrm{A} \sim \mathrm{B} \quad \text{if} \quad I\left(A\right) = I\left(B\right), \text{ and}\\ &\mathrm{A} \prec \mathrm{B} \quad \text{if} \quad I\left(A\right) < I\left(B\right).\end{aligned}$$

The classification criteria using fuzzy multinomial logistic regression analysis are also defined in the same way as the crisp method presented in Sect. 2.2.

3.4 WHR (Weighted Hit Ratio)

WHR (Weighted Hit Ratio) is modified hit ratio which is introduced in [48]. In general, when calculating accuracy, cases where a class is not accurately classified are treated equally. However, if an order is given to the classes, the accuracy must be calculated so that the performance of the model differs when it is classified into a class one level higher or lower and when it is classified into a class that is several levels different. So, in this paper, WHR is defined as follows:

If the original data x_i is included in the kth class C_k,

$$w\left(\hat{C}_i\right) = \begin{cases} 1 & if\ \hat{C}_i = C_k \\ 0.5 & if\ \hat{C}_i = C_{i-1}\ or\ \hat{C}_i = C_{i+1} \\ 0 & o.w \end{cases}$$

where $\hat{C}_i$ is the predicted class of x_i. That is, $\hat{C}_i = C_k$ means the predicted class $\hat{C}_i$ of x_i is C_k, which is the case of accurate prediction.

Then, WHR is defined by

$$\mathrm{WHR} = \frac{\sum_{i=1}^{n} w\left(\hat{C}_i\right)}{n},$$

where n is the number of data to be predicted. The overall flowchart of the proposed method is shown in Fig. 1.

4 Some Machine Learning Techniques for Classification for Fuzzy Data

In this section, several machine learning models are employed to compare the classification accuracy of the proposed crisp and fuzzy multinomial logistic regression models. For the fuzzy model, in order to assess its accuracy, fuzzy machine learning models are introduced by fuzzifying the data as follows, which is then compared with the results of the fuzzy multinomial logistic classification model.

4.1 Decision Tree for Fuzzy Data

A decision tree (DT) is a visual tool for decision making commonly used in machine learning and decision analysis. It consists of nodes representing decisions, branches showing possible outcomes, and leaves indicating final outcomes or classifications. Decision nodes involve choices based on specific criteria or features, leading to different branches and subsequent decisions, ending in a final classification at the leaf nodes. Mathematically, it is represented as a hierarchy, where each node represents a decision based on a particular feature. The set of all nodes is denoted as N, all features as F, all decisions or classifications as D, all samples or instances as S, and the value of a feature for a given sample as X_i. The condition is based on the value of a feature, and $N_k (k = 1, \cdots, r)$ represents the next node to traverse.

Here, fuzzified data as a fuzzy triangular fuzzy number are represented by $\tilde{X}_i = (l_{x_i}, x_i, r_{x_i})$, where $i = 1, \cdots, n$. Then fuzzy DT can be represented by a set of rules of the form:

If l_{x_i} is Condition then go to node N_k, which is denoted by $N_k^{l_{x_i}}$,

If x_i is Condition then go to node N_k, which is denoted by $N_k^{x_i}$,

If l_{x_i} is Condition then go to node N_k, which is denoted by $N_k^{l_{x_i}}$,

where $k = 1, \cdots, r$. Here r is the number of classes. Then, for the final decision for the classification, the predicted class $C(\tilde{X}_i)$ of the given fuzzy data $\tilde{X}_i$ is defined by

$$C(\tilde{X}_i) = \rho\left\{\frac{1}{3}\left(N_k^{l_{x_i}} + N_k^{x_i} + N_k^{l_{x_i}}\right)\right\},$$

where $\rho(\cdot)$ is a rounding function.

4.2 Random Forest for Fuzzy Data

A random forest (RF) is a powerful ensemble learning technique in machine learning, primarily used for classification and regression tasks. It consists of multiple decision trees, each built using a randomly selected subset of training data and features. RF starts by creating numerous bootstrap samples from the original training data, sampling instances with replacement. Each decision tree randomly selects a subset of features at each node to determine the best split, helping to decorrelate the trees and prevent overfitting. The trees grow by recursively splitting the data based on the chosen features until a stopping criterion, such as maximum depth or minimum samples per leaf node, is met. In classification, RF combines tree predictions through majority voting, and in regression, it averages tree predictions to produce the final output. RF mitigates overfitting by averaging predictions across multiple trees. For a fuzzy triangular data $\tilde{X}_i = (l_{x_i}, x_i, r_{x_i})$, where i $= 1, \cdots,$ n, a random forest can be represented as:

$$\hat{y}_{l_{x_i}} = \frac{1}{n}\sum_{i=1}^{n} f_i(l_{x_i});$$

$$\hat{y}_{x_i} = \frac{1}{n}\sum_{i=1}^{n} f_i(x_i);$$

$$\hat{y}_{r_{x_i}} = \frac{1}{n}\sum_{i=1}^{n} f_i(r_{x_i}),$$

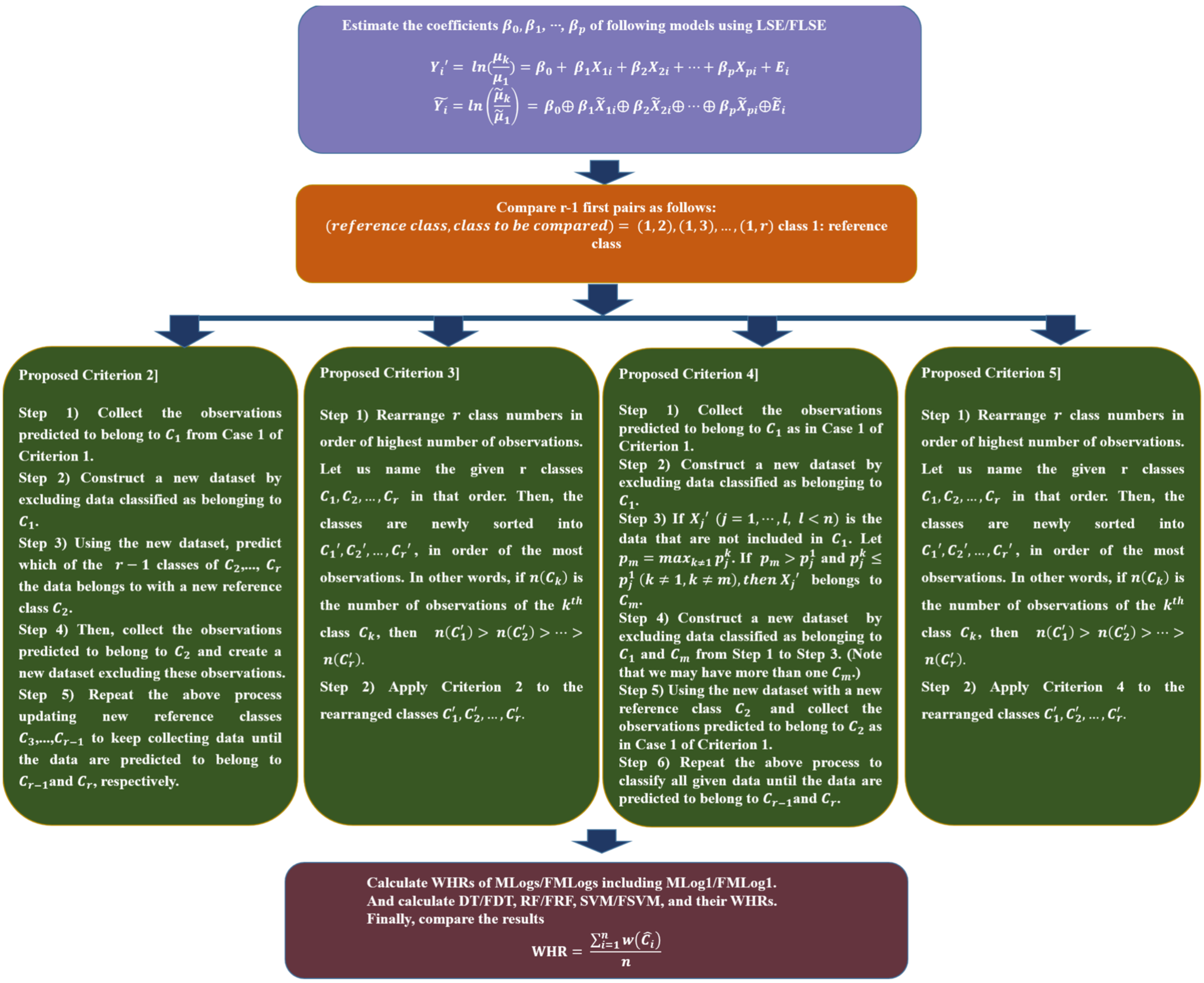

Fig. 1 Overall flowchart of the proposed methods. Note: Proposed MLogs/FMLogs refer to a multinomial logistic model and a fuzzy multinomial logistic model developed based on Criteria 2 through 5, our proposed classification criteria. Similarly, MLog1/FMLog1 correspond to multinomial logistic models using Criterion 1, the conventional classification criterion

is the predicted output, T is the number of trees in the forest, and $f_i(x)$ is the prediction of the ith DT. Then, for the final decision for the classification, the predicted class $C(\tilde{X}_i)$ of the given fuzzy data $\tilde{X}_i$ is defined by

$$C(\tilde{X}_i) = \rho\left\{\frac{1}{3}\left(\hat{y}_{l_{x_i}} + \hat{y}_{x_i} + \hat{y}_{r_{x_i}}\right)\right\},$$

where $\rho(\cdot)$ is a rounding function.

4.3 Support Vector Machine (SVM)

A Support Vector Machine (SVM) is a supervised machine learning algorithm used for classification and regression tasks. Its main goal is to find the optimal hyperplane that separates different classes in the feature space while maximizing the margin, which is the distance between the hyperplane and the nearest data points from each class, known as support vectors. SVM is most effective when classes are linearly separable. In cases where they are not, SVM uses the kernel trick to map input data into a higher-dimensional space to achieve linear separation. Common kernel functions include linear, polynomial, radial basis function (RBF), and sigmoid. For fuzzified data as a fuzzy triangular fuzzy number is represented by $\tilde{X}_i = (l_{x_i}, x_i, r_{x_i})$, where $i = 1, \cdots, n$, the decision function of a linear SVM for binary classification can be represented as follows:

$$f\left(l_{x_i}\right) = sign(\mathrm{w_l} \cdot \mathrm{x_l} + b_l);$$
$$f\left(x_i\right) = sign(\mathrm{w} \cdot \mathrm{x} + b);$$
$$f\left(r_{x_i}\right) = sign(\mathrm{w_r} \cdot \mathrm{x_r} + b_r)$$

where $f(x)$ is the decision function that predicts the class label of x, w is the weight vector perpendicular to the hyperplane, x is the input feature vector, b is the bias term, and $\cdot$ denotes the dot product. The optimization problem of finding the optimal hyperplane can be

$$\min_{\mathrm{w_l},\mathrm{b_l}} \frac{1}{2}\|\mathrm{w_l}\|^2;$$
$$\min_{\mathrm{w},\mathrm{b}} \frac{1}{2}\|\mathrm{w}\|^2;$$
$$\min_{\mathrm{w_r},\mathrm{b_r}} \frac{1}{2} f\left(r_{x_i}\right)\|\mathrm{w_r}\|^2$$

subject to the constraints:

$$y_i^l(\mathrm{w_l}\cdot \mathrm{x_l} + b_l) \geq 1; \quad y_i(\mathrm{w}\cdot \mathrm{x} + b) \geq 1; \quad y_i^r(\mathrm{w_r}\cdot \mathrm{x_r} + b_r) \geq 1$$

for all i =1,…,n, where yi is the class label of the i-th sample and xi is its feature vector.

Then, for the final decision for the classification, the predicted class $C(\tilde{X}_i)$ of the given fuzzy data $\tilde{X}_i$ is defined by $C(\tilde{X}_i) = \rho\{\frac{1}{3}\left(f\left(l_{x_i}\right) + f(x_i) + f\left(r_{x_i}\right)\right)\}$, where $\rho(\cdot)$ is a rounding function.

5 Data Analysis

The sovereign credit-rating data for 61 countries from 1995 to 2022, provided by Moody's, are used in our analysis. As shown in Appendix Table 6, Moody's evaluates sovereign credit in 21 grades, excluding "not rated." For empirical analysis in our fuzzy multinomial logit model, each grade is broadly reclassified again into 8 categories as shown in Appendix Table 6. For example, a country rated A2 by Moody's falls into the third category.

This study utilizes macroeconomic variables as determinants that were used in earlier research [15–18]. These determinants include inflation measured by the percentage change in the Consumer Price Index (CPI), the national debt ratio, the fiscal balance ratio, and the current account balance ratio expressed as a proportion of GDP, GDP per capita, and the economy's GDP growth rate. We employ the Heritage Foundation's Index of Economic Freedom (EFI) as a qualitative determinant. The EFI assesses and ranks 50 items in 10 categories, encompassing rule of law and judicial effectiveness, small government, regulatory effectiveness, and market openness. GDP per capita is normalized to 1 and then multiplied by 100, because it has a relatively large value.

Among the determinants of credit ratings, economic data such as the EFI and the inflation are recorded based on end-of-year values, making it challenging to consider these as representative values for the entire period. Therefore, it is appropriate to apply a fuzzy model to these economic data. In addition, the dependent variable, sovereign credit ratings, exhibits a wide range within each rating level, making it preferable to consider them as fuzzy numbers. Consequently, for the fuzzy multinomial logistic model, sovereign credit ratings, the EFI, and inflation are fuzzified. The descriptive statistics for the data utilized in this study are presented in Table 1.

Here, the fuzzification for a fuzzy data $\tilde{x}_i = (l_{x_i}, x_i, r_{x_i})$ of x_i ($i = 1, \ldots, n$) is conducted as follows:

Case (1) $\quad i = 1$

$$\text{If } x_1 < x_2, \text{ then } l_{x_1} = 0,\ r_{x_1} = \frac{x_2 - x_1}{2}$$
$$\text{If } x_1 > x_2, \text{ then } l_{x_1} = \frac{x_1 - x_2}{2},\ r_{x_1} = 0$$

Case (2)

$$x_{i-1} < x_i < x_{i+1}\ (1 < i < n)$$
$$l_{x_i} = \frac{x_i - x_{i-1}}{2},\ r_{x_i} = \frac{x_{i+1} - x_i}{2}$$

Case (3)

$$x_{i-1} > x_i > x_{i+1}(1 < i < n)$$
$$l_{x_i} = \frac{x_i - x_{i+1}}{2},\ r_{x_i} = \frac{x_{i-1} - x_i}{2}$$

Case (4)

$$x_{i-1} < x_i, x_i > x_{i+1} \quad (1 < i < n)$$
$$l_{x_i} = \frac{x_i - x_{i-1}}{2},\ r_{x_i} = 0$$

Case (5)

$$x_{i-1} > x_i,\ x_i < x_{i+1} \quad (1 < i < n)$$
$$l_{x_i} = 0,\ r_{x_i} = \frac{x_{i-1} - x_i}{2}$$

Case (6) $i=n$

$$\text{If } x_{n-1} < x_n, \text{ then } l_{x_n} = \frac{x_n - x_{n-1}}{2},\ r_{x_n} = 0$$
$$\text{If } x_{n-1} > x_n, \text{ then } l_{x_n} = 0,\ r_{x_n} = \frac{x_{n-1} - x_n}{2}$$

Table 2 shows the prediction power of the multinomial logistic model based on different criteria, as well as machine learning techniques including decision tree (DT), random forest (RF), and SVM. "Exact" represents cases where the estimated class matches the actual class. "Below1" and "Below2" represent the proportions of observations estimated to be one and two classes lower than the actual class, respectively, out of all observations. Similarly, "Above1"

Table 1 Descriptive statistics

	Moody's	Inflation (Percent change of CPI)	Debt/GDP	Fiscal Balance/GDP	Current Account Balance/GDP	Normalized GDP per cap	GDP growth rate	EFI
Average	3.2	4.5	60.5	3.9	4.9	25.3	4.0	66.6
Median	3.0	2.6	54.8	3.1	3.5	24.3	3.5	66.2
Standard deviation	1.8	7.3	37.4	3.4	4.8	17.6	2.7	9.3
Minimum value	1.0	0.0	0.1	0.0	0.0	0.0	0.0	34.3
Maximum value	8.0	121.7	233.3	32.1	36.0	100.0	25.9	90.2
Number of Observation	1560	1560	1560	1560	1560	1560	1560	1560

Table 2 Prediction power of Multinomial logistic model and Machine Learning Methods with crisp data

	Below1	Exact	Above1	Within1	MAE
MLog1	23.9	36.5	4.7	65.2	1.21
Proposed MLog2	18.3	56.6	14.7	89.6	0.56
Proposed MLog3	18.3	55.5	13.5	87.3	0.60
Proposed MLog4	17.8	57.6	12.9	88.3	0.55
Proposed MLog5	8.5	57.7	11.7	77.9	1.21
Decision Tree (DT)	21.5	53.6	9.0	84.1	0.67
Random Forest (RF)	14.1	68.5	7.9	90.5	0.43
SVM	15.1	57.4	15.3	87.8	0.57

The number of observations is 1560. The number of classified observations is 1560 except Proposed MLog4 and Proposed MLog5. The numbers of classified observations are 1,546 and 1,559 Proposed MLog4 and Proposed MLog5, respectively

Table 3 WHR for crisp data

	Method	WHR	Ranking
1	MLog1	50.8	8
2	Proposed MLog2	73.1	2
3	Proposed MLog3	71.4	5
4	Proposed MLog4	72.95	3
5	Proposed MLog5	67.8	7
6	DT	68.85	6
7	RF	79.5	1
8	SVM	72.6	4

and "Above2" indicate the proportions of observations estimated to be one and two classes higher than the actual class, respectively. "Within1" is the sum of "Exact," "Below1," and "Above1," while "Within2" is the sum of "Exact," "Below2," "Above2." MLog1 and Proposed MLog2 through MLog5 indicate multinomial logistic models with Criterion 1 and Criteria 2 through 5 applied to crisp data, respectively.

When using the proposed Criteria 2 through 5, the ratio of the exact predictions (Exact) ranges from 55.5% to 57.7%, and the ratio of the observations where the difference between the predicted rating and the actual rating is 1 or less (Within1) ranges from 77.9% to 89.6%. Both Exact and Within1 of the proposed MLogs are significantly higher than Exact (36.5%) and Within1 (65.2%) of MLog1 using Criterion 1. This confirms that our classification methods have more prediction power than conventional classification method.

The Proposed MLog2 through MLog5 methods demonstrate considerably competitive performance compared to machine learning techniques as well. Proposed MLog4, which overall achieves the best performance among the proposed MLogs with an Exact of 57.6% and Within1 of 88.3%, outperforms SVM (Exact: 57.4%, Within1: 87.8%) and DT (Exact: 53.6%, Within1: 84.1%), though it underperforms RF (Exact: 68.5%, Within1: 90.5%). In addition, Proposed MLog4 records an MAE of 0.55, which is lower than those of DT (0.67) and SVM (0.57).

Table 3 provides the WHR (weighted hit ratio) for eight methods, which are explained in Sect. 3.4, using crisp data. Figure 2 shows the WHR and ranking from Table 3. Here, RF has the highest hit ratio, and proposed MLog2 and MLog4 rank 2nd and 3rd, respectively. However, the conventional classification method (MLog1) has the lowest WHR. The conventional model (MLog1), based on Criterion 1, provides predictions through a relatively simplified approach; however, its accuracy does not reach that of our proposed models. Among our proposed models, Proposed MLog2 and Proposed MLog4 demonstrate superior performance compared to the machine learning models DT and SVM. Although

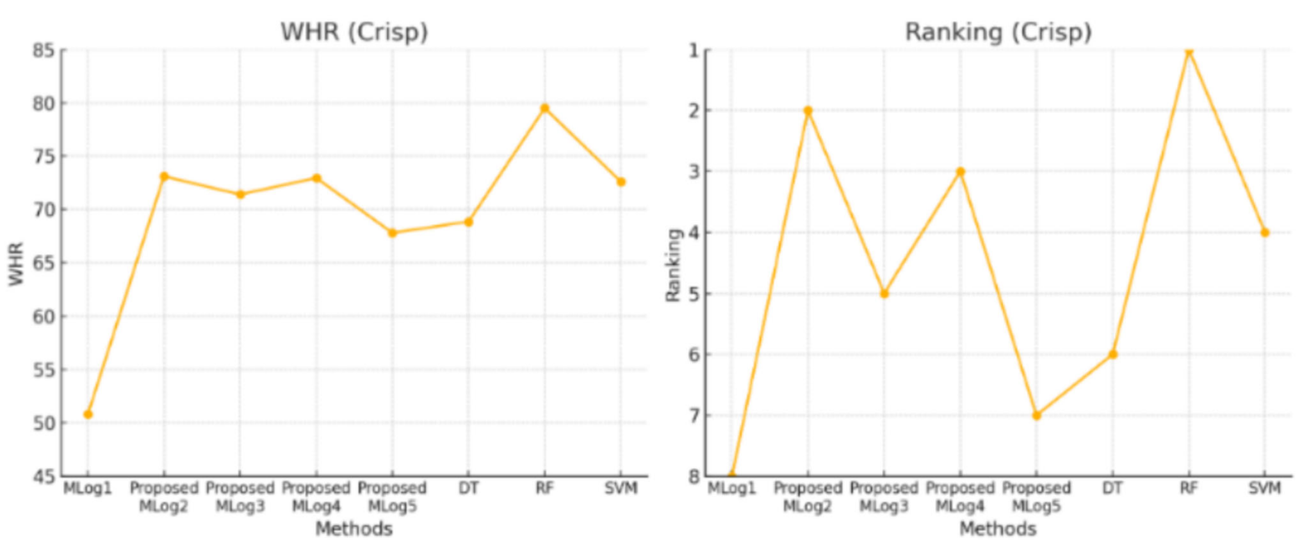

Fig. 2 Ranking of WHR for Crisp Data

Table 4 Prediction power of Multinomial logistic model and Machine Learning Methods with fuzzy data

	Below1	Exact	Above1	Within1	MAE
FMLog1	23.8	36.4	4.7	64.9	1.23
Proposed FMLog2	18.7	55.9	14.9	89.4	0.57
Proposed FMLog3	18.1	54.7	14.8	87.6	0.60
Proposed FMLog4	17.6	57.8	13.3	88.7	0.55
Proposed FMLog5	8.7	57.2	11.5	77.4	0.76
Fuzzy Decision Tree (FDT)	21.6	53.6	9.0	84.2	0.66
Fuzzy Random Forest (FRF)	15.4	67.2	8.7	91.2	0.43
Fuzzy SVM (FSVM)	15.7	57.1	15.9	88.7	0.55

The number of observations is 1560. The number of classified observations is 1560 except Proposed FMLog4 and Proposed FMLog5. The numbers of classified observations are 1,551 and 1,552 Proposed FMLog4 and Proposed FMLog5 respectively

RF shows the highest performance overall, our MLog models offer a significant advantage in terms of interpretability, which is often more challenging with machine learning models.

Table 4 shows the prediction power of the multinomial logistic model and machine learning techniques using fuzzy data. FMLog1 and the Proposed FMLogs represent multinomial logistic models based on Criterion 1 and Criteria 2 through 5 with fuzzy data, respectively. When applying the proposed Criteria 2 through 5, the exact prediction rate (Exact) falls between 54.7% and 57.2%, while the rate of predictions with a difference of 1 or less from the actual rating (Within1) spans from 77.4% to 89.4%. The Exact and Within1 values of the proposed FMLogs show a substantial increase compared to FMLog1 under Criterion 1, where Exact is 36.4% and Within1 is 64.9%. This result highlights the enhanced predictive capability of our classification methods over traditional approaches in the context of the fuzzy model.

Proposed FMLog4 achieves an Exact of 57.8% and Within1 of 88.7%, which are higher than those of the Fuzzy Decision Tree (FDT) (Exact: 53.6%, Within1: 84.2%) and the Fuzzy SVM (FSVM) (Exact: 57.1%, Within1: 88.7%). However, it is lower than that of Fuzzy Random Forest (FRF) (Exact: 67.2%, Within1: 91.2%). In addition, the MAE value of Proposed FMLog4 is 0.55, indicating more accurate predictions compared to those of FDT (0.66) and FSVM (0.55).

Table 5 provides the WHRs for the eight fuzzy methods, calculated as explained in Sect. 3.4. Figure 3 shows the WHR and ranking from Table 5. Here, the FRF has the highest WHR, and Proposed FMLog4, FSVM, and FMLog2 rank 2nd, 3rd, and 4th, respectively. FMLog1, which uses the conventional classification method, has the lowest WHR. In the fuzzy model as well, the conventional model (FMLog1) does not achieve the accuracy of our proposed FMLog models. In addition, among the proposed fuzzy models, Proposed FMLog4 demonstrates superior performance compared to the machine learning models FDT and FSVM.

Fig. 3 Ranking of WHR for Fuzzy Data

Table 5 WHR for fuzzy data

	Method	WHR	Ranking
1	FMLog1	50.65	8
2	Proposed FMLog2	72.7	4
3	Proposed FMLog3	71.15	5
4	Proposed FMLog4	73.25	2
5	Proposed FMLog5	67.3	7
6	FDT	68.9	6
7	FRF	79.25	1
8	FSVM	72.9	3

Figure 4 illustrates the prediction power by class, utilizing both crisp and fuzzy data. Overall, Fig. 4 suggests that our proposed MLogs and FMLogs maintain their prediction power for most classes without significant decreases in specific classes.

To begin, the multinomial logistic model demonstrates similar prediction power regardless of whether crisp or fuzzy data are used, as evidenced by comparing MLog1 with FMLog1 and Proposed MLog2 with Proposed FMLog4. The machine learning techniques, however, exhibit different results depending on the type of data (crisp or fuzzy data), as shown bsy the differences between RF and FRF, particularly for Within1 of Classes 3, 5, and 6.

Second, our proposed classification methods (MLog2, MLog4, FMLog2, FMLog4) exhibit much smaller discrepancies in prediction power among classes. In contrast, the conventional classification criteria using the multinomial logistic model (MLog1, FMLog1) show significantly lower prediction power in both Exact and Within1 measures, especially for classes 4 through 8. This indicates that classes relatively far from the reference class tend to have lower prediction power.

Furthermore, significant differences exist between the prediction power of machine learning techniques and the proposed MLogs across classes. For instance, the Exact of the proposed MLog2 and MLog4 in Class 1 is considerably smaller than that of RF and FRF. However, the Exact of FMLog4 in Class 6 is significantly higher than that of FRF.

6 Conclusions

Sovereign credit ratings are essential for raising funds in international financial markets. Furthermore, they have considerable impact on the economic operations, longevity, and stability of a country. Consequently, predicting the sovereign credit ratings is important to prepare for uncertainties in foreign investments and domestic economic management.

This paper introduces a multinomial logistic model that employs LSE and FLSE with transformed linear expressions of predictors. We also suggest novel classification criteria that use both crisp and fuzzy data. In addition, we address the research gaps in the field of classification methodology and the prediction of sovereign credit ratings by applying our methodology to predicting sovereign credit ratings of 61 countries provided by Moody's. Our proposed multinomial logistic model and fuzzy multinomial logistic model, in conjunction with the proposed classification methods, are shown to outperform the results of conventional classification methods. Additionally, our proposed classification method, compared to conventional classification methods and machine learning methods, does not show fragile predictive power for specific classes; rather, it

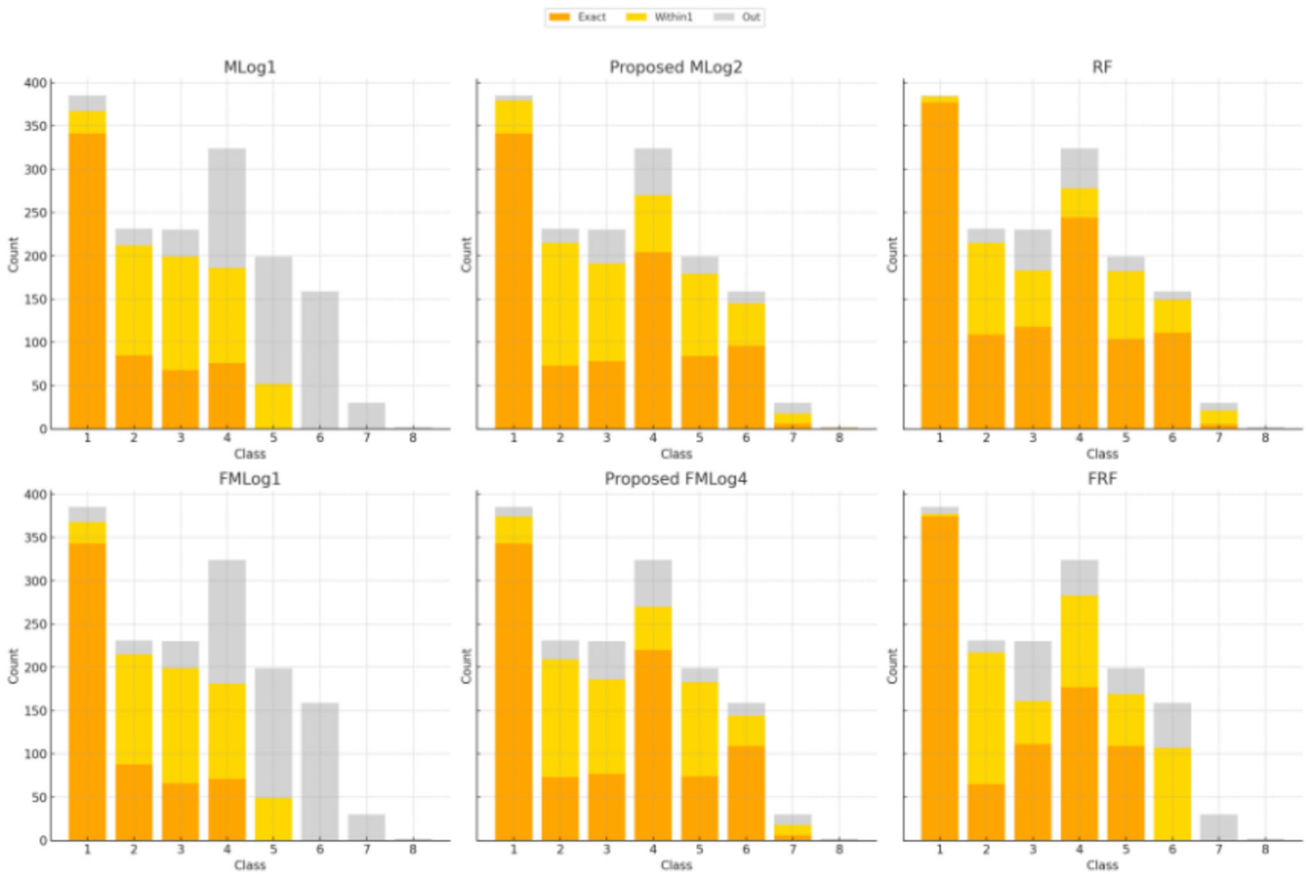

Fig. 4 Prediction Power by Class. We present only six results for the following reasons. First, since the class-specific prediction power of proposed MLog2, 3, 4, and 5 and proposed FMLog2, 3, 4, and 5 have similar patterns, we representatively present proposed MLog2 and proposed FMLog4. Second, our focus is on comparing the results of the proposed MLog and proposed FMLog methods with those of the machine learning methods, so we present the results of RF and FRF, which have the highest WHR among the methods using crisp and fuzzy data, respectively. Although the prediction power varies only for class 4 among machine learning methods, this does not affect our primary comparison. Third, we include the results of MLog1 and FMLog1 to compare the results of the proposed methods with the conventional MLog methods

demonstrates relatively consistent prediction accuracy across various classes. This implies that our methodology can consistently predict credit ratings for both high- and low-rated countries, thereby validating its effectiveness.

Our methodology can be applied to future research that utilizes not only financial data from other credit-rating agencies, such as S&P and Fitch, but also any data requiring classification analysis. Although we applied our methods only to Moody's data due to its availability, extending the analysis to other datasets can improve the generalizability of the results by allowing a thorough comparison and validation of our method across various contexts. Furthermore, by including the qualitative factors as determinants in our model, the prediction power of our methods can be enhanced. In this study, since the size of the credit-rating data is not very large, it does not show good predictive power in classes with a small number of data. If the method proposed in this paper is used in big data with a large size, it can show better predictive power. In particular, with the rapid advancement of data science recently, AI-based text analytics techniques offer new opportunities to quantify qualitative factors that were difficult to measure in the past, such as political instability and disagreements between ruling and opposition parties.

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Data Availibility Data used in this study are obtained from the International Monetary Fund (https://www.imf.org/en/Publications/WEO), the Heritage Foundation (https://www.heritage.org/index), and Moody's ratings processed by Trading Economics (https://tradingeconomics.com) and Moody's Investors Service.

Appendix

See Table 6.

Table 6 The Ratings classified by Moody's

Meaning	Moody's	New Ratings
Prime	Aaa	1
High Grade	Aa1	2
	Aa2	
	Aa3	
Upper Medium Grade	A1	3
	A2	
	A3	
Lower Medium Grade	Baa1	4
	Baa2	
	Baa3	
Non-investment Grade Speculative	Ba1	5
	Ba2	
	Ba3	
Highly Speculative	B1	6
	B2	
	B3	
Substantial Risks	Caa1	7
	Caa2	
	Caa3	
Extremely Speculative	Ca	
In Default with Little Prospect for Recovery		8
In Default		
	C	
Not Rated	NR	

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Jin Hee Yoon received B.S., M.S., and Ph.D. degree in Mathematics from Yonsei University, South Korea. She is currently a faculty of department of Mathematics and Statistics at Sejong University, Seoul, South Korea. Her research interests are fuzzy regression analysis, fuzzy time series, optimizations, intelligent systems, and machine learning. She is a board member of KIIS (Korean Institute of Intelligent Systems) and has been working as an associate editor, guest editor, and editorial board member of several journals including SCI and SCIE journals. In addition, she has been regularly working as an organizer and committee member of several international conferences.

Yoo Young Koo received B.S., M.S., and Ph.D. degree in Mathematics from Yonsei University, South Korea. She is currently a faculty of University college of Yonsei University, Incheon, South Korea. Her research interests are statistical analysis, data analysis, machine learning and deep learning.

Wonkyung Lee received her B.S. and M.S. in Economics from Ewha Womans University, South Korea, and her Ph.D. in Applied Economics from Tsinghua Unive-rsity, China, Beijing. She is currently a faculty of department of Chinese Trade and Commerce at Sejong University, Seoul, South Korea. Her research interests are international trade, applied economics, econometrics and fuzzy regressions.

Dae Jong Kim received B.S., Hankuk University of Foreign Studies, Economics, MBA Korea University, Ph.D. degree in Economics from Sogang Unive-rsity, South Korea. He is currently a faculty of department of Business School at Sejong University, Seoul, South Korea. He is Present, Korea Institute for Business and Economics and is listed on the Marquis/ Who's Who. His research interests are fuzzy regression analysis, fuzzy time series, optimizations, intelligent systems, and applied economics.