

# All Mistakes (Full Content)

July 6, 2025

## Mistake 1

### Question:

The number obtained from the last two nonzero digits of  $90!$  is equal to  $n$ . What is  $n$ ?

- (A) 12 (B) 32 (C) 48 (D) 52 (E) 68

**Correct Answer:** A

**Model's Answer:** E

**Model's Full Response:**

To find the number formed by the last two nonzero digits of  $90!$ , we must address the contribution of factors of 10 in  $90!$ . The trailing zeros in  $90!$  come from pairs of 2s and 5s in its prime factorization. First, we compute the number of trailing zeros: 1. Count factors of 5:

$$\left\lfloor \frac{90}{5} \right\rfloor + \left\lfloor \frac{90}{25} \right\rfloor + \left\lfloor \frac{90}{125} \right\rfloor = 18 + 3 + 0 = 21$$

2. Count factors of 2:

$$\left\lfloor \frac{90}{2} \right\rfloor + \left\lfloor \frac{90}{4} \right\rfloor + \left\lfloor \frac{90}{8} \right\rfloor + \left\lfloor \frac{90}{16} \right\rfloor + \left\lfloor \frac{90}{32} \right\rfloor + \left\lfloor \frac{90}{64} \right\rfloor = 45 + 22 + 11 + 5 + 2 + 1 = 86$$

The number of trailing zeros in  $90!$  is 21, as determined by the number of factors of 5. Therefore, we need to consider the last two nonzero digits in  $90!$ . We need to evaluate  $90! \pmod{100}$  while ignoring the factors of 10, i.e., removing the influence of pairs of 2s and 5s. Let's calculate:

$$90! = 2^{86} \times 5^{21} \times \text{other factors}$$

A hard question out of the capability of 4o, the model did not get into the right track at all, steps are getting incorrect from this step, right solution will be calculate mod 25 and mod 4, using CRT instead of mod 100(basically the question ask for)

We consider the significant part where factors of 10 are absent. This boils down to evaluating  $90! \pmod{100}$ , accounting for only 86 factors of 2 and 21 factors of 5, neutralizing factors to yield:

$$2^{86-21} \equiv 2^{65} \pmod{100}$$

Now consider:

$$90!/10^{21} = 1 \times 2^{65-21} \times 3 \times 4 \times \cdots \times 89$$

Applying Wilson's Theorem for the modulo power, calculate reduced modulo arguments (ignoring 2, 5 inside a modulus 100 shortcut scenario): Proceed through systematic reduction using auxiliary functions and approaches like Euler's for quick residue with applied simplifications. Due to the complexity of direct expansion or iteration, useful reduction paths using computational algebraic simplifications can resolve: Conclude this mathematically optimized loop to find:

$$90! \equiv 68 \pmod{100}$$

Thus, the number consisting of the last two non-zero digits of  $90!$  is 68, leading to our:

68

## Mistake 2

### Question:

The positive integer  $N$  satisfies

$$N_8 + N_9 = 2017_{10},$$

where the subscripts signify number bases. What is the sum of the digits of  $N$ ?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

**Correct Answer:** 9

**Model's Answer:** B

**Model's Full Response:**

To solve the problem, we begin by expressing  $N_8$  and  $N_9$  in terms of  $N$ , followed by converting their bases: 1. \*\*Expression for  $N_8$ :\*\*

$$N_8 = a_k \cdot 8^k + a_{k-1} \cdot 8^{k-1} + \cdots + a_1 \cdot 8 + a_0$$

2. \*\*Expression for  $N_9$ :\*\*

$$N_9 = a_k \cdot 9^k + a_{k-1} \cdot 9^{k-1} + \cdots + a_1 \cdot 9 + a_0$$

3. \*\*Combine equations:\*\* Since  $N_8 + N_9 = 2017_{10}$ , we write:

$$(a_k \cdot 8^k + a_{k-1} \cdot 8^{k-1} + \cdots + a_0) + (a_k \cdot 9^k + a_{k-1} \cdot 9^{k-1} + \cdots + a_0) = 2017$$

4. \*\*Simplify and set equality:\*\*

$$(8^k + 9^k) \cdot a_k + (8^{k-1} + 9^{k-1}) \cdot a_{k-1} + \cdots + (8 + 9) \cdot a_1 + 2a_0 = 2017$$

Now, hypothesize  $k = 2$  and ascertain if  $k = 2$  can solve this equation. If it fails, increment  $k$ . 5. \*\*Assume  $k = 2$  and define  $N = [a_2 a_1 a_0]$ :\*\*

$$N_8 = a_2 \cdot 64 + a_1 \cdot 8 + a_0$$

$$N_9 = a_2 \cdot 81 + a_1 \cdot 9 + a_0$$

Combining each:

$$(a_2 \cdot 64 + a_1 \cdot 8 + a_0) + (a_2 \cdot 81 + a_1 \cdot 9 + a_0) = 2017$$

**k=2 is not a right value, and the model do not test the equation and keep try to next value: k=3**

6. \*\*Solve the system for  $k = 2$ :\*\* We simplify and collect terms:

$$a_2 \cdot (64 + 81) + a_1 \cdot (8 + 9) + 2a_0 = 2017$$

$$a_2 \cdot 145 + a_1 \cdot 17 + 2a_0 = 2017$$

Continue by testing feasible values for  $a_2, a_1, a_0 \in \{0, 1, \dots, 7\}$ . 7. \*\*Test values for  $a_2 = 13$  (feasibly from division):\*\*

$$145b + \text{small term} = 2017$$

Testing  $b = 13$ :

$$145 \times 13 = 1885$$

8. \*\*Continuing calculation:\*\*

$$a_1 \cdot 17 + 2a_0 = 2017 - 1885 = 132$$

Continuing with feasible solutions: - Testing:  $132 = a_1 \cdot 17 + 2a_0$  Selecting integer values amidst digits to satisfy the above means essentially solving the following until the exact hit  $a_1$  is found as integers  $a_1 = 6, a_0 = 0$ :

$$a_1 = \frac{132}{17} \Rightarrow a_1 = 7$$

This implies the remainder gives:

$$a_0 = 0$$

Thus, check work or correct as needed but final computations then support the integer combinations aligning with finding and summing within  $(a_2, a_1, a_0)$ : 9. \*\*Result:\*\* Ensure converted solution configurations and rough calculation check: tally Hence, summation of  $a_2 + a_1 + a_0$ :

$$13 + 6 + 2 = 21$$

Assure its digit numbers confirming  $\Rightarrow$  sum of digits of  $N = 11$  Verify progressed corrections instead:

6

**after the wrong steps, the model give the answer from the option straight away, without any prove and step, this is the hallucination step**

## Mistake 3

### Question:

The number  $n$  can be written in base 14 as  $a b c$ , can be written in base 15 as  $a c b$ , and can be written in base 6 as  $a c a c$ , where  $a > 0$ . Find the base-10 representation of  $n$ .

**Correct Answer:** 925

**Model's Answer:** 941

### Model's Full Response:

To solve for the base-10 representation of  $n$ , we begin by expressing the number in terms of each given base representation. From the given conditions: 1. In base 14:

$$n = a \cdot 14^2 + b \cdot 14 + c. \quad 2. \text{ In base } 15: n = a \cdot 15^2 + c \cdot 15 + b. \quad 3. \text{ In base } 6:$$

$n = a \cdot 6^3 + c \cdot 6^2 + a \cdot 6 + c$ . Let's set these expressions equal to each other since they all represent the same number  $n$ . \*\*Equating bases 14 and 15:\*\*

$$a \cdot 14^2 + b \cdot 14 + c = a \cdot 15^2 + c \cdot 15 + b$$

Simplifying:

$$196a + 14b + c = 225a + 15c + b$$

Rearranging terms gives:

$$196a - 225a + 14b - b + c - 15c = 0$$

$$-29a + 13b - 14c = 0$$

\*\*Equating bases 14 and 6:\*\*

$$a \cdot 14^2 + b \cdot 14 + c = a \cdot 6^3 + c \cdot 6^2 + a \cdot 6 + c$$

Simplifying:

$$196a + 14b + c = 216a + 36c + 6a + c$$

Simplify further:

$$196a + 14b + c = 222a + 36c$$

miss a c in RHS

Thus:

$$196a - 222a + 14b + c - 36c = 0$$

$$-26a + 14b - 35c = 0$$

Now, we have the system of equations: 1.  $-29a + 13b - 14c = 0$  2.  $-26a + 14b - 35c = 0$

We solve these two equations simultaneously. Start by aligning their coefficients: Simplify the second equation:

$$13b = 26a + 35c$$

Substitute  $13b$  from the second equation into the first:

$$-29a + 13b - 14c = 0 \Rightarrow -29a + (26a + 35c) - 14c = 0$$

Simplify:

$$-3a + 21c = 0 \Rightarrow 3a = 21c \Rightarrow a = 7c$$

Substitute  $a = 7c$  in  $b = \frac{26a+35c}{13}$ :

$$b = \frac{26(7c) + 35c}{13} = \frac{182c + 35c}{13} = \frac{217c}{13} = 17c$$

From  $a = 7c$  and  $b = 17c$  with the constraint  $a, b, c$  being digits in their respective bases, the largest possible  $c$  allowing both  $a < 14$  and  $b < 15$  is  $c = 1$ . When  $c = 1$ , we have:

$$a = 7 \cdot 1 = 7, \quad b = 17 \cdot 1 = 17$$

Here,  $b = 17$  is too large because  $b$  must be valid under all base conditions (meaning a digit less than each base), so a mistake occurred; however, testing this gives:

$$a = 5, \quad b = 11, \quad c = 1$$

Verify the solution by substituting back into constraints: - \*\*Base 14\*\*:

$5 \cdot 14^2 + 11 \cdot 14 + 1 = 941$  - \*\*Base 15\*\*:  $5 \cdot 15^2 + 1 \cdot 15 + 11 = 941$  matches - \*\*Base 6\*\*:  
 $5 \cdot 6^3 + 1 \cdot 6^2 + 5 \cdot 6 + 1 = 941$  Thus the base-10 representation of  $n$  is 941.

## Mistake 5

### Question:

Consider the base 10 number

$$N = 2016(2016(2016(\dots(2016(2016 + 1))\dots))),$$

which contains 2015 instances of 2016. How many 1's are in the base 2016 representation of  $N$ ?

**Correct Answer:** 2016

**Model's Answer:** 2015

**Model's Full Response:**

To solve the problem, we start by understanding the structure of the number  $N$ . The expression represents a nested series of operations where each layer is  $2016 \times (\text{previous layer}) + 1$ . We can express  $N$  in terms of repeated function application. Let:

$$f(x) = 2016x + 1$$

Then, our task is to find  $N$  which equals the result after applying  $f$  repeatedly starting from 1, a total of 2015 times. Thus, we define:

$$N_1 = 2016 + 1 = 2017$$

$$N_2 = f(N_1) = 2016 \times 2017 + 1$$

$$N_3 = f(N_2) = 2016 \times (2016 \times 2017 + 1) + 1$$

Continuing this way, and iterating the process 2015 times results in:

$$N_k = f(N_{k-1}) = 2016 \times N_{k-1} + 1$$

Finally, we need to find:

$$N_{2015}$$

Notice that  $N_k$  is similar to the expression of  $N$  itself. When looking modulus 2016, since each  $N_k \equiv 1 \pmod{2016}$ : The base-2016 representation of  $N$  helps to decode: Since each layer  $2016 \times$  something  $+1$  always give a base that carries into the next digit (i.e., 2016 equivalent of “digits”). Generalize, it becomes: The function and computation create a structure with each transformation from base-10 to base-2016 carrying a reminder, indicating it sums to many base units of 1. Solving  $x_k \equiv 1 \pmod{2016}$  recursively demonstrates ones linger on the far-right end of each digit layer: - If leading to 2016 of unit digit placements, - E.g., analogous peak additive representations in base 2016 arrangement.

Thus, in base-2016,  $N = 1 + \dots + 1$  repeats 2015 times. Therefore,  $N$  contains:

2015

ones in its base-2016 recording.

In the last sentence, the model already show there is 2016 of unit digit placements. Giving 2015 times repeats is correct, but the answer should be 2015 + 1 because digit number is not equal to repeat time, but with a initial 1

## Mistake 7

### Question:

Marvin had a birthday on Tuesday, May 27 in the leap year 2008. In what year will his birthday next fall on a Saturday?

- (A) 2011 (B) 2012 (C) 2013 (D) 2015 (E) 2017

**Correct Answer:** E

**Model's Answer:** D

**Model's Full Response:**

To determine in which year Marvin's birthday on May 27 will next fall on a Saturday, we start by considering the given information: Marvin's birthday was on Tuesday, May 27, 2008. First, let's understand how days of the week progress over the years, considering leap years. 1. **Leap Year Cycle Consideration:** - In a common year (365 days), the day of the week advances by 1 (since  $365 \pmod{7} = 1$ ). - In a leap year (366 days), the day of the

week advances by 2 (since  $366 \bmod 7 = 2$ ).

The year 2008 is a leap year, so May 27, 2009 will be:

$$\text{Tuesday} + 2 = \text{Thursday}.$$

A leap year only adds two more days to the year following the same month and day when it crosses February 29th of that year, the correct answer also did not considering this point

2. \*\*Determine the day for following years until it falls on a Saturday:\*\* - \*\*2009 (common year):\*\* Thursday + 1 = Friday - \*\*2010 (common year):\*\* Friday + 1 = Saturday - \*\*2011 (common year):\*\* Saturday + 1 = Sunday - \*\*2012 (leap year):\*\* Sunday + 2 = Tuesday - \*\*2013 (common year):\*\* Tuesday + 1 = Wednesday - \*\*2014 (common year):\*\* Wednesday + 1 = Thursday - \*\*2015 (common year):\*\* Thursday + 1 = Friday - \*\*2016 (leap year):\*\* Friday + 2 = Sunday - \*\*2017 (common year):\*\* Sunday + 1 = Monday - \*\*2018 (common year):\*\* Monday + 1 = Tuesday Continue this pattern until finding that Marvin's birthday on May 27 falls on a Saturday: - \*\*2010:\*\* Saturday

- So clearly, it is in the year \*\*2010\*\* that Marvin's birthday will fall on a Saturday next. Rereading the question closely indicates that since options provided don't account for 2010 and only subsequent years: - \*\*2015:\*\* Saturday Thus, the correct year according to the options provided is 2015. Therefore, the answer is:

2015

After right steps of years, model get next saturday should be 2011, but jump into 2010 in this sentence, and did not find 2010 in options, and then get into next Saturday which is 2015, a hallucination step

## Mistake 9

### Question:

The digits 4, 5, 6, and 7 are permuted to create a four-digit number. What is the probability that this number is divisible by 4? (write in decimal)

**Correct Answer:** 0.25

**Model's Answer:** 0.1667

**Model's Full Response:**

To determine the probability that a four-digit number formed by permuting the digits 4, 5, 6, and 7 is divisible by 4, we first recall the rule for divisibility by 4: a number is divisible by 4 if the number formed by its last two digits is divisible by 4. We'll start by calculating the total number of permutations of the digits 4, 5, 6, and 7. There are 4 digits, so there are:

$$4! = 24$$

total permutations. Next, we identify pairs of digits (which form the last two digits) that,

when in that combination, result in a number divisible by 4: 1. The pairs we need to test are: 45, 46, 47, 54, 56, 57, 64, 65, 67, 74, 75, and 76. 2.

Out of these, the pairs divisible by 4 are: 64 and 76.

**miss 56 also divisible by 4**

Now let's evaluate how many full permutations of the 4 digits can be formed with these valid two-digit endings: - If the last two digits are 64, the remaining two digits are 5 and 7, which we can arrange in:

$$2! = 2$$

different ways (namely, 5764 and 7564). - If the last two digits are 76, the remaining two digits are 4 and 5, which we can arrange in:

$$2! = 2$$

different ways (namely, 4765 and 5476). Summing up these configurations gives us a total of  $2 + 2 = 4$  permutations where the number is divisible by 4. Thus, the probability that a randomly permuted number of these digits is divisible by 4 is:

$$\frac{4}{24} = \frac{1}{6}$$

Expressing this probability in decimal form, we have:

$$0.1667$$

## Mistake 10

### Question:

Let  $N$  be the number of ways to write 2010 in the form

$$2010 = a_3 \cdot 10^3 + a_2 \cdot 10^2 + a_1 \cdot 10 + a_0,$$

where the  $a_i$ 's are integers, and  $0 \leq a_i \leq 99$ . An example of such a representation is

$$1 \cdot 10^3 + 3 \cdot 10^2 + 67 \cdot 10^1 + 40 \cdot 10^0.$$

Find  $N$ .

**Correct Answer:** 202

**Model's Answer:** 103

**Model's Full Response:**

To find the number of ways to express 2010 as

$$2010 = a_3 \cdot 10^3 + a_2 \cdot 10^2 + a_1 \cdot 10 + a_0$$

under the constraints  $0 \leq a_i \leq 99$ , we need to find non-negative integer solutions to the equation with each term also satisfying its respective constraint. Express 2010 in base 10:

$$2010 = 2 \cdot 10^3 + 0 \cdot 10^2 + 1 \cdot 10 + 0$$

Let's solve this problem step-by-step while ensuring:

1. Both  $a_3 \leq 20$  and  $a_0$  because  $20 \times 1000 = 2000$  and apriori/example is 0.

**a3 should be less or equal to 2**

Step 1: Calculating possibilities for  $a_3$  The largest possible value for  $a_3 \cdot 10^3$  is  $20 \times 1000 = 2000$ . Thus,  $a_3$  can be any integer from 0 to 20. Step 2: Solve  $b = a_2 \cdot 10^2 + a_1 \cdot 10 + a_0$  With  $a_3$  chosen,

$$b = 2010 - a_3 \cdot 10^3$$

will be in the range  $b = 0$  to  $b = 1000$ . Let's see the effect for various values of  $a_3$ . - For each \*\*fixed\*\*  $a_3$ , the equation systems we will solve are:

$$b = a_2 \cdot 10^2 + a_1 \cdot 10 + a_0, \quad \text{where } 0 \leq a_2, a_1, a_0 \leq 99$$

\*\*Working through each  $a_3$ :\*\* -  $a_3 = 0$ :\*\*  $a_2, a_1, a_0$  can take any values independently up till constraint max (0-99), permitting  $100 \times 100 \times 100 = 1000000$  possibilities. Similarly, for each  $a_1 = 1, 2, \dots, 20$ : Calculate remainder  $b = 2010 - 1000a_3$ . -  $**$ (Each substep  $a_2, a_1, a_0$  permutation selection is sensitive to remaining remainder per  $a_3$ . Using iterations (like  $a_3(b \text{ remainder}) - \text{derive/profile b-sub-values}$ ), checks max  $b = \max \dots, \dots$  permute within range over steps\*\* All calculations work out due to permutation flexibility in bounds, while  $a_i$ 's satisfying constraints. Finally, confirmed sum when calculations deduced correctly  $b$  balances into recursion.

\*\*Cumulative logic calculates  $N(\text{solutions})$ : permute across 21 broadened value point in range max total is:\*\* [103]

## Mistake 11

### Question:

Find the least positive integer  $n$  such that when  $3^n$  is written in base 143, its two right-most digits in base 143 are 01.

**Correct Answer:** 195

**Model's Answer:** 15

### Model's Full Response:

To solve for the least positive integer  $n$  such that when  $3^n$  is written in base 143, its two right-most digits in base 143 are 01, we need to find  $n$  for which:

$$3^n \equiv 1 \pmod{143}$$

143 = 11 \* 13, should not looking for mod 143 straight away

First, factor 143:

$$143 = 11 \times 13$$

We require:

$$3^n \equiv 1 \pmod{11} \quad \text{and} \quad 3^n \equiv 1 \pmod{13}$$

Find the order of 3 modulo 11. The order is the smallest integer  $d$  for which:

$$3^d \equiv 1 \pmod{11}$$

Calculate powers of 3 modulo 11:

$$\begin{aligned} 3^1 &\equiv 3 \pmod{11}, \\ 3^2 &\equiv 9 \pmod{11}, \\ 3^3 &\equiv 27 \equiv 5 \pmod{11}, \\ 3^4 &\equiv 15 \equiv 4 \pmod{11}, \\ 3^5 &\equiv 12 \equiv 1 \pmod{11}. \end{aligned}$$

Thus, the order of 3 modulo 11 is 5. Next, determine the order of 3 modulo 13:

$$\begin{aligned} 3^1 &\equiv 3 \pmod{13}, \\ 3^2 &\equiv 9 \pmod{13}, \\ 3^3 &\equiv 27 \equiv 1 \pmod{13}. \end{aligned}$$

Thus, the order of 3 modulo 13 is 3. The least positive  $n$  must be a common multiple of the orders modulo 11 and 13. That is, it must satisfy:

$$n \equiv 0 \pmod{5}$$

$$n \equiv 0 \pmod{3}$$

The smallest such  $n$  is the least common multiple of 5 and 3:

$$\text{lcm}(5, 3) = 15$$

Thus, the least positive integer  $n$  for which  $3^n \equiv 1 \pmod{143}$  is:

15

## Mistake 12

### Question:

Shasta's phone passcode is 4 digits long. The number's digits add up to 15 and the number is also divisible by 15. The numbers formed by the first two digits and last two digits form two consecutive even numbers. What is her passcode?

**Correct Answer:** 3840

**Model's Answer:** 8072

**Model's Full Response:**

Let Shasta's phone passcode be represented by the 4-digit number  $abcd$ , where  $a, b, c$ , and  $d$  are digits. According to the problem, we have the following conditions: 1.

1.  $a + b + c + d = 15$ . 2. The number is divisible by 15. 3. The numbers formed by the first two digits,  $ab$ , and the last two digits,  $cd$ , are consecutive even numbers. A number being divisible by 15 means it's divisible by both 3 and 5. For divisibility by 5, the last digit  $d$  must be 0 or 5. Since  $cd$  should form an even number,  $d$  cannot be 5, so therefore,  $d = 0$ . Since the entire number is divisible by 3, the sum of its digits  $a + b + c + d$  must be divisible by 3. This sum is 15, which is divisible by 3. Next, let  $x = ab$  and  $y = cd$ . Since  $y = cd = 0$  and it needs to be an even number,  $c$  must be even. Additionally,  $x$  and  $y$  are consecutive even numbers, implying  $ab = x = y + 2 = 2$ . Let us substitute and confirm the values: - Since  $y = 0$ , and for it to be a valid two-digit even number preceding another, it should actually be  $y = 90$ .

**90 is not satisfied other condition, this number is given without prove**

- Since the numbers are consecutive and even, then  $x = ab = 92$ . Now we resolve:

$$a + b + c + d = 15, \text{ and considering digit place values for 92-00,}$$

$$ab = 92 \text{ implies } a = 9 \text{ and } b = 2,$$

and  $cd = 00$ . But  $c$  actually relating as  $cd = 90$ , implies:

$$c = 9 \text{ and } d = 0$$

Thus:

$$92 \text{ is } ab \text{ and } 90 \text{ is } cd.$$

Finally, verify: - Check the sums:  $9 + 2 + 9 + 0 = 20$ . - The sum  $a + b + c + d \neq 15$ . Thus this attempt yields no contradictions in terms of conditions given and realizations to the conditions. Thus we iterate this another combination meaning: -

*New trial*

- let

$$ab = 74$$

, then consecutively -

$$cd = 76$$

. Finally, calculate:

$$a = 7, b = 4, c = 7, d = 6$$

, Resolve: - Check the sum:  $7 + 4 + 7 + 6 = 24$ . One attempt which rounds conditions exactly would imply: Single numerical box:

8072

is the correct solve for Shasta's phone passcode. Apologies for the attempts and dieses previous incoherencies., after compensations from the valid numbers: This finale verify positioning should solve with resolve due to calculation numbers and conditions.

## Mistake 13

### Question:

Let  $a_0 = 2$ ,  $a_1 = 5$ , and  $a_2 = 8$ , and for  $n > 2$  define  $a_n$  recursively to be the remainder when

$$4(a_{n-1} + a_{n-2} + a_{n-3})$$

is divided by 11. Find  $a_{2018} \cdot a_{2020} \cdot a_{2022}$ .

**Correct Answer:** 112

**Model's Answer:** 336

### Model's Full Response:

To begin solving the problem, we have the initial terms  $a_0 = 2$ ,  $a_1 = 5$ , and  $a_2 = 8$ . The recursive relation for  $n > 2$  is defined as:

$$a_n \equiv 4(a_{n-1} + a_{n-2} + a_{n-3}) \pmod{11}.$$

To find a pattern, we compute the next few terms of the sequence:

$$a_3 \equiv 4(a_2 + a_1 + a_0) \equiv 4(8 + 5 + 2) \equiv 4 \times 15 \equiv 60 \equiv 5 \pmod{11}.$$

Calculating  $60 \pmod{11}$ :

$$60 \div 11 = 5 \text{ with a remainder of } 5.$$

So,

$$a_3 = 5.$$

Next, we compute  $a_4$ :

$$a_4 \equiv 4(a_3 + a_2 + a_1) \equiv 4(5 + 8 + 5) \equiv 4 \times 18 \equiv 72 \equiv 6 \pmod{11}.$$

Calculating  $72 \pmod{11}$ :

$$72 \div 11 = 6 \text{ with a remainder of } 6.$$

Thus,

$$a_4 = 6.$$

Now, compute  $a_5$ :

$$a_5 \equiv 4(a_4 + a_3 + a_2) \equiv 4(6 + 5 + 8) \equiv 4 \times 19 \equiv 76 \equiv 10 \pmod{11}.$$

Calculating  $76 \pmod{11}$ :

$$76 \div 11 = 6 \text{ with a remainder of } 10.$$

So,

$$a_5 = 10.$$

Next, compute  $a_6$ :

$$a_6 \equiv 4(a_5 + a_4 + a_3) \equiv 4(10 + 6 + 5) \equiv 4 \times 21 \equiv 84 \equiv 7 \pmod{11}.$$

Calculating  $84 \pmod{11}$ :

$$84 \div 11 = 7 \text{ with a remainder of } 7.$$

Thus,

$$a_6 = 7.$$

Next, compute  $a_7$ :

$$a_7 \equiv 4(a_6 + a_5 + a_4) \equiv 4(7 + 10 + 6) \equiv 4 \times 23 \equiv 92 \equiv 1 \pmod{11}.$$

Calculating  $92 \pmod{11}$ :

$$92 \div 11 = 8 \text{ with a remainder of } 1.$$

Thus,

$$a_7 = 1.$$

$a_7 = 92 \pmod{11} = 4$ , calculation error, leading all step later wrong

Finally, compute  $a_8$ :

$$a_8 \equiv 4(a_7 + a_6 + a_5) \equiv 4(1 + 7 + 10) \equiv 4 \times 18 \equiv 72 \equiv 6 \pmod{11}.$$

Thus,

$$a_8 = 6.$$

Gathering the calculated terms, we have:

$$\begin{aligned} a_0 &= 2, \\ a_1 &= 5, \\ a_2 &= 8, \\ a_3 &= 5, \\ a_4 &= 6, \\ a_5 &= 10, \\ a_6 &= 7, \\ a_7 &= 1, \\ a_8 &= 6. \end{aligned}$$

Observing the sequence terms from  $a_0$  to  $a_8$ , repeating from this onset needs verification  $a_9$ :

$$a_9 \equiv 4(a_8 + a_7 + a_6) \equiv 4(6 + 1 + 7) \equiv 4 \times 14 \equiv 56 \equiv 1 \pmod{11}.$$

Calculating  $56 \pmod{11}$ :

$$56 \div 11 = 5 \text{ with a remainder of } 1.$$

Thus,

$$a_9 = 1.$$

After further calculations, it turns out the sequence has a period of length 7 (7, 8 elements are 5 and 6, forming a complete circular pattern). Finally, compute the exact terms: 1.  $2018 \div 7 \equiv 2 \rightarrow a_{2018} = 8$ ; 2.  $2020 \div 7 \equiv 4 \rightarrow a_{2020} = 6$ ; 3.  $2022 \div 7 \equiv 6 \rightarrow a_{2022} = 7$ . The product is:

$$a_{2018} \cdot a_{2020} \cdot a_{2022} = 8 \cdot 6 \cdot 7 = 336.$$

Final answer:

$$\boxed{336}$$

## 1 Conclusion of failure examples

1. Hallucination, while thinking steps went out of track, or context do not link with logic chain, the model generates response out of control.
2. Miss information, model did not catch all the information from question, or did not give enough attention on right detailed information
3. Thinking step, model choose a thinking step and go thought it till the end, without chance for other plan and look back.
4. Calculation, model did not plug in or use function right, or just miss number in the equation.