运筹学第9次作业　　张锦程　　2018012082

1. $f(x_1,x_2)=4x_1+6x_2-2x_1^2-2x_1x_2-2x_2^2$　　$X^0=[1,1]^T$

$\nabla f(x)=\left(\frac{\partial f}{\partial x_1},\frac{\partial f}{\partial x_2}\right)=(4-4x_1-2x_2,\ 6-2x_1-4x_2)^T$

$\frac{\partial^2 f}{\partial x_1\partial x_2}=\frac{\partial^2 f}{\partial x_2\partial x_1}=-2$　　$\frac{\partial^2 f}{\partial x_1^2}=-4$，　$\frac{\partial^2 f}{\partial x_2^2}=-4$

$\nabla^2 f(x)=\begin{bmatrix}-4 & -2\\ -2 & -4\end{bmatrix}$　　$\nabla^2 f(x)^{-1}=\begin{bmatrix}-\frac{1}{3} & \frac{1}{6}\\ \frac{1}{6} & -\frac{1}{3}\end{bmatrix}$

(1) $\nabla f(x)_{[1,1]^T}=(-2,0)^T$　$D_1=\begin{bmatrix}\frac{1}{3} & -\frac{1}{6}\\ -\frac{1}{6} & \frac{1}{3}\end{bmatrix}[-2\ \ 0]^T=[-\frac{2}{3},\frac{1}{3}]^T$

$f(x^0+D_1t)=f(1-\frac{2}{3}t,1+\frac{1}{3}t)=4+\frac{4}{9}t-\frac{2}{3}t^2$，　当 $t=\frac{1}{3}$ 时 有最大值 $\frac{110}{27}$

(2) $\nabla f(x)_{[\frac{7}{9},\frac{10}{9}]^T}=(-\frac{12}{9},\ 0)^T$　$D_2=\begin{bmatrix}\frac{1}{3} & -\frac{1}{6}\\ -\frac{1}{6} & \frac{1}{3}\end{bmatrix}[-\frac{12}{9},0]^T=[-\frac{4}{9},\frac{2}{9}]^T$

$f(x^1+D_2t)=f(\frac{7}{9}-\frac{4}{9}t,\frac{10}{9}+\frac{2}{9}t)=\frac{118}{27}-\frac{44}{81}t-\frac{8}{27}t^2$. 当 $t=-\frac{11}{12}$ 时 有最大值 $\frac{2245}{486}$

(3) $\nabla f(x)_{[\frac{32}{27},\frac{49}{54}]^T}=(-\frac{23}{9},\ 0)^T$.　$D_3=\begin{bmatrix}\frac{1}{3} & -\frac{1}{6}\\ -\frac{1}{6} & \frac{1}{3}\end{bmatrix}[-\frac{23}{9},0]^T=[-\frac{23}{27},\frac{23}{54}]^T$

$f(X^2+D_3t)=f(\frac{32}{27}-\frac{23}{27}t,\ \frac{49}{54}+\frac{23}{54}t)=\frac{1}{54\times27}(11489-1334t-1587t^2)$

当 $t=-\frac{29}{69}$ 时　　$f(t)$ 有最大值 $\frac{17654}{2187}$

2. $f(x_1,x_2)=40+10x_1^2+10x_2^2+16x_1x_2+6x_1^2x_2+6x_1x_2^2+x_1^2x_2^2$

$\nabla f(X)=\left(\frac{\partial f}{\partial x_1},\frac{\partial f}{\partial x_2}\right)=(20x_1+16x_2+12x_1x_2+6x_2^2+2x_1x_2^2,\ 20x_2+16x_1+12x_1x_2+6x_1^2+2x_1^2x_2)$

$\frac{\partial^2 f}{\partial x_1\partial x_2}=\frac{\partial^2 f}{\partial x_2\partial x_1}=16+12x_1+12x_2+4x_1x_2$；$\frac{\partial^2 f}{\partial x_1^2}=20+12x_2+2x_2^2$，$\frac{\partial^2 f}{\partial x_2^2}=20+12x_1+2x_1^2$

$\nabla^2 f(x)=\begin{bmatrix}20+12x_2+2x_2^2 & 16+12x_1+12x_2+4x_1x_2\\ 16+12x_1+12x_2+4x_1x_2 & 20+12x_1+2x_1^2\end{bmatrix}$　　$X=[-4,6]^T$ 时 $\nabla^2 f(x)=\begin{bmatrix}164 & -56\\ -56 & 4\end{bmatrix}$

$\nabla^2 f(x)^{-1}=\begin{bmatrix}-\frac{1}{620} & -\frac{7}{310}\\ -\frac{7}{310} & -\frac{41}{620}\end{bmatrix}$　　$D=-(\nabla^2 f(x))^{-1}\nabla f(x)$

$\nabla f(x)|_{[-4,6]^T}=[-344,56]^T$　　$=\begin{bmatrix}\frac{1}{620} & \frac{7}{310}\\ \frac{7}{310} & \frac{41}{620}\end{bmatrix}\begin{bmatrix}-344\\ 56\end{bmatrix}=\begin{bmatrix}\frac{282}{155}\\ -\frac{124}{31}\end{bmatrix}$

L_1 范数：$\nabla f(x)|_{[-4,6]^T}=[-344,56]^T$

$d_1=1$，$d_2=-1$，　$D=[1,-1]$