概率论与数理统计　第十二次作业　张锦程

(1) (a) 第一类错误 $P_1 = P[\bar{x} \ge 2.5 \mid \mu = 2.1] =$

第二类错误 $P_2 = P[\bar{x} < 2.5 \mid \mu = 2.9]$

$\bar{x} \sim N(\mu, \frac{1}{19})$, $y = \sqrt{19}(\bar{x} - \mu) \sim N(0,1)$

$\therefore P_1 = P(y \ge 0.4\sqrt{19}) \approx 1 - u(1.744) = 1 - 0.9595 = 0.0405$

$P_2 = P(y < -0.4\sqrt{19}) \approx u(-1.744) = 0.0405$

(b) 此时: $P_2' = P(y < -0.4\sqrt{n}) \le 0.02$

得 $u(0.4\sqrt{n}) > 0.98$

$\therefore 0.4\sqrt{n} > 2.055 \Rightarrow n > 26.39 \Rightarrow n$ 应取 27

(c) $P_1 + P_2 = P(y \ge 0.4\sqrt{n}) + P(y < -0.4\sqrt{n}) = 2P(y \ge 0.4\sqrt{n}) = 2 - 2u(0.4\sqrt{n})$

n 很大时 $u(0.4\sqrt{n}) \to 1$ $\therefore P_1 + P_2 \to 0$

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(2) (a) $g(\theta) = P\{x_{(n)} \le 2.4 \mid \theta = \theta_0\}$

由题: ~~$f_n(x)$~~ $p(x) = \frac{1}{\theta}$, $F(x) = \frac{x}{\theta}$ $\quad \therefore p_n(x) = \frac{n!}{(n-1)!}(F(x))^{n-1}p(x) = \frac{nx^{n-1}}{\theta^n}$

$\therefore g(\theta_0) = \int_0^{2.4} \frac{nx^{n-1}}{\theta_0^n}dx = \frac{x^n}{\theta_0^n}\Big|_0^{2.4} = (\frac{2.4}{\theta_0})^n$

(b) 第一类错误: $\alpha(\theta) = g(\theta)$, $\theta \in [3, +\infty)$

$= (\frac{2.4}{\theta})^n \le (\frac{2.4}{3})^n = 0.8^n$. 当 $n=1$ 时, $\alpha(\theta)$ 有最大值 0.8

(c) 令 $0.8^n \le 0.05$, 得 $n \ge \log_{0.8} 0.05 = 13.4251$ $\quad \therefore n$ 应取 14

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(3) $g(\theta) =$ ~~$P[x_{(n)} \le 2.4 \mid \theta]$~~ $P[X \le 0.5 \mid \theta] =$

$p(x) = (1+\theta)x^\theta$, $F(x) = x^{\theta+1}$

$\therefore g(\theta) = F(0.5) = 0.5^{\theta+1}$

第一类: $\alpha(\theta) = g(\theta) = 0.5^{\theta+1}$, $\alpha(\theta = 1) = \frac{1}{4}$

第二类: $\beta(\theta) = 1 - g(\theta) = 1 - 0.5^{\theta+1}$ $\quad (\theta < 1)$

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(4) 检验统计量为 $t_0 = (\bar{x} - \bar{y}) / (S_w\sqrt{\frac{1}{m} + \frac{1}{n}}) \sim t(m+n-2) = t(15)$

$S_w^2 = \frac{1}{m+n-2}[mS_x^2 + nS_y^2] = \frac{1}{15}[9 \times 0.14 + 8 \times 0.17] = 0.175$ $\quad S_w \approx 0.418$

$t_0 = (0.25 - 0.28) / (0.418\sqrt{\frac{1}{9} + \frac{1}{8}}) = -0.1477$

$P_{值} = P(|t| \ge |t_0|) = P(t \le -t_0) + 1 - P(t < t_0)$

$= t(-0.1477) + 1 - t(-0.1477) = 2t(-0.1477) \approx 2 \times 0.4423$

$= 0.8846$

所以可以认为它们的均值一样

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(5) $H_0: \mu_1 - \mu_2 = 0$, $H_1: \mu_1 - \mu_2 \ne 0$; $m = 176$, $\bar{x} = 465$, $S_x^2 = \cancel{49}54$; $n = 109$, $\bar{y} = 422$, $S_y^2 = 49$

~~统计量 $t_0 = \frac{\bar{x} - \bar{y}}{S_w\sqrt{\frac{1}{m} + \frac{1}{n}}} = (465 - 422) / (7.2426\sqrt{\frac{1}{176} + \frac{1}{109}}) = 48.71$ $\quad t \sim t(m+n-2) = t(283)$~~

~~$P_{值} = P(|t| \ge |t_0|) = 2P(t \le -48.71) =$~~

统计量 $u = (\bar{x} - \bar{y}) / \sqrt{S_x^2/m + S_y^2/n} = 49.44$ $\quad P_{值} = 2 \cdot (1 - \Phi(|u|)) = 2 - 2\Phi(49.44) \approx 0$

$\therefore$ 不可以认为男女该蛋白含量相同

(6) $m=22$, $\bar{x}=1.3$, $S_x=0.57$, $S_x^2=0.3249$

$n=24$, $\bar{y}=1.5$, $S_y=0.48$, $S_y^2=0.2304$ $\alpha=0.05$

$H_0: \mu_1-\mu_2\le 0$ vs $H_1: \mu_1-\mu_2>0$

~~统计量 $u_0=(\bar{x}-\bar{y})/\sqrt{\frac{\sigma_1^2}{m}+\frac{\sigma_2^2}{n}}=-0.2\times\sqrt{\frac{0.3249}{22}+\frac{0.2304}{24}}\approx-0.031$~~

~~$P_I=1-\Phi(u_0)\approx\Phi(0.03)\approx0.512$ $0.05<P_I$, 可以认为第二种化肥更好~~

统计量 $t_0=(\bar{x}-\bar{y})/(\sqrt{\frac{S_x^2}{m}+\frac{S_y^2}{n}})$, ~~$S_w=\frac{1}{m+n-2}[mS_x^2+nS_y^2]\approx0.288$~~

$\approx$ ~~-1.1917~~ -1.2812

拒绝域: $W_I=\{t\ge t_{1-\alpha}(m+n-2)\}$; P值 $=P(t\ge t_0)=P(t(44)\ge$ ~~-1.1917~~ $-1.2812)$

$\approx 1-P(t(44)\le$ ~~1.1917~~ $1.2812)$ $=P(t(44)\le 1.2812)$

知 $t_{0.95}(44)=1.68>$ ~~-1.1917~~ -1.2812 $\approx$ ~~0.888~~ 0.8965

不在拒绝域, 故可以认为第二种化肥更好

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(7)(a) $m=7$, $\bar{x}=95.7$, $S_x^2=2208$ $\alpha=0.05$ 令$x\sqrt{10}$ $\Rightarrow$ $\bar{x}=95.7\sqrt{10}$, $\bar{y}=97.4\sqrt{10}\approx308$, $S_x^2=2208$, $S_y^2=788$

$n=5$, $\bar{y}=97.4$, $S_y^2=78.8$

$H_0: \sigma_1^2=\sigma_2^2$, vs $H_1: \sigma_1^2\ne\sigma_2^2$

$W_{II}=\{F\le F_{\alpha/2}(m-1,n-1)$ 或 $F\ge F_{1-\alpha/2}(m-1,n-1)\}$

$F=\frac{S_x^2}{S_y^2}=\frac{2208}{788}=2.802$; $F_{0.025}(6,4)=0.1605$; $F_{0.975}(6,4)=9.197$ 不在拒绝域中

假设成立

(b) ~~服从的 $\bar{x}-\bar{y}$: 检验量~~ $\sigma_1^2=10\sigma^2$, $\sigma_2^2=\sigma^2$

$\frac{1}{\sigma^2}[\frac{1}{10}\sum(x_i-\bar{x})^2+\sum(y_i-\bar{y})^2]\sim\chi^2(m+n-2)$

$S_w^2=\frac{1}{m+n-2}[\frac{1}{10}\sum(x_i-\bar{x})^2+\sum(y_i-\bar{y})^2]=\frac{1}{10}[\frac{1}{10}\times7\times2208+5\times78.8]=193.96$

$\therefore t=\frac{(\bar{x}-\bar{y})-(\mu_1-\mu_2)}{S_w\sqrt{\frac{1}{m}+\frac{1}{n}}}$ = ~~$(\bar{x}-\bar{y})$~~ $\sim t(m+n-2)$

检验统计量为 $t=\frac{(\bar{x}-\bar{y})}{S_w\sqrt{\frac{1}{m}+\frac{1}{n}}}=-0.356$

拒绝域: $W_I=\{t\ge t_{1-\alpha}(m+n-2)\}$ $t_{1-\alpha}(m+n-2)=t_{0.95}(10)=1.812$ 不在 W_I 内

则接受 H_0

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(8) $H_0: \frac{1}{\lambda}-\frac{1}{\theta}=0$ vs $H_1: \frac{1}{\lambda}-\frac{1}{\theta}\ne0$ $x\sim Exp(\lambda)$, $y\sim Exp(\theta)$

$n\bar{x}\sim Ga(n,\theta_0)$, $m\bar{y}\sim Ga(m,\lambda_0)$

$2n\bar{x}\theta_0\sim\chi^2(2n)$ $2m\bar{y}\lambda_0\sim\chi^2(2m)$

$=Ga(n,\frac{1}{2})$ $=Ga(m,\frac{1}{2})$

$\therefore\chi^2=2n\bar{x}\theta_0-2m\bar{y}\lambda_0=Ga(n-m,\frac{1}{2})=\chi^2(2n-2m)$

$\therefore$ 拒绝域: $W_{II}=\{\chi^2\le\chi^2_{\alpha/2}(2n-2m)$ 或 $\chi^2\ge\chi^2_{1-\alpha/2}(2n-2m)\}$