Modeling 1–3 Composite Piezoelectrics: Thickness-Mode Oscillations

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***Abstract*—A simple physical model of 1–3 composite piezoelectrics is advanced for the material properties that are relevant to thickness-mode oscillations. This model is valid when the lateral spatial scale of the composite is sufficiently fine that the composite can be treated as an effective homogeneous medium. Expressions for the composite's material parameters in terms of the volume fraction of piezoelectric ceramic and the properties of the constituent piezoelectric ceramic and passive polymer are derived. A number of examples illustrate the implications of using piezocomposites in medical ultrasonic imaging transducers. While most material properties of the composite roughly interpolate between their values for pure polymer and pure ceramic, the composite's thickness-mode electromechanical coupling can exceed that of the component ceramic. This enhanced electromechanical coupling stems from partially freeing the lateral clamping of the ceramic in the composite structure. Their higher coupling and lower acoustic impedance commend composites for medical ultrasonic imaging transducers. Our model also reveals that the composite's material properties cannot all be optimized simultaneously; trade-offs must be made. Of most significance is the trade-off between the desired lower acoustic impedance and the undesired smaller electromechanical coupling that occurs as the volume fraction of piezoceramic is reduced.**

I. Introduction

A VARIETY of composite piezoelectric materials can be made by combining a piezoelectric ceramic with a passive polymer phase [1]–[8]. These new piezoelectrics greatly extend the range of material properties offered by the conventional piezoelectric ceramics and polymers.

Recently, composite piezoelectrics have found fruitful application in transducers for pulse–echo medical ultrasonic imaging [9]. These medical transducers are made from PZT-rod/polymer composites that are shown schematically in Fig. 1. There are several requirements for the piezoelectric used in these transducers. First, for sensitive transducers, the piezoelectric must efficiently convert between electrical and mechanical energy. Second, the piezoelectric must be acoustically matched to tissue so that the acoustic waves in the transducer and tissue couple well both during transmission and reception. Third, the electric properties must be compatible with the driving and receiving electronics. The relevant properties are, respectively, the electromechanical coupling constant, k_t, the specific acoustic impedance, Z, and the dielectric constant, ϵ^S. For sensitive transducers, one must also give attention to electrical ($\tan\delta$) and mechanical (Q_m) losses. Many other technological requirements (shapeability, thermal stability, structural strength, etc.)

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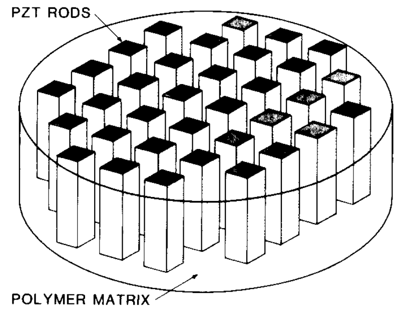

Fig. 1. Schematic representation of 1–3 composite piezoelectric made from PZT rods in a polymer matrix.

must be met, but the primary requirements qua piezoelectric are: high electromechanical coupling ($k_t \rightarrow 1$), acoustic impedance close to tissue ($Z \rightarrow 1.5$ Mrayls), reasonably large dielectric constant ($\epsilon^S \geq 100$) and low electrical ($\tan\delta \leq 0.05$) and mechanical ($Q_m \geq 10$) losses.

Conventional piezoelectric materials only partially meet these needs. Piezoelectric ceramics, such as lead zirconate-titanate, lead metaniobate and modified lead titanates, are the most popular choices for medical ultrasonic transducers. These ceramics offer high electromechanical coupling ($k_t \sim 0.4$–0.5), a wide selection of dielectric constants ($\epsilon^S \sim 100$–2400) and low electrical and mechanical losses ($\tan\delta \leq 0.02$, $Q_m \sim 10$–1000). Their major drawback is their high acoustic impedance ($Z \sim 20$–30 Mrayls). An acoustic matching layer technology has been developed to couple these ceramics to tissue; indeed, relatively broadband, sensitive transducers can be made by this approach.

Piezoelectric polymers, such as polyvinylidene difluoride and its copolymer with trifluroethylene, provide a contrasting set of material properties. Their low acoustic impedance ($Z \sim 4$ Mrayls) makes acoustic matching easy, but their low electromechanical coupling ($k_t \leq 0.3$) and high dielectric losses ($\tan\delta \sim 0.15$) seriously degrade the sensitivity; moreover, their low dielectric constant ($\epsilon^S \sim 10$) places severe demands on the transmitter and receiver.

Composite piezoelectrics provide material properties superior to both the piezoceramics and piezopolymers. Coupling constants can be larger ($k_t \sim 0.6$–0.75) than those of the ceramics, while the acoustic impedance is much lower ($Z < 10$ Mrayls)—almost reaching the range of the piezopolymers. The composites also provide a wide range of dielectric constants ($\epsilon^S \sim 10$–1000) and low dielectric and mechanical losses. Piezocomposites with characteristics superior to those of conventional piezoceramics and piezopolymers have already been demonstrated.

This paper describes a simple physical model for the material properties which govern the thickness-mode oscillations in thin plates of 1-3 composite piezoelectrics. Such composites can be treated as a homogeneous medium with new effective material parameters so long as the rod size and spacing are sufficiently fine compared with all relevant acoustic wavelengths; this model applies only if we stay safely within this regime. Since we focus only on thickness-mode oscillations of large thin plates, with negligible electrical and mechanical losses, substantial simplifications are achieved. Moreover, our sharp focus allows us to treat an important physical effect—the lateral stresses generated within these plates—by means of a simple physical approximation.

Detailed finite element analyses, as well as systematic experimental studies, have been compared with the predictions of this model [10]–[19]. The agreement is rather better than we supposed it might be when we first advanced this model [20].

II. Composite Spatial Scale

We wish to model the 1-3 composites as a homogeneous medium with a set of effective material properties. The detailed microstructure of the composite and the different properties of the constituent phases are subsumed in the averages taken to calculate the behavior of the new effective medium. To do so, we must first consider when we can validly model the composite as a homogeneous medium. While this is a good picture in certain circumstances, it is clearly not universally valid; indeed, the lateral periodicity of the rods gives rise to a rich set of phenomena reflecting the composite's microstructure [21]–[41]. In particular, the composite's microstructure appears whenever the running waves on these composite plates are resonantly reflected by the lateral periodicity of the rod spacing. When the periodicity of these Lamb waves matches that of the composite microstructure stopbands appear in the Lamb wave dispersion curves.

Fig. 2(a) provides a schematic representation of the Lamb wave dispersion curves for an homogeneous medium. These curves show the angular frequency versus wavenumber for running waves on a plate. The heavy black dot on the vertical axis ($k = 0$, $\omega = \omega_0$) represents the fundamental thickness resonance in this plate. Figs. 2(b), (c) and (d) depict schematically the modifications produced by resonant reflections in the rod lattice which has a periodicity d in the direction of wave propagation. The dashed vertical lines identify the wavelengths that are resonantly reflected by the periodic lattice of rods. In Fig. 2(b) the rod periodicity is sufficiently coarse that a lateral stopband occurs at frequencies near that of the fundamental thickness resonance. Figs. 2(c) and (d) illustrate composite plates with successively finer lateral spatial scale compared with the thickness of the plate; increasingly, lateral stop-band resonances are pushed to frequencies much higher than the thickness-mode resonance. In devising our model, we assume that the spatial scale is so fine that the composite is validly represented as a homogeneous medium for the frequencies of interest near the thickness resonance.

In designing a composite material for ultrasonic transducers, it is important to note that the lowest of these stopbands, at $k = \pi/d$, does not couple electrically in a uniformly electroded plate. This lowest resonance corresponds to the rows of rods being separated by one half a wavelength of the Lamb wave; in this case, rods oscillate out of phase with their neighbors [33], [35]. When uniform electrodes cover the opposite surfaces of the plate, such oscillations are not excited electrically. Even when excited by an incident acoustic wave, such resonant waves produce no electric signal; only those stopband resonances at $k = 2n\pi/d$, in which all rods oscillate in phase, are electrically coupled. Because it is not electrically coupled, this lowest stopband does not interfere with the performance of a composite as a thickness-mode transducer. Indeed, positioning this half wavelength stopband near the thickness mode resonance is helpful in suppressing cross talk in undiced arrays [39]. This stopband also suppresses lateral resonances of the transducer structure. These resonances appear as the skirts of the main response and may, indeed, completely disturb the main resonance; avoiding this effect requires following a complicated and delicate design procedure for the case of lower frequency transducers.

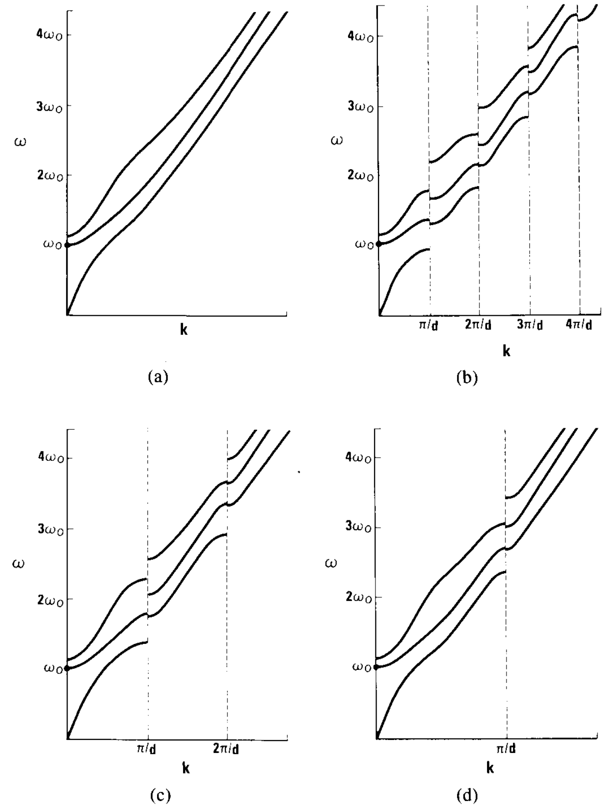

Fig. 2. Schematic representation of the Lamb wave dispersion curve for (a) a homogeneous plate, and (b), (c), (d), composite plates with successively finer lateral periodicity, d, in the direction of Lamb wave propagation. The large dot at $k = 0$ denotes the thickness-mode resonance at angular frequency, ω_0.

III. Constitutive Relations

This section presents a simple physical model for the material parameters that govern the thickness-mode resonance in 1-3 piezoelectric composites. To describe the thickness resonance in these composite plates we calculate the effective constitutive relations for this homogeneous medium. We choose the strain and electric field as the independent coordinates for our analysis and orient the composite plate in the x-y plane with the fundamental repeats of the rod lattice lying along the axes. The constitutive relations for the component phases give the stress and electric displacement at every point (x, y, z) within the plate. The polymer phase is an isotropic homogeneous medium that is piezoelectrically inactive, so for (x, y, z) within the polymer phase:

$$T_1 = c_{11}S_1 + c_{12}S_2 + c_{12}S_3 \tag{1a}$$

$$T_2 = c_{12}S_1 + c_{11}S_2 + c_{12}S_3 \tag{1b}$$

$$T_3 = c_{12}S_1 + c_{12}S_2 + c_{11}S_3 \tag{1c}$$

$$T_4 = c_{44}S_4 \tag{1d}$$

$$T_5 = c_{44}S_5 \tag{1e}$$

$$T_6 = c_{44}S_6 \tag{1f}$$

$$D_1 = \epsilon_{11}E_1 \tag{1g}$$

$$D_2 = \epsilon_{11}E_2 \tag{1h}$$

$$D_3 = \epsilon_{11}E_3. \tag{1i}$$

We take the piezoelectric ceramic to be poled along the rods—perpendicular to the plate, so for (x, y, z) within the ceramic phase,

$$T_1 = c_{11}^E S_1 + c_{12}^E S_2 + c_{13}^E S_3 - e_{31}E_3 \tag{2a}$$

$$T_2 = c_{12}^E S_1 + c_{11}^E S_2 + c_{13}^E S_3 - e_{31}E_3 \tag{2b}$$

$$T_3 = c_{13}^E S_1 + c_{13}^E S_2 + c_{33}^E S_3 - e_{33}E_3 \tag{2c}$$

$$T_4 = c_{44}^E S_4 - e_{15}E_2 \tag{2d}$$

$$T_5 = c_{44}^E S_5 - e_{15}E_1 \tag{2e}$$

$$T_6 = c_{66}^E S_6 \tag{2f}$$

$$D_1 = e_{15}S_5 + \epsilon_{11}^S E_1 \tag{2g}$$

$$D_2 = e_{15}S_4 + \epsilon_{11}^S E_2 \tag{2h}$$

$$D_3 = e_{31}S_1 + e_{31}S_2 + e_{33}S_3 + \epsilon_{33}^S E_3. \tag{2i}$$

These equations serve to define our notation. The elastic and dielectric constants of the ceramic phase are easily distinguished from those of the polymer phase by the superscripts E and S, respectively.

An analysis with these full field equations is quite complex. We introduce six simplifying approximations to extract the essential physics. As our first approximation, we assume that the strain and electric field are independent of x and y throughout the individual phases. This is clearly not true in detail, as finite-element calculations reveal. The expectation is that this approximation captures the physical behavior in an average sense. The stress, strain, field, and displacement in the different phases are then distinguished explicitly by a superscript p in the polymer phase and a superscript c in the ceramic phase, instead of implicitly by their (x, y) coordinates.

Second, we add the usual simplifications made in analyzing the thickness mode oscillations in a large, thin, electroded plate (symmetry in the x-y plane, $E_1 = E_2 = 0$, etc.). The constitutive relations for the individual phases have a compact form, namely, within the polymer (^{p}) phase,

$$T_1^p = (c_{11} + c_{12})S_1^p + c_{12}S_3^p \tag{3a}$$

$$T_3^p = 2c_{12}S_1^p + c_{11}S_3^p \tag{3b}$$

$$D_3^p = \epsilon_{11}E_3^p \tag{3c}$$

and within the ceramic (^{c}) phase,

$$T_1^c = (c_{11}^E + c_{12}^E)S_1^c + c_{13}^E S_3^c - e_{31}E_3^c \tag{3d}$$

$$T_3^c = 2c_{13}^E S_1^c + c_{33}^E S_3^c - e_{33}E_3^c \tag{3e}$$

$$D_3^c = 2e_{31}S_1^c + e_{33}S_3^c + \epsilon_{33}^S E_3^c. \tag{3f}$$

Our third approximation embodies the picture that the ceramic and polymer move together in a uniform thickness oscillation. Thus the vertical strains (in the z direction) are the same in both phases,

$$S_3^p(z) = S_3^c(z) = \bar{S}_3(z). \tag{4}$$

This is clearly not always true as laser probe measurements of the displacements of oscillating composite plates reveal. However this is a reasonably good picture when the composite has such fine spatial scale that stop-band resonances are at much higher frequencies than the thickness resonance.

Fourth, we relate the electric fields in the two phases. Since the faces of the composite plates are electroded and hence equipotentials, we take the electric fields to be the same in both phases, namely,

$$E_3^p(z) = E_3^c(z) = \bar{E}_3(z). \tag{5}$$

This is not a critical approximation. Indeed, since the dielectric constant of piezoceramics is several hundred times that of a polymer, we might even totally neglect the polymer phase for volume fractions of piezoceramic greater than a percent or so. The constitutive relations now simplify to read,

$$T_1^p = (c_{11} + c_{12})S_1^p + c_{12}\bar{S}_3 \tag{6a}$$

$$T_3^p = 2c_{12}S_1^p + c_{11}\bar{S}_3 \tag{6b}$$

$$D_3^p = \epsilon_{11}\bar{E}_3 \tag{6c}$$

$$T_1^c = (c_{11}^E + c_{12}^E)S_1^c + c_{13}^E\bar{S}_3 - e_{31}\bar{E}_3 \tag{6d}$$

$$T_3^c = 2c_{13}^E S_1^c + c_{33}^E\bar{S}_3 - e_{33}\bar{E}_3 \tag{6e}$$

$$D_3^c = 2e_{31}S_1^c + e_{33}\bar{S}_3 + \epsilon_{33}^S\bar{E}_3. \tag{6f}$$

Our fifth approximation addresses the lateral interaction between the phases. We assume that the lateral stresses are equal in both phases and that the ceramic's lateral strain is compensated by a complimentary strain in the polymer so that the composite as a whole is laterally clamped, namely,

$$T_1^p(z) = T_1^c(z) = \bar{T}_1(z) \tag{7}$$

$$\bar{S}_1(z) = \tilde{v}S_1^p(z) + vS_1^c(z) = 0 \tag{8}$$

where v is the volume fraction of piezoceramic in the composite and $\tilde{v} = (1 - v)$ is the volume fraction of polymer. We expect this captures the lateral clamping of the ceramic rods by the polymer in an average sense. This assumption permits us to express the lateral strains in terms of the vertical strain and electric field as

$$S_1^c = \tilde{v}\,\frac{-(c_{13}^E - c_{12})\bar{S}_3 + e_{31}\bar{E}_3}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^E + c_{12}^E)} \tag{9}$$

$$S_1^p = v\,\frac{(c_{13}^E - c_{12})\bar{S}_3 - e_{31}\bar{E}_3}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^E + c_{12}^E)}. \tag{10}$$

We then eliminate the lateral strains from the constitutive relations and obtain for the eliminated coordinate:

$$\bar{T}_1(z) = \bar{c}_{13}^E\bar{S}_3 - \bar{e}_{31}\bar{E}_3, \tag{11}$$

where,

$$\bar{c}_{13}^{E} = \frac{v c_{13}^{E}(c_{11} + c_{12}) + \tilde{v} c_{12}(c_{11}^{E} + c_{12}^{E})}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})} \tag{12}$$

$$\bar{e}_{31} = \frac{v e_{31}(c_{11} + c_{12})}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})} \tag{13}$$

and for the coordinates that remain:

$$T_3^p = \left[c_{11} + \frac{2 v c_{12}(c_{13}^{E} - c_{12})}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})}\right]\bar{S}_3 - \left[\frac{2 v e_{31} c_{12}}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})}\right]\bar{E}_3 \tag{14a}$$

$$D_3^p = \epsilon_{11}\bar{E}_3 \tag{14b}$$

$$T_3^c = \left[c_{33}^{E} - \frac{2\tilde{v} c_{13}^{E}(c_{13}^{E} - c_{12})}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})}\right]\bar{S}_3 - \left[e_{33} - \frac{2\tilde{v} e_{31} c_{13}^{E}}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})}\right]\bar{E}_3 \tag{14c}$$

$$D_3^c = \left[e_{33} - \frac{2\tilde{v} e_{31}(c_{13}^{E} - c_{12})}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})}\right]\bar{S}_3 + \left[\epsilon_{33}^{S} + \frac{2\tilde{v}(e_{31})^2}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})}\right]\bar{E}_3. \tag{14d}$$

Our sixth and final approximation deals with the dependent coordinates. Since the lateral periodicity is sufficiently fine, we get the effective total stress and electric displacement by averaging over the contributions of the constituent phases, namely,

$$\bar{T}_3(z) = v T_3^c(z) + \tilde{v} T_3^p(z) \tag{15a}$$

$$\bar{D}_3(z) = v D_3^c(z) + \tilde{v} D_3^p(z). \tag{15b}$$

So the final constitutive relations are,

$$\bar{T}_3 = \bar{c}_{33}^{E}\bar{S}_3 - \bar{e}_{33}\bar{E}_3 \tag{16a}$$

$$\bar{D}_3 = \bar{e}_{33}\bar{S}_3 + \bar{\epsilon}_{33}^{S}\bar{E}_3, \tag{16b}$$

where,

$$\bar{c}_{33}^{E} = v\left[c_{33}^{E} - \frac{2\tilde{v}(c_{13}^{E} - c_{12})^2}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})}\right] + \tilde{v} c_{11} \tag{17}$$

$$\bar{e}_{33} = v\left[e_{33} - \frac{2\tilde{v} e_{31}(c_{13}^{E} - c_{12})}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})}\right] \tag{18}$$

$$\bar{\epsilon}_{33}^{S} = v\left[\epsilon_{33}^{S} + \frac{2\tilde{v}(e_{31})^2}{v(c_{11} + c_{12}) + \tilde{v}(c_{11}^{E} + c_{12}^{E})}\right] + \tilde{v}\epsilon_{11}. \tag{19}$$

The results of our model are embodied in the expressions for the composite's material parameters, given in (12), (13), (17), (18), and (19), in terms of the material parameters of the constituent phases and their proportions.

IV. Transducer Parameters

Using $\bar{S}_3$ and $\bar{E}_3$ as independent coordinates is convenient for evaluating the composite properties in terms of the properties of the component phases. However, the oscillations in a thin plate are most readily analyzed using $\bar{S}_3$ and $\bar{D}_3$ as independent coordinates. This change is simple to effect and yields,

$$\bar{T}_3 = \bar{c}_{33}^{D}\bar{S}_3 - \bar{h}_{33}\bar{D}_3 \tag{20a}$$

$$\bar{E}_3 = \bar{h}_{33}\bar{S}_3 + \bar{\beta}_{33}^{S}\bar{D}_3 \tag{20b}$$

where

$$\bar{c}_{33}^{D} = \bar{c}_{33}^{E} + (\bar{e}_{33})^2/\bar{\epsilon}_{33}^{S} \tag{21a}$$

$$\bar{h}_{33} = \bar{e}_{33}/\bar{\epsilon}_{33}^{S} \tag{21b}$$

$$\bar{\beta}_{33}^{S} = 1/\bar{\epsilon}_{33}^{S}. \tag{21c}$$

These relations must be supplemented by the expression for the composite density, namely,

$$\bar{\rho} = v\rho^{c} + \tilde{v}\rho^{p}. \tag{22}$$

We have, then, in these equations all the needed parameters of the composite piezoelectric expressed in terms of the properties of the components. We need only insert these effective material parameters into the conventional analysis of thickness-mode oscillations in a thin piezoelectric plate to obtain the quantities of interest for ultrasonic transducers, namely, the electromechanical coupling constant,

$$\bar{k}_t = \bar{h}_{33}/(\bar{c}_{33}^{D}\bar{\beta}_{33}^{S})^{1/2} = \bar{e}_{33}/(\bar{c}_{33}^{D}\bar{\epsilon}_{33}^{S})^{1/2} \tag{23}$$

the specific acoustic impedance,

$$\bar{Z} = (\bar{c}_{33}^{D}\bar{\rho})^{1/2} \tag{24}$$

and the longitudinal velocity,

$$\bar{v}_l = (\bar{c}_{33}^{D}/\bar{\rho})^{1/2}. \tag{25}$$

V. Results

This section illustrates this simple physical model with several examples for different choices of polymer, ceramic and volume fraction. We present both our model's predictions and their implications for medical ultrasonic imaging transducers. Table I contains the material parameters used for all examples in this section.

To show how the composite's properties vary with volume fraction of piezoceramic, we consider a piezocomposite made from PZT5 ceramic [42] and Spurr epoxy.[1]

Fig. 3 shows the variation in the basic material parameters, $\bar{\rho}$, $\bar{\epsilon}_{33}^{S}$, $\bar{c}_{33}^{D}$, and $\bar{e}_{33}$. These quantities vary essentially linearly with volume fraction over most of the range ($\bar{\rho}$ strictly so over the entire range). However, as the volume fraction becomes large, the lateral clamping of the rods by the polymer has an appreciable effect on the elastic and piezoelectric behavior: the elastic stiffness, $\bar{c}_{33}^{D}$, increases and the piezoelectric strain constant, $\bar{e}_{33}$, diminishes. These are the principal effects of the lateral stress exerted by the polymer matrix on the piezoceramic rods. This lateral clamping of the rods also diminishes the dielectric constant in this range, but the effect is only a few percent. The dielectric constant also deviates from strict proportionality to volume fraction at the low end. But this deviation appears only when v is well below a percent. For small volume fractions, the other quantities also vary essentially linearly with v, assuming at zero their value in the polymer phase. At zero, the piezoelectric coupling strictly vanishes. The density and

[1]Spurr Low Viscosity Embedding Medium, Polysciences, Warrington, PA.

TABLE I
MATERIAL PARAMETERS USED IN EXAMPLES

Piezoceramic	(Pb, Ca) TiO_3	PZT5	PZT5H-Type
$c_{11}^E\ (10^{10}\ N/m^2)$	15.0	12.1	15.1
$c_{12}^E\ (10^{10}\ N/m^2)$	3.7	7.54	9.8
$c_{13}^E\ (10^{10}\ N/m^2)$	3.2	7.52	9.6
$c_{33}^E\ (10^{10}\ N/m^2)$	12.7	11.1	12.4
$e_{31}\ (C/m^2)$	1.61	−5.4	−5.1
$e_{33}\ (C/m^2)$	8.50	15.8	27
$\epsilon_{33}^S/\epsilon_0$	145	830	1700
$\rho\ (gm/cm^3)$	6.94	7.75	7.75
$k_{33}\ (\%)$	54	70	75
$k_t\ (\%)$	52	49	52

Polymer	Polyethylene	Spurr	Stycast	Dow
$c_{11}\ (10^{10}\ N/m^2)$	0.34	0.53	1.25	1.78
$c_{12}\ (10^{10}\ N/m^2)$	0.29	0.31	0.57	0.86
$\rho\ (gm/cm^3)$	0.90	1.10	1.59	1.76

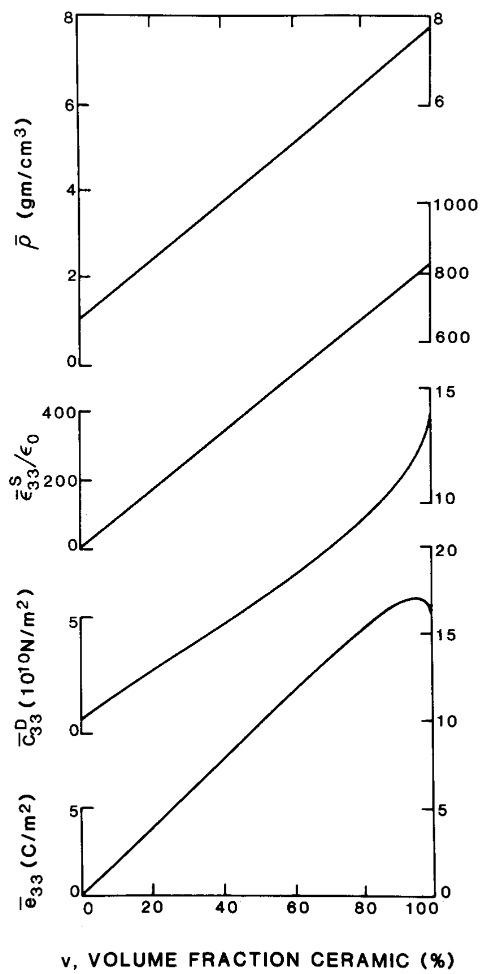

Fig. 3. Variation with volume fraction of piezoelectric ceramic, v, of a composite's basic material parameters: density, $\bar{\rho}$; clamped dielectric constant, $\bar{\epsilon}_{33}^S$, elastic stiffness, $\bar{c}_{33}^D$; and piezoelectric constant, $\bar{e}_{33}$. Material parameters for PZT5 ceramic and Spurr epoxy are used.

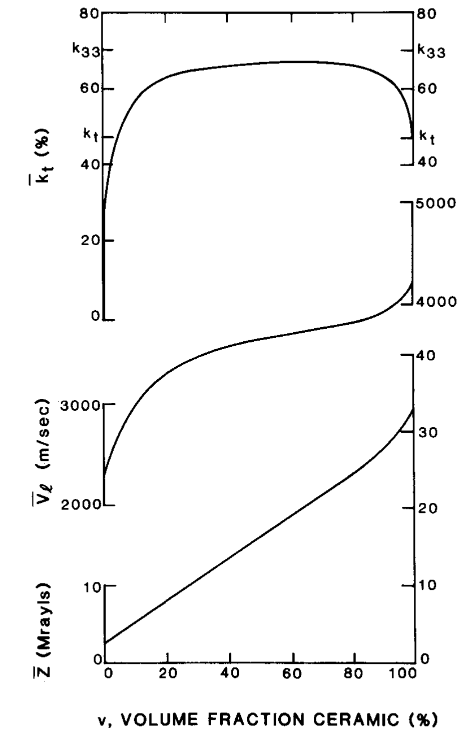

Fig. 4. Variation with volume fraction of piezoelectric ceramic, v, of a composite's transducer parameters: specific acoustic impedance, $\bar{Z}$; longitudinal velocity, $\bar{v}_l$; and thickness-mode electromechanical coupling constant, $\bar{k}_t$. Material parameters are for PZT5 ceramic and Spurr epoxy.

elastic stiffness of the polymer, though much smaller than the ceramic's (~ 10%), are not totally negligible as is the case with the dielectric constant ($\ll$ 1%).

Fig. 4 illustrates the behavior of the device parameters, the acoustic impedance, $\bar{Z}$, the longitudinal velocity, $\bar{v}_l$, and the thickness-mode electromechanical coupling constant, $\bar{k}_t$. These variations with volume fraction follow directly from those of the basic material parameters. The acoustic impedance increases essentially linearly with volume fraction, except at the high end where the clamping of the rods causes it to sweep up. The velocity also sweeps up at the high end due to stiffening of the rods by lateral forces from the polymer. For low volume fractions also, the velocity exhibits an interesting variation. As the volume fraction increases, the stiffening effect of adding more ceramic is overcome by the concomitant mass loading (increased average density). The electromechanical coupling constant is nearly the k_{33} of free ceramic rods except for deviations at low, and high, volume fractions. As the high end is approached, the lateral clamping of the rods by the polymer causes the sharp decrease in $\bar{k}_t$ to the value of the coupling constant for a solid ceramic disk, k_t. For small volume fractions, the large amount of surrounding polymer stiffens the thin rods; this elastic loading causes the diminution in $\bar{k}_t$. For intermediate volume fractions of piezoceramic, $\bar{k}_t$ remains below k_{33} partially due to elastic loading and partially due to lateral clamping.

To make a sensitive, broadband ultrasonic transducer, one wants a piezoelectric with low acoustic impedance and high electromechanical coupling. These calculations show that composite piezoelectrics can be superior to solid ceramic piezoelec-

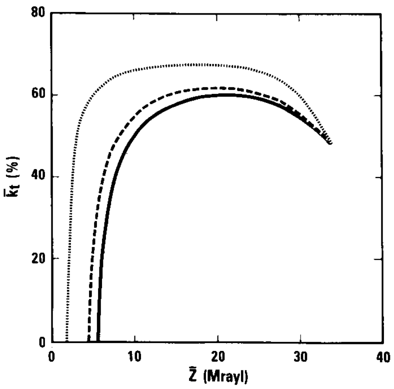

Fig. 5. Trade-off between high electromechanical coupling and low acoustic impedance in piezocomposites made from PZT5 ceramic and three different polymers: polyethylene, a soft, low-density polymer (dotted line); Stycast, a firm, medium-density polymer (dashed line); and Dow epoxy resin loaded with alumina, a stiff, high-density polymer (solid line).

trics in both respects. The optimum material can be achieved by adjusting the volume fraction of piezoceramic. Lowering the volume fraction always lowers the acoustic impedance but eventually causes a deterioration in the electromechanical coupling. A trade-off then must be made between minimizing the impedance and maximizing the coupling, as illustrated in Fig. 5.

To illustrate how the composite's properties vary with the stiffness of the polymer phase, we consider combining PZT5 ceramic with three polymers: polyethylene, [43] a soft, low-density epoxy; Stycast,[2] a denser, stiffer epoxy; and Dow epoxy resin loaded with alumina, [43] an even denser, stiffer choice. Fig. 6 shows the behavior of the device parameters, the acoustic impedance, $\bar{Z}$, the longitudinal velocity, $\bar{v}_l$, and the thickness-mode electromechanical coupling constant, $\bar{k}_t$. Clearly, softer, lower density polymers are preferable as they yield higher electromechanical coupling constants, and both velocity and acoustic impedance closer to those of tissue. A softer polymer will yield a more favorable $\bar{k}_t - \bar{Z}$ trade-off, as Fig. 5 shows. However, the softer polymer will require finer lateral spatial scales in the composite structure to remain in the long-wavelength region. Going out of this long-wavelength limit will deteriorate the improvements in both impedance matching and electromechanical conversion efficiency of the composite transducer.

To illustrate the effect of varying the piezoceramic we show in Fig. 7 our model's predictions for the composite's thickness mode electromechanical coupling constant for three different ceramics: a calcium modified lead titanate,[3] PZT5, [42] and a PZT5H-type[4]. In these calculations, the polymer phase is taken to be Stycast, a firm, medium-density epoxy. Calcium modified lead titanate has a k_{33} only marginally larger than its k_t, resulting in no enhancement of the electromechanical coupling when forming a composite; this is a poor choice for making a composite to use in thickness-mode transducers. The higher k_{33} of the PZT5H-type ceramic offers significant benefits over standard PZT5 in terms of higher thickness-mode coupling. Moreover, this material has a higher dielectric constant that facilitates

[2]Stycast 2057, Emerson and Cummings, Gardena, CA.
[3]Toshiba C-24, Toshiba Ceramics Company, Tokyo, Japan.
[4]Honeywell 278, Honeywell Ceramics Center, New Hope, MN.

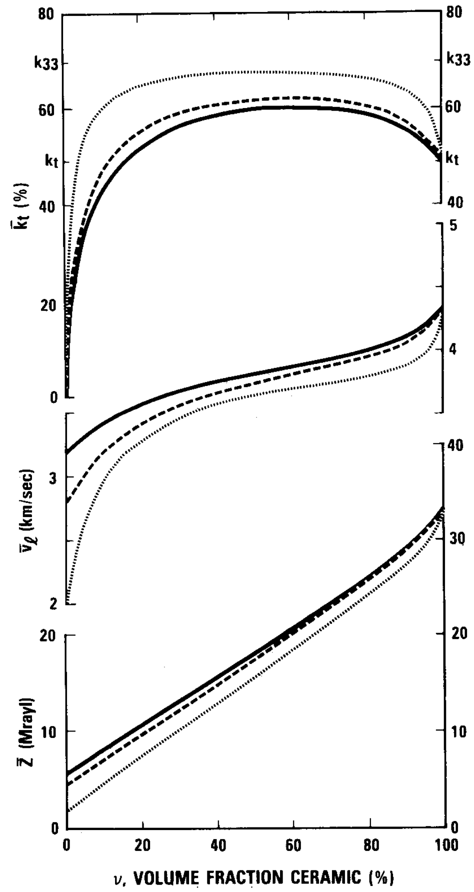

Fig. 6. Variation with PZT5 volume fraction of a composite's acoustic impedance, $\bar{Z}$, longitudinal velocity, $\bar{v}_l$, and thickness-mode electromechanical coupling constant, $\bar{k}_t$ for three different polymers: polyethylene, a soft, low-density polymer (dotted lines); Stycast, a firm, medium-density polymer (dashed lines); and Dow epoxy resin loaded with alumina, a stiff, high-density polymer (solid lines).

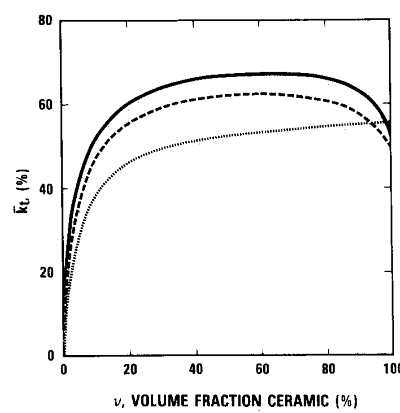

Fig. 7. Variation of the thickness-mode electromechanical coupling constant with volume fraction of piezoelectric ceramic for piezocomposites made from Stycast, a firm, medium-density epoxy, and three different ceramics: calcium modified lead titanate (dotted line), PZT5 (dashed line), and a PZT5H-type material (solid line).

the electric impedance match between the transducer and the drive/receive electronics.

VI. Conclusion

Using simple physical reasoning, we have devised a model for 1–3 composite piezoelectrics that provides the material parameters important for thickness mode transducers in terms of the volume fraction of piezoceramic and the properties of the component piezoelectric ceramic and passive polymer. Reliable results are obtained only if the lateral spatial scale of the composite is finer than all relevant acoustic wavelengths so that the composite can be treated as a homogeneous medium with new effective material parameters. This model reveals how the material parameters important for making medical ultrasonic transducers can be tailored by choice of the constituents and by varying the volume fraction of PZT in the composite structure. In particular, the possibility of higher electromechanical coupling and lower acoustic impedance commend composites for use in medical imaging transducers. Our model reveals a trade-off between higher thickness-mode coupling and lower acoustic impedance in these composite materials. Larger thickness-mode electromechanical coupling than the component piezoceramic is achievable because the lateral clamping of the ceramic is partially released in the composite structure. The composite's acoustic impedance can be reduced by lowering the volume fraction of piezoceramic; values much below conventional piezoceramics, approaching that of tissue, can be achieved. However, the electromechanical coupling of the piezocomposite falls off at very low ceramic content because of elastic loading by the polymer. This $\bar{k}_t - \bar{Z}$ trade-off and other design choices [39], provide a rich set of options for the transducer designer to engineer a piezocomposite for each particular application.

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