# ELEC 4700 Assignment 2

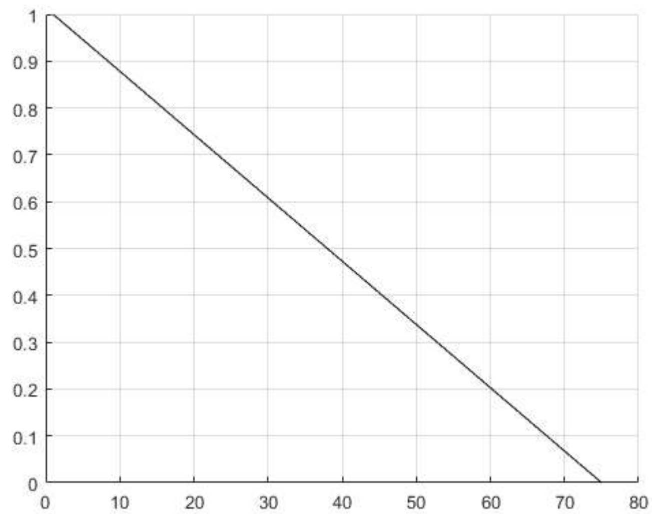
Finite Difference Method - Jinseng Vanderkloot 101031534 - Due: March 1, 2022

## Contents



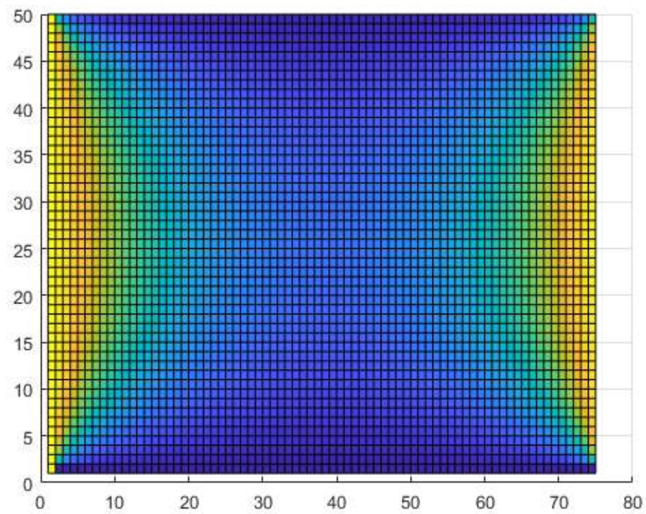
## Part 1A: Define the area and see what happens when left side is 1V and right side is 0V while top and bottom isolate.

```
nx = 75; % # of colums
ny = 50; % # of rows
G = sparse(nx*ny,ny*nx);
F = zeros(nx*ny,1);

for i = 1:nx
    for j = 1:ny
        n = j + (i-1) * ny;     % middle
        nxm = j + (i-2) * ny;   % right
        nxp = j + i * ny;       % left
        nym = j-1 + (i-1) * ny; % top
        nyp = j+1 + (i-1) * ny; % down
        if i == 1 %Left Boundary V=Vo
            G(n,n) = 1;
            F(n,1) = 1;
        elseif i == nx %Right Boundary V=0
            G(n,n) = 1;
        elseif j == 1 %Bottom
            G(n,n) = -3;
            G(n,nxm) = 1;
            G(n,nxp) = 1;
            G(n,nyp) = 1;
        elseif j == ny %Top
            G(n,n) = -3;
            G(n,nxm) = 1;
            G(n,nxp) = 1;
            G(n,nym) = 1;
        else %Middle
            G(n,n) = -4;
            G(n,nxm) = 1;
            G(n,nxp) = 1;
            G(n,nym) = 1;
            G(n,nyp) = 1;
        end
    end
end

V = G\F;
Vmap = reshape(V, [ny, nx]); % Reshaping Vector to a matrix
figure('name', 'Solution 1A'), title('Simulation Solution'), surf(Vmap'), view(90,0);
```



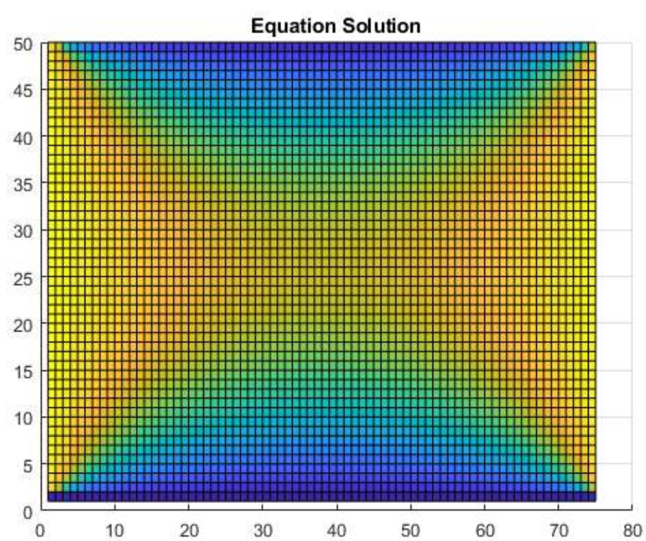

**Part 1B: Both left and right have 1V and sides and top and bottom are 0V, get the finite difference solution and compare the mathmatical solution for the shape.**

```
nx = 75; % # of colums
ny = 50; % # of rows
G = sparse(nx*ny,ny*nx);
F = zeros(nx*ny,1);
for i = 1:nx
    for j = 1:ny
        n = j + (i-1) * ny;     % middle
        nxm = j + (i-2) * ny;   % right
        nxp = j + i * ny;       % left
        nym = j-1 + (i-1) * ny; % top
        nyp = j+1 + (i-1) * ny; % down
        if i == 1 %Left Boundary V=Vo
            G(n,n) = 1;
            F(n,1) = 1;
        elseif i == nx %Right Boundary V=Vo
            G(n,n) = 1;
            F(n,1) = 1;
        elseif j == 1 %Bottom Boundary V=0
            G(n,n) = 1;
            F(n,1) = 0;
        elseif j == ny %Top Boundary V=0
            G(n,n) = 1;
            F(n,1) = 0;
        else %Middle
            G(n,n) = -4;
            G(n,nxm) = 1;
            G(n,nxp) = 1;
            G(n,nym) = 1;
            G(n,nyp) = 1;
        end
    end
end

V = G\F;
Vmap = reshape(V, [ny, nx]); % Reshaping Vector to a matrix
figure('name', 'Solution 1B'), surf(Vmap), view(0,90);
```



### Analytical Solution

```
a = nx;
b = ny/2;
x = linspace(-b, b, nx); % needs to be +b and -b for both sides of surface
y = linspace(0, a, ny);
V2 = zeros(ny, nx);

figure('name', 'Equation Solution')
[X,Y] = meshgrid(x,y);
for n = 1:2:99 %1,3,5,7...99
    V2 = V2 + ( (1/n) * (cosh((n*pi*X)/a)/cosh((n*pi*b)/a)).* sin((n*pi*Y)/a) );
    surf(4/pi*V2), title('Equation Solution'), view(0,90);
    pause(0.01);
end
% The simulated solution matches closely the analytical solution, the analytical solution wont complete at the corners becuase it is infinite and hard to solve.
```




### Part 2 - made a function to easily change the parameters of the area.

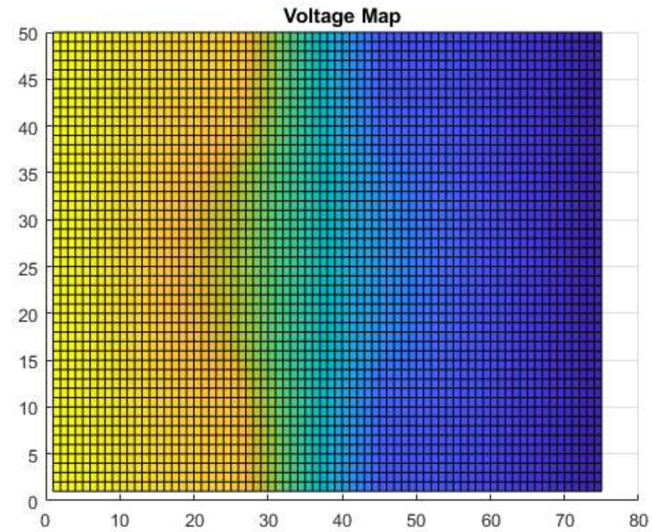
```
%Make this into a function like in the intro to the lab which inputs size;
% of area, size of boxes (placed into the middle x and bottom and top).;
% and box conduction values;
%function [V] = A2_Function(nx, ny, xBox, yBox,boxCond,x0,x1);
%Inputs:;
%Area x dimension, Area y dimension, box x dimension in middle of area,;
%Box y dimension from bottom to high and from top down, box conductivity,;
%x0 = volatge at left side, x1 = volatge at right side.;
%
% global Carea %NEEDS TO BE GLOBAL - a lot of issues when not global;
%
% % Add bottleneck;
% Carea = ones(nx,ny); %set conduction area to 1;
% % In area, place boxes with new conduction (faster than for loop) ;
```

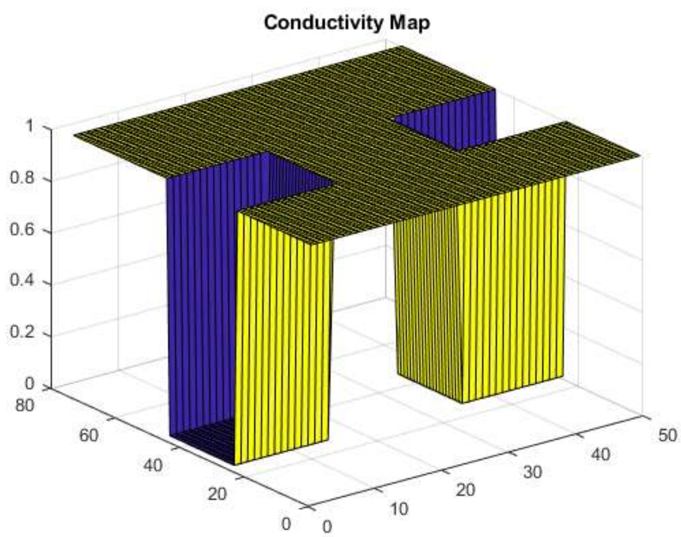
```
% Carea(nx/2 - xBox/2:nx/2 + yBox/2,1:yBox) = boxCond; %Bottom Box;
% Carea(nx/2 - xBox/2:nx/2 + yBox/2,ny-yBox:ny) = boxCond; %Top Box;
%
% G = sparse(nx*ny,ny*nx);
% F = zeros(nx*ny,1);
%
% for i = 1:nx;
%     for j = 1:ny;
%         n = j + (i-1) * ny;     % middle;
%         nxm = j + (i-2) * ny;   % right;
%         nxp = j + i * ny;       % left;
%         nym = j-1 + (i-1) * ny; % top;
%         nyp = j+1 + (i-1) * ny; % down;
%         if i == 1 %Left Boundary V=Vo
%             G(n,n) = 1;
%             F(n,1) = x0;
%         elseif i == nx %Right Boundary V=Vo
%             G(n,n) = 1;
%             F(n,1) = x1;
%         elseif j == 1 %Bottom Boundary (Free)
%             bxm = (Carea(i,j) + Carea(i-1,j)) / 2;
%             bxp = (Carea(i,j) + Carea(i+1,j)) / 2;
%             byp = (Carea(i,j) + Carea(i,j+1)) / 2;
%
%             G(n,n) = -(bxm + bxp + byp);
%             G(n,nxm) = bxm;
%             G(n,nxp) = bxp;
%             G(n,nyp) = byp;
%         elseif j == ny %Top Boundary (Free)
%             bxm = (Carea(i,j) + Carea(i-1,j)) / 2;
%             bxp = (Carea(i,j) + Carea(i+1,j)) / 2;
%             bym = (Carea(i,j) + Carea(i,j-1)) / 2;
%
%             G(n,n) = -(bxm + bxp + bym);
%             G(n,nxm) = bxm;
%             G(n,nxp) = bxp;
%             G(n,nym) = bym;
%         else %Middle
%             bxm = (Carea(i,j) + Carea(i-1,j)) / 2;
%             bxp = (Carea(i,j) + Carea(i+1,j)) / 2;
%             byp = (Carea(i,j) + Carea(i,j+1)) / 2;
%             bym = (Carea(i,j) + Carea(i,j-1)) / 2;
%
%             G(n,n) = -(bxm + bxp + bym + byp);
%             G(n,nxm) = bxm;
%             G(n,nxp) = bxp;
%             G(n,nym) = bym;
%             G(n,nyp) = byp;
%         end
%     end
% end
% V = G\F;
% end
```

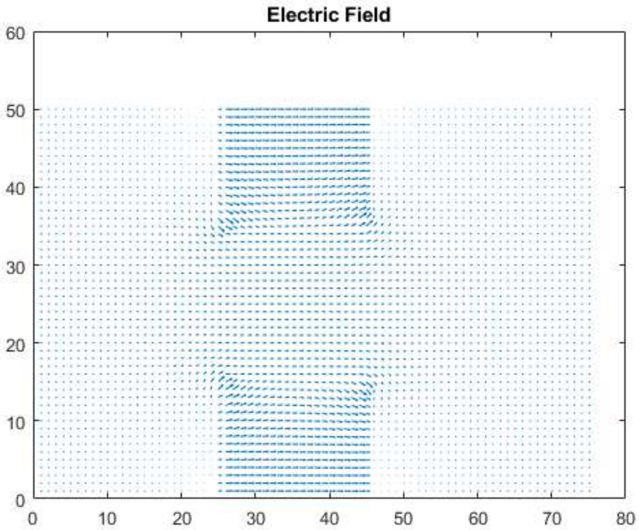
### A2_2A - Get Current Flow through area and use function.

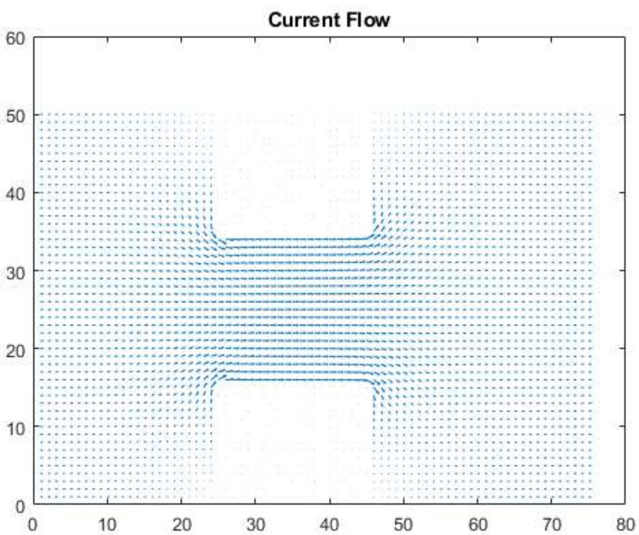
```
nx = 75; % # of colums
ny = 50; % # of rows
xBox = 25; %Width of box
yBox = 15; %Hight of box
boxCond = 0.01;
x0 = 1; %voltage at right side of area
x1 = 0; %Voltage at left side of area
global Carea %Must declare global for both in and out of function


V=A2_Function(nx, ny, xBox, yBox, boxCond, x0, x1);

Vmap = reshape(V, [ny, nx]); % Reshaping Vector to a matrix
figure('name', 'Voltage Solution')
surf(Vmap),title('Voltage Map'),view(2);

% Conductivity Map
figure('name', 'Conductivity Map');
surf(Carea), title('Conductivity Map');

% Electric Field
[Ex,Ey] = gradient(-Vmap);
figure('name', 'Electric Field');
quiver(Ex,Ey), title('Electric Field');

% Current Flow
Jx = Carea'.* Ex;
Jy = Carea'.* Ey;
figure('name', 'Current Flow');
quiver(Jx,Jy), title('Current Flow');
```

```
%When size of area increases, there is more space for current to flow
%therefor the current density if less.
```



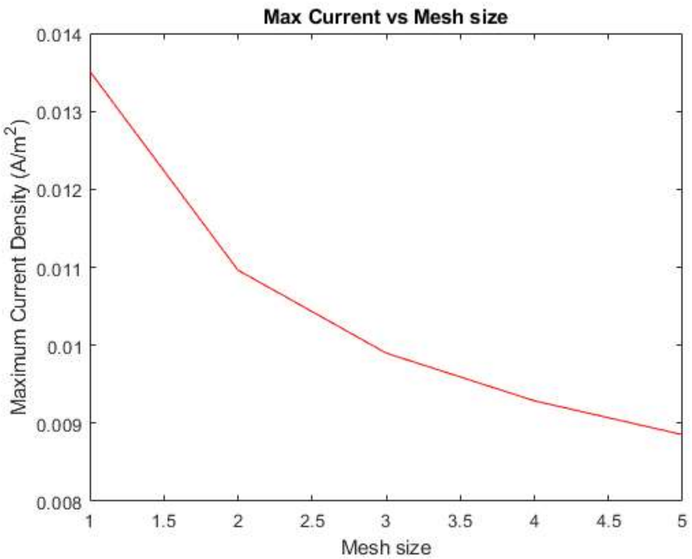





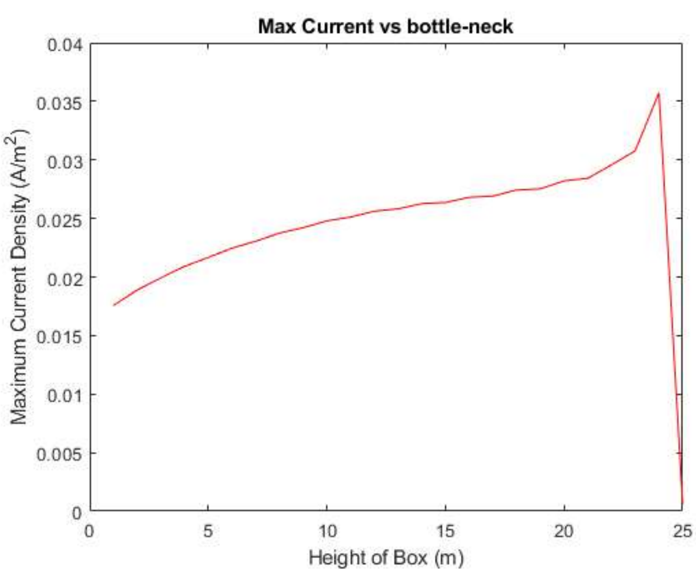

### A2_2B - Change Mesh Density

```
warning ('off')
mesh = 1:1:5; %increase mesh 5 times in steps of 1 and see effect
x0 = 1; %voltage at right side of area
x1 = 0; %Voltage at left side of area
cur = zeros(size(mesh,2),1);
global Carea %Must declare global for both in and out of function

for a = 1:size(mesh,2)
    %Size of area and box changes to increase mesh
    nx = 75*a; % # of colums
    ny = 50*a; % # of rows

    xBox = 25*a; %Width of box
    yBox = 15*a; %Length of box

    V=A2_Function(nx, ny, xBox, yBox, a, x0, x1);
    Vmap = reshape(V, [ny, nx]);
    J = Carea'.*gradient(-Vmap);
    cur(a,1) = max(J,[],"all");
end

figure('name', 'Max Current vs Mesh size');
plot(mesh,cur, 'r');
xlabel('Mesh size');
ylabel('Maximum Current Density (A/m^2)');
title('Max Current vs Mesh size');
```




### A2_2C - Narrow the Bottleneck

```
nx = 75; % # of colums
ny = 50; % # of rows
xBox = 25; %Width of box
yBox = 1:1:25; %Length of box
boxCond = 0.01;
x0 = 1; %voltage at right side of area
x1 = 0; %Voltage at left side of area
cur = zeros(25,1);
global Carea %Must declare global for both in and out of function

for a = 1:25
    V=A2_Function(nx, ny, xBox, a, boxCond, x0, x1);
    Vmap = reshape(V, [ny, nx]);
    J = Carea'.*gradient(-Vmap);
    cur(a,1) = max(J,[],"all");
end

figure('name', 'Max Current vs bottle-neck');
plot(yBox,cur, 'r');
xlabel('Height of Box (m)');
ylabel('Maximum Current Density (A/m^2)');
title('Max Current vs bottle-neck');

%Current Density increases as the current squeezes though a smaller area
%until it is completely cut off.
```




### A2_2D - Varrying conductivity in the box

```
nx = 75; % # of colums
ny = 50; % # of rows
xBox = 25; %Width of box
```

```
yBox = 15; %Length of box
boxCond = logspace(0.0001,100); %increase in increments of x10
x0 = 1; %voltage at right side of area
x1 = 0; %Voltage at left side of area
cur = zeros(size(boxCond,2),1);
global Carea %Must declare global for both in and out of function

for a = 1:size(boxCond,2)
    V=A2_Function(nx, ny, xBox, yBox, a, x0, x1);
    Vmap = reshape(V, [ny, nx]);
    J = Carea'.*gradient(-Vmap);
    cur(a,1) = max(J,[],"all");
end

figure('name', 'Max Current vs Box Conductivity');
plot(boxCond,cur, 'r');
set(gca, 'XScale', 'log');
xlabel('Box Conductivity (S/m)');
ylabel('Maximum Current Density (A/m^2)');
title('Max Current vs Box Conductivity');

%When increasing the conductivity of the boxes, more current will flow
%through them reducing the maximum current density
```