

Fractional Multiset Coherent Super-Resolution Representation for Low Resolution Face Recognition

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Abstract: In this paper, we address the problem of multiple resolution simultaneous learning in the limited training samples or noise disturbance cases and propose a novel fractional multiset partial least squares (FMPLS) approach for simultaneously dealing with multiset high dimensional data. The proposed FMPLS reconstructs the sample covariance matrices by fractional order spectral decomposition. Through using this FMPLS as a tool, we further present a new fractional multiset coherent super-resolution representation (FMCSR) method for low-resolution face recognition. Experimental results on two benchmark face databases demonstrate the effectiveness of the proposed FMCSR method.

Keywords: Face recognition; Multiset partial least squares; Super resolution; Low resolution

1 Introduction

In numerous real applications, human faces captured are usually in low resolution (LR) due to various restrictions such as inadequate lighting conditions and long distance. In general, low resolution faces make the performance of conventional face recognition algorithms seriously degraded. Thus, it is necessary in the LR scenarios to develop the discriminating facial feature representation for LR face recognition. During the last two decades, there have been many dedicated LR face recognition methods; see, for example, [1-6]. Despite this, low resolution face recognition remains quite challenging due to its unsatisfactory performance in practice.

In LR face recognition, a feasible and popular manner is to super-resolve the face images of LR inputs, which is termed as face super-resolution (FSR) [7] (or face hallucination [8]). FSR is a domain-specific problem of image super-resolution (SR), which takes advantage of face prior information. Until now, a large number of FSR techniques have been developed; see, for example, [9-19]. In recent years, a dominant direction in FSR is machine learning-based methods, the goal of which is to reconstruct high-resolution (HR) faces through learning the rich relationship between pairs of LR faces and HR faces.

Although FSR is a hard problem where it is possible that an LR face has multiple distinct HR counterparts, the super-resolved face images can be well predicted via training a machine learning model. Patch-based learning

methods [7, 20-22] are one of the most common super-resolution ideas, which divide LR faces and HR faces into many image patches. In such methods, the final super-resolved images can be obtained by merging all the image patches. Many experiments have shown that this category of methods are promising, particularly for facial image texture reconstruction.

Over the past decade, with successful use of deep neural networks (DNNs) in numerous fields, DNNs have also been utilized for image SR [23-28] to learn the HR counterparts of LR images. For example, Dong et al. [25] proposed a deep convolutional network for image SR. Kim et al. [26] put forward a novel single image SR method using a very deep convolutional neural network, called VDSR, which can significantly improve the SR performance as the depth of network increases. In addition, VDSR has an remarkable advantage that it can handle multi-scale-factor image SR only with a single network. Zhuang et al. [27] put forward a multi-level landmark-guided DNN for FSR, which is able to achieve impressive SR performance.

It should be pointed out that most of the aforementioned FSR methods focus on two resolution views, i.e., one target HR view and its corresponding LR view during the whole super-resolution procedure. But, the same objects often have multiple different resolutions in real world, such as one target HR view and multiple LR views. In this scenario, it is intractable for classical FSR methods to simultaneously deal with more than two resolutions. To solve the issue, Yuan et al. [29] proposed a novel FSR approach via utilizing multiset partial least squares (MPLS), which can jointly learn the relationship among multiple different image resolutions.

However, LR and HR training face images may be noisy or not enough in practice. This results in an obvious problem that the covariance matrices of training data in MPLS deviate from the population covariance matrices. It makes MPLS incapable of generating superior facial feature for LR face recognition. To address this problem, we incorporate the idea from fractional order embedding [30, 31] into MPLS and thus propose a novel fractional MPLS (FMPLS) method. With the proposed FMPLS as a tool, we further present a fractional multiset coherent super-resolution representation (FMCSR) method for LR face recognition. Numerous experimental results on benchmarks demonstrate that FMCSR is promising.

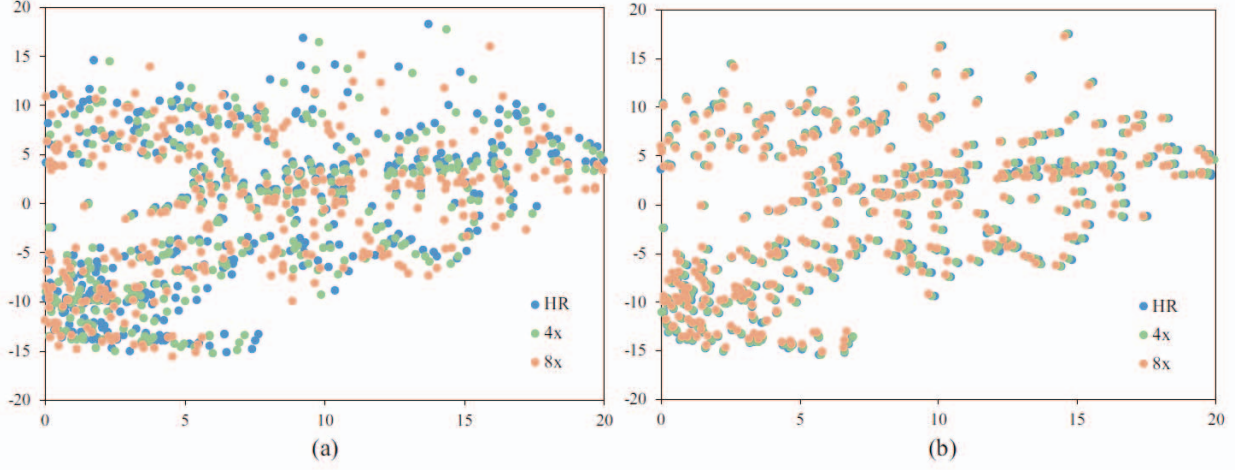


Figure 1 A consistency test example, where (a) does not utilize FMPLS and (b) uses FMPLS

2 Multiset Partial Least Squares (MPLS)

Let $\{x^i \in R^{d_i}\}_{i=1}^h$ denote h zero-mean random vectors. The goal of MPLS [29] is to seek h transformations, i.e., $U^i = [u_1^i, \dots, u_d^i] \in R^{d_i \times d}$, to maximize the sum of the covariance between $y_i^j = (u_i^j)^T x^i$ and $y_j^i = (u_j^i)^T x^j$ pairs, where $d \leq \min(d_1, \dots, d_h)$, $t=1, \dots, d$ and $i, j=0, 1, \dots, h$. Concretely, MPLS calculates the projection matrices, U^1, \dots, U^h , by the next maximization problem:

$$\begin{aligned} \max_{U^1, \dots, U^h} \sum_{i,j=1}^h \text{Tr}[(U^i)^T C_{ij} U^j] \\ \text{s.t. } (U^1)^T U^1 = \dots = (U^h)^T U^h = I, \end{aligned} \quad (1)$$

where $C_{ij} \in R^{d_i \times d_j}$ ($i \neq j$) denotes the inter-set covariance matrix of random vectors x^i and x^j , $C_{ii} \in R^{d_i \times d_i}$ represents intra-set covariance matrix of vector x^i , $\text{Tr}(\cdot)$ denotes the matrix trace, and I represents the identity matrix. In reality, intra-set and inter-set covariance matrices in (1) are computed by given samples.

3 Fractional MPLS

For each within-set covariance matrix C_{ii} , its eigenvalue decomposition is:

$$C_{ii} = Q_i \Sigma_i Q_i^T, \quad (2)$$

where Q_i denotes the matrix of eigenvectors of C_{ii} and $Q_i^T Q_i = I$, $\Sigma_i = \text{diag}(\lambda_{i,1}, \dots, \lambda_{i,r_i})$ with diagonal entries as descending eigenvalues, and $r_i = \text{rank}(C_{ii})$. Let α_i be a fraction and α_i meets $0 \leq \alpha_i \leq 1$, $i=1, \dots, h$. Then, fractional C_{ii} can be expressed as

$$C_{ii}^{\alpha_i} = Q_i \Sigma_i^{\alpha_i} Q_i^T, \quad (3)$$

where $\Sigma_i^{\alpha_i} = \text{diag}(\lambda_{i,1}^{\alpha_i}, \dots, \lambda_{i,r_i}^{\alpha_i})$.

Similarly, for each between-set covariance matrix C_{ij} ($i \neq j$), its singular value decomposition is:

$$C_{ij} = U_{ij} \Sigma_{ij} V_{ij}^T, \quad (4)$$

where $\Sigma_{ij} = \text{diag}(\sigma_{ij,1}, \dots, \sigma_{ij,r_{ij}})$ with diagonal entries as

descending singular values, $U_{ij}^T U_{ij} = I$, $V_{ij}^T V_{ij} = I$, and $r_{ij} = \text{rank}(C_{ij})$. Let β_{ij} ($i \neq j$) be a fraction and $0 \leq \beta_{ij} \leq 1$. Similar to (3), fractional C_{ij} is able to be defined as

$$C_{ij}^{\beta_{ij}} = U_{ij} \Sigma_{ij}^{\beta_{ij}} V_{ij}^T, \quad (5)$$

where $\Sigma_{ij}^{\beta_{ij}} = \text{diag}(\sigma_{ij,1}^{\beta_{ij}}, \dots, \sigma_{ij,r_{ij}}^{\beta_{ij}})$.

For the convenience of expression, (3) and (5) are able to be rewritten as a unified form, as follows:

$$F_{ij} = \begin{cases} Q_i \Sigma_i^{\alpha_i} Q_i^T, & i = j, \\ U_{ij} \Sigma_{ij}^{\beta_{ij}} V_{ij}^T, & i \neq j. \end{cases} \quad (6)$$

Replacing C_{ij} in (1) with F_{ij} in (6) leads to the optimization model of FMPLS:

$$\begin{aligned} \max_{U^1, \dots, U^h} \sum_{i,j=1}^h \text{Tr}[(U^i)^T F_{ij} U^j] \\ \text{s.t. } (U^1)^T U^1 = \dots = (U^h)^T U^h = I. \end{aligned} \quad (7)$$

It is obvious that when fractional order parameters $\alpha_i = 1$ and $\beta_{ij} = 1$, FMPLS is the same as MPLS. In other words, MPLS is a special case of FMPLS.

Similar to MPLS, our proposed FMPLS method is able to be reduced as a multivariate eigenvalue problem [32]. We use the Horst algorithm [32] to solve (7). In addition, we verify the effectiveness of FMPLS for resolution view consistency using CMU PIE dataset (please see Section 5.1 for more details), as shown in Fig. 1. As can be seen, the 2D feature points of each face image are not consistent before FMPLS transformation. In contrast, these points become much closer and almost have the same distributions after FMPLS transformation. It indicates that our proposed FMPLS method is effective for learning resolution consistency.

4 The FMCSR Method

Assume that the training face images with target HR are given as

$$X^0 = [x_1^0, \dots, x_n^0] \in R^{d_0 \times n}$$

and multi-LR face sets are

$$X^i = [x_1^i, \dots, x_n^i] \in R^{d_i \times n}, \quad i = 1, \dots, h,$$

where d_0 and d_i are the dimension of HR/LR image vectors, h is the LR view number, and n is the training sample size. All the LR face images in each resolution view are amplified to the size of HR images through bicubic interpolation method. In each resolution view, we center X^i by

$$\tilde{x}_j^i = x_j^i - \mu^i, \quad i = 0, 1, \dots, h \text{ and } j = 1, \dots, n,$$

where μ^i is the mean of all face vectors in the i -th resolution view.

After centering, we compute the principal components of \tilde{x}_j^i via

$$\hat{x}_j^i = (P^i)^T \tilde{x}_j^i, \quad (8)$$

where P^i denotes the transformation matrix of principal component analysis (PCA) [33] in the i -th resolution view. Then, we employ our FMPLS to model the relationship among all LR/HR facial features. Note that, to simplify the parameters, F_{ij} in (7) is computed by

$$F_{ij} = \begin{cases} C_{ii}^\alpha, & i = j, \\ C_{ij}^\beta, & i \neq j, \end{cases} \quad (9)$$

where α and β are two fractional order parameters with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$, $i, j = 0, 1, \dots, h$. Let U^0, U^1, \dots, U^h be the FMPLS transformations and $\hat{X}^i = [\hat{x}_1^i, \dots, \hat{x}_n^i]$.

The coherent features of \hat{X}^i is able to be obtained by

$$O^i = (U^i)^T \hat{X}^i, \quad (10)$$

where $O^i = [o_1^i, \dots, o_n^i]$, $i = 0, 1, \dots, h$.

Considering an LR input $l^i \in R^{d_i}$, we amplify it to the size of training HR faces with bicubic interpolation algorithm, thus obtaining $\tilde{l}^i \in R^{d_0}$. Then, the principal components of \tilde{l}^i are extracted by

$$\hat{l}^i = (P^i)^T (\tilde{l}^i - \mu^i). \quad (11)$$

The coherent features o_l^i of \hat{l}^i can be obtained by

$$o_l^i = (U^i)^T \hat{l}^i. \quad (12)$$

Next, we reconstruct the HR coherent features o_h^i of l^i through neighborhood search. First, we search for the k nearest neighbors of o_l^i in O^i with Euclidean distance metric, i.e., $\{o_{j_v}^i\}_{v=1}^k$. Then, we compute reconstruction coefficients $\{s_v^i\}_{v=1}^k$ by the next minimization problem:

$$\begin{aligned} \min_{s_1^i, \dots, s_k^i} & \left\| o_l^i - \sum_{v=1}^k s_v^i o_{j_v}^i \right\| \\ \text{s.t.} & \sum_{v=1}^k s_v^i = 1, \end{aligned} \quad (13)$$

where $\|\cdot\|$ represents the 2-norm. According to [34], it is easy to know that the optimal $\{s_v^i\}_{v=1}^k$ can be obtained by

$$s_v^i = \frac{\sum_{q=1}^k G_{vq}^{-1}}{\sum_{m=1}^k \sum_{q=1}^k G_{mq}^{-1}}, \quad (14)$$

where $G_{vq} = (o_l^i - o_v^i)^T (o_l^i - o_q^i)$ denotes the Gram matrix.

Using the alignment information of different resolution views, we apply the reconstruction coefficients $\{s_v^i\}_{v=1}^k$ to the corresponding HR features $\{o_{j_v}^0\}_{v=1}^k$ and thus the HR facial features of l^i can be obtained by the following form of

$$o_h^i = \sum_{v=1}^k s_v^i o_{j_v}^0, \quad (15)$$

which will be utilized for LR face recognition tasks.

5 Experiment

In this section, we evaluate the effectiveness of our proposed FMCSR method in LR face recognition tasks. We perform several experiments using two popular face databases and compare the FMCSR method with the related methods including CLLR-SR [20], SRDCCA [35], LINE [7], VDSR [26], TLcR [21], and the baseline algorithm Bicubic-PCA (Bic-PCA for short).

5.1 Data Preparation

In our test, we use two benchmark face databases. One is the CMU PIE face image database [36], and the other is the AT&T face image database¹. The CMU PIE database includes the face images with various illumination conditions, poses, and expressions. In our experiments, we choose 24 frontal faces of each of 68 individuals. The size of all the HR images is 64×64 pixels. The corresponding LR faces are separately in size of 32×32 , 16×16 , 8×8 with downsampling factors 2, 4 and 8.

The AT&T database contains 400 face images of 40 distinct individuals. Each individual has 10 images. For some individuals, the images were taken at different times. The lighting, facial expressions (open/closed eyes, smiling/not smiling) and facial details (glasses/no glasses) also vary. On this database, the size of HR images is 112×92 pixels. The corresponding LR faces are, respectively, in size of 56×46 (2×), 28×23 (4×) and 14×12 (8×).

5.2 Parameter Setting

In CLLR-SR, the neighborhood size is set to 30. In SRDCCA and VDSR, we utilize the same parameters as those in [35] and [26]. In the LINE method, the locality regularization parameter is 10^{-3} and the patch size is 16×16 with an 8-pixel overlapping. In TLcR, the patch size is 24×24 with a 12-pixel overlapping. The locality constraint parameter and window size are set to 0.04 and 28, respectively. In addition, the number of nearest neighbors in LINE and the thresholding parameter in TLcR are, respectively, set to 100 and 300 in the case of three training samples of each person on AT&T database

¹<http://cam-orl.co.uk/facedatabase.html/>

Table I Recognition rates (percent) of different methods under NN classifier on CMU PIE

Factor	FMCSR	CLLR-SR	SRDCCA	LINE	VDSR	TLcR	Bic-PCA
2×	95.96	93.63	94.73	95.71	95.71	95.71	87.50
4×	96.20	94.12	94.24	95.59	94.49	95.83	85.42
8×	95.22	93.75	92.52	91.30	86.15	93.26	74.88

Table II Recognition rates (percent) of different methods under NN classifier on AT&T

Factor	# / class	FMCSR	CLLR-SR	SRDCCA	LINE	VDSR	TLcR	Bic-PCA
2×	3	86.79	78.57	83.57	85.71	85.00	85.00	75.36
	5	90.50	84.00	81.00	90.00	90.00	90.00	84.00
	7	96.67	93.33	95.00	95.00	95.00	95.00	94.17
4×	3	88.21	79.64	80.00	85.36	85.71	85.71	75.36
	5	92.00	88.50	85.50	89.50	89.50	89.50	84.50
	7	96.67	95.00	95.00	95.00	95.00	95.00	95.00
8×	3	88.21	84.29	83.93	85.36	84.29	85.36	75.36
	5	93.50	90.50	88.50	89.50	88.50	90.00	83.00
	7	97.50	94.17	95.00	95.83	95.83	95.83	95.00

and to 150 and 500 in all the other cases. We use the source code of TLcR² and VDSR³ on GitHub to perform them. In our proposed FMCSR method, fractional order parameters α and β are separately set to 0.4 and 0.8 on the CMU PIE database and the neighborhood size k is set to 40. On the AT&T database, when the number of training samples of each individual is 3, 5, 7, α and β are set to 0.2 and 0.8, 0.4 and 1, 0.2 and 1, respectively. In addition, the neighborhood size is set to 50.

5.3 Results

On the CMU PIE database, all even face images of each person are used for training and the remaining faces for testing. On the AT&T database, the first l (l is 3, 5, and 7, respectively) images per individual are selected to form the training set, and the rest are used for testing. In all our experiments, the nearest neighbor (NN) classifier is used for performance test. Table I shows the recognition rates of each method with NN classifier under different resolution views on the CMU PIE face database. Table II reports the recognition rates of each method with NN classifier under different resolutions on the AT&T face database.

As can be seen clearly from Table I, our FMCSR method performs better than the other methods, regardless of the variation of downsampling factor. On the AT&T dataset, it can be seen from Table II that FMCSR outperforms the other six methods on all the cases, no matter how many training samples per person are used. These results suggest that our proposed FMCSR method is effective for low resolution face recognition.

6 Conclusion

In this paper, we present a novel FMPLS method based on the idea of fractional order embedding for analyzing the relationship among multiset random variables. By

using FMPLS as a tool, we further propose an FMCSR method for LR face recognition tasks. A number of experiments demonstrate that the proposed FMCSR method is effective.

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²<https://github.com/junjun-jiang/TLcR-RL>

³<https://github.com/twtygqyy/pytorch-vdsr>

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